Electricity Market - Equilibrium - Strategic Models - Strategic Equilibrium

Developed by: João Augusto Silva Lêdo

NEOS Server

NEOS SERVER

- The NEOS Server is a free internet-based service for solving numerical optimization problems. Hosted by the Wisconsin Institute for Discovery at the University of Wisconsin in Madison.

- The NEOS Server provides access to more than 60 state-of-the-art solvers in more than a dozen optimization categories. Solvers hosted by the University of Wisconsin in Madison run on distributed high-performance machines enabled by the HTCondor software;

- Remote solvers run on machines at Arizona State University, the University of Klagenfurt in Austria, and the University of Minho in Portugal.

- Official website: https://neos-server.org/neos/

Market Clearing Model for Electricity Markets

Market Clearing Model for Electricity Markets

This Section aims to introduce the Optimization Model used as foundation along the whole class:

- What is a Market Clearing Model and who operates it?
- What Perfect Competition Means?
- What is KKT Optimality Conditions and what that means?

Market Clearing Model for Electricity Markets

- What is a Market Clearing Model and who operates it:
  - The Market Operator get the energy price/quantities offers and bids from the Producers and Consumers;
  - The market operator clears the electricity market using a Market Clearing Model by collecting the provided offers and biddings by the Producer and Consumer agents (players).
  - The most common Objective function for those models is the Social-Welfare, which embodies the maximization difference between the bilinear bidding price times demanded energy and offering price times power dispatched terms.
- Perfect Competition:
  - The perfect competition in electricity markets happens when the producer player submit their energy offers by its marginal costs and the consumer player submit their biddings to its maximum utility in which they are willing to purchase the energy.

Market Clearing Model for Electricity Markets

- Bellow we have the Market Clearing Optimization Model:

- $\underset{p_{iu},d_{jc}}{Max} \quad \sum_{j\in J}\sum_{c\in C_j} b_{jc}d_{jc} - \sum_{i\in I}\sum_{u\in U_i} o_{iu}p_{iu}$

  - S.t.:

    - $\sum_{i\in I}\sum_{u\in U_i} p_{iu} - \sum_{j\in J}\sum_{c\in C_j} d_{jc} = 0 : \quad (\lambda)$

    - $0 \le p_{iu} \le \overline{p}_{iu} : \quad \left(\underline{\mu}^p_{iu}, \overline{\mu}^p_{iu}\right), \forall i \in I, \forall u \in U_i$

    - $0 \le d_{jc} \le \overline{d}_{jc} : \quad \left(\underline{\mu}^d_{jc}, \overline{\mu}^d_{jc}\right), \forall j \in J, \forall c \in C_j$

Primal Variables

Dual Variables

Market Clearing Model for Electricity Markets

- KKT Optimality Conditions:
  - It is a set of constraints that configures the optimality conditions of a convex problem:
  - The mathematicians Karush-Kuhn-Thucker (KKT) have proved that an optimization model can be represented through a set of constraints that represents a local and/or global optimal conditions of a convex optimization problem.
    - KKT conditions of a linear (convex) optimization problem always find the global optimal solution of a problem.
    - The set of constraints that encompasses the KKT conditions are:
      - The original constraints of the problem;
      - The partial derivatives (gradient) of the Lagrangian Function of the Optimization problem in respect to the problems' variables;
      - Complementarities Constraints.

Market Clearing Model for Electricity Markets

- Lagrangian Function of the Market Clearing Model:

- $$L = \sum_{i \in I} \sum_{u \in U_i} o_{iu} p_{iu} - \sum_{j \in J} \sum_{c \in C_j} b_{jc} d_{jc} - \lambda \left(\sum_{i \in I} \sum_{u \in U_i} p_{iu} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} \right) - \sum_{i \in I} \sum_{u \in U_i} \underline{\mu}_{iu}^{p} p_{iu} - \sum_{i \in I} \sum_{u \in U_i} \overline{\mu}_{iu}^{p} \left(\overline{p}_{iu} - p_{iu} \right) - \sum_{j \in J} \sum_{c \in C_j} \underline{\mu}_{jc}^{d} d_{jc} - \sum_{j \in J} \sum_{c \in C_j} \overline{\mu}_{jc}^{d} \left(\overline{d}_{jc} - d_{jc} \right)$$

Market Clearing Model for Electricity Markets

- Partial derivatives (gradient) of the Lagrangian function in respect of its primal variables:
  - $\dfrac{\partial L}{\partial p_{iu}} : o_{iu} - \lambda - \underline{\mu}^{p}_{iu} + \overline{\mu}^{p}_{iu} = 0 : \quad \forall i \in I, \forall u \in U_i$
  - $\dfrac{\partial L}{\partial d_{jc}} : - b_{jc} + \lambda - \underline{\mu}^{d}_{jc} + \overline{\mu}^{d}_{jc} = 0 : \quad \forall j \in J, \forall c \in C_j$

Market Clearing Model for Electricity Markets

- Complementarities Constraints:
  - $0 \leq p_{iu} \perp \underline{\mu}^{p}_{iu} \geq 0 : \forall i \in I, \forall u \in U_i$
  - $p_{iu}\underline{\mu}^{p}_{iu} = 0 : \forall i \in I, \forall u \in U_i$
  - $0 \leq \overline{p}_{iu} - p_{iu} \perp \overline{\mu}^{p}_{iu} \geq 0 : \forall i \in I, \forall u \in U_i$
  - $\left(\overline{p}_{iu} - p_{iu}\right) \overline{\mu}^{p}_{iu} = 0 : \forall i \in I, \forall u \in U_i$

Market Clearing Model for Electricity Markets

- Complementarities Constraints:
  - $0 \leq \underbrace{d_{jc} \perp \underline{\mu}_{jc}^{d}} \geq 0 : \forall j \in J, \forall c \in C_j$
  - $d_{jc}\underline{\mu}_{jc}^{d} = 0 : \forall j \in J, \forall c \in C_j$
  - $0 \leq \underbrace{\bar{d}_{jc} - d_{jc} \perp \bar{\mu}_{jc}^{d}} \geq 0 : \forall j \in J, \forall c \in C_j$
  - $\left(\bar{d}_{jc} - d_{jc}\right)\bar{\mu}_{jc}^{d} = 0 : \forall j \in J, \forall c \in C_j$

Market Clearing Model for Electricity Markets

- $\sum_{i \in I} \sum_{u \in U_i} p_{iu} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} = 0$
- $0 \leq p_{iu} \leq \overline{p}_{iu} : \forall i \in I, \forall u \in U_i$
- $0 \leq d_{jc} \leq \overline{d}_{jc} : \forall j \in J, \forall c \in C_j$

Primal Constraints

- $o_{iu} - \lambda - \underline{\mu}^p_{iu} + \overline{\mu}^p_{iu} = 0 : \forall i \in I, \forall u \in U_i$
- $-b_{jc} + \lambda - \underline{\mu}^d_{jc} + \overline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$

Partial derivatives Constraints (Gradient)

- $p_{iu} \underline{\mu}^p_{iu} = 0 : \forall i \in I, \forall u \in U_i$
- $\left(\overline{p}_{iu} - p_{iu}\right) \overline{\mu}^p_{iu} = 0 : \forall i \in I, \forall u \in U_i$
- $d_{jc} \underline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
- $\left(\overline{d}_{jc} - d_{jc}\right) \overline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
- $\underline{\mu}^p_{iu}, \overline{\mu}^p_{iu} \geq 0 : \forall i \in I, \forall u \in U_i$
- $\underline{\mu}^d_{jc}, \overline{\mu}^d_{jc} \geq 0 : \forall j \in J, \forall c \in C_j$

Complementarities Constraints

Market Clearing Model for Electricity Markets

Market Clearing

- $\underset{p_{iu},d_{jc}}{Max} \quad \sum_{j\in J}\sum_{c\in C_j} b_{jc}d_{jc} - \sum_{i\in I}\sum_{u\in U_i} o_{iu}p_{iu}$
- S.t.:
  - $\sum_{i\in I}\sum_{u\in U_i} p_{iu} - \sum_{j\in J}\sum_{c\in C_j} d_{jc} = 0 : (\lambda)$
  - $0 \le p_{iu} \le \overline{p}_{iu} : \left(\underline{\mu}^p_{iu}, \overline{\mu}^p_{iu}\right), \forall i \in I, \forall u \in U_i$
  - $0 \le d_{jc} \le \overline{d}_{jc} : \left(\underline{\mu}^d_{jc}, \overline{\mu}^d_{jc}\right), \forall j \in J, \forall c \in C_j$

