Resolução - Lista 2 - Sistema de Equações Lineares

1º a)

$$A = \begin{bmatrix} 1 & 1 & 2 & 0 \\ 2 & -1 & -1 & 3 \\ 1 & -3 & -2 & 4 \end{bmatrix}_{3\times 4} \begin{matrix} L_2 \leftarrow L_2 - 2L_3 \\ L_3 \leftarrow L_3 - L_1 \\ \sim \end{matrix} \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 5 & 3 & -5 \\ 0 & -4 & -4 & 4 \end{bmatrix}_{3\times 4}$$

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 5 & 3 & -5 \\ 0 & -4 & -4 & 4 \end{bmatrix} \begin{matrix} L_3 \leftarrow L_3 * \frac{1}{4} \\ \sim \end{matrix} \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 5 & 3 & -5 \\ 0 & -1 & -1 & 1 \end{bmatrix}_{3\times 4}$$

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 5 & 3 & -5 \\ 0 & -1 & -1 & 1 \end{bmatrix} \begin{matrix} L_1 \leftarrow L_1 + L_3 \\ L_2 \leftarrow L_2 + 4L_3 \\ \sim \end{matrix} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -9 & -1 \\ 0 & -1 & -1 & 1 \end{bmatrix}_{3\times 4}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -9 & -1 \\ 0 & -1 & -1 & 1 \end{bmatrix}_{3\times4} \underset{\sim}{L_3 \leftarrow L_3 + L_2} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -9 & -1 \\ 0 & 0 & -10 & 0 \end{bmatrix}_{3\times4}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -9 & -1 \\ 0 & 0 & -10 & 0 \end{bmatrix} \underset{\sim}{L_3 \leftarrow L_3 * (-10)} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -9 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3\times4}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -9 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underset{\sim}{L_2 \leftarrow L_2 + 9L_3} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3\times4}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underset{\sim}{L_1 \leftarrow L_1 - L_3} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{3\times4}$$

Logo, A é linha equivalente a B.

b) Sim, B é matriz linha reduzida à forma escada, pois os 4 pontos abaixo são atendidos:

1- O primeiro elemento não-nulo de cada linha é 1;

2- Os outros elementos da coluna que contém o elemento não-nulo são 0;

3- Não há linha nula, mas se houvesse, deveria estar no final;

4- Os elementos não nulos aparecem formando uma escada, os primeiros elementos não-nulos.

c) O posto de A é 3. A nulidade é 4-3 = 1.

2º a)

$$A = \begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -4 & 3 \\ 2 & 3 & 2 & -1 \end{bmatrix} \begin{matrix} L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 - 3L_1 \\ \sim \end{matrix} \begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 0 & -7 & 5 \\ 2 & 0 & -7 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 0 & -7 & 5 \\ 2 & 0 & -7 & 5 \end{bmatrix} \begin{matrix} L_3 \leftarrow L_3 + (-L_2) \\ \sim \end{matrix} \begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 0 & -7 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 0 & -7 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} L_2 \leftarrow L_2 * \frac{1}{2} \\ \sim \end{matrix} \begin{bmatrix} 0 & 1 & 3 & -2 \\ 1 & 0 & -7/2 & 5/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 3 & -2 \\ 1 & 0 & -7/2 & 5/2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} L_1 \leftrightarrow L_2 \\ \sim \end{matrix} \begin{bmatrix} 1 & 0 & -7/2 & 5/2 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

O posto de A é 2. A nulidade é $4-2=2$.

$$\begin{bmatrix} & & 2 \\ & 1 & 3 \\ 3 & -4 & 2 \\ 2 & -3 & 1 \end{bmatrix} \begin{matrix} L_3 \leftarrow L_3 - 3L_2 \\ L_4 \leftarrow L_4 - 2L_2 \\ \sim \end{matrix} \begin{bmatrix} 0 & 2 & 2 \\ 1 & 1 & 3 \\ 0 & -7 & -7 \\ 0 & -5 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 2 \\ 1 & 1 & 3 \\ 0 & -7 & -7 \\ 0 & -5 & -5 \end{bmatrix} \begin{matrix} L_1 \leftarrow L_1 * 1/2 \\ L_3 \leftarrow L_3 * 1/_{-7} \\ L_4 \leftarrow L_4 * 1/_{-5} \end{matrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{matrix} L_1 \leftarrow L_1 - L_3 \\ L_1 \leftrightarrow L_4 \end{matrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \underset{\sim}{L_1 \leftrightarrow L_2} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \underset{\sim}{L_3 \leftarrow L_3 - L_2} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \underset{\sim}{L_1 \leftarrow L_1 - L_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

b. O posto de A é 2. A nulidade é $3 - 2 = 1$.

