

Resolução - Lista 2 - Sistema de Equações Lineares

1ª a)

$$A = \begin{bmatrix} 1 & 1 & 2 & 0 \\ 2 & -1 & -1 & 3 \\ 1 & -3 & -2 & 4 \end{bmatrix}_{3 \times 4} \begin{array}{l} L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 - L_1 \end{array} \sim \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 5 & 3 & -5 \\ 0 & -4 & -4 & 4 \end{bmatrix}_{3 \times 4}$$

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 5 & 3 & -5 \\ 0 & -4 & -4 & 4 \end{bmatrix} \begin{array}{l} L_3 \leftarrow L_3 * \frac{1}{4} \\ \sim \end{array} \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 5 & 3 & -5 \\ 0 & -1 & -1 & 1 \end{bmatrix}_{3 \times 4}$$

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 5 & 3 & -5 \\ 0 & -1 & -1 & 1 \end{bmatrix} \begin{array}{l} L_1 \leftarrow L_1 + L_3 \\ L_2 \leftarrow L_2 + 4L_3 \\ \sim \end{array} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & -1 & -9 & -1 \\ 0 & -1 & -1 & 1 \end{bmatrix}_{3 \times 4}$$

Continuação
da 1ª Letra
a)

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -9 & -1 \\ 0 & -1 & -1 & 1 \end{bmatrix} \begin{matrix} L_3 \leftarrow L_3 + L_2 \\ \sim \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -9 & -1 \\ 0 & 0 & -10 & 0 \end{bmatrix}_{3 \times 4}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -9 & -1 \\ 0 & 0 & -10 & 0 \end{bmatrix} \begin{matrix} L_3 \leftarrow L_3 \times (-10) \\ \sim \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -9 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3 \times 4}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -9 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} L_2 \leftarrow L_2 + 9L_3 \\ \sim \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3 \times 4}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} L_1 \leftarrow L_1 - L_3 \\ \sim \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3 \times 4}$$

Logo, A é linha equivalente a B.

1ª b) Sim, B é matriz linha reduzida à forma escada, pois as 4 pontos abaixo são atendidas:

- 1- O primeiro elemento não-nulo de cada linha é 1;
- 2- Os outros elementos da coluna que contém o elemento não-nulo são 0;
- 3- Não há linha nula, mas se houvesse, deveria estar no final;
- 4- Os elementos não nulos aparecem formando uma escada, o primeiro elemento não-nulo.

c) O posto de A é 3. A nulidade é $4 - 3 = 1$.

2ª Letra a)

$$A = \begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -4 & 3 \\ 2 & 3 & 2 & -1 \end{bmatrix} \begin{array}{l} L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 - 3L_1 \\ \sim \end{array} \begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 0 & -7 & 5 \\ 2 & 0 & -7 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 0 & -7 & 5 \\ 2 & 0 & -7 & 5 \end{bmatrix} \begin{array}{l} L_3 \leftarrow L_3 + (-L_2) \\ \sim \end{array} \begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 0 & -7 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 0 & -7 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} L_2 \leftarrow L_2 \cdot \frac{1}{2} \\ \sim \end{array} \begin{bmatrix} 0 & 1 & 3 & -2 \\ 1 & 0 & -7/2 & 5/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 3 & -2 \\ 1 & 0 & -7/2 & 5/2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} L_1 \leftarrow L_1 \leftrightarrow L_2 \\ \sim \end{array} \begin{bmatrix} 1 & 0 & -7/2 & 5/2 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b) O posto de A é 2. A nulidade é $4-2=2$.