KKT

$\xrightarrow{\text{KKT}}$

KKT

- $0 \le p_{iu} \le \overline{p}_{iu} : \forall i \in I, \forall u \in U_i$
- $o_{iu} - \lambda - \underline{\mu}^p_{iu} + \overline{\mu}^p_{iu} = 0 : \forall i \in I, \forall u \in U_i$
- $0 \le p_{iu} \perp \underline{\mu}^p_{iu} \ge 0 : \forall i \in I, \forall u \in U_i$
- $0 \le \overline{p}_{iu} - p_{iu} \perp \overline{\mu}^p_{iu} \ge 0 : \forall i \in I, \forall u \in U_i$
- $0 \le d_{jc} \le \overline{d}_{jc} \forall j \in J, \forall c \in C_j$
- $-b_{jc} + \lambda - \underline{\mu}^d_{jc} + \overline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
- $0 \le d_{jc} \perp \underline{\mu}^d_{jc} \ge 0 : \forall j \in J, \forall c \in C_j$
- $0 \le \overline{d}_{jc} - d_{jc} \perp \overline{\mu}^d_{jc} \ge 0 : \forall j \in J, \forall c \in C_j$
- $\sum_{i\in I}\sum_{u\in U_i} p_{iu} - \sum_{j\in J}\sum_{c\in C_j} d_{jc} = 0$

Market Clearing Model for Electricity Markets

- Primal-dual Optimality Condition Constraints (PDOC):
  - Embodies a set of Primal Constraints (PC), Dual Constraints (DC) and Strong Duality Equality Constraint (SDE);
  - From the Lagrangian function of an optimization model, it is possible to find its respectively dual problem:
    - By taking the gradient of the Lagrangian function in respect of the primal variables of the problem we find the dual constraints;
    - After taking off the gradiaent, what is left in the Lagrangian function is the dual objective function;
  - If we put together, all the primal constraints of the problem, all the dual constraints of the problem, and enforce the optimality between the primal and the dual problem by equating the primal objective function with the dual objective function, we obtain the strong duality equality.

Market Clearing Model for Electricity Markets

PDOC

- $\sum_{i\in I}\sum_{u\in U_i} p_{iu} - \sum_{j\in J}\sum_{c\in C_j} d_{jc} = 0$
- $0 \le p_{iu} \le \overline{p}_{iu} : \forall i \in I, \forall u \in U_i$
- $0 \le d_{jc} \le \overline{d}_{jc}\ \forall j \in J, \forall c \in C_j$
- $o_{iu} - \lambda - \underline{\mu}^p_{iu} + \overline{\mu}^p_{iu} = 0 : \forall i \in I, \forall u \in U_i$
- $-b_{jc} + \lambda - \underline{\mu}^d_{jc} + \overline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
- $\sum_{i\in I}\sum_{u\in U_i} o_{iu}p_{iu} - \sum_{j\in J}\sum_{c\in C_j} b_{jc}d_{jc} = -\sum_{i\in I}\sum_{u\in U_i} \overline{\mu}^p_{iu}\overline{p}_{iu} - \sum_{j\in J}\sum_{c\in C_j} \overline{\mu}^d_{jc}\overline{d}_{jc}$
- $\underline{\mu}^p_{iu}, \overline{\mu}^p_{iu} \ge 0 : \forall i \in I, \forall u \in U_i$
- $\underline{\mu}^d_{jc}, \overline{\mu}^d_{jc} \ge 0 : \forall j \in J, \forall c \in C_j$

PCs
DCs
SDE

Market Clearing
(Primal Problem)

- $\underset{p_{iu}, d_{jc}}{Min} \quad \sum_{i\in I}\sum_{u\in U_i} o_{iu}p_{iu} - \sum_{j\in J}\sum_{c\in C_j} b_{jc}d_{jc}$
- S.t.:
  - $\sum_{i\in I}\sum_{u\in U_i} p_{iu} - \sum_{j\in J}\sum_{c\in C_j} d_{jc} = 0 : (\lambda)$
  - $0 \le p_{iu} \le \overline{p}_{iu} : \left(\underline{\mu}^p_{iu}, \overline{\mu}^p_{iu}\right), \forall i \in I, \forall u \in U_i$
  - $0 \le d_{jc} \le \overline{d}_{jc} : \left(\underline{\mu}^d_{jc}, \overline{\mu}^d_{jc}\right), \forall j \in J, \forall c \in C_j$

Dual

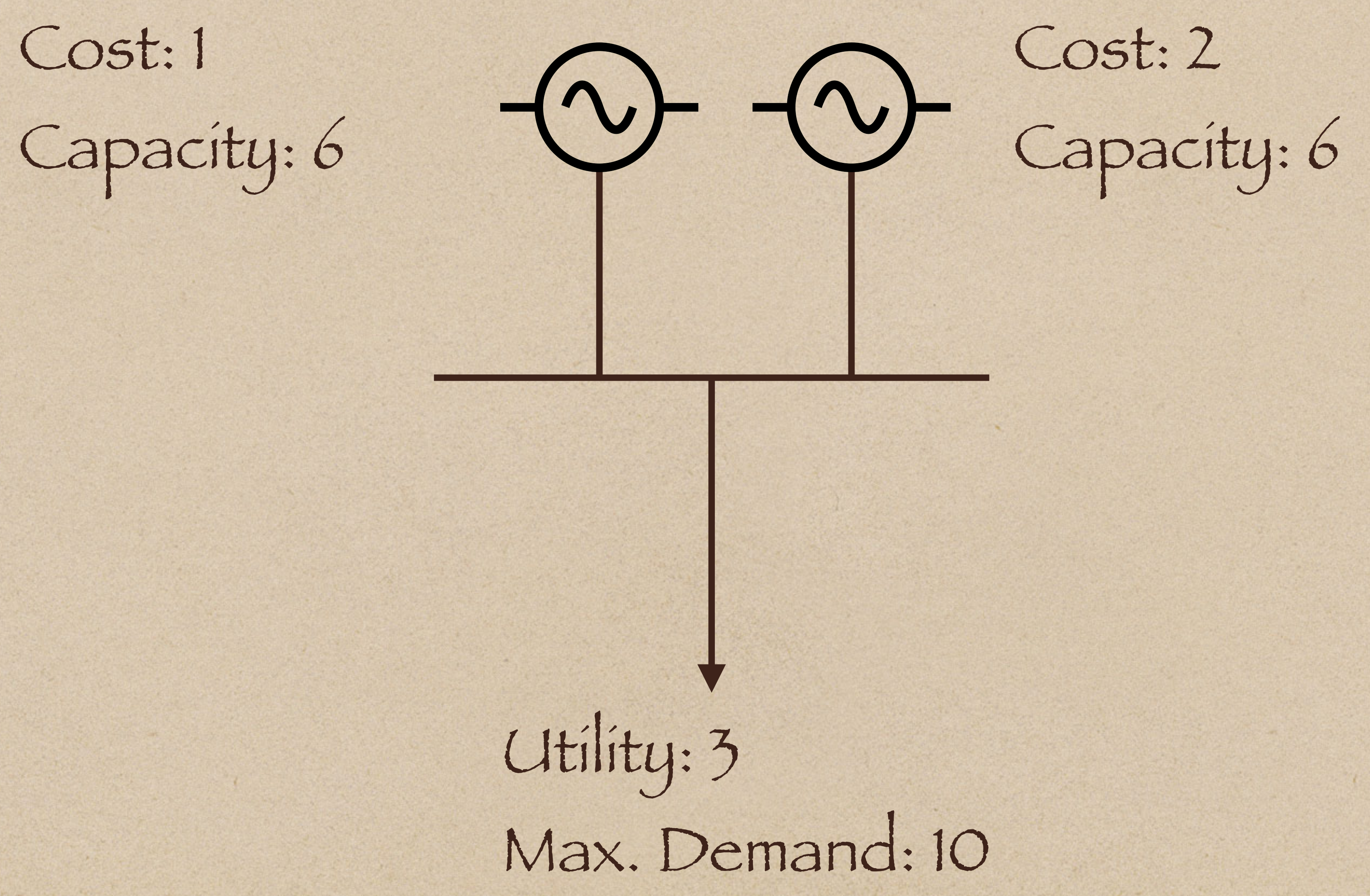
Market Clearing
(Dual Problem)

- $\underset{\lambda, \underline{\mu}^p_{iu}, \overline{\mu}^p_{iu}, \underline{\mu}^d_{jc}, \overline{\mu}^d_{jc}}{Max} \quad -\sum_{i\in I}\sum_{u\in U_i} \overline{\mu}^p_{iu}\overline{p}_{iu} - \sum_{j\in J}\sum_{c\in C_j} \overline{\mu}^d_{jc}\overline{d}_{jc}$
- S.t.:
  - $o_{iu} - \lambda - \underline{\mu}^p_{iu} + \overline{\mu}^p_{iu} = 0 : \forall i \in I, \forall u \in U_i$
  - $-b_{jc} + \lambda - \underline{\mu}^d_{jc} + \overline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
  - $\underline{\mu}^p_{iu}, \overline{\mu}^p_{iu} \ge 0 : \forall i \in I, \forall u \in U_i$
  - $\underline{\mu}^d_{jc}, \overline{\mu}^d_{jc} \ge 0 : \forall j \in J, \forall c \in C_j$

Time to Code!

