RESOLUÇÃO INTEGRAIS POR SUBST.

1º $\displaystyle\int \frac{2x-9}{\sqrt{x^2-9x+1}}\,dx$

$u = x^2 - 9x + 1$

$du = 2x - 9\,dx$

$$\int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}}\,du = \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = 2\sqrt{u} + C$$

$$= 2\sqrt{x^2-9x+1} + C$$

2º $\displaystyle\int \frac{dx}{\sqrt{8x-x^2}}$

· Complementando o quadrado do denominador:

$$8x - x^2 = -(x^2 - 8x)$$
$$= -(x^2 - 8x + 16 - 16)$$
$$= -(x-4)^2 + 4^2$$
$$= 4^2 - (x-4)^2$$

· Então: $\displaystyle\int \frac{dx}{\sqrt{8x-x^2}} = \int \frac{dx}{\sqrt{\underbrace{4}_{a}{}^2 - \underbrace{(x-4)}_{x}{}^2}}$

· Lembre que: $\displaystyle\int \frac{dx}{\sqrt{a^2-x^2}}$, $a \in \mathbb{R}$ $= \text{arc sen}\left(\frac{x}{a}\right) + C$

· Logo: $\displaystyle\int \frac{dx}{\sqrt{4^2-(x-4)^2}} = \text{arc sen}\left(\frac{x-4}{4}\right) + C$

3º $\displaystyle\int (\sec x + tg\, x)^2\, dx = \int (\sec^2 x + 2\cdot\sec x\cdot tg\, x + tg^2 x)\, dx$

$$= \int \sec^2 x\, dx + 2\int \sec x \cdot tg\, x + \int tg^2 x\, dx$$

$$= (tang\, x + C) + (2\cdot \sec x + C) + (tg\, x - x + C)$$

$$= tg\, x + tg\, x + 2\sec x - x + C$$

$$= 2\, tg\, x + 2\sec x - x + C$$

º $\displaystyle\int_0^{\pi/4} \sqrt{1 + \cos 4x}\, dx$

Lembre que:

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$x = 2x$

$$\cos^2 2x = \frac{1 + \cos 4x}{2}$$

$$2\cos^2 2 = 1 + \cos 4x$$

Logo, $\displaystyle\int_0^{\frac{\pi}{4}} \sqrt{1 + \cos 4x}\, dx = \int_0^{\frac{\pi}{4}} \sqrt{2\cos^2 2x}\, dx = \int_0^{\pi/4} \sqrt{2}\cdot\sqrt{\cos^2 2x}\, dx$

$$= \sqrt{2}\int_0^{\pi/4} \cos 2x\, dx = \sqrt{2}\cdot\left[\frac{\text{sen}\, 2x}{2}\right]_0^{\pi/4}$$

$$\sqrt{2}\cdot\left[\frac{\text{sen}\,\frac{2\pi}{4}}{2} - \text{sen}\, 0\right] = \sqrt{2}\cdot\frac{1}{2} = \frac{\sqrt{2}}{2}$$

5°
$$\int \frac{3x^2-7x}{3x+2}\,dx$$

$$\begin{array}{r|l} 3x^2-7x & 3x+2 \\ \cline{2-2} -3x^2-2x & x-3 \\ \hline -9x & \\ +9x+6 & \\ \hline (6) & \end{array}$$

$$\frac{3x^2-7x}{3x+2} = (3x+2)(x-3) + \frac{6}{3x+2}$$

$$= \int \frac{3x^2-7x}{3x+2}\,dx = \int (x-3) + \frac{6}{3x+2}\,dx$$

$$= \int x\,dx - 3\int dx + 6\int \frac{dx}{3x+2}$$

$$= \frac{x^2}{2} - 3x + \frac{6 \cdot \ln|3x+2|}{3} + C$$

6°
$$\int \sqrt{t^2-2t^4}\,dt = \int \sqrt{t^2(1-2t^2)}\,dt = \int \sqrt{t^2}\,\sqrt{1-2t^2}\,dt = \int t\,\sqrt{1-2t}$$

$u = 1-2t^2$

$du = -4t\,dt$

$-\frac{du}{4} = t\,dt$

$$\int \sqrt{u}\left(-\frac{du}{4}\right) = -\frac{1}{4}\int \sqrt{u}\,du$$

$$= -\frac{1}{4} \cdot \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} = -\frac{1}{4} \cdot \frac{2\sqrt{u^3}}{3} = -\sqrt{}$$

$$= -\frac{\sqrt{(1-2t^2)^3}}{6} + C$$

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7º $\int (x^3+x)^9(3x^2+1)\,dx$

