RESOLUÇÃO - PROVA 2022.1

1ª

a) $B = [b_{ij}]_{4\times4}$, sendo que $b_{ij} = i \cdot j$

$b_{11} = 1.1 = 1$ $\quad b_{12} = 1.2 = 2$ $\quad b_{13} = 1.3 = 3$ $\quad b_{14} = 1.4 = 4$

$b_{21} = 2.1 = 2$ $\quad b_{22} = 2.2 = 4$ $\quad b_{23} = 2.3 = 6$ $\quad b_{24} = 2.4 = 8$

$b_{31} = 3.1 = 3$ $\quad b_{32} = 3.2 = 6$ $\quad b_{33} = 3.3 = 9$ $\quad b_{34} = 3.4 = 12$

$b_{41} = 4.1 = 4$ $\quad b_{42} = 4.2 = 8$ $\quad b_{43} = 4.3 = 12$ $\quad b_{44} = 4.4 = 16$

$$B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}_{4\times4}$$

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b)
$$\begin{pmatrix} 0 & 0 \\ x & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & x^2 \end{pmatrix}$$

$$\begin{pmatrix} x-y & 0 \\ x & z \end{pmatrix} + \begin{pmatrix} z-4 & 0 \\ y-z & 0 \end{pmatrix} = \begin{pmatrix} x-y+z-4 & 0 \\ x+y-z & z \end{pmatrix}$$

$$\begin{cases} x - y + z - 4 = 0 \\ x + y + z = 0 \\ z = x^2 \Rightarrow \boxed{z = 2^2 = 4} \end{cases} \Rightarrow \begin{array}{r} x - \cancel{y} + \cancel{z} = 4 \\ x + \cancel{y} - \cancel{z} = 0 \\ \hline 2x = 4 \\ \boxed{x = 2} \end{array} \Rightarrow 2 + y - 4 = 0 \quad \boxed{y = 4 - 2 = 2}$$

$$S = \{x = 2,\ y = 2,\ z = 4\}$$

c)

$$A = \begin{bmatrix} 5 & 6x-9 \\ x^2 & 10 \end{bmatrix}$$

$x^2 = 6x - 9$

$x^2 - 6x + 9 = 0$

$\text{Soma} = \frac{-b}{a} = \frac{6}{1} = 6$

$x_I + x_{II} = 6$

$\text{Produto} = \frac{c}{a} = \frac{9}{1} = 9$

$x_I \cdot x_{II} = 9$

$\boxed{x_I = 3 \quad e \quad x_{II} = 3}$

d.
$$\left[\begin{array}{cc|c} 2 & -1 & k \\ 5 & -4 & 0 \\ -4 & 3 & 2 \end{array}\right] \underset{\sim}{L_1 \to L_1 \cdot \frac{1}{2}} \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{k}{2} \\ 5 & -4 & 0 \\ -4 & 3 & 2 \end{array}\right] \begin{array}{l} L_2 \to L_2 - 5L_1 \\ L_3 \to L_3 + 4L_1 \\ \sim \end{array}$$

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{k}{2} \\ 0 & -\frac{3}{2} & -\frac{5k}{2} \\ 0 & 1 & 2+2k \end{array}\right] \underset{\sim}{L_2 \leftrightarrow L_3} \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{k}{2} \\ 0 & 1 & 2+2k \\ 0 & -\frac{3}{2} & -\frac{5k}{2} \end{array}\right] \begin{array}{l} L_1 \to L_1 + \frac{1}{2}L_2 \\ L_3 \to L_3 + \frac{3}{2}L_2 \\ \sim \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 0 & \frac{k+2+2k}{2} \\ 0 & 1 & 2+2k \\ 0 & 0 & \frac{6+k}{2} \end{array}\right]$$

$P_c = 2$ $\qquad \dfrac{6+k}{2} = 0$

$6+k=0$

$\boxed{k=-6}$

É preciso que $k=-6$, pois só assim $P_c = P_A$.

2°

$$det = 0 \cdot \Delta_{15} + 0 \cdot \Delta_{25} + 0 \cdot \Delta_{3.5} + 0 \cdot \Delta_{4.5} + (-1) \cdot \Delta_{55}$$

$$det = -1 \cdot \Delta_{55}$$

$$\Delta_{55} = (-1)^{10} \cdot \begin{vmatrix} 3 & 0 & 0 & 0 \\ 19 & 18 & 0 & 0 \\ -6 & \pi & -5 & 0 \\ 4 & \sqrt{2} & \sqrt{3} & 1/54 \end{vmatrix} = 1 \cdot (3 \cdot 18 \cdot [-5] \cdot 1/54)$$

$$= -15 \cdot 18 \cdot \frac{1}{54}$$

$$= -5$$

Logo, $det = -1 \cdot \Delta_{55}$

$$= -1 \cdot (-5) = 5$$

3.

a) $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 4 & 0 & 1 & 0 \\ 1 & 3 & 9 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} L_2 \to L_2 - L_1 \\ L_3 \to L_3 - L_1 \\ \sim \end{array} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 2 & 8 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} L_1 \to L_1 - L_2 \\ \sim \end{array}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 2 & -1 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 2 & 8 & -1 & 0 & 1 \end{array}\right] \begin{array}{l} L_3 \to L_3 - 2L_2 \\ \sim \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 2 & -1 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 & -2 & 1 \end{array}\right] \begin{array}{l} L_1 \to L_1 + L_3 \\ \sim \end{array}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -3 & 1 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 & -2 & 1 \end{array}\right] \begin{array}{l} L_3 \to L_3 * \frac{1}{2} \\ \sim \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -3 & 1 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1/2 & -1 & 1/2 \end{array}\right]$

$\begin{array}{l} L_2 \to L_2 - 3L_3 \\ \sim \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -3 & 1 \\ 0 & 1 & 0 & -5/2 & 4 & -3/2 \\ 0 & 0 & 1 & 1/2 & -1 & 1/2 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} 3 & -3 & 1 \\ -5/2 & 4 & -3/2 \\ 1/2 & -1 & 1/2 \end{bmatrix}$

b. $AX = B$

$$A^{-1} \cdot A \cdot \cancel{X} = A^{-1} \cdot B$$

$$X = A^{-1} \cdot B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underset{3\times3}{\begin{bmatrix} 3 & -3 & 1 \\ -5/2 & 4 & -3/2 \\ 1/2 & -1 & 1/2 \end{bmatrix}} \cdot \underset{3\times1}{\begin{bmatrix} 1 \\ 4 \\ 9 \end{bmatrix}} = \underset{3\times1}{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$$

$$3 \cdot 1 + (-3 \cdot 4) + 1 \cdot 9 = 3 - 12 + 9 = 0$$

$$-\frac{5}{2} \cdot 1 + 4 \cdot 4 + \left(-\frac{3}{2}\right) \cdot 9 = -\frac{5}{2} + 16 - \frac{27}{2}$$

$$= -\frac{32}{2} + 16 = 0$$

$$\frac{1}{2} \cdot 1 + (-1 \cdot 4) + \frac{1}{2} \cdot 9 = \frac{1}{2} - 4 + \frac{9}{2} = 5 - 4 = 1$$

$$S = \{0, 0, 1\}$$

4:

a) F. Veja um contra-exemplo:

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$ // A matriz ao lado é não-nula, mas não tem inversa.

b) Falso. O correto seria: $(AB)^T = B^T . A^T$

c) Verdadeiro.

d) Verdadeiro.