Screening Costly Information*

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Abstract

We study screening with endogenous information acquisition. A monopolist offers a menu of quality-differentiated products. After observing the offer, a consumer can costly learn about which product is right for them. We characterize the optimal menu and how it is distorted: typically, all types receive lower-than-efficient quality, and distortions are more intense than under standard screening, even when no information is acquired in equilibrium. The additional distortion is due to the *threat* from the buyer to obtain information that is not optimal for the seller. This threat interacts with the division of surplus: profits are U-shaped in the level of information costs and, when such costs are low, the consumer may be better off than when information is free. Among several applications, we show (i) transparency policies may harm consumers by lowering their strategic advantage, and (ii) an analyst who empirically measures distortions ignoring information acquisition could severely underestimate the level of inefficiency in the market.

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1 Introduction

Consumers are often unsure about which product is right for them. Investors struggle to choose which financial instruments to purchase (Gargano and Rossi, 2018; Sicherman et al., 2016); and potential buyers grapple with deciding which health insurance plan to contract, incurring large losses for selecting the wrong product (Abaluck and Gruber, 2011; Brown and Jeon, 2020; Handel and Kolstad, 2015). Frequently, additional information that would aid the agent's decision is available, but acquiring and processing it requires effort. In the health insurance example, one could look at experience in previous years and research family history to forecast coverage needs, or compare how different plans cover various possible conditions. Whether the buyer expends effort to gauge their own preference or to assess how product characteristics match their personal taste, information acquisition is costly. We study how this costly information acquisition in demand affects product supply and, therefore, equilibrium outcomes.

This paper considers a model of vertical product differentiation in which consumers need to pay a cost to discover their type. Understanding how supply responds to information frictions is important for several reasons. First, extensive evidence shows costly information acquisition significantly affects consumer behavior.[1] Second, supply responses shed light on producers' incentives to aid or hinder consumer learning. Indeed, network provider and insurance company websites alike offer tools that help consumers compare plans, suggesting that sellers act to influence information acquisition. Finally, information frictions justify public interventions; for example, a health insurance policy may include instruments that help customers choose the right product for them (Brown, 2019). However, the consequences of these interventions on consumer welfare cannot be ascertained without considering equilibrium effects. In fact, we show that helping buyers choose often *reduces* consumer welfare.

The Model As in Mussa and Rosen (1978), a monopolist (she) offers goods of different quality to screen different types of the agent (he). In standard screening, the buyer knows his taste for quality, but the seller does not. Here, we assume the agent's valuation is unknown to both players at the beginning of the game. After observing the menu of available goods, but before making a purchase decision, the buyer can costly acquire information about how much he values quality; that is, the agent is rationally inattentive about his type. Information acquisition can be interpreted as the buyer learning either about his own preferences, or about product characteristics that affect his personal match value with the good. Because learning takes place *after* the seller makes her offer, the menu of goods will affect information choices: the monopolist can influence which information is acquired by choosing which contracts to provide. This interaction between

[1] See Abaluck and Gruber (2011), Brown and Jeon (2020), and Taubinsky and Rees-Jones (2018), for example for evidence on consumer behavior.

contracts and costly learning is central to our analysis.

Results Information acquisition brings a new dimension to the monopolist's problem: contracts must induce the buyer to acquire information that is optimal for the seller. This requirement implies two additional constraints to optimal screening. First, the principal must fine-tune goods' quality for the agent to acquire specific information. If quality levels are far apart, selecting the wrong product implies large utility losses, and the buyer has considerable incentives to learn. If quality levels are close together, mistakes are less consequential, and the buyer will not choose to learn much. Thus, which information is acquired depends on the extent of product differentiation. Second, prices must be low enough for the consumer to buy. The more expensive products are, the higher the incentives for the buyer to learn: if he learns his value for quality is low, he does not purchase any goods, and thus he avoids paying too much for quality he does not enjoy. To discourage this behavior, the principal must offer price discounts.

We characterize the solution to the principal's problem and derive two key results. First, we show information acquisition adds to, rather than mitigates, the inefficiencies generated by information asymmetry alone. Quality distortions are both more widespread and larger than in standard screening. They are more widespread because agents with *all* valuations typically receive below-efficient quality levels — in contrast to the standard screening result, where the agent with the highest valuation is assigned efficient quality. Distortions are larger in the sense that, on average, quality levels are further away from the efficient ones, when compared with the standard screening benchmark. The reason is that the consumer shifts the balance of power in the agency relationship by *threatening* to learn what the seller does not want him to. The buyer receives rents not only due to the private information he actually bears, but also because of the information he could have obtained.

For an intuition, consider the following simple case. When acquisition costs are high enough, we prove no learning takes place, so information is symmetric in equilibrium. In standard screening, if information is symmetric, the seller offers a single good to the buyer, with efficient quality tailored to his valuation; prices are such that the consumer is indifferent between buying or not. Here, this offer does not work. If the agent is indifferent between buying and not buying *before* acquiring information, he has incentives to learn whether his value is just a little smaller than his ex-ante belief. In such a case, the good is too expensive for his type, and he does not purchase it. To dissuade the buyer from learning, the seller gives the product a price discount and, to save on costs, degrades its quality. Thus, the contract is inefficient even when information is symmetric. This argument highlights the distinction between this model and standard screening: the key distortion here is the *threat* of obtaining information, rather than the presence of private information.

The second main result is that profits and consumer surplus are non-monotonic in the level of information costs. Profits are U-shaped, whereas consumer surplus often increases for low acquisition costs, and always decreases when costs are high. The reason is that the credibility of the agent's threat varies as costs change. When information costs are small, a fair amount of information is acquired in equilibrium. In that setting, the agent earns rents by threatening the principal not to learn as much as she wants. This threat gets more credible as acquisition costs rise and learning becomes harder. Thus, in that range, an increase in costs benefits the agent at the expenses of the principal. By contrast, when acquisition costs are high, information acquisition is very limited in equilibrium, and the agent extracts rents from the threat of learning *more* than the principal wants. Then, an increase in costs reduces the credibility of the threat, benefiting the seller and hurting the buyer.

Implications These results have several implications. The first is that information acquisition is fundamental for quantifying inefficiency. As previously mentioned, quality levels are lower on average in our model than they would be under standard screening. Thus, a researcher who estimates efficiency losses while ignoring the role of information acquisition would believe demand to be lower than it actually is. To rationalize that lower demand, the researcher would underestimate the taste for quality in the economy and, thus, the level of quality distortions. The estimation error could be substantial. We illustrate this error using the closed-form solution for the case in which both production and information costs are quadratic. We assume the researcher observes the quality of signed contracts but uses the results in Mussa and Rosen (1978) to quantify inefficiency. In a specific parameterization, the researcher may believe contracts are virtually efficient, whereas in reality, quality levels are about 45% lower than the optimal level.