$u = x^3+x$

$du = 3x^2+1\,dx$

$$\int u^9\,du = \frac{u^{10}}{10} + C = \frac{(x^3+x)^{10}}{10} + C$$

8º $\int \frac{\ln x}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int u\,du = \frac{u^2}{2} + C$$

$$= \frac{\ln^2 x}{2} + C$$

9º $\int \frac{\operatorname{arc\,tg} x}{1+x^2}\,dx$

$u = \operatorname{arc\,tg} x$

$du = \frac{1}{1+x^2}\,dx$

$$\int u\,du = \frac{u^2}{2} + C$$

$$= \frac{(\operatorname{arc\,tg} x)^2}{2} + C$$

º $\int \frac{3\cos x\,dx}{\sqrt{1+3\operatorname{sen} x}} = 3\int \frac{\cos x\,dx}{\sqrt{1+3\operatorname{sen} x}}$

$u = 1+3\operatorname{sen} x$

$du = 3\cos x\,dx$

$\frac{du}{3} = \cos x\,dx$

$$= 3\int \frac{\frac{du}{3}}{\sqrt{u}} = 3\int \frac{du}{3\sqrt{u}} = \int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}}\,du$$

$$= 2\sqrt{u} + C$$

$$= 2\sqrt{1+3\operatorname{sen} x} + C$$

$$11^{\circ}\quad \int \frac{2\,dx}{x\sqrt{1-4\ln^2 x}} = 2\int \frac{dx}{x\sqrt{1-4\ln^2 x}} \qquad u = \ln x \qquad du = \frac{1}{x}\,dx$$

$$= 2\int \frac{du}{\sqrt{1-4u^2}} = 2\int \frac{du}{\sqrt{4\left(\frac{1}{4}-u^2\right)}}$$

$$= 2\int \frac{du}{2\sqrt{\left(\frac{1}{2}\right)^2-u^2}} = \int \frac{du}{\sqrt{\left(\frac{1}{2}\right)^2-u^2}} = \text{arc sen}\left(\frac{2\ln x}{1}\right) + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \text{arc sen}\left(\frac{x}{a}\right) + C$$

$$2^{\circ}\quad \int \frac{dx}{\sqrt{x}\,(\sqrt{x}+1)} \qquad u = \sqrt{x}+1 \qquad du = \frac{1}{2\sqrt{x}}\,dx \qquad 2\,du = \frac{1}{\sqrt{x}}\,dx$$

$$\int \frac{2\,du}{u} = 2\int u^{-1}\,du = 2\ln|u|$$

$$= 2\ln|\sqrt{x}+1|$$

$$3^{\circ}\quad \int \frac{dx}{x-\sqrt{x}} = \int \frac{dx}{\sqrt{x}\left(\frac{x}{\sqrt{x}}-1\right)} = \int \frac{dx}{\sqrt{x}\,(\sqrt{x}-1)}$$

$$u = \sqrt{x}-1 \qquad du = \frac{1}{2\sqrt{x}}\,dx \qquad 2\,du = \frac{1}{\sqrt{x}}\,dx$$

$$= \int \frac{2\,du}{u} = 2\int u^{-1} = 2\ln|u| + C$$

$$= 2\ln|\sqrt{x}-1| + C$$

14º $\displaystyle\int \frac{dx}{(x-2)\sqrt{x^2-4x+3}} = \int \frac{dx}{(x-2)\sqrt{(x-2)^2-1}}$

$u = x-2$

$du = dx$

$\displaystyle = \int \frac{du}{u\sqrt{u^2-1}} = \operatorname{arc\,sec} u + C$

$= \operatorname{arc\,sec}(x-2) + C$

15: $\displaystyle\int_0^{1/2} \frac{2-8x}{1+4x^2}\,dx = \int \left(\frac{2}{1+4x^2} - \frac{8x}{1+4x^2}\right) dx$

$$= 2\underbrace{\int \frac{1}{1+(2x)^2}\,dx} - \underbrace{\int \frac{8x}{1+(2x)^2}\,dx}$$

$u = 2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$u = 1+4x^2$

$du = 8x\,dx$

$$= \cancel{2} \int \frac{1}{1+u^2}\,\frac{du}{\cancel{2}} - \int \frac{du}{u}$$

$$= \int \frac{1}{1+u^2}\,du - \int \frac{du}{u}$$

$$= \operatorname{arc\,tg} u - \ln|u|$$

$$= \Big[\operatorname{arc\,tg}(2x) - \ln|1+4x^2|\Big]_0^{1/2}$$

$$= \big(\operatorname{arc\,tg}(1) - \ln|2|\big) - \big(\operatorname{arc\,tg}(0) - \ln|1|\big)$$

Ou seja, resolve normalmente e após substituir o u pelo valor original, é só realizar o F(b) - F(a).