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INTEGRAÇÃO POR PARTES:

1º a) $\int x \cdot sen\left(\frac{x}{2}\right) dx$

$$u = x \qquad v = \int sen \frac{x}{2} dx = -\cos\frac{x}{2} \cdot 2$$
$$du = 1 \qquad dv = sen \frac{x}{2}$$

$$\int u\, dv = u \cdot v - \int v\, du$$

$$= x \cdot (-2) \cos\left(\frac{x}{2}\right) - \int -2 \cos\left(\frac{x}{2}\right) dx$$

$$= -2x \cdot \cos\left(\frac{x}{2}\right) + 2 \int \cos\left(\frac{x}{2}\right) dx$$

$$= -2x \cdot \cos\left(\frac{x}{2}\right) + 2 \frac{sen\left(\frac{x}{2}\right)}{\frac{1}{2}} + C$$

$$\boxed{= -2x \cdot \cos\left(\frac{x}{2}\right) + 4 \cdot sen\left(\frac{x}{2}\right) + C}$$

b) $\int x \cdot e^{3x}\,dx$

$$\boxed{\begin{array}{ll} u = x & v = \int e^{3x}\,dx = \dfrac{e^{3x}}{3} \\ du = 1 & dv = e^{3x} \end{array}}$$

$$\int u\,dv = v \cdot u - \int v\,du$$

$$= \frac{e^{3x}}{3} \cdot x - \int \frac{e^{3x}}{3}\,dx$$

$$= \frac{e^{3x}}{3} \cdot x - \frac{1}{3} \int e^{3x}$$

$$= \frac{e^{3x}}{3} \cdot x - \frac{1}{3} \cdot \frac{e^{3x}}{3} + C$$

$$= \frac{e^{3x}}{3} \cdot x - \frac{e^{3x}}{9} + C$$

$$\boxed{= e^{3x}\left(\frac{x}{3} - \frac{1}{9}\right) + C}$$

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C) $\int x^2 \cdot e^{-x} \, dx$

· Neste caso, como temos uma função fácil de DERIVAR (x^2) e uma fácil de integrar (e^{-x}), vamos usar o método tabular.

x^2 e suas DERIVADAS		e^{-x} e suas INTEGRAIS
x^2	$+$	e^{-x}
$2x$	$-$	$-e^{-x}$
2	$+$	e^{-x}
0	$-$	$-e^{-x}$

$$\int x^2 \cdot e^{-x} \, dx = -x^2 \cdot e^{-x} - 2x e^{-x} + (-2e^{-x}) + C$$

d) $\int x \cdot \ln x \, dx$

$$\boxed{\begin{array}{ll} u = x & v = \int \ln x = \frac{1}{x} \\ du = 1 & dv = \ln x \end{array}}$$

$$\int u \, dv = u \cdot v - \int v \, du$$

$$= x \cdot \frac{1}{x} - \int \frac{1}{x} \, dx$$

$$= - \ln |x| + C$$

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e) $\int e^x \cdot \operatorname{sen} x \, dx$

$$\boxed{\begin{array}{ll} u = \operatorname{sen} x & v = \int e^x dx = e^x \\ du = \cos x & dv = e^x \end{array}}$$

$$\int u \, dv = u \cdot v - \int v \, du$$

$$= \operatorname{sen} x \cdot e^x - \int e^x \cdot \cos x \, dx \quad (\mathrm{I})$$

* Vamos usar partes em $\int e^x \cdot \cos x$:

$$\boxed{\begin{array}{ll} u' = \cos x & v' = e^x \\ du' = -\operatorname{sen} x & dv' = e^x \end{array}}$$

$$\int u' \, dv' = u' \cdot v' - \int v' \, du'$$

$$= \cos x \cdot e^x - \int e^x (-\operatorname{sen} x) \cdot dx$$

$$= \cos x \cdot e^x + \int e^x \cdot \operatorname{sen} x \, dx \quad (\mathrm{II})$$

* Substituindo II em I:

$$\int e^x \cdot \operatorname{sen} x \, dx = \operatorname{sen} x \cdot e^x - \left(\cos x \cdot e^x + \int e^x \cdot \operatorname{sen} x \, dx \right)$$

$$= \operatorname{sen} x \cdot e^x - \cos x \cdot e^x - \int e^x \cdot \operatorname{sen} x \, dx$$

$$2 \int e^x \operatorname{sen} x = \operatorname{sen} x \cdot e^x - \cos x \cdot e^x$$

$$\int e^x \cdot \operatorname{sen} x = (\operatorname{sen} x - \cos x) \cdot \frac{e^x}{2}$$

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f) $\int (x^2+x+1)\cdot e^x \, dx$

$\frac{d}{dx} x^2+x+1$ $\qquad\qquad$ $\int e^x \, dx$

$\downarrow$ $\qquad\qquad$ $\downarrow$

x^2+x+1	$+$	e^x
$2x+1$	$-$	e^x
2	$+$	e^x
0	$-$	e^x

$$= e^x(x^2+x+1) - e^x(2x+1) + e^x \cdot 2 + C$$