Our results also shed light on sellers' incentives to manipulate learning. These incentives vary with the level of information costs. If costs are low, it tends to be optimal for the principal to facilitate learning, by favoring transparency and providing tools that ease information acquisition. If acquisition costs are high, sellers may benefit from hiding product information and dissuading learning. Importantly, the seller may benefit from manipulating learning even if her action does not change information choices in equilibrium. Rather, changing the *credibility* of the buyer's threat is what drives the redistribution of surplus. This rationale suggests a possible explanation for the observed heterogeneity in how much companies help consumer learning. In the last section, we discuss this rationale at length, as well as how these implications complement the literature on product obfuscation.[2] In that literature, firms dissuade consumer learning in order to profit against competitors. Conversely, in our model, the seller aims to reduce the buyer's strategic advantage and may even help buyers acquire information.

[2]See Ellison (2005), Ellison and Wolitzky (2012), and Petrikaitė (2018).

Finally, the non-monotonicity of consumer welfare has implications for the design of policies that facilitate consumer learning. These transparency policies are often used in practice (Brown, 2019; Hackethal et al., 2012). The rationale supporting such interventions is based on how information costs affect consumers by causing choice mistakes: buyers fail to purchase the product that is right for them. Thus, when supply is fixed, consumer welfare increases if acquisition costs decrease. We show this argument may fail when supply is allowed to respond. The seller benefits from more informed consumers, because she is then able to tailor products to consumer types. Hence, she is willing to provide incentives for the agent to learn and make fewer mistakes. When information costs are not too high, incentive provision more than compensates the buyer for his choice mistakes. Thus, transparency policies could have unintended equilibrium consequences, harming consumers, instead of benefiting them.

Related literature This paper contributes to the large body of work applying rational inattention models to different strategic interactions (Bloedel and Segal, 2020; Matějka and Tabellini, 2016; Ravid, 2020; Yang, 2019).[3] In particular, it relates to the literature on product market equilibrium when consumers are inattentive (Boyacı and Akçay, 2018; Hefti, 2018; Martin, 2017; Matějka and McKay, 2012). In these papers, firms offer a single good to agents who are rationally inattentive with respect to quality, price, or both. Closest to ours is the concurrent work of Mensch and Ravid (2022). They study a similar model of monopolistic screening with endogenous information acquisition, and also prove that quality distortions are more widespread than under standard screening. We differ in two dimensions. First, we provide additional results on the effect of these distortions on surplus. In particular, we show information costs affect profits and consumers' surplus non-monotonically, with implications for firms' incentives to manipulate learning, as well as for the design of transparency policies. Second, we model information costs distinctly. While their model allows for an uncountable state space and focuses on mean-measurable information costs, ours restricts attention to a finite state space, but permits more general cost functions.

Our paper complements a literature that studies how information acquisition interacts with mechanism design. Roesler and Szentes (2017) model a bilateral trade setting in which the buyer obtains information *before* the seller offers a menu. Similarly, Ravid et al. (2022) study the same setting when acquisition decisions and price setting occur simultaneously. These papers differ from ours in the timing of information acquisition: here, the monopolist commits to the menu beforehand, providing incentives for the information she wants to be acquired, which gives rise to the key distortion in our model. This timing distinction is critical: the main takeaway of Ravid et al. (2022) is that the consumer can be significantly better off when information is freely available, versus when it is extremely cheap. Our timing assumption reverses

[3]See Mackowiak et al. (2020) for a thorough review of applications of rational inattention by field.

that message, in that the buyer can be better off with costly information than when information is free. In section 5, we discuss further this reversal and the role of timing.

A large literature on behavioral industrial organization studies how sellers can exploit attention frictions. Examples are Piccione and Spiegler (2012), De Clippel et al. (2014), and Bordalo et al. (2015). In these papers, consumers are inattentive to observable product characteristics and firms use different instruments to manipulate consumers' attention in a competitive framework. We complement that literature in that our model features one firm, and agents are inattentive with respect to their *value* for quality. Thus, the monopolist uses product differentiation to control the buyer's attention to their type. Our applications are connected to other results in industrial organization: in particular, we discuss seller incentives to dissuade consumer learning, which is the focus of the obfuscation literature (Ellison and Ellison, 2009; Ellison and Wolitzky, 2012; Petrikaitė, 2018).

Li and Shi (2017) and Guo et al. (2018) study information disclosure by a principal in a screening setting. In contrast to our model, information is controlled by the seller, not by the buyer. Our work is thematically related to an older literature on mechanisms with information acquisition (Bergemann and Välimäki, 2002; Crémer and Khalil, 1992; Shi, 2012; Szalay, 2009). We differ from these papers by focusing on multiple products and allowing for flexible information acquisition. The techniques we apply are connected with works on contracting with information design (Boleslavsky and Kim, 2018; Doval and Skreta, 2020; Georgiadis and Szentes, 2020; Ostrizek, 2020). We differ from these papers in that our problem is not one of information design, but rather of information acquisition. Additionally, none of those papers study screening. Finally, we are indebted to tools and ideas developed in decision problems under rational inattention, for example Caplin et al. (2017) and Matějka and McKay (2015). In particular, our solution to the acquisition problem adapts the method developed by Caplin et al. (2017), based on concavification techniques (Aumann and Maschler, 1995; Kamenica and Gentzkow, 2011).

2 A Model of Menu Design with Costly Information Acquisition

A monopolist (the principal, she) aims to sell indivisible goods to a potential buyer (the agent, he). She produces goods of different quality levels $q \geq 0$ at cost $c(q)$, where the cost function $c \in \mathcal{C}^2$ is increasing and strongly convex.[4] The buyer's valuation for quality is $\vartheta \in \{\underline{\theta}, \overline{\theta}\}$, with $0 < \underline{\theta} < \overline{\theta}$. In Online Appendix D, we extend the model to an arbitrary finite number of valuations and show the main results continue to hold, as further discussed in section 5. If an agent with valuation ϑ purchases a good of quality q at price t, he receives utility $\vartheta q - t$. The monopolist offers the buyer a menu of quality-price pairs. We assume

[4] A function $f \in \mathcal{C}^2$ is strongly convex if $\min_x f''(x) > 0$.

throughout that $c(0) = c'(0) = 0$.