Time to Code!

- Given Input data:



Time to Code!

- Homework:
    - For the next class, code in GAMS the KKT Optimality Condition of the Market Clearing Model using an Unitary Auxiliary Objective Function.
- Next class - How to code a set of KKT optimality condition constraints constraints using PATH solver.

Equilibrium

Market Clearing with Perfect Competition for Electricity Markets

This current section aims to introduce the EQUILIBRIUM idea between the mathematical models:

- What is an Equilibrium;
- The equilibrium among agents in an Electricity Markets;
- How can an Equilibrium among the agents be represented similarly to the Market Clearing Model;

Equilibrium

- An Equilibrium model happens when different agents with different objective simultaneously agree to the equilibrium;
- It is possible to see an equivalent equilibrium model to the market clearing model with perfect competition setup. Where the market agents submit their offers/bids to the market at its marginal cost and maximum utility.

Equilibrium

- The producer agent has the intention to maximize its profit having the dispatched energy (p_{iu}) as variable to its problem. While, in a perfect competition context o_{iu} (offer) represent its marginal cost:
  - $\underset{p_{iu}}{Max} \quad \sum_{i \in I} \sum_{u \in U_i} \lambda p_{iu} - o_{iu} p_{iu}$
  - S.t.:
    - $0 \leq p_{iu} \leq \overline{p}_{iu} : \quad \left(\underline{\mu}_{iu}^{p}, \overline{\mu}_{iu}^{p}\right), \forall i \in I, \forall u \in U_i$

Equilibrium

- The consumer agent has the intention to maximize its utility having the demanded energy (d_{jc}) as its variable and b_{jc} (bid) as its maximum purchase willingness (maximum utility) for the perfect competition setup:

- $\underset{d_{jc}}{Max} \quad \sum_{j \in J} \sum_{c \in C_j} b_{jc} d_{jc} - \lambda d_{jc}$

- S.t.:

  - $0 \leq d_{jc} \leq \bar{d}_{jc} : \quad \left(\underline{\mu}^{d}_{jc}, \overline{\mu}^{d}_{jc} \right), \forall j \in J, \forall c \in C_j$

Equilibrium

- The network transmission agent has the intention to get the power offered and the demand in order to dispatch accordingly . Due to the given problem having no transmission network modeled, this agent variable (power flow - *f*) is not in the model. Also to this agent its Objective function is imaterial, since its only purpose is to reassure the power supply balance between producers and consumers.

  - $Max \quad 1$
  - S.t.:
    - $\sum_{i \in I} \sum_{u \in U_i} p_{iu} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} = 0 : \quad (\lambda)$

Equilibrium

Producer →

- $\underset{p_{iu}}{Max} \quad \sum_{i\in I}\sum_{u\in U_i} \lambda p_{iu} - o_{iu}p_{iu}$
- S.t.:
  - $0 \le p_{iu} \le \overline{p}_{iu} : \quad \left(\underline{\mu}^{p}_{iu}, \overline{\mu}^{p}_{iu}\right), \forall i \in I, \forall u \in U_i$

Consumer →

- $\underset{d_{jc}}{Max} \quad \sum_{j\in J}\sum_{c\in C_j} b_{jc}d_{jc} - \lambda d_{jc}$
- S.t.:
  - $0 \le d_{jc} \le \overline{d}_{jc} : \quad \left(\underline{\mu}^{d}_{jc}, \overline{\mu}^{d}_{jc}\right), \forall j \in J, \forall c \in C_j$

Network Agent →

- $Max \quad 1$
- S.t.:
  - $\sum_{i\in I}\sum_{u\in U_i} p_{iu} - \sum_{j\in J}\sum_{c\in C_j} d_{jc} = 0 : \quad (\lambda)$

Equilibrium

Equilibrium

- $\underset{p_{iu}}{Max} \sum_{i\in I}\sum_{u\in U_i} \lambda p_{iu} - o_{iu}p_{iu}$
- S.t.:
  - $0 \le p_{iu} \le \bar{p}_{iu} : \left(\underline{\mu}^p_{iu}, \bar{\mu}^p_{iu}\right), \forall i \in I, \forall u \in U_i$
- $\underset{d_{jc}}{Max} \sum_{j\in J}\sum_{c\in C_j} b_{jc}d_{jc} - \lambda d_{jc}$
- S.t.:
  - $0 \le d_{jc} \le \bar{d}_{jc} : \left(\underline{\mu}^d_{jc}, \bar{\mu}^d_{jc}\right), \forall j \in J, \forall c \in C_j$
- $\underset{f}{Max} \; 1$
- S.t.:
  - $\sum_{i\in I}\sum_{u\in U_i} p_{iu} - \sum_{j\in J}\sum_{c\in C_j} d_{jc} = 0 : (\lambda)$

KKT →

KKT →

KKT →

KKT

- $0 \le p_{iu} \le \bar{p}_{iu} : \forall i \in I, \forall u \in U_i$
- $o_{iu} - \lambda - \underline{\mu}^p_{iu} + \bar{\mu}^p_{iu} = 0 : \forall i \in I, \forall u \in U_i$
- $0 \le p_{iu} \perp \underline{\mu}^p_{iu} \ge 0 : \forall i \in I, \forall u \in U_i$
- $0 \le \bar{p}_{iu} - p_{iu} \perp \bar{\mu}^p_{iu} \ge 0 : \forall i \in I, \forall u \in U_i$
- $0 \le d_{jc} \le \bar{d}_{jc} \forall j \in J, \forall c \in C_j$
- $-b_{jc} + \lambda - \underline{\mu}^d_{jc} + \bar{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
- $0 \le d_{jc} \perp \underline{\mu}^d_{jc} \ge 0 : \forall j \in J, \forall c \in C_j$
- $0 \le \bar{d}_{jc} - d_{jc} \perp \bar{\mu}^d_{jc} \ge 0 : \forall j \in J, \forall c \in C_j$
- $\sum_{i\in I}\sum_{u\in U_i} p_{iu} - \sum_{j\in J}\sum_{c\in C_j} d_{jc} = 0$