4°

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 11 \\ 4 & 3 & 2 & 0 \\ 1 & 1 & 1 & 6 \\ 3 & 1 & 1 & 4 \end{array}\right]$$

λ.

$$\begin{bmatrix} 2 & -1 & 3 & 11 \\ 4 & 3 & 2 & 0 \\ 1 & 1 & 1 & 6 \\ 3 & 1 & 1 & 4 \end{bmatrix} \begin{array}{l} L_1 \leftarrow L_1 - L_3 \\ L_2 \leftarrow L_2 - 4L_3 \\ L_4 \leftarrow L_4 - 3L_3 \\ \sim \end{array} \begin{bmatrix} 1 & -2 & 2 & 5 \\ 0 & -1 & -2 & -24 \\ 1 & 1 & 1 & 6 \\ 0 & -2 & -2 & -14 \end{bmatrix} \begin{array}{l} L_3 \leftarrow L_3 - L_1 \\ L_4 \leftarrow L_4 * 1/2 \end{array}$$

$$\begin{bmatrix} 1 & -2 & 2 & 5 \\ 0 & -1 & -2 & -24 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & 7 \end{bmatrix} \begin{array}{l} L_2 \leftarrow L_2 * (-1) \\ L_4 \leftarrow L_4 - L_3 \\ \sim \end{array} \begin{bmatrix} 1 & -2 & 2 & 5 \\ 0 & 1 & 2 & 24 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 2 & 6 \end{bmatrix} \begin{array}{l} L_3 \leftarrow L_3 - L_2 \\ L_1 \leftarrow L_1 + 2L_2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 6 & 53 \\ 0 & 1 & 2 & 24 \\ 0 & 0 & -3 & -23 \\ 0 & 0 & 2 & 6 \end{bmatrix} \begin{matrix} L_3 \leftarrow L_3 \cdot (-1) \\ L_1 \leftarrow L_1 - 3L_4 \\ L_2 \leftarrow L_2 - L_4 \\ \sim \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 35 \\ 0 & 1 & 0 & 18 \\ 0 & 0 & 3 & 23 \\ 0 & 0 & 2 & 6 \end{bmatrix} \begin{matrix} L_3 \leftarrow L_3 - L_4 \\ L_4 \leftarrow L_4 \cdot \frac{1}{2} \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 35 \\ 0 & 1 & 0 & 18 \\ 0 & 0 & 1 & 17 \\ 0 & 0 & 1 & 3 \end{bmatrix} \begin{matrix} L_4 \leftarrow L_4 - L_3 \\ L_4 \leftarrow L_4 \cdot \frac{1}{-14} \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 35 \\ 0 & 1 & 0 & 18 \\ 0 & 0 & 1 & 17 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} L_1 \leftarrow L_1 - 35L_4 \\ L_2 \leftarrow L_2 - 18L_4 \\ L_3 \leftarrow L_3 - 17L_4 \end{matrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

c) Posto da matriz ampliada : 4

Posto da matriz dos coeficientes : 3

d) Sistema Impossível

e) Não há solução.

5: a)

$$\begin{array}{cccccc} x_I & x_{II} & x_{III} & x_{IV} & & \\ \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 1 \end{array}\right]$$

$P_A = 1$ $P_C = 1$ Número de incógnitas = 4

Sistema Possível Indeterminado

$$S = \{ x_I = 1 - 2x_{II} + x_{III} - 3x_{IV} \ ; \text{ com } x_{II}, x_{III}, x_{IV} \in \mathbb{R} \}$$

5º b.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 2 & 5 & -2 & 3 \end{array}\right] \underset{\sim}{L_2 \leftarrow L_2 - 2L_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & -4 & -9 \end{array}\right] \underset{\sim}{L_2 \leftarrow L_2 * \frac{1}{3}}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & -\frac{4}{3} & -3 \end{array}\right] \underset{\sim}{L_1 \leftarrow L_1 - L_2} \left[\begin{array}{ccc|c} 1 & 0 & 7/3 & 7 \\ 0 & 1 & -\frac{4}{3} & -3 \end{array}\right]$$