The model departs from traditional screening in two ways. First, information is symmetric at the beginning of the game: originally, ϑ is unknown to *both* players, who share a prior with mean μ. Second, before making his purchasing decision, but after observing the menu, the agent can acquire information about his valuation. Formally, he can choose an information structure $(\mathcal{S}, P)$, which consists of a set of signal realizations $\mathcal{S}$ and a function $P : \{\underline{\theta}, \overline{\theta}\} \to \Delta(\mathcal{S})$ that assigns a distribution over signals for each state. Although the agent is free to choose any information, learning is costly, as described below.

The timing of the game is as follows. The principal acts first, offering a schedule of quality-price pairs. After observing the offer, the agent decides which information to acquire. Upon observing a signal realization, the agent decides whether to buy one of the options from the menu or none — in which case he obtains a utility of zero. Importantly, the choice of information happens only after the menu is observed and, hence, may depend on the menu of options offered by the monopolist.

Information and Acquisition Costs Because the agent's utility is linear, it depends on information only through the mean of posterior beliefs. Following standard practice, we associate each signal to the posterior mean it generates (Dworczak and Martini, 2019; Gentzkow and Kamenica, 2016). Formally, a signal realization is $\theta \in [\underline{\theta}, \overline{\theta}] \equiv \Theta$, and, upon observing signal θ, the buyer's payoff is $\theta q - t$. We refer to the realization of the agent's private information, θ, as his type or, abusing notation, his posterior. Then, any information structure is a distribution over realizations $F \in \Delta(\Theta)$ satisfying a Bayesian consistency constraint (Kamenica and Gentzkow, 2011). Because the state is binary, this constraint can be written as:

$$\mathbb{E}_F[\theta] = \mu. \tag{BC}$$

We assume information-acquisition costs are posterior-separable (Caplin et al., 2017). A convex function over posterior means, $H : \Theta \to \mathbb{R}$, exists, with $H(\mu) = 0$, and a scalar $k \geq 0$ such that the cost of an information structure F is defined as:

$$K(F) = k\mathbb{E}_F[H(\theta)].$$

This class of cost functions generalizes mutual information-based acquisition costs, widely used in works on rational inattention, and encompasses almost all of the costs contained in the flexible information acquisition literature.[5] By obtaining information, the agent moves his posterior away from the prior.

[5] The standard formulation for posterior-separable information costs is in terms of posterior distributions. Because the state is binary, we can write it as a function of posterior means only. To embed mutual information, $H(\theta) = \sum_{\vartheta \in \{\underline{\theta}, \overline{\theta}\}} \left\{ \frac{|\vartheta - \theta|}{\overline{\theta} - \underline{\theta}} \log \frac{|\vartheta - \theta|}{\overline{\theta} - \underline{\theta}} - \frac{|\vartheta - \mu|}{\overline{\theta} - \underline{\theta}} \log \frac{|\vartheta - \mu|}{\overline{\theta} - \underline{\theta}} \right\}$. Part of the literature has been devoted to finding costs of acquisition that are alternative to mutual information, for example Hébert and Woodford (2020) and Pomatto et al. (2020). The cost specifications in these papers satisfy posterior-separability. Applications have either used Shannon entropy or other posterior-separable costs. Examples are Matějka and McKay (2015), Hébert and La'O (2020) and Yang (2019).

Heuristically, H defines this informal notion of distance between prior and posterior. The cost K reflects the expectation of this distance across all possible signal realizations according to information structure F. Note K is monotonic in the Blackwell order, in the sense that more informative structures are costlier. The parameter k scales the cost, and it is later used for comparative statics. We assume $H \in C^3$ and strongly convex.

Two subclasses of posterior-separable cost functions are of particular relevance. We say that a cost function has the unbounded marginal costs (UMC) property if increasing the precision of a signal becomes arbitrarily costly as the signal becomes more precise. Formally, H is UMC if $\lim_{\theta \to \vartheta} |H'(\theta)| = \infty$, for $\vartheta \in \{\underline{\theta}, \overline{\theta}\}$. Mutual-information costs belong to this class. Conversely, we say a function is of bounded marginal costs (BMC) if $|H'|$ is bounded. A prominent example is quadratic costs, $H(\theta) = \frac{(\theta-\mu)^2}{2}$, which measure the expected reduction in prior variance obtained by observing information F.

The Principal's Problem. We study the mechanism that maximizes expected profits for the monopolist. When choosing a mechanism, the seller takes into account that her offer affects the decisions of the buyer in two ways. First, it determines the buyer's choice given his information. In standard screening, the revelation principle guarantees that it suffices for the seller to offer one contract to each type of agent. However, here the agent type is determined by the information he decides to acquire, which is endogenous. As a consequence, the traditional revelation principle does not apply directly in this setting. Nonetheless, we show in the Online Appendix that the principal can restrict attention to menus of quality-transfer pairs $\mathcal{M} = \{q(\theta), t(\theta)\}_{\theta \in \Theta}$, in which one contract is offered to each possible posterior of the agent, including those that have zero probability in equilibrium.[6]

The second way in which the menu affects consumers' choices is through information choices. Because information is acquired after contracts are offered, acquisition depends on the terms of those contracts. By designing the menu, the principal indirectly controls information acquisition. Therefore, the seller maximizes profits knowing both which information will be acquired in equilibrium and how each type selects across contracts. Defining the outside option for the consumer as $C_o \equiv \{q(o), t(o)\} = \{0, 0\}$, we can

[6]This revelation principle in the presence of information acquisition is proved in Theorem OA 1 in Online Appendix B. For related, yet distinct, results, see Hellwig (2010) and Skreta (2006).

write the principal's problem:

$$\max_{\mathcal{M},F\in\Delta(\Theta)} \quad \mathbb{E}_F\left[t(\theta)-c\left(q\left(\theta\right)\right)\right]:$$

$$\text{s.t.} \quad \theta\in\arg\max_{\omega\in\Theta\cup\{o\}} \theta q(\omega)-t(\omega), \quad \theta\in\Theta \tag{IC}$$

$$F\in\arg\max_{\mathbb{E}_G[\theta]=\mu} \mathbb{E}_G\left[\max_{\omega\in\Theta\cup\{o\}}\{\theta q(\omega)-t(\omega)\}-kH(\theta)\right]. \tag{IA}$$

The monopolist maximizes expected profits subject to two sets of constraints. The first one, IC, subsumes both traditional incentive compatibility and individual rationality constraints. It says that, given a posterior, $\theta\in\Theta$, the buyer chooses from the contracts in the menu and the outside option to maximize utility, obtaining interim rents $U(\theta)\equiv\max_{\omega\in\Theta\cup\{o\}}\{\theta q(\omega)-t(\omega)\}$. Note IC depends only on the offered menu, and not on which information is acquired. Thus, this constraint can be characterized following standard practice in mechanism design (Myerson, 1981). Maximizing expected profits subject to IC is the classic monopolistic screening problem, given a distribution over types F (Maskin and Riley, 1984; Mussa and Rosen, 1978). Throughout this paper, we denote this optimization as the standard screening problem, and we use it as a benchmark.