3ª Letra a)

$$\begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & 3 \\ 0 & -4 & 2 \\ 0 & 2 & -3 \end{bmatrix} \begin{array}{l} L_3 \leftarrow L_3 - 3L_2 \\ L_4 \leftarrow L_4 - 2L_2 \\ \sim \end{array} \begin{bmatrix} 0 & 2 & 2 \\ 1 & 1 & 3 \\ 0 & -7 & -7 \\ 0 & -5 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 2 \\ 1 & 1 & 3 \\ 0 & -7 & -7 \\ 0 & -5 & -5 \end{bmatrix} \begin{array}{l} L_1 \leftarrow L_1 \times 1/2 \\ L_3 \leftarrow L_3 \times 1/-7 \\ L_4 \leftarrow L_4 \times 1/-5 \end{array} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{array}{l} L_1 \leftarrow L_1 - L_3 \\ L_1 \leftarrow L_1 - L_4 \end{array} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Continuação da 3ª a)

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{L_1 \leftrightarrow L_2} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{L_3 \leftarrow L_3 - L_2} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{L_1 \leftarrow L_1 - L_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

b) O posto de A é 2. A multiplicidade é $3 - 2 = 1$.

4ª Letra a)

$$4^a \quad \left[\begin{array}{ccc|c} 2 & -1 & 3 & 11 \\ 4 & 3 & 2 & 0 \\ 1 & 1 & 1 & 6 \\ 3 & 1 & 1 & 4 \end{array} \right]$$

letra b)

$$L_1 \quad \left[\begin{array}{ccc|c} 2 & -1 & 3 & 11 \\ 4 & 3 & 2 & 0 \\ 1 & 1 & 1 & 6 \\ 3 & 1 & 1 & 4 \end{array} \right] \quad \begin{array}{l} L_1 \leftarrow L_1 - L_3 \\ L_2 \leftarrow L_2 - 4L_3 \\ L_4 \leftarrow L_4 - 3L_3 \end{array} \quad \sim \quad \left[\begin{array}{ccc|c} 1 & -2 & 2 & 5 \\ 0 & -1 & -2 & -24 \\ 1 & 1 & 1 & 6 \\ 0 & -2 & -2 & -14 \end{array} \right] \quad \begin{array}{l} L_3 \leftarrow L_3 - L_1 \\ L_4 \leftarrow L_4 + 2L_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 5 \\ 0 & -1 & -2 & -24 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & 7 \end{array} \right] \quad \begin{array}{l} L_2 \leftarrow L_2 * (-1) \\ L_4 \leftarrow L_4 - L_3 \end{array} \quad \sim \quad \left[\begin{array}{ccc|c} 1 & -2 & 2 & 5 \\ 0 & 1 & 2 & 24 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 2 & 6 \end{array} \right] \quad \begin{array}{l} L_3 \leftarrow L_3 - L_2 \\ L_1 \leftarrow L_1 + 2L_2 \end{array}$$

Continuação letra b)

$$\begin{bmatrix} 1 & 0 & 6 & 53 \\ 0 & 1 & 2 & 24 \\ 0 & 0 & -3 & -23 \\ 0 & 0 & 2 & 6 \end{bmatrix} \begin{array}{l} L_3 \leftarrow L_3 + (-1)L_4 \\ L_1 \leftarrow L_1 - 3L_4 \\ L_2 \leftarrow L_2 - L_4 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 35 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 3 & 23 \\ 0 & 0 & 2 & 6 \end{bmatrix} \begin{array}{l} L_3 \leftarrow L_3 - L_4 \\ L_4 \leftarrow L_4 + \frac{1}{2} \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 35 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 17 \\ 0 & 0 & 1 & 3 \end{bmatrix} \begin{array}{l} L_4 \leftarrow L_4 - L_3 \\ L_4 \leftarrow L_4 + \frac{1}{-14} \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 35 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 17 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} L_1 \leftarrow L_1 - 35L_4 \\ L_2 \leftarrow L_2 - 11L_4 \\ L_3 \leftarrow L_3 - 17L_4 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

c) Posto da matriz ampliada : 4

Posto da matriz dos coeficientes : 3

d) Sistema Impossível

e) Não há solução pois o posto da matriz ampliada é diferente do posto da matriz dos coeficientes.