$Posto_A = 2$ $\quad$ $Posto_C = 2$ $\quad$ Número de incógnitas: 3

Sistema Possível e Indeterminado

$= 7 - 7/3 \cdot z \quad , z \in \mathbb{R}$

$-3 + 4/3 \cdot z \quad , z \in \mathbb{R}$

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$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 2 & 5 & -2 & 3 \\ 1 & 7 & -7 & 5 \end{array}\right] \begin{array}{l} L_2 \longrightarrow L_2 - 2L_1 \\ L_3 \rightarrow L_3 - L_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & -4 & -5 \\ 0 & 6 & -8 & 1 \end{array}\right] \begin{array}{l} L_1 \leftarrow L_1 - \frac{1}{3} L_2 \\ L_2 \leftarrow L_2 \cdot \frac{1}{3} \\ \sim \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 + 4/3 & 4 + 5/3 \\ 0 & 1 & -4/3 & -5/3 \\ 0 & 6 & -8 & 1 \end{array}\right] \begin{array}{c} L_3 \leftarrow L_3 - 6L_2 \\ \sim \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 7/3 & 17/3 \\ 0 & 1 & -4/3 & -5/3 \\ 0 & 0 & 0 & 11 \end{array}\right]$$

$P_A = 3$ $\qquad$ $P_C = 2$

Sistema Impossível

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 2 & 5 & 6 & 0 \end{array}\right] \underset{\sim}{L_2 \leftarrow L_2 - 2L_1} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 9 & 0 & 0 \end{array}\right] \; L_2 \leftarrow L_2 \cdot \frac{1}{9}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \underset{\sim}{L_1 \leftarrow L_1 + 2L_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

$\rho_A = 2$ $\quad \rho_c = 2$ $\quad$ Número de incógnitas $= 3$

Sistema Possível Indeterminado

$$S = \left\{ \begin{array}{l} x = -3z \, , \; z \in \mathbb{R} \\ y = 0 \\ z \in \mathbb{R} \end{array} \right\}$$

2)

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & -1 & 4 \\ 1 & 1 & -1 & 1 & -4 \\ 1 & -1 & 1 & 1 & 2 \end{array}\right] \begin{array}{l} L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 - L_1 \\ L_4 \leftarrow L_4 - L_1 \\ \sim \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 4 \\ 0 & 0 & -2 & 0 & -4 \\ 0 & -2 & 0 & 0 & 2 \end{array}\right] \begin{array}{l} L_2 \leftarrow L_2 * \left(-\frac{1}{2}\right) \\ L_3 \leftarrow L_3 * \left(-\frac{1}{2}\right) \\ L_4 \leftarrow L_4 * \left(-\frac{1}{2}\right) \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \end{array}\right] \begin{array}{c} L_2 \leftrightarrow L_4 \\ \sim \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \begin{array}{l} L_1 \leftarrow L_1 - L_2 \\ L_1 \leftarrow L_1 - L_4 \\ L_1 \leftarrow L_1 - L_3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$P_A = 4$ $\quad P_C = 4$ $\quad$ Nulidade $= 4 - 4 = 0$ $\quad$ Nº de incógnitas $= 4$

Sistema Possível e Determinado

$S = (-1, -1, 2, -2)$

2)

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & -1 & 4 \\ 1 & 1 & -1 & 1 & -4 \\ 1 & -1 & 1 & 1 & 2 \end{array}\right] \begin{array}{l} L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 - L_1 \\ L_4 \leftarrow L_4 - L_1 \\ \sim \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 4 \\ 0 & 0 & -2 & 0 & -4 \\ 0 & -2 & 0 & 0 & 2 \end{array}\right] \begin{array}{l} L_2 \leftarrow L_2 * (-\frac{1}{2}) \\ L_3 \leftarrow L_3 * (-\frac{1}{2}) \\ L_4 \leftarrow L_4 * (-\frac{1}{2}) \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \end{array}\right] \begin{array}{c} L_2 \leftrightarrow L_4 \\ \sim \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right] \begin{array}{l} L_1 \leftarrow L_1 - L_2 \\ L_1 \leftarrow L_1 - L_4 \\ L_1 \leftarrow L_1 - L_3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$P_A = 4$ $P_c = 4$ Nulidade $= 4 - 4 = 0$ Nº de incógnitas $= 4$