← KKT

Market Clearing with Perfect Competition

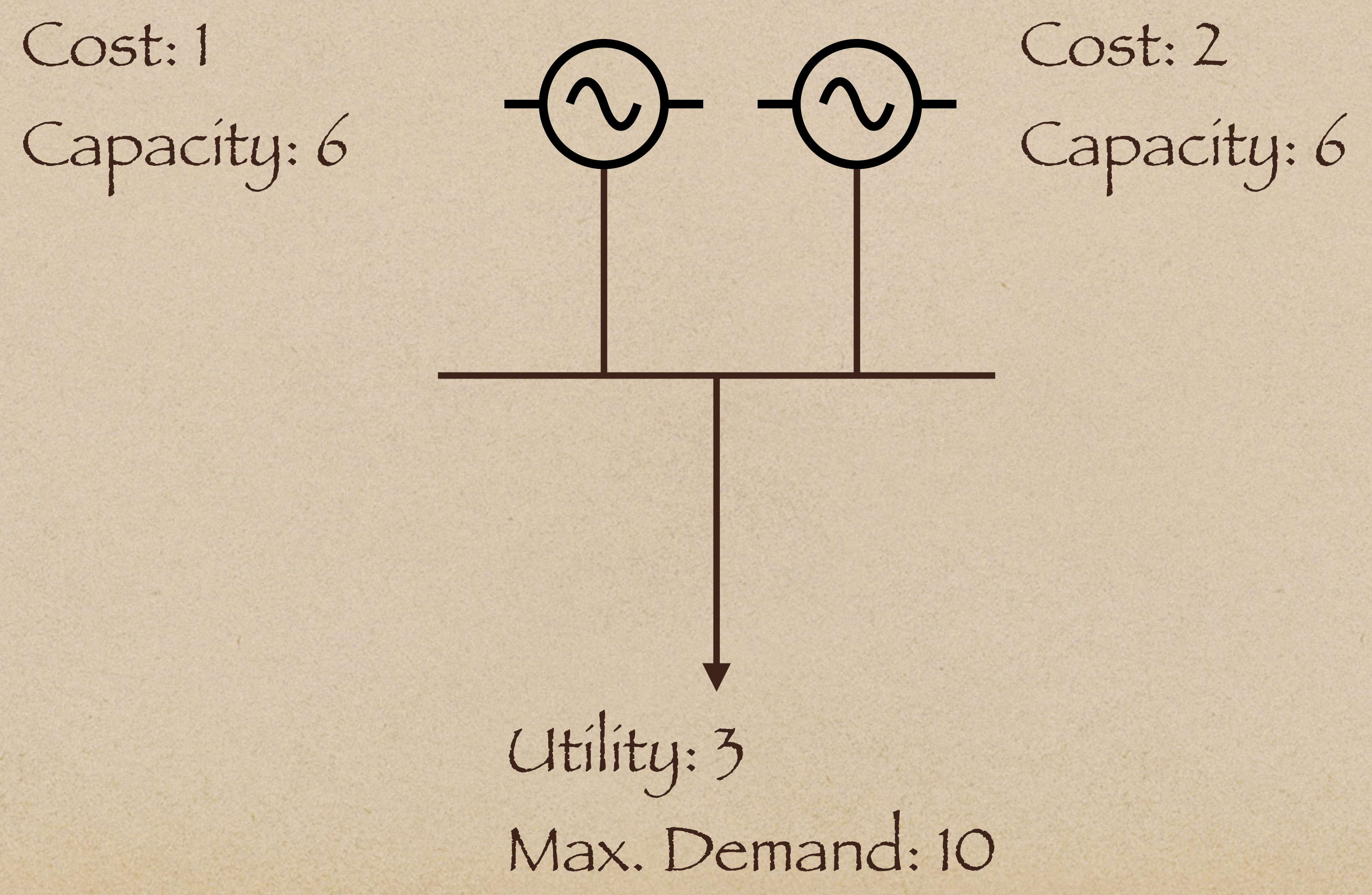
- $\underset{p_{iu}, d_{jc}}{Max} \sum_{j\in J}\sum_{c\in C_j} b_{jc}d_{jc} - \sum_{i\in I}\sum_{u\in U_i} o_{iu}p_{iu}$
- S.t.:
  - $\sum_{i\in I}\sum_{u\in U_i} p_{iu} - \sum_{j\in J}\sum_{c\in C_j} d_{jc} = 0 : (\lambda)$
  - $0 \le p_{iu} \le \bar{p}_{iu} : \left(\underline{\mu}^p_{iu}, \bar{\mu}^p_{iu}\right), \forall i \in I, \forall u \in U_i$
  - $0 \le d_{jc} \le \bar{d}_{jc} : \left(\underline{\mu}^d_{jc}, \bar{\mu}^d_{jc}\right), \forall j \in J, \forall c \in C_j$

Same KKT!

Time to Code!

Time to Code!

- Code the equilibrium using the Extended Mathematical Programming (EMP):
  - Given input data:



Mathematical Programming with Equilibrium Constraints (MPEC)

Stackelberg Game - Bilevel Models

Developed by: João Augusto Silva Lêdo

Bilevel Models

This Section aims to introduce bilevel models idea:

- What bilevel models, Stackelberg Game and MPECs are;
- Introduce the strategic agents in an Electricity Market;
- How to solve bilevel strategic problems;

Bilevel Models

- Stackelberg Game:
  - Can be defined as a sequential game where players are divided into leaders and followers. Leaders make the first move to maximize their objective function, and followers respond accordingly to also maximize their own objective function.
  - From the perspective of mathematical modeling, the Stackelberg Game can be seeing as a bilevel model.

Bilevel Models

- Bilevel model structure:
    - Leader's Objective Function
        - Leader's constraints
        - Follower's objective function
            - Follower's constraints
- Notice that, from the leader's model perspective (upper-level model) the entire follower's model (lower-level model) are seen as constraint to the Leader's model (upper-level model). Where leader have a full sight and take a straight decision into the follower model. At the same time, from the follower's perspective, the follower can only respond accordingly to the leader's actions, being unable to see the whole picture like the leader does!
- This way, the Stackelberg Game - bilevel model, can be used for modeling agents' models that seeks to behave strategically over another agent or entity.

Bilevel Models

- To the Electricity Market context, a producer company can behave strategically by modeling an entire market clearing model as one of its constraints as a lower-level model to its own bilevel strategic offering model:
  - The upper-level model: Producer's Strategic offering model, subject to:
    - Lower-level model: Market Clearing Model, that takes such strategic offers from the upper-level strategic agent and from the no strategic agents and further clear the price and dispatch the energy.
- At this kind of strategic behavior, generally the strategic producer agent have the ability to change the market behavior to its will, to the point of increasing the cleared price and sell less energy in comparison to the perfect competition, therefore granting higher profits to the strategic agent!

Bilevel Models

- Bellow we have the bilevel model of a Strategic Producer i, the non-strategic producer k and the non-strategic consumers j:
- $\underset{o_{iu}}{Max} \quad \sum_{u \in U_i} \lambda p_{iu} - c_{iu} p_{iu}$
- S.t.:
  - $o_{iu} \geq c_{iu} : \forall u \in U_i$

Upper-level

  - $\underset{p_{ku}, d_{jc}}{Max} \quad \sum_{j \in J} \sum_{c \in C_j} b_{jc} d_{jc} - \sum_{k \in I} \sum_{u \in U_k} o_{ku} p_{ku}$
  - S.t.:
    - $\sum_{k \in I} \sum_{u \in U_k} p_{ku} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} = 0 : \quad (\lambda)$
    - $0 \leq p_{ku} \leq \overline{p}_{ku} : \quad \left(\underline{\mu}^p_{ku}, \overline{\mu}^p_{ku}\right), \forall k \in I, \forall u \in U_k$
    - $0 \leq d_{jc} \leq \overline{d}_{jc} : \quad \left(\underline{\mu}^d_{jc}, \overline{\mu}^d_{jc}\right), \forall j \in J, \forall c \in C_j$

Lower-Level

The non-strategic agents offers at their margina costs $(o_{ku} : \forall k \in I : k \neq i, \forall u \in U_k)$ and bids at Maximum utility $(d_{jc} : \forall j \in J, \forall c \in C_j)$

Bilevel Models

- A bilevel model can be remodeled as an equivalent single-level model!
  - What could be a good replacement to an entire optimization model?
  - Could be an equivalent set of constraints?

Bilevel Models

- A KKT set of optimality conditions constraints can be equivalent to an entire model!
- If we replace a believe's lower-level model by its KKT optimality constraints, the model that previously was bilevel becomes an equivalent single-level model!
- Such single-level model equivalent to a bilevel formulation has a name: Mathematical Programming with Equilibrium Constraints (MPEC).
- The MPEC can be defined as constrained optimization problem where the constraints include variational inequality or complementarities! In another words, it is a optimization model where among its constraints there are a set of KKT constraints.

Bilevel Models

- Bellow we have the bilevel model of a Strategic Producer i and the non-strategic producer k and non-strategic consumers j:
- $\underset{o_{iu}}{Max} \sum_{u \in U_i} \lambda p_{iu} - c_{iu} p_{iu}$
- S.t.:
  - $o_{iu} \geq c_{iu} : \forall u \in U_i$
  - $\underset{p_{ku}, d_{jc}}{Max} \sum_{j \in J} \sum_{c \in C_j} b_{jc} d_{jc} - \sum_{k \in I} \sum_{u \in U_k} o_{ku} p_{ku}$
  - S.t.:
    - $\sum_{k \in I} \sum_{u \in U_k} p_{ku} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} = 0 : \quad (\lambda)$
    - $0 \leq p_{ku} \leq \overline{p}_{ku} : \quad \left(\underline{\mu}^p_{ku}, \overline{\mu}^p_{ku}\right), \forall k \in I, \forall u \in U_k$
    - $0 \leq d_{jc} \leq \overline{d}_{jc} : \quad \left(\underline{\mu}^d_{jc}, \overline{\mu}^d_{jc}\right), \forall j \in J, \forall c \in C_j$