5ª Letra a)

$$\begin{array}{cccc|c} x_1 & x_{II} & x_{III} & x_{IV} & \\ \hline 1 & 2 & -1 & 3 & 1 \end{array}$$

$$P_A = 1$$

$$P_C = 1$$

Número de inequantes = 4

Sistema Posível Indeterminado

$$S = \left\{ x_1 = 1 - 2x_{II} + x_{III} - 3x_{IV} ; \text{ com } x_{II}, x_{III}, x_{IV} \in \mathbb{R} \right\}$$

5ª b)

$$\begin{bmatrix} 1 & 1 & 1 & | & 4 \\ 2 & 5 & -2 & | & 3 \end{bmatrix} \xrightarrow{L_2 \leftarrow L_2 - 2L_1} \begin{bmatrix} 1 & 1 & 1 & | & 4 \\ 0 & 3 & -4 & | & -5 \end{bmatrix} \xrightarrow{L_2 \leftarrow L_2 \cdot \frac{1}{3}} \sim$$

$$\begin{bmatrix} 1 & 1 & 1 & | & 4 \\ 0 & 1 & -\frac{4}{3} & | & -\frac{5}{3} \end{bmatrix} \xrightarrow{L_1 \leftarrow L_1 - L_2} \begin{bmatrix} 1 & 0 & \frac{7}{3} & | & \frac{17}{3} \\ 0 & 1 & -\frac{4}{3} & | & -\frac{5}{3} \end{bmatrix}$$

$\text{Rango}_A = 2$

$\text{Rango}_C = 2$

Número de incógnitas: 3

Sistema Positivo e Indeterminado

$$x = 7 - \frac{7}{3}z, \quad z \in \mathbb{R}$$

$$y = -3 + \frac{4}{3}z, \quad z \in \mathbb{R}$$

$$5^a \text{ c)} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 2 & 5 & -2 & 3 \\ 1 & 7 & -7 & 5 \end{array} \right] \begin{array}{l} L_2 \rightarrow L_2 - 2L_1 \\ L_3 \rightarrow L_3 - L_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & -4 & -5 \\ 0 & 6 & -8 & 1 \end{array} \right] \begin{array}{l} L_1 \leftarrow L_1 - \frac{1}{3}L_2 \\ L_3 \leftarrow L_3 + \frac{1}{3}L_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 + \frac{4}{3} & 4 + \frac{5}{3} \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} \\ 0 & 0 & -3 & 1 \end{array} \right] \begin{array}{l} L_3 \leftarrow L_3 - 6L_2 \\ \sim \end{array} \left[\begin{array}{ccc|c} 1 & 0 & \frac{7}{3} & \frac{17}{3} \\ 0 & 1 & -\frac{4}{3} & -\frac{5}{3} \\ 0 & 0 & 0 & 11 \end{array} \right]$$

$$P_A = 3$$

$$P_C = 2$$

Sistema Incompatível

5ª d)

$$\begin{bmatrix} 1 & -2 & 3 & | & 0 \\ 2 & 5 & 6 & | & 0 \end{bmatrix} \xrightarrow{L_2 \leftarrow L_2 - 2L_1} \begin{bmatrix} 1 & -2 & 3 & | & 0 \\ 0 & 9 & 0 & | & 0 \end{bmatrix} \xrightarrow{L_2 \leftarrow L_2 \cdot \frac{1}{9}}$$

$$\begin{bmatrix} 1 & -2 & 3 & | & 0 \\ 0 & 1 & 0 & | & 0 \end{bmatrix} \xrightarrow{L_1 \leftarrow L_1 + 2L_2} \begin{bmatrix} 1 & 0 & 3 & | & 0 \\ 0 & 1 & 0 & | & 0 \end{bmatrix}$$

$P_A = 2$ $P_C = 2$ Número de incógnitas = 3

Sistema Parcial Indeterminado

$$S = \left\{ \begin{array}{l} x = -3z, \quad z \in \mathbb{R} \\ y = 0 \\ z \in \mathbb{R} \end{array} \right\}$$

e)