The information-acquisition constraint, IA, is the departure from standard screening. It says information must be a solution to the agent's acquisition problem. In that problem, the buyer chooses an information structure — namely, a Bayesian-consistent distribution of types — to maximize the expectation of utility net of acquisition costs, $U-kH$. Costs are posterior-separable, and utility is given by interim rents: the maximum payoff of the contracting choice for each type realization. Finally, if multiple information structures solve the acquisition problem, we allow the principal to select his favorite; that is, we consider the principal-optimal equilibrium of the game.

Remark. The model above assumes the agent obtains information statically. In section 5, we argue this framework is equivalent to one in which the principal sets the menu first, and then the agent acquires information continuously until he decides to stop and make a purchasing decision. Online Appendix C proves this equivalence formally, using the results in Hébert and Woodford (2021).

3 Results

3.1 Simplifying the Principal's Problem

In this section, we turn the problem of the principal into a simple optimization that can be solved by standard techniques. We show two features of the problem that connect contracting with information acquisition. These properties allow for rewriting the principal's optimization into a tractable, finite-dimensional problem.

Information acquisition brings two novel features to the contracting problem, which we call (i) marginal incentives for acquisition and (ii) the threat point. Recall that the principal controls information indirectly, by designing the menu. She must provide incentives for any information she wants to be chosen. (i) and (ii) characterize how the principal can provide such incentives: by controlling the marginal benefits of learning through product differentiation; and by adjusting the level of rents. In the discussion below, we explain these properties and their implications in constraining the space of contracts available to the principal.

Henceforth, we follow the standard practice of denoting menus as rent-quality pairs, $\{U(\theta), q(\theta)\}_{\theta\in\Theta}$. In the following discussion, we assume an optimal information has at most two posteriors. As a consequence of binary states, any such information structure can be identified by the two posterior means in its support: $\operatorname{supp} F = \{\theta_L, \theta_H\}$, $\theta_L \leq \mu \leq \theta_H$. Because only two posteriors are chosen, at most two contracts are signed with positive probability, $\{C_L, C_H\}$, with rents and quality levels $\{U_i, q_i\}_{i\in\{L,H\}}$, where $U_i \equiv U(\theta_i)$ and $q_i \equiv q(\theta_i)$. At the end of this subsection, Proposition 1 shows the optimal information structure is indeed binary. We characterize properties (i) and (ii) in terms of these equilibrium contracts.

Marginal incentives. The first property ties together product differentiation and the endogenously acquired information. Because product quality affects the value of learning, it affects information acquisition. Intuitively, agents benefit from information because they can adopt the contract that better reflects their tastes. As quality levels grow apart, this benefit increases, because purchasing the wrong contract has larger consequences. To induce a specific information strategy, the principal must fine-tune the quality of contracts to the information she wants to be acquired. To see this argument heuristically, recall that the consumer chooses information to maximize the expectation of utility net of costs, $U - kH$. Then, marginal net utilities $U' - kH'$ can be understood as the marginal value of information. But by the classic characterization of Myerson (1981), incentive compatibility implies $U'(\theta) = q(\theta)$, so marginal net utilities are affected by product quality. Thus, quality influences information decisions.

The marginal-incentives property makes this interplay between product differentiation and information acquisition precise. Roughly, it states that *marginal net utilities must be equated in the support of an optimal* *structure*. When this property fails, the buyer prefers to acquire information that is not predicated by the information structure, either because quality levels are too far apart and incentives for learning are excessive, or vice versa. This argument works only partially if an optimal structure fully reveals a state. In that case, because the agent cannot learn more about the fully revealed state, an inequality of marginal utilities holds instead. The following result formalizes the discussion above and establishes the marginal-incentives constraint formally.

Lemma 1. *Let F and $\{U(\theta), q(\theta)\}_{\theta\in\Theta}$ satisfy IC and IA, with $\operatorname{supp} F = \{\theta_L, \theta_H\}$. Then, there is $\psi \in \mathbb{R}$ such that, for $i \in L, H$:*

$$\begin{aligned} q_i - kH'(\theta_i) &= \psi, \quad \text{if } \theta_i \in (\underline{\theta}, \overline{\theta}) \\ q_L - kH'(\theta_L) &\leq \psi, \quad \text{if } \theta_L = \underline{\theta} \\ \psi &\leq q_H - kH'(\theta_H), \quad \text{if } \theta_H = \overline{\theta} \end{aligned} \tag{M}$$

Figure 1 illustrates this result graphically. The figure plots the net utility obtained at different posteriors, which are depicted in the horizontal axis. For each posterior, the agent chooses the best contract from $\{C_L, C_H\}$ or the outside option C_o, as shown in the bottom of the picture. The standard concavification argument implies that any optimal information structure, F, is represented by a segment tangent to the graph of $U - kH$.[7] We identify this segment with its slope ψ. The support of F are the types at which tangency happens — that is, $\operatorname{supp} F = \{\theta_L, \theta_H\}$ —, and the ex-ante utility of the agent is the height of the line segment at the prior mean. Note ψ plays an important role in our analysis. In particular, as we further emphasize later, ψ adjusts the value of information $U - kH$ to take into account the Bayesian-consistency constraint.

An immediate consequence of Lemma 1 is that, insofar as no posterior in the support of F is fully revealing, the difference between quality levels is pinned down by the marginal-incentives property:

$$q_H - q_L = kH'(\theta_H) - kH'(\theta_L)$$

Threat Point. The second property concerns the level, rather than the margin, of acquisition incentives: it is reminiscent of a participation constraint. Recall that the principal wants the agent to pick a contract from the menu she designs. However, the agent can always choose the outside option C_o after information realizes. He does so if he finds out that both contracts are too expensive given his information, that is, if he gets a signal that his type is too low. To guarantee participation, the principal must discourage the acquisition of any information that induces the agent to opt out of the menu. Figure 2a depicts a situation of non-participation. In that example, the agent rejects the principal's prescription, F, deviating to the information structure denoted by the dotted segment, with support $\{\theta_o, \theta_H\}$. If signal θ_o realizes, he opts-

[7] For concavification in general, see Aumann and Maschler (1995) and Kamenica and Gentzkow (2011). For the argument applied to information acquisition, see Caplin et al. (2017).

