Integrais com Secante e Tangente

a) $\int tg^4 x \, dx$

$$\boxed{\begin{aligned} tg^2 x + 1 &= sec^2 x \\ tg^2 x &= sec^2 x - 1 \end{aligned}}$$

$$\int tg^2 x \cdot tg^2 x \, dx = \int (sec^2 x - 1) \cdot tg^2 x \, dx = \int sec^2 x \cdot tg^2 x \, dx - \int tg^2 x \, dx$$

* Agora, usamos substituição:

$$u = tg\, x$$
$$du = sec^2 x \, dx$$

$$\int u^2 \, du - \int (sec^2 x - 1) \, dx = \int u^2 \, du - \int sec^2 x \, dx + \int dx$$

$$\frac{u^3}{3} - tg\, x + x + C = \frac{tg^3 x}{3} - tg\, x + x + C$$

b. $\int sec^3 x \, dx$

$\boxed{sec^2 x = tg^2 x + 1}$

$$\int sec^2 x \cdot sec\, x \, dx = \int (tg^2 x + 1) \cdot sec\, x \, dx = \int tg^2 x \cdot sec\, x + \int sec\, x \, dx$$

$\longrightarrow$ * Não deu certo desta forma

* Vamos tentar integrar por partes:

$$\int sec^3 x \, dx = \int sec\, x \cdot sec^2 x \, dx$$

$u = sec\, x$ $\quad$ $v = tg\, x$

$du = sec\, x \cdot tg\, x$ $\quad$ $dv = sec^2 x$

$$\int u \, dv = u \cdot v - \int v \, du = sec\, x \cdot tg\, x - \int tg\, x \cdot sec\, x \cdot tg\, x = sec\, x \cdot tg\, x - \int tg^2 x \cdot sec\, x \, dx$$

$$= sec\, x \cdot tg\, x - \int (sec^2 x - 1) \cdot sec\, x = sec\, x \cdot tg\, x \boxed{- \int sec^3 x \, dx} + \int sec\, x \, dx$$

$$2 \int sec^3 x \, dx = sec\, x \cdot tg\, x + \int sec\, x \, dx$$

$$\int sec^3 x \, dx = \frac{1}{2} \left(sec\, x \cdot tg\, x + \frac{\ln |sec\, x + tg\, x|}{2} + C \right)$$

d) $\int_0^{\pi} \operatorname{sen} 2x \cdot \cos^2 2x \, dx$

$$\boxed{\begin{aligned} u &= \cos 2x \\ du &= -\frac{\operatorname{sen} 2x}{2} \, dx \\ -\frac{du}{2} &= \operatorname{sen} 2x \, dx \end{aligned}}$$

$$\int_0^{\pi} u^2 - \frac{du}{2} = -\frac{1}{2}\int_0^{\pi} u^2 \, du = \frac{-u^2}{2} + C = \frac{-\cos^2 2x}{2} + C$$

$$\left[\frac{-\cos^2 2x}{2}\right]_0^{\pi} = -\frac{1}{2}\left(\cos^2 2\pi - \cos^2 2\cdot 0\right) = -\frac{1}{2}\left(1 - 1\right) = 0$$

$$\int_0^1 8\cos^4 2\pi x\,dx$$

$$\boxed{\begin{array}{ll} sen^2 x + \cos^2 x = 1 & sen^2 x = \dfrac{1-\cos 2x}{2} \\ \cos^2 x = \dfrac{1+\cos 2x}{2} & \end{array}}$$

$$8\int_0^1 \cos^4 2\pi\, x\,dx = 8\int_0^1\left(\frac{1+\cos 4\pi x}{2}\right)^2 dx = \frac{8}{4}\int_0^1 (1+\cos 4\pi x)^2\,dx$$

$$= \frac{8}{4}\int_0^1 (1 + 2\cos 4\pi + \cos^2 4\pi x)\,dx$$

$$= 2\int_0^1 \left(1 + 2\cos 4\pi x + \frac{1+\cos 8\pi x}{2}\right)dx$$

$$= 2\int_0^1 dx + 4\int_0^1 \cos 4\pi x\,dx + \int_0^1 dx + 2\int \cos 8\pi x\,dx$$

$$= 2x + \frac{4\cdot sen\, 4\pi x}{4\pi} + x + \frac{2\cdot sen\, 8\pi x}{8\pi}$$

$$= \left[3x + \frac{sen\, 4\pi x}{4\pi} + \frac{sen\, 8\pi x}{8\pi}\right]_0^1$$

$$= [3.1 + 0 + 0] - [0 + 0 + 0] = 3$$

* $$\int \operatorname{sen}(m\,x)\cdot\cos(N\,x)\,dx = \frac{1}{2}\int\left[\operatorname{sen}(m+N)x + \operatorname{sen}(m-N)x\right]dx$$

* $$\int \operatorname{sen}(m\,x)\cdot\operatorname{sen}(N\,x)\,dx = \frac{1}{2}\int\left[\cos(m+N)x - \cos(m-N)x\right]dx$$

* $$\int \cos(m\,x)\cdot\cos(N\,x)\,dx = \frac{1}{2}\int\left[\cos(m+N)x + \cos(m-N)x\right]dx$$