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & -1 & 4 \\ 1 & 1 & -1 & 1 & -4 \\ 1 & -1 & 1 & 1 & 2 \end{bmatrix} \begin{array}{l} L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 - L_1 \\ L_4 \leftarrow L_4 - L_1 \\ \sim \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 4 \\ 0 & 0 & -2 & 0 & -4 \\ 0 & -2 & 0 & 0 & 2 \end{bmatrix} \begin{array}{l} L_2 \leftarrow L_2 + (-\frac{1}{2}) \\ L_3 \leftarrow L_3 + (-\frac{1}{2}) \\ L_4 \leftarrow L_4 + (-\frac{1}{2}) \end{array}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{array}{l} L_2 \leftrightarrow L_4 \\ \sim \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \begin{array}{l} L_1 \leftarrow L_1 - L_2 \\ L_1 \leftarrow L_1 - L_4 \\ L_1 \leftarrow L_1 - L_3 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

$P_A = 4$ $P_C = 4$ $\text{Nulidade} = 4 - 4 = 0$ $N^\circ \text{ de incógnitas} = 4$

Sistema Possível e Determinado

$S = (-1, -1, 2, -2)$

f)

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & -1 & 4 \\ 1 & 1 & -1 & 1 & -4 \\ 1 & -1 & 1 & 1 & 2 \end{bmatrix} \begin{array}{l} L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 - L_1 \\ L_4 \leftarrow L_4 - L_1 \\ \sim \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 4 \\ 0 & 0 & -2 & 0 & -4 \\ 0 & -2 & 0 & 0 & 2 \end{bmatrix} \begin{array}{l} L_2 \leftarrow L_2 + (-\frac{1}{2}) \\ L_3 \leftarrow L_3 + (-\frac{1}{2}) \\ L_4 \leftarrow L_4 + (-\frac{1}{2}) \end{array}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{array}{l} L_2 \leftrightarrow L_4 \\ \sim \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \begin{array}{l} L_1 \leftarrow L_1 - L_2 \\ L_1 \leftarrow L_1 - L_4 \\ L_1 \leftarrow L_1 - L_3 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

$P_A = 4$ $P_C = 4$ $\text{Nulidade} = 4 - 4 = 0$ $N^\circ \text{ de incógnitas} = 4$

Sistema Possível e Determinado

$S = (-1, -1, 2, -2)$

g)

$$\left[\begin{array}{ccc|c} 3 & 2 & -4 & 1 \\ 1 & -1 & 1 & 3 \\ 1 & -1 & -3 & -3 \\ 3 & 3 & -5 & 0 \\ -1 & 1 & 1 & 1 \end{array} \right] \begin{array}{l} L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 + L_1 \\ L_4 \leftarrow L_4 - L_1 \\ L_5 \leftarrow 3L_5 + L_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 3 & 2 & -4 & 1 \\ 0 & 0 & 4 & 6 \\ 0 & 0 & -2 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 5 & -1 & 4 \end{array} \right] \begin{array}{l} L_2 \leftarrow L_2 \cdot \frac{1}{2} \\ L_3 \leftarrow L_3 \cdot \frac{1}{2} \\ L_4 \leftarrow L_4 \cdot (-1) \end{array}$$

$$\left[\begin{array}{ccc|c} 3 & 2 & -4 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 4 & -1 \\ 0 & 5 & -1 & 4 \end{array} \right] \begin{array}{l} L_1 \leftarrow L_1 + 2L_2 \\ L_4 \leftarrow 5L_4 + L_5 \\ \sim \end{array}$$

$$\left[\begin{array}{ccc|c} 3 & 0 & -2 & 3 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 4 & 9 \\ 0 & 5 & -1 & 4 \end{array} \right] \begin{array}{l} L_1 \leftarrow L_1 + L_2 \\ L_3 \leftarrow L_3 - L_5 \\ L_4 \leftarrow L_4 + (-2L_2) \end{array}$$

$$\left[\begin{array}{ccc|c} 3 & 0 & 0 & 6 \\ 0 & 0 & 2 & 3 \\ 0 & -5 & 0 & -3 \\ 0 & 0 & 0 & 3 \\ 0 & 5 & -1 & 4 \end{array} \right] \begin{array}{l} L_2 \leftarrow L_2 + 2L_3 \\ L_5 \leftarrow L_5 + L_3 \\ \sim \end{array}$$

$$\left[\begin{array}{ccc|c} 3 & 0 & 0 & 6 \\ 0 & -10 & 0 & -11 \\ 0 & -5 & 0 & -3 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & -1 & 1 \end{array} \right] \begin{array}{l} L_3 \leftarrow L_3 + L_2 \\ L_5 \leftarrow L_5 \cdot (-1) \end{array}$$

continuação da g)