Variables

Parameters

KKT

- $\sum_{k \in I} \sum_{u \in U_k} p_{ku} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} = 0$
- $0 \leq p_{ku} \leq \overline{p}_{ku} : \forall k \in I, \forall u \in U_k$
- $0 \leq d_{jc} \leq \overline{d}_{jc} : \forall j \in J, \forall c \in C_j$
- $o_{iu} - \lambda - \underline{\mu}^p_{iu} + \overline{\mu}^p_{iu} = 0 : \forall u \in U_i$
- $o_{ku} - \lambda - \underline{\mu}^p_{ku} + \overline{\mu}^p_{ku} = 0 : \forall k \in I : k \neq i, \forall u \in U_k$
- $-b_{jc} + \lambda - \underline{\mu}^d_{jc} + \overline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
- $p_{ku} \underline{\mu}^p_{ku} = 0 : \forall k \in I, \forall u \in U_k$
- $\left(\overline{p}_{ku} - p_{ku}\right) \overline{\mu}^p_{ku} = 0 : \forall k \in I, \forall u \in U_k$
- $d_{jc} \underline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
- $\left(\overline{d}_{jc} - d_{jc}\right) \overline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
- $\underline{\mu}^p_{ku}, \overline{\mu}^p_{ku} \geq 0 : \forall k \in I, \forall u \in U_k$
- $\underline{\mu}^d_{jc}, \overline{\mu}^d_{jc} \geq 0 : \forall j \in J, \forall c \in C_j$

$MPEC_i$

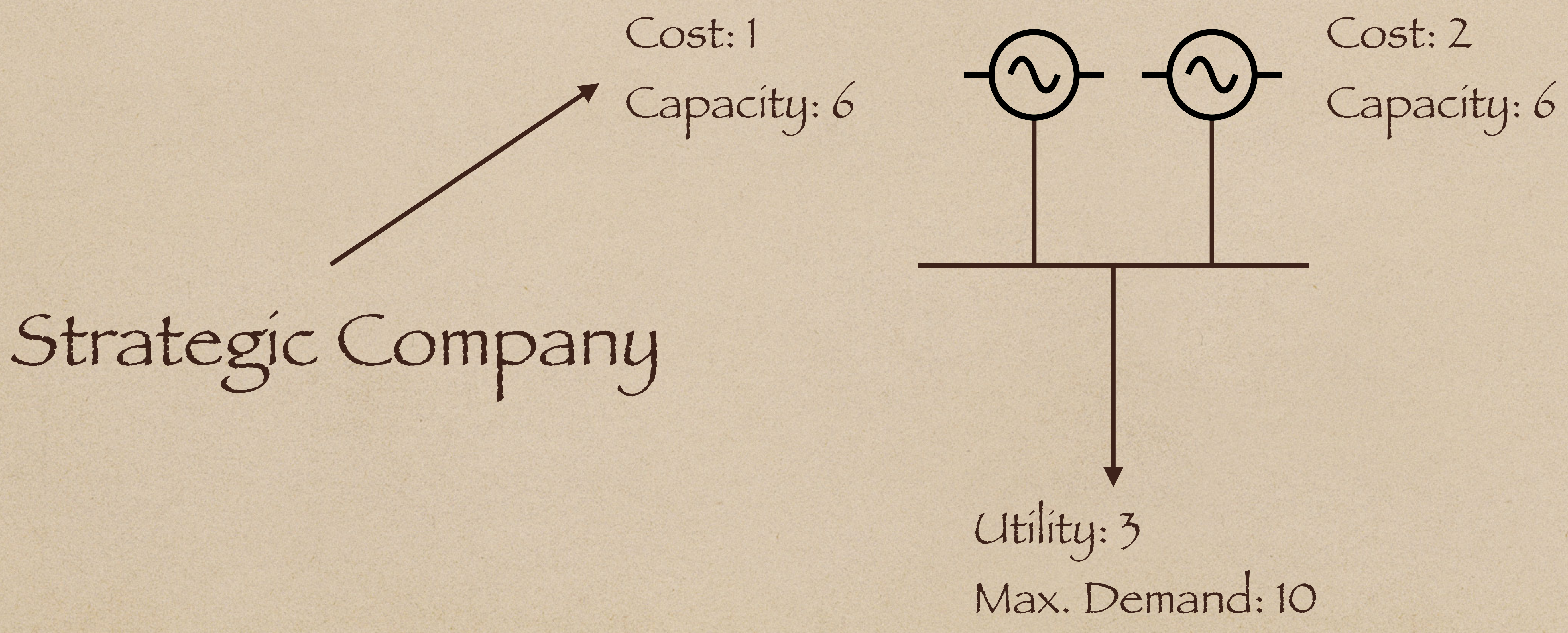
- $\underset{o_{iu},p_{ku},d_{jc},\lambda,\overline{\mu}^p_{ku},\overline{\mu}^d_{jc},\underline{\mu}^p_{ku},\underline{\mu}^d_{jc}}{Max} \quad \sum_{u \in U_i} \left(\lambda p_{iu} - c_{iu} p_{iu}\right)$
- S.t.:
  - $o_{iu} \geq c_{iu} : \forall u \in U_i$
  - $\sum_{k \in I} \sum_{u \in U_k} p_{ku} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} = 0$
  - $0 \leq p_{ku} \leq \overline{p}_{ku} : \forall k \in I, \forall u \in U_k$
  - $0 \leq d_{jc} \leq \overline{d}_{jc} : \forall j \in J, \forall c \in C_j$
  - $o_{iu} - \lambda - \underline{\mu}^p_{iu} + \overline{\mu}^p_{iu} = 0 : \forall u \in U_i$
  - $o_{ku} - \lambda - \underline{\mu}^p_{ku} + \overline{\mu}^p_{ku} = 0 : \forall k \in I : k \neq i, \forall u \in U_k$
  - $-b_{jc} + \lambda - \underline{\mu}^d_{jc} + \overline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
  - $p_{ku} \underline{\mu}^p_{ku} = 0 : \forall k \in I, \forall u \in U_k$
  - $\left(\overline{p}_{ku} - p_{ku}\right) \overline{\mu}^p_{ku} = 0 : \forall k \in I, \forall u \in U_k$
  - $d_{jc} \underline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
  - $\left(\overline{d}_{jc} - d_{jc}\right) \overline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
  - $\underline{\mu}^p_{ku}, \overline{\mu}^p_{ku} \geq 0 : \forall k \in I, \forall u \in U_k$
  - $\underline{\mu}^d_{jc}, \overline{\mu}^d_{jc} \geq 0 : \forall j \in J, \forall c \in C_j$

The primal and dual variables from the lower-level model joins to the upper-level variables

Time to Code!

Time to Code!

- Given input data:



Time to Code

- Homework:
  - Run the presented algorithm in class where producer company 2 is the Strategic one.
- Challenge:
  - Find the Strategic Consumer Mathematical Model, code it and run its algorithm to the given example with Strategic Consumer agent!

Bilevel Models

- Bellow we have the bilevel model of a Strategic Consumer j, the non-strategic producer i and non-strategic consumers l:

- $\underset{b_{jc}}{Max} \quad \sum_{c \in C_j} \left(u_{jc} d_{jc} - \lambda d_{jc} \right)$
- S.t.:
  - $b_{jc} \leq u_{jc} : \forall c \in C_j$

Upper-level

  - $\underset{p_{iu}, d_{lc}}{Max} \quad \sum_{l \in J} \sum_{c \in C_l} b_{lc} d_{lc} - \sum_{i \in I} \sum_{u \in U_i} o_{iu} p_{iu}$
  - S.t.:
    - $\sum_{i \in I} \sum_{u \in U_i} p_{iu} - \sum_{l \in J} \sum_{c \in C_l} d_{lc} = 0 : \quad (\lambda)$
    - $0 \leq p_{iu} \leq \overline{p}_{iu} : \quad \left(\underline{\mu}_{iu}^{p}, \overline{\mu}_{iu}^{p} \right), \forall i \in I, \forall u \in U_i$
    - $0 \leq d_{lc} \leq \overline{d}_{lc} : \quad \left(\underline{\mu}_{lc}^{d}, \overline{\mu}_{lc}^{d} \right), \forall l \in J, \forall c \in C_l$

Lower-Level

The non-strategic agents offers at their margina costs
$(o_{iu} : \forall i \in I, \forall u \in U_i)$
and bids at Maximum utility
$(d_{lc} : \forall l \in J : l \neq j, \forall c \in C_l)$