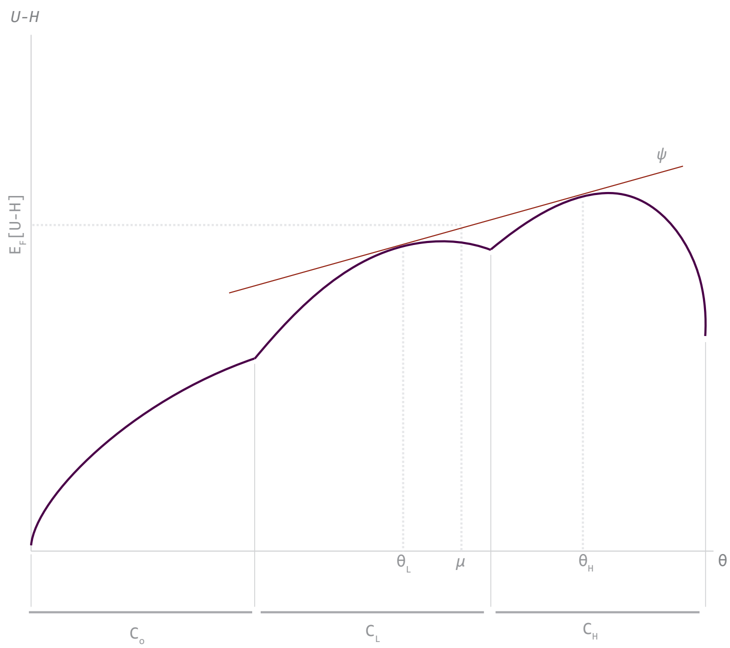
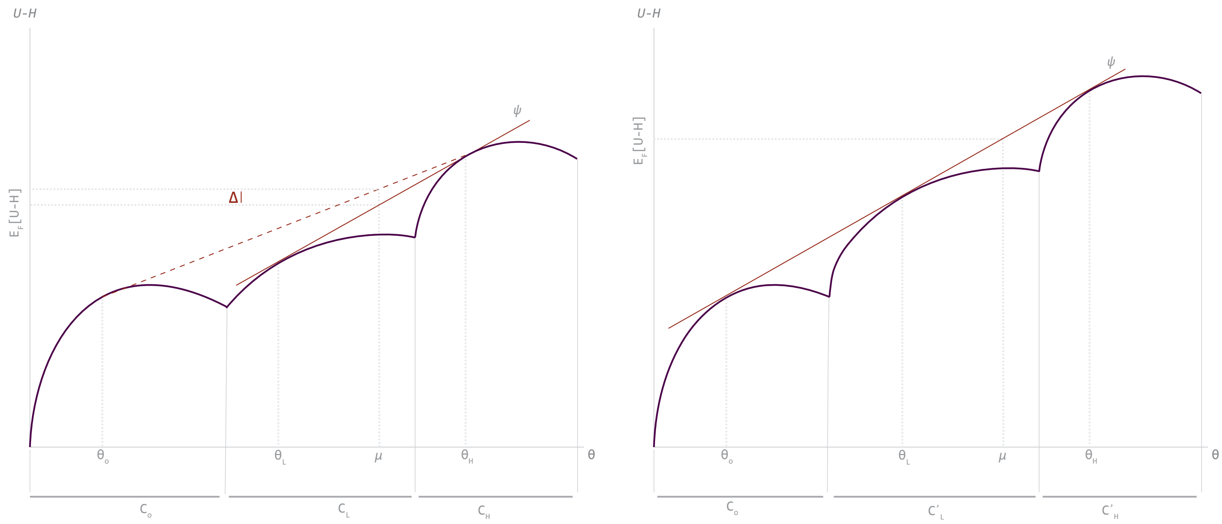


Figure 1: An Optimal Information Structure

out of the menu. That deviation generates an expected gain of $\Delta > 0$ for the buyer. In the figure, F satisfies the marginal-incentives property, but the deviation is still attractive to the buyer: the seller must resort to a different set of tools to guarantee participation.

The principal can avoid such deviations by making the menu more attractive. In particular, she can affect the level of net utilities by reducing the prices of C_L and C_H, obtaining new contracts $\{C_L', C_H'\}$, as can be seen in Figure 2b. By reducing transfers in each of these contracts, the principal raises rents — and, thus, net utility — for types that accept those contracts. Additionally, she increases the interval of types that participate in the menu, that is, the interval of posteriors choosing the outside option, C_o, shrinks. This change in contracts renders the prescribed information structure F optimal to the buyer by ruling out the deviation to θ_o.

This intuition generalizes. Given the quality levels and prescribed information, we identify a type that plays a central role in the opting-out deviations: we call it the threat point. The threat point is the posterior that maximizes the ex-ante value of purchasing C_o for the buyer. Recall that the value of information is given by the net utility $U - kH$. By trying to maximize this value, the buyer must take into account the Bayesian consistency constraint. To do so, the buyer maximizes the adjusted value of information $U - kH - \psi\theta$. The threat point, $\theta_o(\psi)$, is a posterior under which the agent chooses not to buy any contract



(a) A profitable deviation

(b) No profitable deviation

Figure 2: Net Utilities and the Threat Point

and that maximizes the adjusted value of information:

$$\theta_o(\psi) \equiv \arg\max_{v\in\Theta}\{-kH(v) - \psi v\}.$$

As in Figure 2b, the threat-point property establishes that, at an optimal information structure, the adjusted value of information in the support is equated with the adjusted value of information at the threat point. The next result states that property formally.

Lemma 2. *Let $F, \{U(\theta), q(\theta)\}$ solve the principal's problem with* $\operatorname{supp} F = \{\theta_L, \theta_H\}$*. Furthermore, assume ψ is as in Lemma 1. Then, the following holds:*

$$U_i - kH(\theta_i) - \psi\theta_i = -kH(\theta_o(\psi)) - \psi\theta_o(\psi) \quad \textit{for all } \; i \in \{L, H\} \tag{TP}$$

Lemma 2 shows the relation between contracts in the support and the threat point is tight: the consumer is exactly indifferent between learning $\theta_o(\psi)$ and following F. The expression on the left-hand side is the adjusted value of obtaining posterior θ_i, that is in the support of information structure F, prescribed by the principal. The right-hand side is the adjusted value of learning the threat point. This indifference implies the threat point regulates the level of rents. By determining the level of rents, quality is pinned down because rents are linked to quality by incentive compatibility. A higher threat point reduces quality and rents. Formally, if the principal wants to implement information structure $\operatorname{supp} F = \{\theta_L, \theta_H\}$ such that no type is fully revealing of a state, quality levels must satisfy:

$$q_i = kH'(\theta_i) - kH'(\theta_o(\psi)), \quad i \in \{L, H\}.$$

Together, marginal incentives for acquisition and the threat point tightly constrain the set of contracts the principal can choose from. Marginal incentives for acquisition tie the differences in quality levels for any two contracts to the prescribed information structure, whereas the threat point fixes the quality level. An implication of this discussion is that product differentiation is pinned down by the information structure that the principal wants to induce. Given the choice of information, the only instrument remaining for the principal is the choice of the threat point or, equivalently, the adjustment term ψ.