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 1 & 3 & 0 \\ 3 & 2 & 1 & 0 \end{array} \right] \quad \begin{array}{l} L_1 \leftarrow L_1 - 2L_2 \\ L_3 \leftarrow L_3 - 3L_2 \\ \sim \end{array} \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & -4 & -8 & 0 \end{array} \right] \quad \begin{array}{l} L_1 \leftarrow L_1 + \frac{1}{-3} \\ L_3 \leftarrow L_3 + \frac{1}{-4} \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right] \quad \begin{array}{l} L_3 \leftarrow L_3 - L_2 \\ L_1 \leftarrow L_1 - 2L_2 \\ \sim \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \quad \begin{array}{l} L_1 \leftarrow L_1 - L_3 \\ L_2 \leftarrow L_2 - L_3 \\ \sim \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$P_A = 3 \quad P_C = 3 \quad \text{Índice} = 3$$

Sistema Possível Determinado

$$S = (0, 0, 0)$$

última letra da 5ª - h)

$$\left[\begin{array}{ccc|c} 3 & 0 & 0 & 6 \\ 0 & 10 & 0 & 11 \\ 0 & 0 & 0 & 17 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

$$L_2 \leftarrow L_2 \cdot \frac{1}{10}$$

$$L_5 \leftarrow L_5 - L_2$$

$$\left[\begin{array}{ccc|c} 3 & 0 & 0 & 6 \\ 0 & 1 & 0 & \frac{11}{10} \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 17 \end{array} \right]$$

$$L_1 \leftarrow L_1 \cdot \frac{1}{3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 11/10 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 17 \end{array} \right]$$

$$P_A = 5$$

$$P_C = 3$$

Sistema Impensável

6a

$$\left[\begin{array}{cc|c} R & 2 & 6 \\ 3 & -1 & -2 \\ 1 & 1 & 0 \end{array} \right] \begin{array}{l} L_2 \leftarrow L_2 - 3L_3 \\ \sim \end{array} \left[\begin{array}{cc|c} R & 2 & 6 \\ 0 & -4 & -2 \\ 1 & 1 & 0 \end{array} \right] \begin{array}{l} L_2 \leftarrow L_2 \cdot \frac{1}{(-4)} \\ L_3 \leftarrow kL_3 - L_1 \\ \sim \end{array}$$

$$\left[\begin{array}{cc|c} R & 2 & 6 \\ 0 & 2 & 1 \\ 0 & R-1 & -6 \end{array} \right] \begin{array}{l} L_1 \leftarrow L_1 - L_2 \\ L_2 \leftarrow L_2 + L_3 \\ L_3 \leftarrow L_3 - L_2 \end{array} \left[\begin{array}{cc|c} R & 0 & 5 \\ 0 & R & -5 \\ 0 & -2 & -11 \end{array} \right] \begin{array}{l} L_3 \leftarrow kL_3 + 2L_2 \\ \sim \end{array}$$

$$\left[\begin{array}{cc|c} R & 0 & 5 \\ 0 & R & -5 \\ 0 & 0 & -11k + 10 \end{array} \right]$$

Continuação da 6ª

$$\left[\begin{array}{cc|c} R & 0 & 5 \\ 0 & R & -5 \\ 0 & 0 & -11R + 10 \end{array} \right]$$

Para o Sistema ser em SPD, é preciso que $-11R + 10 = 0$, logo:

$$\begin{aligned} -11R + 10 &= 0 \\ R &= \frac{-10}{-11} = \frac{10}{11} \end{aligned}$$

Para $R \neq \frac{10}{11}$, então o sistema é Impossível.