$MPEC_j$

- $\underset{b_{jc},p_{iu},d_{lc},\lambda,\overline{\mu}^p_{iu},\overline{\mu}^d_{lc},\underline{\mu}^p_{iu},\underline{\mu}^d_{lc}}{Max} \sum_{c\in C_j}\left(u_{jc}d_{jc} - \lambda d_{jc}\right)$
- S.t.:
  - $b_{jc} \leq u_{jc} : \forall c \in C_j$
  - $\sum_{i\in I}\sum_{u\in U_i} p_{iu} - \sum_{l\in J}\sum_{c\in C_l} d_{lc} = 0$
  - $0 \leq p_{iu} \leq \overline{p}_{iu} : \forall k \in I, \forall u \in U_i$
  - $0 \leq d_{lc} \leq \overline{d}_{lc} : \forall l \in J, \forall c \in C_l$
  - $o_{iu} - \lambda - \underline{\mu}^p_{iu} + \overline{\mu}^p_{iu} = 0 : \forall i \in I, \forall u \in U_i$
  - $-b_{lc} + \lambda - \underline{\mu}^d_{lc} + \overline{\mu}^d_{lc} = 0 : \forall l \in J : l \neq j, \forall c \in C_l$
  - $-b_{jc} + \lambda - \underline{\mu}^d_{jc} + \overline{\mu}^d_{jc} = 0 : \forall c \in C_j$
  - $p_{iu}\underline{\mu}^p_{iu} = 0 : \forall i \in I, \forall u \in U_i$
  - $\left(\overline{p}_{iu} - p_{iu}\right)\overline{\mu}^p_{iu} = 0 : \forall i \in I, \forall u \in U_i$
  - $d_{lc}\underline{\mu}^d_{lc} = 0 : \forall l \in J, \forall c \in C_l$
  - $\left(\overline{d}_{lc} - d_{lc}\right)\overline{\mu}^d_{lc} = 0 : \forall l \in J, \forall c \in C_l$
  - $\underline{\mu}^p_{iu}, \overline{\mu}^p_{iu} \geq 0 : \forall i \in I, \forall u \in U_i$
  - $\underline{\mu}^d_{lc}, \overline{\mu}^d_{lc} \geq 0 : \forall l \in J, \forall c \in C_l$

The primal and dual variables from the lower-level model joins to the upper-level variables

Parameter

Variable

Question

- Now, after we already covered isolated strategic behaviors from the producers and consumers players into the electricity market, what would happen if ALL players becomes strategic at the same time?
  - Is there any mathematical model for this?
  - Or maybe an iteratively algorithm able to solve MPECs?

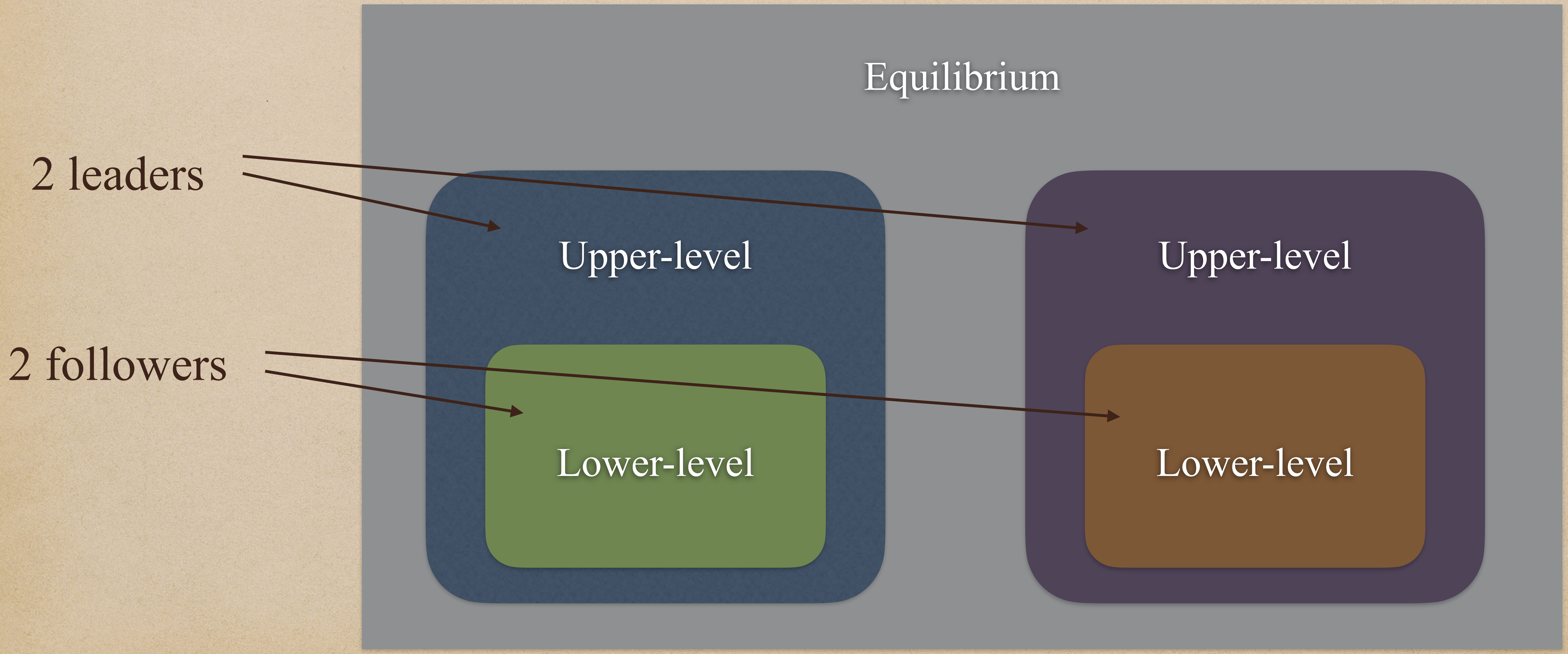
Equilibrium Problem with Equilibrium Constraints (EPEC)

EPEC

- Basically an EPEC is nothing more than an EQUILIBRIUM among strategic agents. This kind of equilibrium can happen between:
  - Multi-leader-multi-follower Game: When many strategic agents are in equilibrium and each of them has a different lower-level follower model!
  - Multi-leader-single-follower Game: When many strategic agents are in equilibrium and all of them have the same lower-level follower model!
  - Notice that the difference between the EQUILIBRIUM previously tough and the EPEC is that in the first EQUILIBRIUM all agents play nice without strategy (Perfect Competition) and to the EPEC equilibrium case all the agents behaves strategically!