Sistema Possível e Determinado

$S = (-1, -1, 2, -2)$

4)

$$\left[\begin{array}{ccc|c} 3 & 2 & -4 & 1 \\ 1 & -1 & 1 & 3 \\ 1 & -1 & -3 & -3 \\ 3 & 3 & -5 & 0 \\ -1 & 1 & 1 & 1 \end{array}\right] \begin{array}{l} L_2 \leftarrow L_2 - L_3 \\ L_3 \leftarrow L_3 + L_5 \\ L_4 \leftarrow L_4 - L_1 \\ L_5 \leftarrow 3L_5 + L_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 3 & 2 & -4 & 1 \\ 0 & 0 & 4 & 6 \\ 0 & 0 & -2 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 5 & -1 & 4 \end{array}\right] \begin{array}{l} L_2 \leftarrow L_2 \cdot \frac{1}{2} \\ L_3 \leftarrow L_3 \cdot \frac{1}{2} \\ L_4 \leftarrow L_4 \cdot (-1) \end{array}$$

$$\left[\begin{array}{ccc|c} 3 & 2 & -4 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & +1 & +1 \\ 0 & 5 & -1 & 4 \end{array}\right] \begin{array}{l} L_1 \leftarrow L_1 + 2L_4 \\ L_4 \leftarrow 5L_4 + L_5 \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 3 & 0 & -2 & 3 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 4 & 9 \\ 0 & 5 & -1 & 4 \end{array}\right] \begin{array}{l} L_1 \leftarrow L_1 + L_2 \\ L_3 \leftarrow L_3 - L_5 \\ L_4 \leftarrow L_4 + (-2L_2) \end{array}$$

$$\left[\begin{array}{ccc|c} 3 & 0 & 0 & 6 \\ 0 & 0 & 2 & 3 \\ 0 & -5 & 0 & -3 \\ & 0 & 0 & 3 \\ & 5 & -1 & 4 \end{array}\right] \begin{array}{l} L_2 \leftarrow L_2 + 2L_5 \\ L_5 \leftarrow L_5 + L_3 \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 3 & 0 & 0 & 6 \\ 0 & 10 & 0 & 11 \\ 0 & -5 & 0 & -3 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & -1 & 1 \end{array}\right] \begin{array}{l} L_3 \leftarrow 2L_3 + L_2 \\ L_5 \leftarrow L_5 \cdot (-1) \end{array}$$

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$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 1 & 3 & 0 \\ 3 & 2 & 1 & 0 \end{array}\right] \begin{array}{l} L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 - 3L_1 \\ \sim \end{array} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & -4 & -8 & 0 \end{array}\right] \begin{array}{l} L_2 \leftarrow L_2 \cdot \frac{1}{-3} \\ L_3 \leftarrow L_3 \cdot \frac{1}{-4} \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \begin{array}{l} L_3 \leftarrow L_3 - L_2 \\ L_1 \leftarrow L_1 - 2L_2 \\ \sim \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} L_1 \leftarrow L_1 - L_3 \\ L_2 \leftarrow L_2 - L_3 \\ \sim \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\rho_A = 3$ $\rho_C = 3$ Incógnitas $= 3$

Sistema Possível Determinado

$S = (0, 0, 0)$

$$\left[\begin{array}{ccc|c} 0 & 5 & -1 & 4 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 0 & 0 & -1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 3 & 6 & 0 & 6 \\ 0 & 10 & 0 & 11 \\ 0 & 0 & 0 & 17 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right] \begin{array}{l} L_2 \leftarrow L_2 * \frac{1}{10} \\ L_5 \leftrightarrow L_3 \\ \sim \end{array} \left[\begin{array}{ccc|c} 3 & 0 & 0 & 6 \\ 0 & 1 & 0 & \frac{11}{10} \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 17 \end{array}\right] L_1 \leftarrow L_1 * \frac{1}{3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 11/10 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 17 \end{array}\right]$$