A simpler problem. Lemma 1 and Lemma 2 show M and TP are constraints to the principal's optimization. The following result proves they are the *only* constraints the principal must satisfy in addition to Bayesian consistency, BC. We say two optimization problems are equivalent if they have the same value and the solution to one can be used to construct a solution to the other.

Proposition 1. *The principal's problem is equivalent to:*

$$\max_{\{\theta_i, U_i, q_i\}_{i\in\{L,H\}}, \psi} \Big\{ \sum_{i\in\{L,H\}} p_i^F \left[\theta_i q_i - c(q_i) - U_i\right] : M, \;\; TP \quad and \quad BC \Big\}, \tag{P}$$

where p_i^F is the probability of θ_i under $\operatorname{supp} F = \{\theta_L, \theta_H\}$.

Furthermore, this problem has a solution.

Proposition 1 shows the principal's problem can be greatly simplified to a finite dimensional optimization with equality and inequality constraints. The objective function is the principal's profits rewritten as surplus minus rents, as standard, and assuming F is at-most-binary. Essentially, the result states that the principal can focus on binary information structures and on equilibrium variables: posteriors and contracts that are signed with positive probability in equilibrium. In the original principal's problem, the seller must take into account the off-equilibrium behavior of the buyer: the information they could have chosen to obtain and the contracts they would pick if they acquired different information. In Proposition 1, instead, the principal only needs to track the slope ψ, which, in addition to equilibrium variables, guarantees that the agent will follow the plan of action she prescribes.

For an intuition, note our discussion thus far has proved that M and TP must hold in a solution to the principal's problem. Thus, provided the restriction to binary distributions, the profits attained in the problem in Proposition 1 are at least as large as the principal's profits. The majority of the proof consists in showing the converse, namely, that any solution to the problem above can be extended to satisfy IC and IA without loss of profits. This result is obtained in several steps. First, we show contracts are incentive compatible in $\{\theta_L, \theta_H\}$, which is guaranteed because M implies quality is monotonic, and TP allows us to compare interim rents. TP also guarantees individual rationality. In fact, IC holds strictly: the principal needs to provide more incentives for the agent to acquire information, than for him to simply reveal it. Then, we show we can extend contracts to satisfy IC to posteriors that have zero probability in equilibrium. Finally, a concavification argument guarantees IA is also satisfied.

A tractable problem Note Proposition 1 delivers a tractable optimization problem suitable for applications. For an illustration, consider the case in which H is UMC; that is, the marginal cost of learning grows towards infinity as the precision of any signal increases. Thus, the optimal information structure contains no fully revealed state. As a consequence, M holds with equality, allowing us to write quality, q_i, as a function of θ_i and ψ. Similarly, one can use the threat-point property, TP, to solve for interim rents, U_i. Finally, Bayesian-consistency can be used to write p_i^F as a function of $\operatorname{supp} F$ and μ.

These three observations allow for substitution of q_i, U_i, and p_i in the seller's optimization. The following remark summarizes this discussion:

Remark. *When H is UMC the problem of the principal can be rewritten as*[8]*:*

$$\max_{\theta_L \le \mu \le \theta_H, \psi} \sum_i \frac{|\theta_j - \mu|}{\theta_H - \theta_L} \Big\{ \theta_i \left(kH'(\theta_i) - kH'(\theta_o(\psi))\right) - c\left(kH'(\theta_i) - kH'(\theta_o(\psi))\right) \\ - \left[kH(\theta_i) - kH(\theta_o(\psi)) + \psi(\theta_i - \theta_o(\psi))\right] \Big\}.$$

The problem above is a simple optimization over three variables. Using calculus, the solution can be characterized directly from this tractable version of the seller's optimization. However, any particular solution will depend on the specific form of the cost functions, so this exercise does not deliver clear insights. Instead, we use the simplified problem to characterize general properties of the optimal menu and of the surplus, which are the main results of the paper. In the following subsections, we describe these results and discuss how they are guided by the two key forces above: the marginal-incentives property, M, and the threat point, TP.

3.2 Menu Design and Information Acquisition

This section focuses on the efficiency of optimal contracts. In particular, we compare them with the first-best allocations and to the ones obtained in standard screening. This comparison is not straightforward. Here, the agent is uninformed to begin with, and all the information is obtained endogenously by acquisition choices. By contrast, in the standard screening model, information is exogenous. If we aim to evaluate quality distortions, we need to keep information constant across models. To this end, we solve for the optimal information structure in our model and then take it as the exogenous information in a standard screening problem.

[8]The simplified optimization problem for the non UMC case can be found in the proof of Proposition 2 in the Appendix.

The efficient (or first-best) quality for type θ_i is the quality that maximizes the production surplus for that type, namely, q_i^{f} such that $c'(q_i^{\mathrm{f}}) = \theta_i$. Under symmetric, exogenous information, each type of buyer is assigned a contract with efficient quality. In the second-best problem, when information is exogenous but asymmetric, the principal wants to dissuade the high-type agent from purchasing contracts aimed toward low type. The optimal way for her to achieve that goal is to make the low contract less attractive. Thus, in the standard screening model, the principal underprovides quality to the low type, $q_L^s < q_L^{\mathrm{f}}$. On the other hand, the quality assigned to the high type, q_H^s, is efficient. Note the sole driver of this distortion is private information: the principal needs to avoid attracting high types to the low contract, because she is uninformed about consumer's value.

For any information structure F and menu of contracts with quality levels q, we use $\tau(F,q) \equiv \mathbb{E}_F\left[\theta - c'(q(\theta))\right]$ to denote the expected wedge: it measures how distorted from efficiency the quality levels in the menu are. In particular, $\tau(F,q^{\mathrm{f}}) = 0$ because the first-best menu maximizes production surplus, and $\tau(F,q^s) > 0$. We now show that when information is endogenously acquired, in contrast to the pure screening case, both types receive below-efficient quality. Additionally, the wedge is larger than in the second-best solution.