7a

$$\left[\begin{array}{cc|c} -4 & 3 & 2 \\ 5 & -4 & 0 \\ 2 & -1 & R \end{array} \right] \begin{array}{l} L_1 \quad L - L_1 + 2L_3 \\ \sim \end{array} \left[\begin{array}{cc|c} 0 & 0 & 2+2R \\ 5 & -4 & 0 \\ 2 & -1 & R \end{array} \right] \begin{array}{l} L_2 \quad L - 2L_2 - 5L_3 \\ \sim \end{array}$$

$$\left[\begin{array}{cc|c} 0 & 0 & 2+2R \\ 0 & -3 & -5R \\ 2 & -1 & R \end{array} \right] \begin{array}{l} L_2 \quad L - L_2 - 3L_3 \\ \sim \end{array} \left[\begin{array}{cc|c} 0 & 0 & 2+2R \\ -6 & 0 & -5R-3R = -2R \\ 2 & -1 & R \end{array} \right] \begin{array}{l} L_2 \quad L - L_2 + R \\ \sim \end{array}$$

$$\left[\begin{array}{cc|c} 0 & 0 & 2+2R \\ 0 & -3 & R \\ 2 & -1 & R \end{array} \right] \begin{array}{l} L_1 \quad L - L_1 + L_3 \\ L_3 \quad L - L_3 - L_1 \\ \sim \end{array} \left[\begin{array}{cc|c} 2 & -1 & 2+3R \\ 0 & -3 & R \\ 0 & 0 & 2+2R \end{array} \right] \begin{array}{l} L_1 \quad L - L_1 \times \frac{1}{2} \\ L_2 \quad L - L_2 \times \frac{1}{-3} \\ \sim \end{array}$$

Continuação da 7ª

$$\left[\begin{array}{cc|c} 0 & 0 & 2+2R \\ 0 & -3 & R \\ 2 & -1 & R \end{array} \right] \begin{array}{l} L_1 \quad L - L_1 + L_3 \\ L_3 \quad L - L_3 - L_1 \\ \sim \end{array} \left[\begin{array}{cc|c} 2 & -1 & 2+3R \\ 0 & -3 & R \\ 0 & 0 & 2+2R \end{array} \right] \begin{array}{l} L_1 \quad L - L_1 = \frac{1}{2} \\ L_2 \quad L - L_2 = \frac{1}{-3} \\ \sim \end{array}$$

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{2+3R}{2} \\ 0 & 1 & R/-3 \\ 0 & 0 & 2+2R \end{array} \right] \begin{array}{l} L_1 \quad L - L_1 + \frac{1}{2} L_2 \\ \sim \end{array} \left[\begin{array}{cc|c} 1 & 0 & \frac{6+8R}{6} \\ 0 & 1 & -R/3 \\ 0 & 0 & 2+2R \end{array} \right]$$

Para que o sistema acima tenha solução, é preciso que

$$2+2R=0$$

$$2=-2R$$

$$\boxed{R=-1}$$

8°

8ª

$$\left[\begin{array}{ccccc|c} 1 & 3 & 2 & 3 & -7 & 14 \\ 2 & 6 & 1 & -2 & 5 & -2 \\ 1 & 3 & -1 & 0 & 2 & -1 \end{array} \right] \begin{array}{l} L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 - L_1 \\ \sim \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 3 & 2 & 3 & -7 & 14 \\ 0 & 0 & -3 & -8 & 19 & -30 \\ 0 & 0 & -3 & -3 & 9 & -15 \end{array} \right] \begin{array}{l} L_3 \leftarrow L_3 \cdot \left(\frac{1}{-3} \right) \\ L_2 \leftrightarrow L_3 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 3 & 2 & 3 & -7 & 14 \\ 0 & 0 & 1 & 1 & -3 & 5 \\ 0 & 0 & -3 & -8 & 19 & -30 \end{array} \right] \begin{array}{l} L_1 \leftarrow L_1 - 2L_2 \\ L_3 \leftarrow L_3 + 3L_2 \\ \sim \end{array}$$

