INTEGRAÇÃO POR PARTES:

1º a) $\int x \cdot \operatorname{sen}\left(\frac{x}{2}\right) dx$

$u = x$ $\qquad$ $v = \int \operatorname{sen} \frac{x}{2} \, dx = -\cos\frac{x}{2} \cdot 2$

$du = 1$ $\qquad$ $dv = \operatorname{sen} \frac{x}{2}$

$$\int u \, dv = u \cdot v - \int v \, du$$

$$= x \cdot (-2) \cos\left(\frac{x}{2}\right) - \int -2 \cos\left(\frac{x}{2}\right) dx$$

$$= -2x \cdot \cos\left(\frac{x}{2}\right) + 2 \int \cos\left(\frac{x}{2}\right) dx$$

$$= -2x \cdot \cos\left(\frac{x}{2}\right) + 2 \frac{\operatorname{sen}\left(\frac{x}{2}\right)}{\frac{1}{2}} + C$$

$$\boxed{= -2x \cdot \cos\left(\frac{x}{2}\right) + 4 \cdot \operatorname{sen}\left(\frac{x}{2}\right) + C}$$

b) $\int x \cdot e^{3x}\, dx$

$$u = x \qquad v = \int e^{3x}\, dx = \frac{e^{3x}}{3}$$
$$du = 1 \qquad dv = e^{3x}$$

$$\int u\, dv = v \cdot u - \int v\, du$$

$$= \frac{e^{3x}}{3} \cdot x - \int \frac{e^{3x}}{3}\, dx$$

$$= \frac{e^{3x}}{3} \cdot x - \frac{1}{3} \int e^{3x}$$

$$= \frac{e^{3x}}{3} \cdot x - \frac{1}{3} \cdot \frac{e^{3x}}{3} + C$$

$$= \frac{e^{3x}}{3} \cdot x - \frac{e^{3x}}{9} + C$$

$$\boxed{= e^{3x} \left(\frac{x}{3} - \frac{1}{9} \right) + C}$$

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C) $\int x^2 \cdot e^{-x} \, dx$

· Neste caso, como temos uma função fácil de DERIVAR (x^2) e uma fácil de integrar (e^{-x}), vamos usar o método tabular.

x^2 e suas DERIVADAS		e^{-x} e suas INTEGRAIS
x^2	$+$	e^{-x}
$2x$	$-$	$-e^{-x}$
2	$+$	e^{-x}
0	$-$	$-e^{-x}$

$$\int x^2 \cdot e^{-x} \, dx = -x^2 \cdot e^{-x} - 2x e^{-x} + (-2e^{-x}) + C$$

d) $\int x \cdot \ln x \, dx$

$$\boxed{\begin{array}{ll} u = x & v = \int \ln x = \frac{1}{x} \\ du = 1 & dv = \ln x \end{array}}$$

$$\int u \, dv = u \cdot v - \int v \, du$$

$$= x \cdot \frac{1}{x} - \int \frac{1}{x} \, dx$$

$$= - \ln |x| + C$$

/ /

e) $\int e^x \cdot \operatorname{sen} x \, dx$

$$\boxed{\begin{array}{ll} u = \operatorname{sen} x & v = \int e^x dx = e^x \\ du = \cos x & dv = e^x \end{array}}$$

$$\int u \, dv = u \cdot v - \int v \, du$$

$$= \operatorname{sen} x \cdot e^x - \int e^x \cdot \cos x \, dx \quad (\text{I})$$

* Vamos usar partes em $\int e^x \cdot \cos x$:

$$\boxed{\begin{array}{ll} u' = \cos x & v' = e^x \\ du' = -\operatorname{sen} x & dv' = e^x \end{array}}$$

$$\int u' \, dv' = u' \cdot v' - \int v' \, du'$$

$$= \cos x \cdot e^x - \int e^x (-\operatorname{sen} x) \cdot dx$$

$$= \cos x \cdot e^x + \int e^x \cdot \operatorname{sen} x \, dx \quad (\text{II})$$

