Lista 3 - Determinantes e Matriz Inversa

1º a) $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$ $|A| = (1.0) - (1.2) = 0 - 2 = -2$

$|A| = -2$

b) $B = \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}$ $|B| = (3.1) - (-1.0) = 3 - 0 = 3$

$|B| = 3$

c) $\det A + \det B = |A| + |B| = -2 + 3 = 1$

d) $\det (A + B) = 3$

$$\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 4 & 1 \\ 1 & 1 \end{vmatrix} = (4.1) - (1.1) = 4 - 1 = 3$$

2º a) $t - 7 = 0$

$t = 7$

b) $(t-1).(t+1) = 15$

$t^2 + \cancel{t} - \cancel{t} + 1 = 15$

$t^2 = 14$

$t = \sqrt{14}$

c) $t = 2$, pois torna a 1ª coluna nula.

3°

$$A = \begin{bmatrix} 2 & 3 & 1 & -2 \\ 5 & 3 & 1 & 4 \\ 0 & 1 & 2 & 2 \\ 3 & -1 & -2 & 4 \end{bmatrix}$$

---

a) $$A_{23} = \begin{bmatrix} 2 & 3 & -2 \\ 0 & 1 & 2 \\ 3 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 & -2 \\ 0 & 1 & 2 \\ 3 & -1 & 4 \end{bmatrix}$$

---

b) $$\Delta_{11} = (-1)^2 \cdot \left| \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} \right| = (1 \cdot 4) - (2 \cdot [-1]) = 4 - (-2) = 6$$

$$\Delta_{21} = (-1)^3 \cdot \left| \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \right| = (3 \cdot 4) - (-2 \cdot (-1)) = 12 - (2) = 10$$

$$\Delta_{31} = (-1)^4 \cdot \left| \begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix} \right| = (3 \cdot 2) - (-2 \cdot 1) = 6 - (-2) = 8$$

$$|A_{23}| = 2 \cdot 6 + 0 \cdot 10 + 3 \cdot 8 = 12 + 0 + 24 = 36$$

---

c) $$\Delta_{23} = (-1)^5 \cdot \left| \begin{bmatrix} 2 & 3 & -2 \\ 0 & 1 & 2 \\ 3 & -1 & 4 \end{bmatrix} \right| = -1 \cdot 36 = -36$$

$$\begin{bmatrix} 2 & 3 & -2 & 2 & 3 \\ 0 & 1 & 2 & 0 & 1 \\ 3 & -1 & 4 & 3 & -1 \end{bmatrix} \quad 26 - (-10) = 36$$

$$-6 \quad -4 \quad 0 \quad 8 \quad 18$$

d) $0 \cdot \Delta_{31} + 1 \cdot \Delta_{32} + 2 \cdot \Delta_{33} + 2 \cdot \Delta_{34} = 0 + (-82) + 36 + (-15)$

$= -82 + 21 = -61$

$$\Delta_{32} = (-1)^{3+2} \cdot \left| \begin{bmatrix} 2 & 1 & -2 \\ 5 & 1 & 4 \\ 3 & -2 & 4 \end{bmatrix} \right| = -1 \cdot 82 = -82$$

$$\Delta_{33} = (-1)^{3+3} \cdot \left| \begin{bmatrix} 2 & 3 & -2 \\ 5 & 3 & 4 \\ 3 & -1 & 4 \end{bmatrix} \right| = 1 \cdot 36 = 36$$

$$\Delta_{34} = (-1)^{3+4} \cdot \left| \begin{bmatrix} 2 & 3 & 1 \\ 5 & 3 & 1 \\ 3 & -1 & -2 \end{bmatrix} \right| = -1 \cdot 15 = -15$$

$$\left| \begin{bmatrix} 2 & 1 & -2 \\ 5 & 1 & 4 \\ 3 & -2 & 4 \end{bmatrix} \begin{matrix} 2 & 1 \\ 5 & 1 \\ 3 & -2 \end{matrix} \right|$$

$-6 \quad -16 \quad 20 \quad 8 \quad 12 \quad 20$

$(8 + 12 + 20) - (-6 - 16 - 20)$

$40 - (-42)$

$40 + 42$

82

$$\left| \begin{bmatrix} 2 & 3 & -2 \\ 5 & 3 & 4 \\ 3 & -1 & 4 \end{bmatrix} \begin{matrix} 2 & 3 \\ 5 & 3 \\ 3 & -1 \end{matrix} \right|$$