Proposition 2 (Distortion Patterns)**.** *Let $F, \{U, q^*\}$ solve the principal's problem. Then:*

> ***Quality is underprovided.*** $c'(q_L^*) < \theta_L$ *and* $c'(q_H^*) \leq \theta_H$
>
> ***Aggregate distortions.*** $\tau(F,q^s) \leq \tau(F,q^*)$

All inequalities are strict if either (i) k is high enough; (ii) $k > 0$ and H is UMC; or (iii) whenever $\operatorname{supp} F = \{\mu\}$.[9]

Proposition 2 shows that in the presence of information acquisition, distortions also happen at the top; moreover, they are larger than under standard screening. The reason is that our model adds a new distortion to the screening problem: the *threat* from the agent to obtain information. The buyer could obtain information on whether his type is low and opt-out of the menu if it is the case. By the threat-point property, the principal reduces prices to guarantee no such deviation is profitable for the buyer. The efficient way to do reduce prices, from the production perspective, is to also degrade quality. However, degrading the quality of a contract affects learning incentives again. The marginal-incentives property then guarantees the quality levels of both contracts move together, because it pins down their difference, implying distortions for all types. This new source of distortions adds to the inefficiencies due to information asymmetry. As discussed in the Introduction, the inefficiency persists even when information is symmetric: distortions are determined by the threat of information acquisition, rather than by private information itself.

Note the inequalities in Proposition 2 are not always strict. In particular, for low k, acquiring complete information can be optimal. In this case, it is always profit maximizing for the principal to provide the

[9] Lemma OA1 in Online Appendix A shows that $\overline{k}$ exists such that $\operatorname{supp} F = \{\mu\}$ whenever $k \geq \overline{k}$.

same quality levels as under standard screening — although not the same prices. However, for sufficiently large costs, the inequalities are strict and the difference between this model and standard screening is sharp. Similarly, UMC cost functions guarantee that information cannot be fully acquired and thus that inequalities are always strict. The remainder of this section illustrates these results with a special case in which we completely characterize the optimal solution.

3.2.1 Quadratic Costs

Assume $c(q) = \frac{q^2}{2}$ and $H(\theta) = \frac{(\theta-\mu)^2}{2}$. Under this specification, the cost of acquiring an information structure is proportional to the reduction in prior variance obtained by observing this information.[10] As a function of k, we denote the equilibrium information structure as $F(k)$ and equilibrium quality as $q^*(k) = \{q_L^*(k), q_H^*(k)\}$. We also denote $q^s(k)$ as the second-best quality obtained when the information is exogenously set at $F(k)$. The next proposition characterizes $F(k)$ and compares $q^*(k)$ with $q^s(k)$.

Proposition 3 (Quadratic Costs)**.** *Under quadratic costs, $\underline{k} \leq \overline{k}$ exist such that the optimal structure $F(k)$ satisfies:*

$$\operatorname{supp} F(k) = \begin{cases} \{\underline{\theta}, \overline{\theta}\}, & \text{if } k < \underline{k} \\ \{\omega(k), \overline{\theta}\}, & \text{if } \underline{k} \leq k < \overline{k} \\ \{\mu\}, & \text{if } k > \overline{k} \end{cases}$$

where ω is a strictly increasing, continuous function with $\omega(\underline{k}) = \underline{\theta}$. $F(k)$ is unique except, possibly, at $\overline{k}$. Moreover, $q_H^*(k) = q_H^{\mathrm{f}}(k) = \overline{\theta}$, for all $k < \overline{k}$ and $q_L^*(k) \leq q_L^s(k)$.

In words, $F(k)$ has a very simple form: it contains three regions depending on k. For low k, full information is acquired and the state is revealed. For high k, no information is acquired at all. For intermediate levels of costs, the information consists of one posterior that fully reveals the high state, $\overline{\theta}$, and of a low posterior that is partially informative. In that range, as k increases, the low posterior is increasing; that is, the precision of the low signal deteriorates monotonically. Although the specific form of this optimal structure is special, the existence of $\overline{k}$ such that no information is acquired for $k \geq \overline{k}$ holds for any information and production costs. Similarly, the existence of $\underline{k}$ such that the states are perfectly revealed for $k \leq \underline{k}$ is guaranteed by any BMC information costs.[11]

The high state $\overline{\theta}$ is fully revealed because of the marginal-incentives property, M. By M, quality levels must be fine-tuned not to give the agent incentives to make one of his posteriors more precise. However, $\theta_H = \overline{\theta}$ is as precise as a posterior can be, so the principal does not have to worry about that kind of

[10] Specifically, $K(F) = \frac{k}{2}\mathbb{V}_F\left[\mathbb{E}[\vartheta|\theta]\right]$. Then, by the Variance Decomposition Formula, $K(F) \propto \mathbb{V}[\vartheta] - \mathbb{E}_F\left[\mathbb{V}[\vartheta|\theta]\right]$.

[11] These results are proved in Lemma OA1 and Lemma OA2 in Online Appendix A.

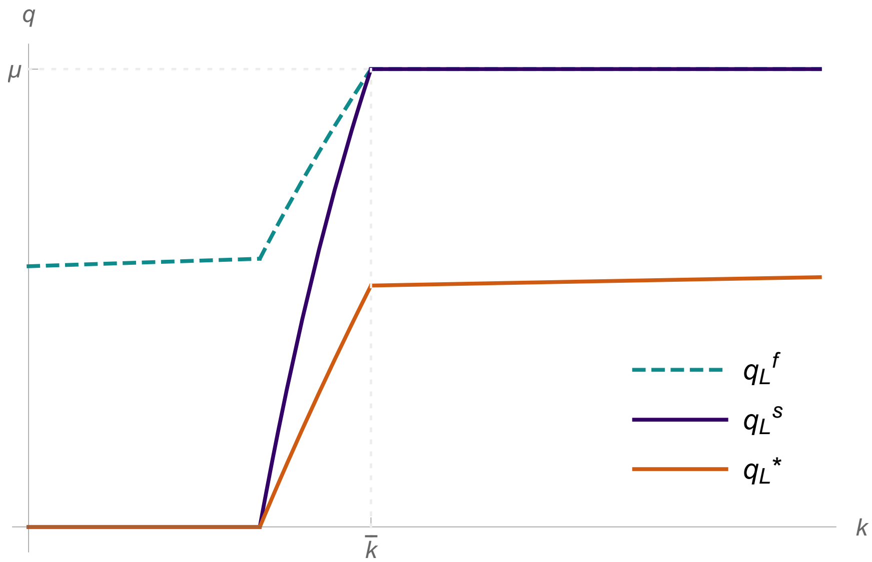


Figure 3: Low Quality Under Quadratic Costs

Notes: This figure plots the quality of the low type under quadratic costs as k varies, q_L^*. For each k, we use the information structure $F(k)$ to solve for the first- and second-best contracts q_L^{f} and q_L^s. For ease of visualization, we start the horizontal axis from $k > \underline{k}$.

deviation. Thus, the seller is free to provide quality to the high agent that is discontinuously larger than what she could offer if the high posterior were all but perfectly informative. She seizes that opportunity, providing the high type with his efficient quality. As k grows, maintaining full revelation at the top becomes costlier and the seller degrades the precision of the low posterior, increasing the quality of the product sold to the low type.