Continuação da 8ª

$$\left[\begin{array}{ccccc|c} 1 & 3 & 0 & 1 & 13 & 4 \\ 0 & 0 & 1 & 1 & -3 & 5 \\ 0 & 0 & 0 & -5 & 10 & -15 \end{array} \right] \quad L_3 \leftarrow L_3 + \begin{pmatrix} 1 \\ -5 \end{pmatrix}$$

$$\left[\begin{array}{ccccc|c} 1 & 3 & 0 & 1 & 13 & 4 \\ 0 & 0 & 1 & 1 & -3 & 5 \\ 0 & 0 & 0 & 1 & -2 & 3 \end{array} \right] \quad \begin{array}{l} L_1 \leftarrow L_1 - L_3 \\ L_2 \leftarrow L_2 - L_3 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 3 & 0 & 0 & 15 & 1 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & -2 & 3 \end{array} \right]$$

$$S = \left\{ \begin{array}{l} x_I = 1 - 15x_V - 3x_{III} \\ x_{III} = 2 + x_V \\ x_{IV} = 3 + 2x_V \\ x_{II} \in \mathbb{R} \\ x_V \in \mathbb{R} \end{array} \right\}$$

9ª a) Toda as inequações equivalem a 0.

$$b_1 \quad \begin{array}{c} x \quad y \quad z \\ \left[\begin{array}{ccc|c} 2 & -5 & 2 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 0 & k & 0 \end{array} \right] \end{array} \quad \begin{array}{l} L_1 \leftrightarrow L_2 \\ L_2 \leftrightarrow L_2 - L_3 \\ L_3 \leftrightarrow L_3 - 2L_1 \end{array} \quad \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -5 & 2-k & 0 \\ 0 & -2 & k-4 & 0 \end{array} \right] \end{array} \quad \begin{array}{l} L_2 \leftrightarrow L_2 \cdot \left(\frac{1}{-5}\right) \end{array}$$

$$\begin{array}{c} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{2-k}{-5} & 0 \\ 0 & -2 & k-4 & 0 \end{array} \right] \end{array} \quad \begin{array}{l} L_3 \leftrightarrow L_3 \cdot \left(\frac{1}{-2}\right) \\ \sim \end{array} \quad \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{-2+k}{5} & 0 \\ 0 & 1 & \frac{-k+4}{2} & 0 \end{array} \right] \end{array} \quad \begin{array}{l} L_1 \leftrightarrow L_1 - L_2 \\ L_3 \leftrightarrow L_3 - L_2 \\ \sim \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 + \frac{2+k}{5} & 0 \\ 0 & 1 & \frac{-2+k}{5} & 0 \\ 0 & 0 & \frac{-k+4}{2} + \frac{2-k}{5} & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 + \frac{2+K}{5} & 0 \\ 0 & 1 & \frac{-2+K}{5} & 0 \\ 0 & 0 & \frac{-K+4}{2} + \frac{2-K}{5} & 0 \end{array} \right]$$

Continuação da 9ª

Para que o sistema acima tenha solução, é preciso que:

$$\frac{5 \cdot (-K+4)}{5 \cdot 2} + \frac{(2-K)2}{5 \cdot 2} = 0$$

$$\frac{-5K+20}{10} + \frac{4-2K}{10} = 0$$

$$-7K + 24$$

$$7K = 24$$

$$K = \frac{24}{7}$$

Solução:

$$S = \left\{ \begin{array}{l} x = - \left(1 + \frac{2+K}{5} \right) \\ y = - \left(\frac{-2+K}{5} \right) \\ z = 0 \end{array} \right\}$$

Última questão - 10ª

$$\begin{bmatrix} 1 & 1 & 0 & -3 \\ 2 & -4 & -1 & 0 \\ 3 & -2 & -1 & 2 \end{bmatrix}_{3 \times 4} \begin{bmatrix} x \\ y \\ z \\ v \end{bmatrix}_{4 \times 1} = \begin{bmatrix} x + y + 0 - 3v \\ 2x - 4y - z + 0 \\ 3x - 2y - z + 2v \end{bmatrix}_{3 \times 1} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}_{3 \times 1}$$

Agora basta resolver o sistema!