$-18 \quad -8 \quad 60 \quad 24 \quad 36 \quad 10$

$(24 + 36 + 10) - (-18 - 8 + 60)$

$70 - 34$

36

$$\left| \begin{bmatrix} 2 & 3 & 1 \\ 5 & 3 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{matrix} 2 & 3 \\ 5 & 3 \\ 3 & -1 \end{matrix} \right|$$

$9 \quad -2 \quad -30 \quad -12 \quad 9 \quad -5$

$(-12 + 9 - 5) - (9 - 2 - 30)$

$-8 - (7 - 30)$

-8 [illegible]

-8 [illegible]

15

4º

$$a1 \quad A = \begin{bmatrix} 3 & -1 & 5 & 0 \\ 0 & 2 & 0 & 1 \\ 2 & 0 & -1 & 3 \\ 1 & 1 & 2 & 0 \end{bmatrix}$$

$$\det(A) = \Delta_{21} \cdot 0 + \Delta_{22} \cdot 2 + \Delta_{23} \cdot 0 + \Delta_{24} \cdot 1$$

$$\det(A) = \Delta_{22} \cdot 2 + \Delta_{24} \cdot 1$$

$$\det(A) = -3 \cdot 2 + 18 \cdot 1 = -6 + 18 = 12$$

$$\Delta_{22} = (-1)^4 \cdot \begin{vmatrix} 3 & 5 & 0 \\ 2 & -1 & 3 \\ 1 & 2 & 0 \end{vmatrix} = 1 \cdot (-3) = -3$$

$$\Delta_{24} = (-1)^6 \cdot \begin{vmatrix} 3 & -1 & 5 \\ 2 & 0 & -1 \\ 1 & 1 & 2 \end{vmatrix} = 1 \cdot 18 = 18$$

$$\det(A_{22}) = \begin{bmatrix} 3 & 5 & 0 & 3 & 5 \\ 2 & -1 & 3 & 2 & -1 \\ 1 & 2 & 0 & 1 & 2 \end{bmatrix} \quad (0 + 0 + 15) - (0 + 0 + 18)$$

$0 \quad 18 \quad 0 \quad\quad 0 \quad 15 \quad 0$

$15 - 18$

-3

$$\det(A_{24}) = \begin{bmatrix} 3 & -1 & 5 & 3 & -1 \\ 2 & 0 & -1 & 2 & 0 \\ 1 & 1 & 2 & 1 & 1 \end{bmatrix} \quad (1 + 10 + 0) - (0 - 3 - 4)$$

$0 \quad -3 \quad -4 \quad\quad 0 \quad 1 \quad 10$

$11 - (-7)$

$11 + 7$

18

b) A matriz d[illegible]ra b, é triangular, logo o seu determinante é o produto da diagonal.

$$\det(A) = 3 \cdot 18 \cdot (-5) \cdot 0 \cdot (-1) = 0$$

5º

a) $A = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$

. A matriz adjacente é a transposta da matriz dos cofatores.

$$\Delta_{11} = (-1)^2 \cdot \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 1 \cdot (6-1) = 5$$

$$\text{adj } A = \begin{bmatrix} 5 & 5 & -10 \\ 0 & 21 & 3 \\ 7 & -2 & 4 \end{bmatrix}^t = \begin{bmatrix} 5 & 0 & 7 \\ 5 & 21 & -2 \\ -10 & 3 & 4 \end{bmatrix}$$

$$\Delta_{21} = (-1)^3 \cdot \begin{vmatrix} 1 & -3 \\ 1 & 3 \end{vmatrix} = -1 \cdot (3-3) = 0$$

$$\Delta_{31} = (-1)^4 \cdot \begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix} = 1 \cdot (1+6) = 7$$

$$\Delta_{12} = (-1)^3 \cdot \begin{vmatrix} 0 & 1 \\ 5 & 3 \end{vmatrix} = -1 \cdot (0-5) = -1 \cdot (-5) = 5$$

$$\Delta_{22} = (-1)^4 \cdot \begin{vmatrix} 2 & -3 \\ 5 & 3 \end{vmatrix} = 1 \cdot (6+15) = 21$$

$$\Delta_{32} = (-1)^5 \cdot \begin{vmatrix} 2 & -3 \\ 0 & 1 \end{vmatrix} = -1 \cdot (2-0) = -2$$

$$\Delta_{13} = (-1)^4 \cdot \begin{vmatrix} 0 & 2 \\ 5 & 1 \end{vmatrix} = 1 \cdot (0-10) = -10$$

$$\Delta_{23} = (-1)^5 \cdot \begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix} = -1 \cdot (2-5) = -1 \cdot (-3) = 3$$

$$\Delta_{33} = (-1)^6 \cdot \begin{vmatrix} 2 & 1 \\ 0 & 2 \end{vmatrix} = 1 \cdot (4-0) = 4$$