* Substituindo II em I:

$$\int e^x \cdot \operatorname{sen} x \, dx = \operatorname{sen} x \cdot e^x - \left(\cos x \cdot e^x + \int e^x \cdot \operatorname{sen} x \, dx \right)$$

$$= \operatorname{sen} x \cdot e^x - \cos x \cdot e^x - \int e^x \cdot \operatorname{sen} x \, dx$$

$$2 \int e^x \operatorname{sen} x = \operatorname{sen} x \cdot e^x - \cos x \cdot e^x$$

$$\int e^x \cdot \operatorname{sen} x = (\operatorname{sen} x - \cos x) \cdot \frac{e^x}{2}$$

f) $\int (x^2+x+1) \cdot e^x \, dx$

$$\frac{d}{dx}\, x^2+x+1 \qquad\qquad \int e^x \, dx$$

$$\begin{array}{ccc}
\downarrow & & \downarrow \\
x^2+x+1 & + & e^x \\
2x+1 & - & e^x \\
2 & + & e^x \\
0 & - & e^x
\end{array}$$

$$= e^x(x^2+x+1) - e^x(2x+1) + e^x \cdot 2 + C$$

RESOLUÇÃO LISTA - 1

2º

a) $\displaystyle\int_0^{\pi} x \cdot \text{sen}\, x \, dx$

X e suas DERIVADAS		sen x e suas INTEGRAIS
x	$+$	$\text{sen}\, x$
1	$-$	$-\cos x$
0	$+$	$-\text{sen}\, x$

$$\int_0^{\pi} x \cdot \text{sen}\, x \, dx = -x \cos x + \text{sen}\, x + C$$

$$= \Big[\text{sen}\, x - x \cos x\Big]_0^{\pi}$$

$$= \Big[\text{sen}\, \pi - \pi \cdot \cos \pi\Big] - \Big[\text{sen}\, 0 - 0 \cdot \cos 0\Big]$$

$$= \Big[0 - \pi \cdot (-1)\Big] - \Big[0 - 0\Big]$$

$$= +\pi \ u.m.^2$$

↳ UNIDADE DE MEDIDA

b) $\left[\text{sen } x - x \cdot \cos x \right]_{\pi}^{2\pi} = \left[\text{sen } 2\pi - 2\pi \cdot \cos 2\pi \right] - \left[\text{sen } \pi - \pi \cdot \cos \pi \right]$

$= \left[0 + (-2\pi) \right] - \left[0 + \pi \right]$

$= -2\pi - \pi$

$= -3\pi$ um²

c) $\left[\text{sen } x - x \cdot \cos x \right]_{2\pi}^{3\pi} = \left[\text{sen } 3\pi - 3\pi \cdot \cos 3\pi \right] - \left[\text{sen } 2\pi - 2\pi \cdot \cos 2\pi \right]$

$= \left[0 + 3\pi \right] - \left[0 - 2\pi \right]$

$= 3\pi + 2\pi$

$= 5\pi$ um²

4:

a) $\int \cos 2x \, dx = \dfrac{\text{sen}\, 2x}{2} + C$

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b) $\int \cos^2 x \, dx$

$$\boxed{\cos^2 x = \frac{1 + \cos 2x}{2}}$$

$$\int \cos^2 x \, dx = \int \frac{1 + \cos 2x}{2} \, dx$$

$$= \frac{1}{2} \int dx + \frac{1}{2} \int \cos 2x \, dx$$

$$= \frac{x}{2} + \frac{\text{sen}\, 2x}{4} + C$$

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c) $\int \cos^3 x \cdot \text{sen}\, x \, dx$

$$\boxed{\begin{array}{l} \text{sen}^2 x + \cos^2 x = 1 \\ \cos^2 x = 1 - \text{sen}^2 x \end{array}} \qquad \boxed{\begin{array}{l} u = \text{sen}\, x \\ du = \cos x \, dx \end{array}}$$

$$\int \cos^2 x \cdot \cos x \cdot \text{sen}\, x \, dx = \int (1 - \text{sen}^2 x) \cdot \cos x \cdot \text{sen}\, x \, dx = \int (1 - u^2) \cdot u \, du = \int u - u^3 \, d$$