EPEC

Multi-leader-multi-follower

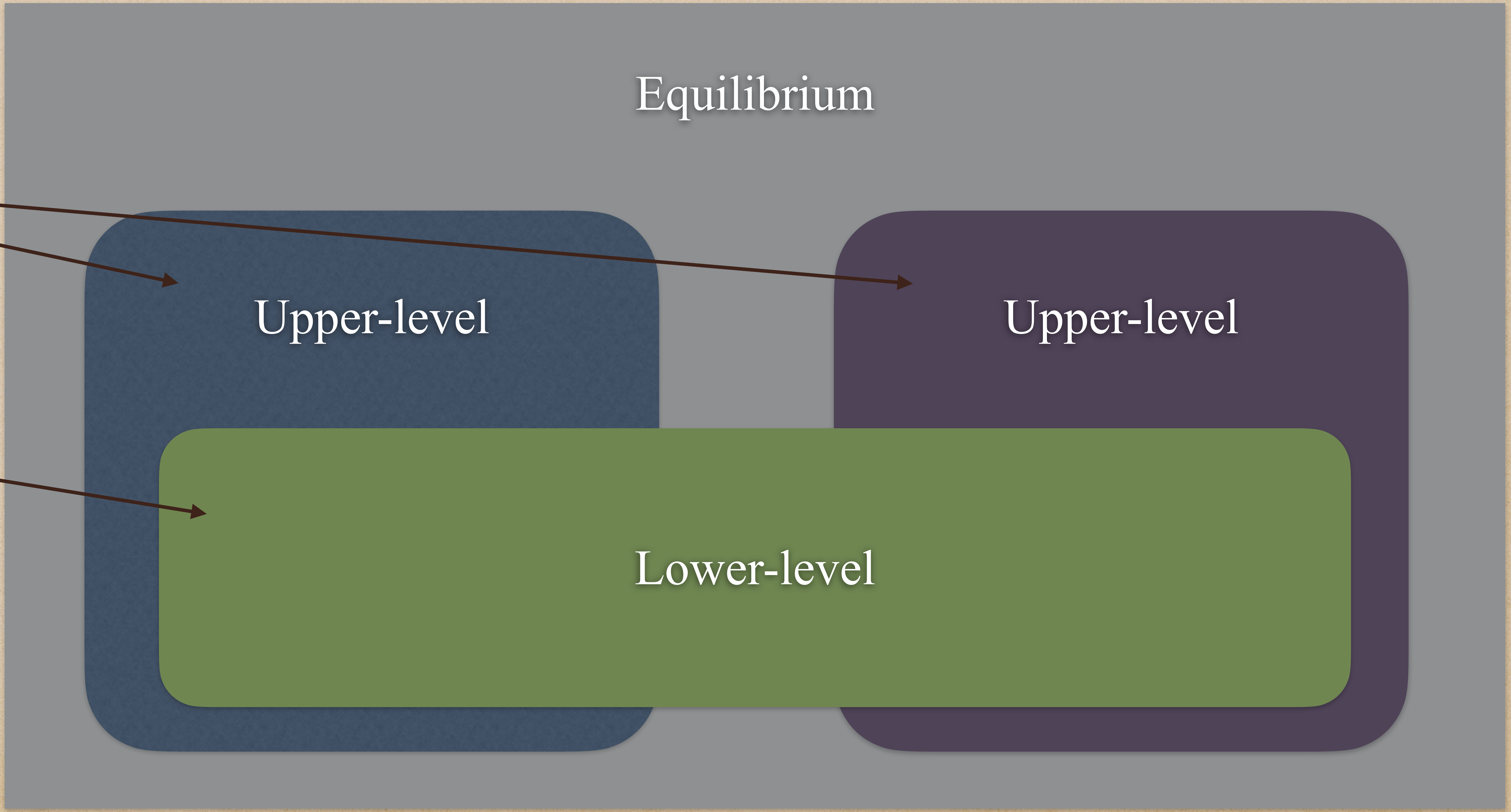


EPEC

Multi-leader-single-follower

2 leaders

1 follower



EPEC

- If we put together the given Strategic Offering models from the past section in equilibrium, could they be labeled as multi-leader-single-follower or multi-leader-multi-follower structures?
- Since EPEC is still a set of MPECs in EQUILIBRIUM, how can this be solved?

EPEC

- Some EPEC models can be solved as in the Equilibrium section. Replacing all the MPECs in equilibrium by its respectively set of KKT optimality condition constraints. However, taking KKTs a second time from a mathematical model that already embodies KKTs in its formulation, is not always possible because model with KKT constraints:
  - Generally does not has a feasible convex region;
  - The complementarity constraints that partly defining this feasible region are not regular at optimal solutions (yes, multiple optimal solutions, EPEC can have many of them!).

EPEC

- So, if it's not always possible to build an EPEC that relies on a single optimization model, using KKT conditions a second time from each individual MPEC in equilibrium, how can you solve it?

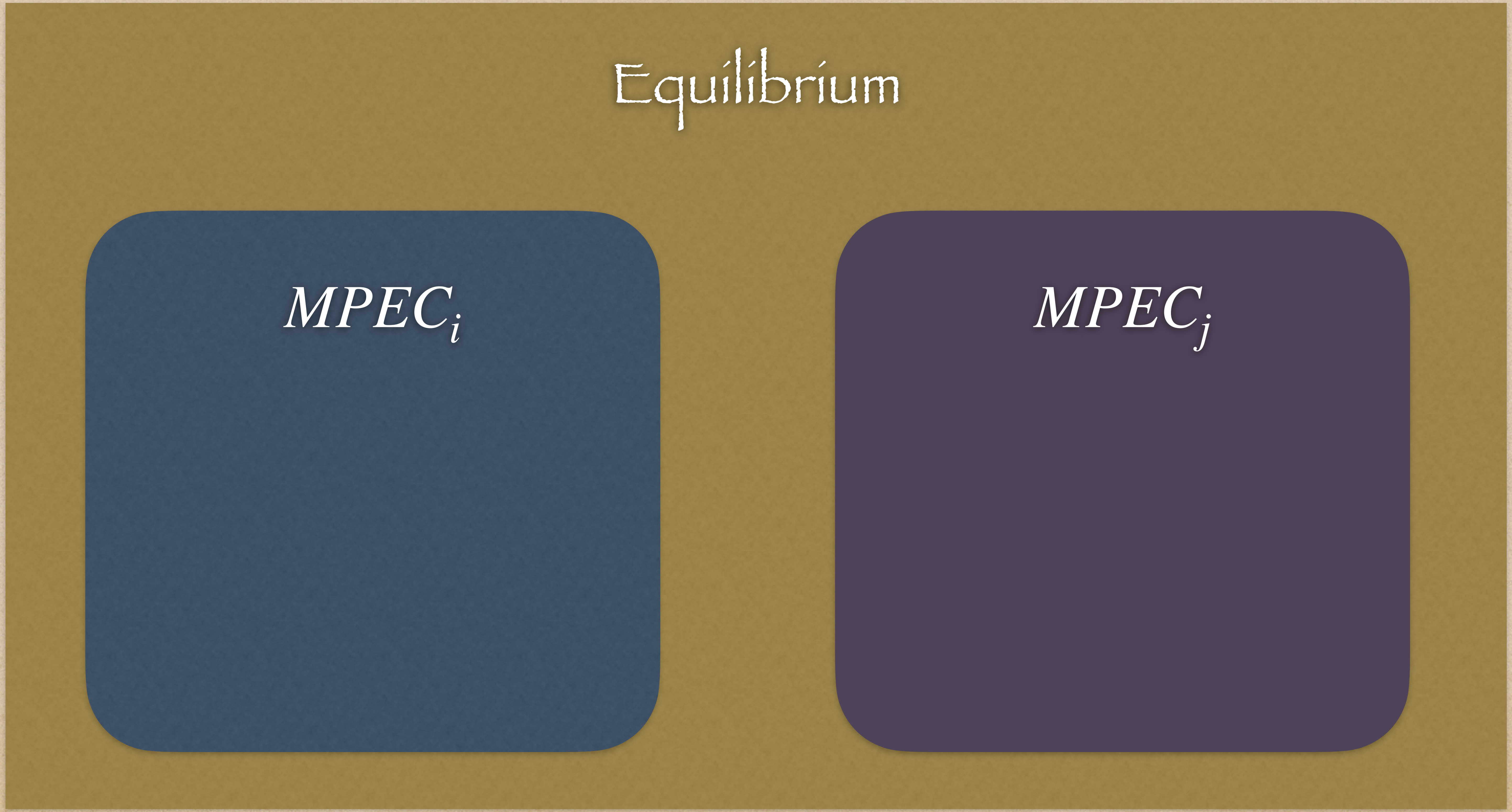
EPEC

- Solving a correlated collection of MPECs iteratively through the DIAGONALIZATION algorithm!
  - In the diagonalization algorithm, each MPEC is solved separately and independently, but the output of an MPEC becomes the input of the next MPEC that is about to solved!
  - Finding a solution where nobody wants to change its strategic offers/bids, because it could entail losses for them, means that we found the Nash Equilibrium!
    - Suggestion: Watch A Beautiful Mind movie. It tells the story of John Nash's life.

EPEC

- The following structure depicts our given example of strategic agents MPEC models, from last section, in equilibrium:

Multi-leader-single-follower



$MPEC_i$

- $\underset{o_{iu},p_{ku},d_{jc},\lambda,\overline{\mu}^p_{ku},\overline{\mu}^d_{jc},\underline{\mu}^p_{ku},\underline{\mu}^d_{jc}}{Max} \quad \sum_{u \in U_i} \left(\lambda p_{iu} - c_{iu} p_{iu}\right)$
- S.t.:
  - $o_{iu} \geq c_{iu} : \forall u \in U_i$
  - $\sum_{k \in I} \sum_{u \in U_k} p_{ku} - \sum_{j \in J} \sum_{c \in C_j} d_{jc} = 0$
  - $0 \leq p_{ku} \leq \overline{p}_{ku} : \forall k \in I, \forall u \in U_k$
  - $0 \leq d_{jc} \leq \overline{d}_{jc} : \forall j \in J, \forall c \in C_j$
  - $o_{iu} - \lambda - \underline{\mu}^p_{iu} + \overline{\mu}^p_{iu} = 0 : \forall u \in U_i$
  - $o_{ku} - \lambda - \underline{\mu}^p_{ku} + \overline{\mu}^p_{ku} = 0 : \forall k \in I : k \neq i, \forall u \in U_k$
  - $-b_{jc} + \lambda - \underline{\mu}^d_{jc} + \overline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
  - $p_{ku} \underline{\mu}^p_{ku} = 0 : \forall k \in I, \forall u \in U_k$
  - $\left(\overline{p}_{ku} - p_{ku}\right) \overline{\mu}^p_{ku} = 0 : \forall k \in I, \forall u \in U_k$
  - $d_{jc} \underline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
  - $\left(\overline{d}_{jc} - d_{jc}\right) \overline{\mu}^d_{jc} = 0 : \forall j \in J, \forall c \in C_j$
  - $\underline{\mu}^p_{ku}, \overline{\mu}^p_{ku} \geq 0 : \forall k \in I, \forall u \in U_k$
  - $\underline{\mu}^d_{jc}, \overline{\mu}^d_{jc} \geq 0 : \forall j \in J, \forall c \in C_j$