6º

$$A = \begin{bmatrix} t & 0 & 1 \\ 0 & 1 & 5 \\ 3 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 2 & 2 \\ 0 & 5 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

* FIZ APENAS OS CÁLCULOS PARA ENCONTRAR A INVERSA, TANTO DE "A" QUANTO DE "B"

$$B^t = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 5 & 1 \\ 2 & 2 & 2 \end{bmatrix} \qquad A^2 = \begin{bmatrix} t^2 & 0 & 1 \\ 0 & 1 & 25 \\ 9 & 0 & 1 \end{bmatrix}$$

01

$$A^{-1} = \left[\begin{array}{ccc|ccc} t & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 5 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$L_1 \to L_1 * \frac{1}{t}$ // Não posso, pois pode ser que $t = 0$

$L_1 \leftrightarrow L_3$ // Irei realizar esta operação

02

$$\left[\begin{array}{ccc|ccc} 3 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 5 & 0 & 1 & 0 \\ t & 0 & 1 & 1 & 0 & 0 \end{array}\right]$$

$L_1 \to L_1 * \frac{1}{3}$ $\sim$

03

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1/3 & 0 & 0 & 1/3 \\ 0 & 1 & 5 & 0 & 1 & 0 \\ t & 0 & 1 & 1 & 0 & 0 \end{array}\right]$$

$L_3 \to L_3 - tL_1$ $\sim$

04

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1/3 & 0 & 0 & 1/3 \\ 0 & 1 & 5 & 0 & 1 & 0 \\ 0 & 0 & 1-\frac{t}{3} & 1 & 0 & -\frac{t}{3} \end{array}\right]$$

$L_2 \to L_2 - 15L_1$

05

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1/3 & 0 & 0 & 1/3 \\ -15 & 1 & 0 & 0 & 1 & -5 \\ 0 & 0 & \frac{3-t}{3} & 1 & 0 & -\frac{t}{3} \end{array}\right]$$

$L_3 \to L_3 * \frac{3}{3-t}$ $\sim$

06

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1/3 & 0 & 0 & 1/3 \\ -15 & 1 & 0 & 0 & 1 & -5 \\ 0 & 0 & 1 & \frac{3}{3-t} & 0 & \frac{-3t}{9-3t} \end{array}\right]$$

$L_1 \to L_1 - \frac{1}{3}L_3$

$L_2 \to L_2 + 15L_1$

$\sim$

07

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3/9-3t & 0 & 3t/27-9t \\ 0 & 1 & 0 & -45/9-3t & 1 & \\ 0 & 0 & 1 & \frac{3}{3-t} & 0 & \frac{-3\ldots}{9-\ldots} \end{array}\right]$$

b) $\det(A) = \Delta_{21} \cdot 0 + \Delta_{22} \cdot 2 + \Delta_{23} \cdot 1$

$= 0 + 21.2 + 3.1$

$= 45$

c) $A^{-1} = \frac{1}{\det A} \cdot \operatorname{adj} A$

$$\frac{1}{45} \cdot \operatorname{adj} A = \frac{1}{45} \cdot \begin{bmatrix} 5 & 0 & 7 \\ 5 & 21 & -2 \\ -10 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1/9 & 0 & 7/45 \\ 1/9 & 21/45 & -2/45 \\ -10/45 & 3/45 & 4/45 \end{bmatrix}$$

* Uma das formas de encontrar a inversa de uma matriz é:

$$\frac{1}{\det A} \cdot \operatorname{adj} A$$

. Em que $\frac{1}{\det A}$ é o inverso do determinante de A

. adj A é a matriz adjacente de A.