$$\int u - u^3 \, du = \frac{u^2}{2} - \frac{u^4}{4} + C = \frac{\text{sen}^2 x}{2} - \frac{\text{sen}^4 x}{4} + C$$

d) $\int \operatorname{sen}^2 x \, dx$

$$\boxed{\operatorname{sen}^2 x = \frac{1 - \cos 2x}{2}}$$

$$\int \operatorname{sen}^2 x \, dx = \int \frac{1 - \cos 2x}{2} \, dx = \frac{1}{2} \int dx - \frac{1}{2} \int \cos 2x \, dx$$

$$= \frac{x}{2} - \frac{\operatorname{sen} 2x}{4} + C$$

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e) $\int \operatorname{sen}^3 x \, dx$

$\operatorname{sen}^2 x + \cos^2 x = 1$

$\operatorname{sen}^2 x = 1 - \cos^2 x$

$u = \cos x$

$du = -\operatorname{sen} x \, dx$

$-du = \operatorname{sen} x \, dx$

$$\int \operatorname{sen}^2 x \cdot \operatorname{sen} x \, dx = \int (1 - \cos^2 x) \cdot \operatorname{sen} x \, dx = -\int (1 - u^2) \, du = -\left(u - \frac{u^3}{3}\right)$$

$$-\left(u - \frac{u^3}{3}\right) = -\left(\cos x - \frac{\cos^3 x}{3}\right) + C = \frac{\cos^3 x}{3} - \cos x + C$$

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f) $\int_0^{\pi/2} \operatorname{sen}^2 \theta \cdot \cos^3 2\theta \, d\theta$

$\int$:(

5:

a) $$\int_0^{2\pi} \sqrt{\frac{1-\cos x}{2}}\, dx$$

$$\operatorname{sen}^2 x = \frac{1-\cos 2x}{2}$$

$$\boxed{\operatorname{sen}^2\left(\frac{x}{2}\right) = \frac{1-\cos x}{2}}$$

$$\hookrightarrow \int_0^{2\pi} \sqrt{\operatorname{sen}^2(x/2)}\, dx = \int_0^{2\pi} |\operatorname{sen}(x/2)|\, dx = \left|-2\cos(x/2)\right|_0^{2\pi}$$

$$= \left|-2\cos(\pi) + 2\cos(0)\right|_0^{2\pi}$$

$$= |2+2|$$

$$= 4$$

* CUIDADO COM O MÓDULO AO RETIRAR A RAIZ.

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b) $$\int_0^{\pi} \sqrt{1-\operatorname{sen}^2 t}\, dt$$

$$\boxed{\cos^2 t = 1-\operatorname{sen}^2 t}$$

$$\hookrightarrow \int_0^{\pi} \sqrt{\cos^2 t}\, dt = \left|\operatorname{sen} t\right|_0^{\pi}$$

$$= |\operatorname{sen}\pi - \operatorname{sen} 0|$$

$$= |0-0|$$

$$= 0$$

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c) $$\int_{\pi/2}^{3\pi/4} \sqrt{1-\operatorname{sen}^2 x}\, dx$$

? :(

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6:

$$\text{sen}(Mx) \cdot \text{sen}(Nx) = \frac{1}{2}\left[\cos(m+N)x - \cos(m-N)x\right]$$

$$\text{sen}(mx) \cdot \cos(Nx) = \frac{1}{2}\left[\text{sen}(m+N)x + \text{sen}(m-N)x\right]$$

$$\cos(mx) \cdot \cos(Nx) = \frac{1}{2}\left[\cos(m+N)x + \cos(m-N)x\right]$$

a) $$\int \text{sen}\,3x \cdot \cos 2x\,dx = \int \frac{1}{2}\left[\text{sen}(5x) + \text{sen}\,x\right]dx$$