$\rho_A = 5 \qquad \rho_C = 3$

Sistema Impossível

6.º

$$\left[\begin{array}{cc|c} k & 2 & 6 \\ 3 & -1 & -2 \\ 1 & 1 & 0 \end{array}\right] \underset{\sim}{L_2 \leftarrow L_2 - 3L_3} \left[\begin{array}{cc|c} k & 2 & 6 \\ 0 & -4 & -2 \\ 1 & 1 & 0 \end{array}\right] \begin{array}{l} L_2 \leftarrow L_2 * \dfrac{1}{(-2)} \\ L_3 \leftarrow k L_3 - L_1 \end{array} \sim$$

$$\left[\begin{array}{cc|c} k & 2 & 6 \\ 0 & 2 & 1 \\ 0 & k-2 & -6 \end{array}\right] \begin{array}{l} L_1 \leftarrow L_1 - L_2 \\ L_2 \leftarrow L_2 + L_3 \\ L_3 \leftarrow L_3 - L_2 \end{array} \left[\begin{array}{cc|c} k & 0 & 5 \\ 0 & k & -5 \\ 0 & -2 & -11 \end{array}\right] \underset{\sim}{L_3 \leftarrow k L_3 + 2L_2}$$

$$\left[\begin{array}{cc|c} k & 0 & 5 \\ 0 & k & -5 \\ 0 & 0 & -11k+10 \end{array}\right]$$

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$$\left[\begin{array}{cc|c} k & 0 & 5 \\ 0 & k & -5 \\ 0 & 0 & -11k+10 \end{array}\right]$$

Para o sistema acima ser SPD, é preciso que $-11k+10=0$, logo:

$$\boxed{\begin{aligned} -11k+10 &= 0 \\ k &= \frac{-10}{-11} = \frac{10}{11} \end{aligned}}$$

Para $k \neq \frac{10}{11}$, então o sistema é Impossível.

7°

$$\left[\begin{array}{cc|c} -4 & 3 & 2 \\ 5 & -4 & 0 \\ 2 & -1 & k \end{array}\right] \underset{\sim}{L_1 \leftarrow L_1 + 2L_3} \left[\begin{array}{cc|c} 0 & 0 & 2+2k \\ 5 & -4 & 0 \\ 2 & -1 & k \end{array}\right] \underset{\sim}{L_2 \leftarrow 2L_2 - 5L_3}$$

$$\left[\begin{array}{cc|c} 0 & 0 & 2+2k \\ 0 & -3 & -5k \\ 2 & -1 & k \end{array}\right] \underset{\sim}{L_2 \leftarrow L_2 - 3L_3} \left[\begin{array}{cc|c} 0 & 0 & 2+2k \\ -6 & 0 & -5k-3k = -2k \\ 2 & -1 & k \end{array}\right] \underset{\sim}{L_2 \leftarrow L_2 + 3L}$$

$$\left[\begin{array}{cc|c} 0 & 0 & 2+2k \\ 0 & -3 & k \\ 2 & -1 & k \end{array}\right] \underset{\sim}{\begin{array}{l} L_1 \leftarrow L_1 + L_3 \\ L_3 \leftarrow L_3 - L_1 \end{array}} \left[\begin{array}{cc|c} 2 & -1 & 2+3k \\ 0 & -3 & k \\ 0 & 0 & 2+2k \end{array}\right] \underset{\sim}{\begin{array}{l} L_1 \leftarrow L_1 \times \frac{1}{2} \\ L_2 \leftarrow L_2 \times \frac{1}{-3} \end{array}}$$

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$$\left[\begin{array}{cc|c} 0 & 0 & 2+2R \\ 0 & -3 & R \\ 2 & -1 & R \end{array}\right] \begin{array}{l} L_1 \leftarrow -L_1 + L_3 \\ L_3 \leftarrow -L_3 - L_1 \\ \sim \end{array} \left[\begin{array}{cc|c} 2 & -1 & 2+3R \\ 0 & -3 & R \\ 0 & 0 & 2+2R \end{array}\right] \begin{array}{l} L_1 \leftarrow L_1 \cdot \frac{1}{2} \\ L_2 \leftarrow L_2 \cdot \frac{1}{-3} \\ \sim \end{array}$$

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{2+3R}{2} \\ 0 & 1 & R/-3 \\ 0 & 0 & 2+2R \end{array}\right] \begin{array}{l} L_1 \leftarrow L_1 + \frac{1}{2} L_2 \\ \sim \end{array} \left[\begin{array}{cc|c} 1 & 0 & \frac{6+8R}{6} \\ 0 & 1 & -R/3 \\ 0 & 0 & 2+2R \end{array}\right]$$

