RESOLUÇÃO INTEGRAIS POR SUBST.

1º $\displaystyle\int \frac{2x-9}{\sqrt{x^2-9x+1}}\,dx$

$u = x^2 - 9x + 1$

$du = 2x - 9\,dx$

$$\int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}}\,du = \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = 2\sqrt{u} + C$$

$$= 2\sqrt{x^2-9x+1} + C$$

2º $\displaystyle\int \frac{dx}{\sqrt{8x-x^2}}$

· Completando o quadrado do denominador:

$$8x - x^2 = -(x^2 - 8x)$$

$$= -(x^2 - 8x + 16 - 16)$$

$$= -(x-4)^2 + 4^2$$

$$= 4^2 - (x-4)^2$$

· Então: $\displaystyle\int \frac{dx}{\sqrt{8x - x^2}} = \int \frac{dx}{\sqrt{\underbrace{4}_{a}{}^2 - \underbrace{(x-4)}_{x}{}^2}}$

· Lembre que: $\displaystyle\int \frac{dx}{\sqrt{a^2 - x^2}}$, $a \in \mathbb{R}$ $= \text{arc sen}\left(\frac{x}{a}\right) + C$

· Logo: $\displaystyle\int \frac{dx}{\sqrt{4^2-(x-4)^2}} = \text{arc sen}\left(\frac{x-4}{4}\right) + C$

3º $\int (\sec x + tg\, x)^2 dx = \int (\sec^2 x + 2 \cdot \sec x \cdot tg x + tg^2 x)\, dx$

$= \int \sec^2 x \, dx + 2 \int \sec x \cdot tg\, x + \int tg^2 x \, dx$

$= (tang\, x + C) + (2 \cdot \sec x + C) + (tg\, x - x + C)$

$= tg\, x + tg\, x + 2 \sec x - x + C$

$= 2\, tg\, x + 2 \sec x - x + C$

º $\int_0^{\pi/4} \sqrt{1 + \cos 4x}\, dx$

Lembre que:

$\cos^2 x = \dfrac{1 + \cos 2x}{2}$

$x = 2x$

$\cos^2 2x = \dfrac{1 + \cos 4x}{2}$

$2 \cos^2 2 = 1 + \cos 4x$

Logo, $\int_0^{\frac{\pi}{4}} \sqrt{1 + \cos 4x}\, dx = \int_0^{\frac{\pi}{4}} \sqrt{2 \cos^2 2x}\, dx = \int_0^{\pi/4} \sqrt{2} \cdot \sqrt{\cos^2 2x}\, dx$

$= \sqrt{2} \int_0^{\pi/4} \cos 2x \, dx = \sqrt{2} \cdot \left[\dfrac{\sin 2x}{2} \right]_0^{\pi/4}$

$\sqrt{2} \cdot \left[\dfrac{\sin \frac{2\pi}{4}}{2} - \sin 0 \right] = \sqrt{2} \cdot \dfrac{1}{2} = \dfrac{\sqrt{2}}{2}$

7º $\int (x^3+x)^9 (3x^2+1)\,dx$

$u = x^3 + x$

$du = 3x^2 + 1\,dx$

$$\int u^9\,du = \frac{u^{10}}{10} + C = \frac{(x^3+x)^{10}}{10} + C$$

8º $\int \frac{\ln x}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int u\,du = \frac{u^2}{2} + C$$

$$= \frac{\ln^2 x}{2} + C$$

9º $\int \frac{\operatorname{arc\,tg} x}{1+x^2}\,dx$

$u = \operatorname{arc\,tg} x$

$du = \frac{1}{1+x^2}\,dx$

$$\int u\,du = \frac{u^2}{2} + C$$

$$= \frac{(\operatorname{arc\,tg} x)^2}{2} + C$$

1º $\int \frac{3\cos x\,dx}{\sqrt{1+3\operatorname{sen} x}} = 3\int \frac{\cos x\,dx}{\sqrt{1+3\operatorname{sen} x}}$

$u = 1 + 3\operatorname{sen} x$

$du = 3\cos x\,dx$

$\frac{du}{3} = \cos x\,dx$

$$= 3\int \frac{\frac{du}{3}}{\sqrt{u}} = 3\int \frac{du}{3\sqrt{u}} = \int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}}\,du$$

$$= 2\sqrt{u} + C$$

$$= 2\sqrt{1+3\operatorname{sen} x} + C$$

$11°$ $\displaystyle\int \frac{2\,dx}{x\sqrt{1-4\ln^2 x}} = 2\int \frac{dx}{x\sqrt{1-4\ln^2 x}}$ $\qquad u = \ln x$, $\quad du = \frac{1}{x}\,dx$.

$$= 2\int \frac{du}{\sqrt{1-4u^2}} = 2\int \frac{du}{\sqrt{4\left(\frac{1}{4}-u^2\right)}}$$

$$= 2\int \frac{du}{2\sqrt{\left(\frac{1}{2}\right)^2-u^2}} = \int \frac{du}{\sqrt{\left(\frac{1}{2}\right)^2-u^2}} = \text{arc sen}\left(\frac{2\ln x}{1}\right) + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \text{arc sen}\left(\frac{x}{a}\right) + C$$

$2°$ $\displaystyle\int \frac{dx}{\sqrt{x}\,(\sqrt{x}+1)}$

$u = \sqrt{x} + 1$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$$\int \frac{2\,du}{u} = 2\int u^{-1}\,du = 2\ln|u|$$
$$= 2\ln|\sqrt{x}+1|$$

$3°$ $\displaystyle\int \frac{dx}{x-\sqrt{x}} = \int \frac{dx}{\sqrt{x}\left(\frac{x}{\sqrt{x}}-1\right)} = \int \frac{dx}{\sqrt{x}\,(\sqrt{x}-1)}$

$u = \sqrt{x} - 1$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$$= \int \frac{2\,du}{u} = 2\int u^{-1} = 2\ln|u| + C$$
$$= 2\ln|\sqrt{x}-1| + C$$

15: $$\int_0^{1/2} \frac{2-8x}{1+4x^2}\,dx = \int\left(\frac{2}{1+4x^2} - \frac{8x}{1+4x^2}\right)dx$$

$$= 2\underbrace{\int \frac{1}{1+(2x)^2}\,dx}_{} - \underbrace{\int \frac{8x}{1+(2x)^2}\,dx}_{}$$

$u = 2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$u = 1+4x^2$

$du = 8x\,dx$

$$= \cancel{2}\int \frac{1}{1+u^2}\,\frac{du}{\cancel{2}} - \int \frac{du}{u}$$

$$= \int \frac{1}{1+u^2}\,du - \int \frac{du}{u}$$

$$= \text{arc tg}\, u - \ln|u|$$

$$= \Big[\text{arc tg}\,(2x) - \ln|1+4x^2|\Big]_0^{1/2}$$

$$= \big(\text{arc tg}\,(1) - \ln|2|\big) - \big(\text{arc tg}\,(0) - \ln|1|\big)$$

Ou seja, resolve normalmente e após substituir o u pelo valor original, é só realizar o $F(b) - F(a)$.