$$= \frac{1}{2}\int \text{sen}\,5x\,dx + \frac{1}{2}\int \text{sen}\,x\,dx$$

$$= -\frac{\cos 5x}{10} - \frac{1}{2}\cos x + C$$

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b) $$\int_{-\pi}^{\pi} \text{sen}\,3x \cdot \text{sen}\,3x\,dx = \int \frac{1}{2}\left[\cos(6x) - \cos(0x)\right]dx = \frac{1}{2}\int \cos 6x\,dx - \frac{1}{2}\int \underbrace{\cos 0}_{=1}\,dx$$

$$= \left[\frac{\text{sen}\,6x}{12} - \frac{x}{2}\right]_{-\pi}^{\pi}$$

$$\left[\frac{\text{sen}\,\cancel{6}x}{\cancel{12}} - \frac{x}{2}\right]_{-\pi}^{\pi} = \left[\frac{\text{sen}\,\pi}{2} - \frac{\pi}{2}\right] - \left[\frac{\text{sen}\,-\pi}{2} + \frac{\pi}{2}\right]$$

$$= -\frac{\pi}{2} - \frac{\pi}{2} = -\frac{2\pi}{2} = -\pi$$

c) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \cdot \cos 7x \, dx = \int \frac{1}{2}\left[\cos(8x) + \cos(6x)\right] dx$

$$= \frac{1}{2}\int \cos 8x \, dx + \frac{1}{2}\int \cos 6x \, dx$$

$$= \left[\frac{\text{sen } \cancel{8}x}{\cancel{16}} + \frac{\text{sen } \cancel{6}x}{\cancel{12}}\right]_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}$$

$$= \left[\frac{\text{sen}\left(\frac{\pi}{2}\right)}{2} + \frac{\text{sen}\left(\frac{\pi}{2}\right)}{2}\right] - \left[\frac{\text{sen}\left(-\frac{\pi}{2}\right)}{2} + \frac{\text{sen}\left(-\frac{\pi}{2}\right)}{2}\right]$$

$$= \left[\frac{1}{2} + \frac{1}{2}\right] - \left[-\frac{1}{2} - \frac{1}{2}\right]$$

$$= 1 - [-1]$$

$$= 2$$

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7°

a) $\displaystyle\int \frac{3\,dx}{\sqrt{1+9x^2}} = 3\int \frac{dx}{\sqrt{1+9x^2}} = 3\int \frac{dx}{\sqrt{1^2+(3x)^2}}$

$$\boxed{\begin{aligned} &* \sqrt{1^2+(3x)^2} = 1.\sec\theta \\ &x = 1.\,tg\,\theta \\ &dx = 1.\sec^2\theta\,d\theta \end{aligned}}$$

$$3\int \frac{dx}{\sqrt{1+9x^2}} = 3\int \frac{\sec^{\cancel{2}}\theta\,d\theta}{\cancel{\sec\theta}} = 3\ln|\sec\theta + tg\,\theta| + C$$

$$= 3\ln\left|\sqrt{1+9x^2} + x\right| + C$$

Desfazer a substituição

b) $\displaystyle\int \sqrt{1-9t^2}\,dt$

$$\boxed{\begin{aligned} &\sqrt{1^2-(3t)^2} = \cos\theta \\ &x = \operatorname{sen}\theta \\ &dx = \cos\theta\,d\theta \end{aligned}}$$

$$\int \cos\theta\,d\theta = -\operatorname{sen}\theta + C$$

$$= -x + C$$

Desfazer a substituição

c) $\displaystyle\int \frac{2\,dx}{x^2\sqrt{x^2-1}} = 2\int \frac{dx}{x^2\sqrt{x^2-1}}$

$$\boxed{\begin{gathered}\sqrt{x^2-1} = tg\,\theta \\ x = sec\,\theta \\ dx = sec\,\theta \cdot tg\,\theta\,d\theta\end{gathered}}$$

$$2\int \frac{sec\,\theta \cdot tg\,\theta\,d\theta}{sec^2\theta \cdot tg\,\theta} = 2\int \frac{d\theta}{sec\,\theta}$$

$$= 2\,arc\,sec\,\theta + C$$

É o inverso da secante