Para que o sistema acima tenha solução, é preciso que

$$2 + 2R = 0$$

$$2 = -2R$$

$$\boxed{R = -1}$$

8:

$$\left[\begin{array}{ccccc|c} 1 & 3 & 2 & 3 & -7 & 14 \\ 2 & 6 & 1 & -2 & 5 & -2 \\ 1 & 3 & -1 & 0 & 2 & -1 \end{array}\right] \begin{array}{l} L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 - L_1 \\ \sim \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 3 & 2 & 3 & 7 & 14 \\ 0 & 0 & -3 & -8 & 19 & -30 \\ 0 & 0 & -3 & -3 & 9 & -15 \end{array}\right] \begin{array}{l} L_3 \leftarrow L_3 \cdot \left(\frac{1}{-3}\right) \\ L_2 \leftrightarrow L_3 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 3 & 2 & 3 & 7 & 14 \\ 0 & 0 & 1 & 1 & -3 & 5 \\ 0 & 0 & -3 & -8 & 19 & -30 \end{array}\right] \begin{array}{l} L_1 \leftarrow L_1 - 2L_2 \\ L_3 \leftarrow L_3 + 3L_2 \\ \sim \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 3 & 0 & 1 & 13 & 4 \\ 0 & 0 & 1 & 1 & -3 & 5 \\ 0 & 0 & 0 & -5 & 10 & -15 \end{array}\right] \quad L_3 \leftarrow L_3 \cdot \left(\frac{1}{-5}\right)$$

$$\left[\begin{array}{ccccc|c} 1 & 3 & 0 & 1 & 13 & 4 \\ 0 & 0 & 1 & 1 & -3 & 5 \\ 0 & 0 & 0 & 1 & -2 & 3 \end{array}\right] \quad \begin{array}{l} L_1 \leftarrow L_1 - L_3 \\ L_2 \leftarrow L_2 - L_3 \\ \sim \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 3 & 0 & 0 & 15 & 1 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & -2 & 3 \end{array}\right]$$

$$S = \left\{ \begin{array}{l} x_1 = 1 - 15x_V - 3x_{II} \\ x_{III} = 2 + x_V \\ x_{IV} = 3 + 2x_V \\ x_{II} \in \mathbb{R} \\ x_V \in \mathbb{R} \end{array} \right\}$$

9: a) Todas as incógnitas iguais a 0.

$$k_1 \quad \begin{array}{cccc} x & y & z & \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & -5 & 2 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 0 & k & 0 \end{array}\right] \begin{array}{l} L_1 \leftrightarrow L_2 \\ L_2 \leftarrow L_2 - L_3 \\ L_3 \leftarrow L_3 - 2L_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -5 & 2-k & 0 \\ 0 & -2 & k-4 & 0 \end{array}\right] L_2 \leftarrow L_2 * \left(\frac{1}{-5}\right)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{2-k}{-5} & 0 \\ 0 & -2 & k-4 & 0 \end{array}\right] L_3 \leftarrow L_3 * \left(\frac{1}{-2}\right) \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{-2+k}{5} & 0 \\ 0 & 1 & \frac{-k+4}{2} & 0 \end{array}\right] \begin{array}{l} L_1 \leftarrow L_1 - L_2 \\ L_3 \leftarrow L_3 - L_2 \end{array} \sim$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 + \frac{2+k}{5} & 0 \\ 0 & 1 & \frac{-2+k}{5} & 0 \\ 0 & 0 & \frac{-k+4}{2} + \frac{2-k}{5} & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1+\frac{2+k}{5} & 0 \\ 0 & 1 & \frac{-2+k}{5} & 0 \\ 0 & 0 & \frac{-k+4}{2}+\frac{2-k}{5} & 0 \end{array}\right]$$

Para que o sistema acima tenha solução, é preciso que:

$$\frac{5}{5}\cdot\frac{(-k+4)}{2}+\frac{(2-k)}{5}\frac{2}{2}=0$$

$$\frac{-5k+20}{10}+\frac{4-2k}{10}=0$$

$$-7k+24$$

$$7k=24$$

$$\boxed{k=\frac{24}{7}}$$

Solução:

$$S=\left\{\begin{array}{l} x=-\left(1+\frac{2+k}{5}\right) \\ y=-\left(\frac{-2+k}{5}\right) \\ z=0 \end{array}\right\}$$

10:

$$\begin{bmatrix}1&1&0&-3\\2&-4&-1&0\\3&-2&-1&2\end{bmatrix}_{3\times4}\begin{bmatrix}x\\y\\z\\v\end{bmatrix}_{4\times1}=\left[\begin{array}{c|c} x+y+0-3v & 1\\ 2x-4y-z+0 & 0\\ 3x-2y-z+2v & 2\end{array}\right]_{3\times1}$$

Agora basta resolver o sistema!