Logo, A^{-1}:
$$\begin{bmatrix} \frac{-3}{9-3t} & 0 & \frac{t}{9-3t} \\ \frac{-15}{3-t} & 1 & 0 \\ \frac{2}{3-t} & 0 & \frac{-t}{3-t} \end{bmatrix}$$

. Agora, vamos calcular B^{-1}:

$$\left[\begin{array}{ccc|ccc} 2 & 2 & 2 & 1 & 0 & 0 \\ 0 & S & 2 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} L_1 \Rightarrow L_1 * \frac{1}{2} \\ L_2 \to L_2 * \frac{1}{S} \\ L_3 \to L_3 - L_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 2/S & 0 & 1/S & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & 1 \end{array}\right] \begin{array}{l} L_2 \to L_2 - \frac{2}{S} L_3 \\ L_1 \to L_1 - L_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & \frac{1}{S} & \frac{1}{S} & -\frac{2}{S} \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & 1 \end{array}\right] \begin{array}{l} L_1 \to L_1 - L_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{S-1}{S} & -\frac{1}{S} & \frac{-S+2}{S} \\ 0 & 1 & 0 & \frac{1}{S} & \frac{1}{S} & \frac{-2}{S} \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & 1 \end{array}\right]$$

Logo, $B^{-1} = \begin{bmatrix} (S-1)/S & -1/S & (-S+2)/S \\ 1/S & 1/S & -2/S \\ -1/2 & 0 & 1 \end{bmatrix}$

7º

$$|A| = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 1 & -1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$0 \quad 0 \quad 0 \qquad -1 \quad 0 \quad 0$

$-1 - 0 = -1$

Sim, é inversível.

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} L_2 \leftrightarrow L_2 - L_1 \\ L_3 \to L_3 - L_1 \\ \sim \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$L_2 \to L_2 * (-1) \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

* Logo, $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

8:

$$A = \begin{bmatrix} 0 & 2 & -1 & 3 \\ 1 & 0 & 2 & 2 \\ 0 & -1 & 1 & 1 \\ 1 & 2 & 1 & 5 \end{bmatrix}$$

* Irei calcular o determinante da matriz acima usando COFATORES. A Fila escolhida é a coluna 1.

$$|A| = 0 . \Delta_{11} + 1 . \Delta_{21} + 0 . \Delta_{31} + 1 . \Delta_{41} = -8 + (8) = 0$$

$$\Delta_{21} = (-1)^3 . \begin{vmatrix} 2 & -1 & 3 \\ -1 & 1 & 1 \\ 2 & 1 & 5 \end{vmatrix} = -8$$

$$\Delta_{41} = (-1)^5 . \begin{vmatrix} 2 & -1 & 3 \\ 0 & 2 & 2 \\ -1 & 1 & 1 \end{vmatrix} = 8$$

$$\det |A_{21}| = \begin{bmatrix} 2 & -1 & 3 & 2 & -1 \\ -1 & 1 & 1 & -1 & 1 \\ 2 & 1 & 5 & 2 & 1 \end{bmatrix} = (10 - 2 - 3) - (6 + 2 + 5) = 5 - 13 = -8$$

6 2 5 10 -2 -3

$$\det |A_{41}| = \begin{bmatrix} 2 & -1 & 3 & 2 & -1 \\ 0 & 2 & 2 & 0 & 2 \\ -1 & 1 & 1 & -1 & 1 \end{bmatrix} = (4 + 2 + 0) - (-6 + 4 + 0) = 6 - (-2) = 8$$

-6 4 0 4 2 0

9º Para que a matriz A não seja invertível, é preciso que o seu determinante seja 0.

$$\left[\begin{array}{ccc|cc} 3 & 2 & 2 & 3 & 2 \\ 1 & 2 & a & 1 & 2 \\ 0 & 1 & 4 & 0 & 1 \end{array}\right]$$

$0 \quad 3a \quad 8 \quad 24 \quad 0 \quad 2$

$$(24+0+2)-(3a+8)=0$$

$$26-3a-8=0$$

$$18-3a=0$$

$$3a=18$$

$$a=6$$

10º

$$\left[\begin{array}{ccc|cc} 2 & 5 & 0 & 2 & 5 \\ 2 & 2 & 1 & 2 & 2 \\ 2 & a & 4 & 2 & a \end{array}\right]$$

$0 \quad 2a \quad 40 \quad 16 \quad 10 \quad 0$

$$(16+10+0)-(0+2a+40)\neq 0$$

$$26-2a-40\neq 0$$

$$-2a-14\neq 0$$

$$-2a\neq 14$$

$$a\neq -7$$

* As próximas questões envolvem REGRA DE CRAMER.

* Para a REGRA DE CRAMER ser aplicável, é preciso que o sistema linear seja quadrado (número de equações = número de incógnitas).

* $AX = B$

$A^{-1} . A . X = A^{-1} . B$

$X = A^{-1} . B$

Ao lado, está a regra de cramer.
Basta encontrar a inversa da matriz dos cofatores e multiplicar pela matriz coluna dos termos independentes.