The primal and dual variables from the lower-level model joins to the upper-level variables

Variable

Parameter

$MPEC_j$

- $\underset{b_{jc},p_{iu},d_{lc},\lambda,\overline{\mu}^p_{iu},\overline{\mu}^d_{lc},\underline{\mu}^p_{iu},\underline{\mu}^d_{lc}}{Max} \sum_{c \in C_j} \left(u_{jc} d_{jc} - \lambda d_{jc} \right)$
- S.t.:
  - $b_{jc} \leq u_{jc} : \forall c \in C_j$
  - $\sum_{i \in I} \sum_{u \in U_i} p_{iu} - \sum_{l \in J} \sum_{c \in C_l} d_{lc} = 0$
  - $0 \leq p_{iu} \leq \overline{p}_{iu} : \forall k \in I, \forall u \in U_i$
  - $0 \leq d_{lc} \leq \overline{d}_{lc} : \forall l \in J, \forall c \in C_l$
  - $o_{iu} - \lambda - \underline{\mu}^p_{iu} + \overline{\mu}^p_{iu} = 0 : \forall i \in I, \forall u \in U_i$
  - $-b_{lc} + \lambda - \underline{\mu}^d_{lc} + \overline{\mu}^d_{lc} = 0 : \forall l \in J : l \neq j, \forall c \in C_l$
  - $-b_{jc} + \lambda - \underline{\mu}^d_{jc} + \overline{\mu}^d_{jc} = 0 : \forall c \in C_j$
  - $p_{iu} \underline{\mu}^p_{iu} = 0 : \forall i \in I, \forall u \in U_i$
  - $\left(\overline{p}_{iu} - p_{iu} \right) \overline{\mu}^p_{iu} = 0 : \forall i \in I, \forall u \in U_i$
  - $d_{lc} \underline{\mu}^d_{lc} = 0 : \forall l \in J, \forall c \in C_l$
  - $\left(\overline{d}_{lc} - d_{lc} \right) \overline{\mu}^d_{lc} = 0 : \forall l \in J, \forall c \in C_l$
  - $\underline{\mu}^p_{iu}, \overline{\mu}^p_{iu} \geq 0 : \forall i \in I, \forall u \in U_i$
  - $\underline{\mu}^d_{lc}, \overline{\mu}^d_{lc} \geq 0 : \forall l \in J, \forall c \in C_l$

The primal and dual variables from the lower-level model joins to the upper-level variables

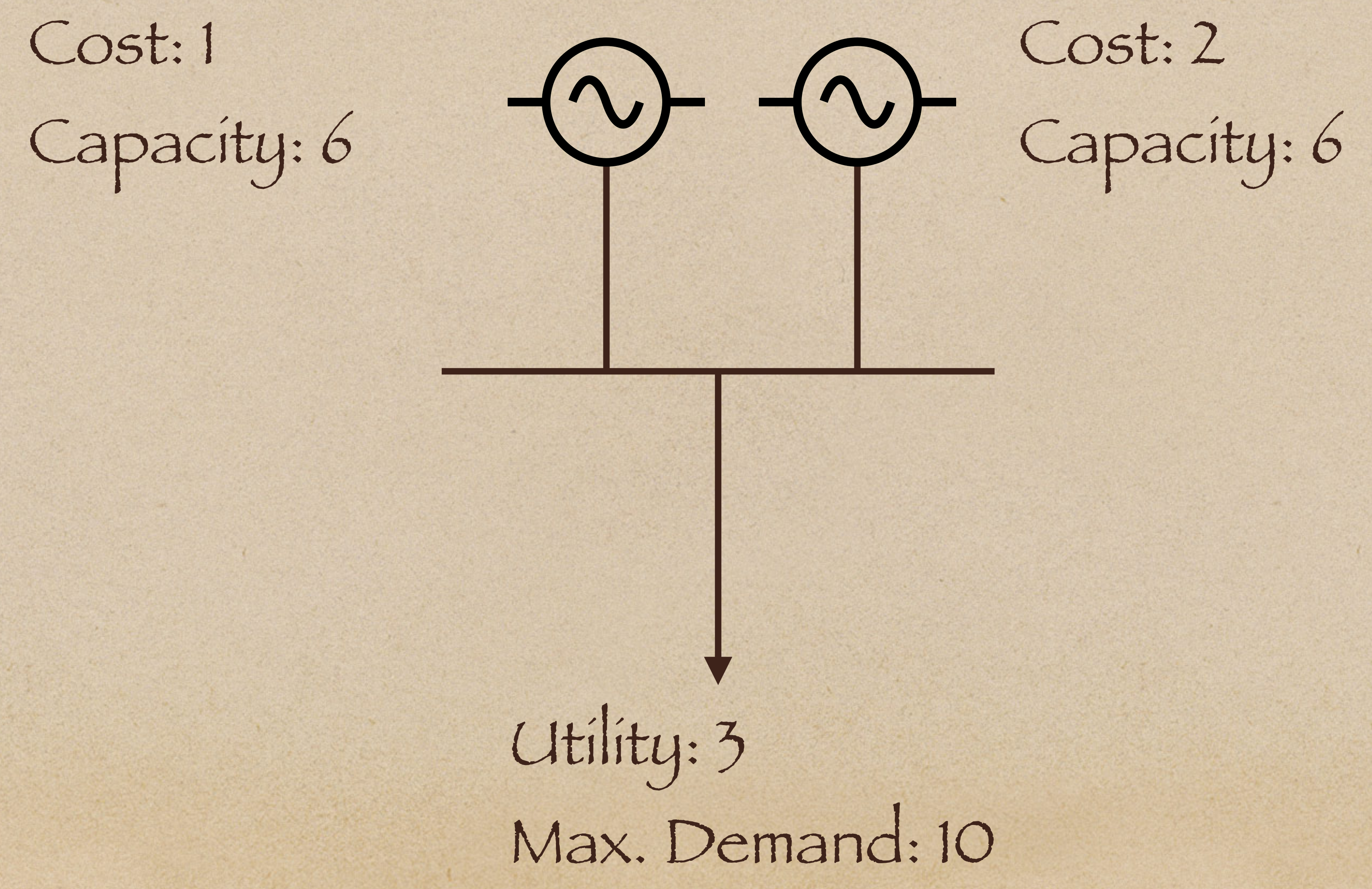
Parameter

Variable

Time to Code

Time to Code

- Code the EPEC using the DIAGONALIZATION algorithm:
  - Given input data:
  - All agents are strategic and in equilibrium!



EPEC

- This same EPEC problem given as an example to this class can be solve differently:
  - Each bilevel model in equilibrium can be reformulated as an MPEC through its Primal Constraints, Dual Constraints, Strong Duality Equality Constraint (PDOC), where it's still and instance of a less general case of the KKT optimality condition constraints.
    - This way can only be implemented for convex problems, which is the case of the bilevel's lower-level model (linear market clearing model).
  - Having MPECs built in this way. Each of them can be replaced by its KKT optimality condition set of constraints and then put together in equilibrium, giving birth to the solvable EPEC!
    - This way of solving this problem can be found in:
      [1] A. J. Conejo and C. Ruiz, "Complementarity, Not Optimization, is the Language of Markets," IEEE Open Access Journal of Power and Energy, vol. 7, pp. 344–353, 2020, doi: 10.1109/OAJPE.2020.3029134.

Conclusion

Conclusion

- By the end of this classe you all learned:
  - How to code in GAMS in an strategic and organized manner from the beginning. By planning ahead its input SETs;
    - Learned how to solve using PATH solver in GAMS, tailored to solve a set of constraints instead of optimization models;
    - Learned how to solve models in equilibrium using EMP solver;
    - Learned how to solve more than one mathematical model in looping;
  - In the scope of mathematical modeling, briefly learned about:
    - Electricity Market Models;
    - Optimality Conditions;
    - Equilibrium;
    - Strategic Offering, Stackelberg Games, Bilevel models, MPEC formulation;
    - Equilibrium of strategic agents, EPEC formulation;

Thank you all for your attention!

Developed by: João Augusto Silva Lêdo

End