Because the high type receives efficient quality, all the distortions in this example come from the contract given to the low type. Figure 3 plots $q_L^*(k)$ and compares it with $q_L^s(k)$ and $q_L^{\mathrm{f}}(k)$. Because production costs are quadratic, $q_L^{\mathrm{f}}(k) = \omega(k)$. The distance between q_L^{f} and q_L^s is the standard screening distortion. It stems from asymmetric information exclusively; therefore, it vanishes for $k \geq \bar{k}$, when information is symmetric in equilibrium. The difference between q_L^s and q_L^* is the distortion given by information acquisition: it is a result of the agent's threat of acquiring information that is not prescribed by the principal. This difference does not disappear when information is symmetric: the distortion persists and only vanishes asymptotically, as k approaches infinity and the agent loses his ability to threaten the principal.

3.3 Nonmonotonic Surpluses

We now turn to discussing how the level of acquisition costs, k, affects profits and consumer surplus. This impact is not clear from the outset, because several mechanisms are at play. First, information has a productive role in this model, helping agents match with contracts of appropriate quality, generating higher surplus. Thus, higher information costs, by constraining the production frontier, could have a negative effect on surpluses. Second, acquired information is costly and must be paid for. To the extent that these expenses vary with the level of information costs, this level affects the surpluses. Finally, costs affect the balance of power in the principal-agent relationship in two ways. One is direct, as k maps into the value to the buyer of deviating from the prescribed information strategy. Because, in equilibrium, he must be compensated for not deviating, k affects the division of surplus. The other is indirect: the level of costs affects which information is acquired and therefore affects information asymmetry. Proposition 4 below describes the end result of all these mechanisms on surpluses. The following assumption is relevant.

Assumption 1. *At least one of the following holds:*

1. *H is BMC;*

2. *$(\overline{\theta} - \underline{\theta})^2 \geq (\overline{\theta} - \mu)\overline{\theta}$.*

Assumption 1 describes a class of economies. It includes all bounded marginal costs of information. Condition 2 in the assumption does not depend on information costs. It requires, instead, that, under full information, the second-best contract excludes the low type, that is, $q_L^s = 0$.

Proposition 4 (Nonmonotonicity)**.** *The principal's profits are U-shaped in k. The consumer's surplus is decreasing for sufficiently large k. Under Assumption 1, consumer's surplus increases at $k = 0$.*

Proposition 4 shows that as the level of acquisition costs changes, profits respond non-monotonically. Under Assumption 1, consumer surplus is also non-monotonic and roughly in the opposite way. Profits are U-shaped in that the initial effect of information costs is to make the principal worse off. By contrast, when k is high enough, the principal benefits from an increase in acquisition costs. Under Assumption 1, the opposite holds for consumer surplus. The initial increase in costs improves their expected utility. However, when k becomes too large, a further increase in acquisition costs generates losses to the buyer. This latter effect does not depend on the assumption.

The intuition for this result relies on how the cost of learning affects the balance of power between the players through the threat of the agent. Changing k affects both the value and the credibility of the agent's deviations. When k is small and thus the prescribed information structure is particularly revealing, the most valuable threat for the agent is acquiring too little information. As k increases, that threat becomes more credible: the agent has fewer incentives to learn as learning gets costlier. This increase initially works in his favor and at the expense of the seller. By contrast, when k is very high and the prescribed information structure is extremely opaque, the most valuable threat is learning too much. However, as costs grow, that deviation becomes less credible. Then, further increases benefits the principal at the cost of the buyer.

This explanation oversimplifies the complex dynamics of acquisition costs. Indeed, all mechanisms mentioned earlier are at play. Nonetheless, the interaction between the level of costs and threats is the most prominent. The easiest way to see that is to consider again the case of quadratic costs. Recall that, in that case, the agent acquires a fully informative information structure and contracts have the second-best quality for $k \leq \underline{k}$. Over this interval, production is just as efficient as in the second-best, so the fact that profits are falling — and consumer surplus increasing — over that range is unrelated to the productive role of information. Additionally, even if the principal is reimbursed for acquisition costs, one can show profits are decreasing: the result is not driven by information expenses. A similar rationale works for $k \geq \overline{k}$. In both of these cases — for sufficiently low or high costs — the only force at play is the one emphasized in the previous paragraph.

4 Implications

In this section, we outline three implications of our main results. First, we study the problem of a researcher trying to quantify the impact of market power in welfare. We argue that ignoring the presence of costly information acquisition may lead to a sizable underestimation of efficiency losses. Then, as mentioned in the introduction, we explore firms' incentives to aid or dissuade consumer learning, and show how these incentives depend on the level of acquisition costs. Finally, we review our model's implications for transparency policies. In particular, policies that facilitate consumer learning may lead to losses to buyers. The same rationale points out that some level of inattention may be beneficial to consumers.

4.1 Estimating Efficiency Losses in Monopolistic Screening

Screening models capture important characteristics of several relevant markets, from cable television to health care to mobile phone providers. A sizable literature empirically studies such models in general, and nonlinear pricing specifically; see Perrigne and Vuong (2019) for a recent survey. In particular, applying these models to real data allows researchers to measure quality degradation (Crawford et al., 2019; Crawford and Shum, 2007), quantifying the extent to which firms with market power generate inefficiencies in environments with asymmetric information. This literature applies the first-order conditions of the principal (Luo et al., 2018) to identify variables of interest, therefore relying heavily on the model specification.

Our results suggest that, by ignoring information acquisition, this approach may underestimate the real level of quality degradation.[12] To make that point concrete, we focus on the simplest possible case. We

[12]Quality degradation could also be understood as quantity degradation in a Maskin and Riley (1984) version of the model. See Luo et al. (2018).

assume contract quality levels q are observable and costs are quadratic, as in the example we discussed in the previous section. Additionally, we assume costly information acquisition exists in reality, but a researcher ignores it and tries to estimate quality degradation using the standard screening results of Mussa and Rosen (1978). The researcher observes the distribution of signed contracts and their quality and knows production costs are quadratic. Her goal is to estimate the distribution of types, F, in the economy. Given F, the level of quality distortion can be measured by the expected wedge, which reads $\tau(q,F) = \mathbb{E}_F[\theta] - \mathbb{E}_F[q]$ in this case. Therefore, under quadratic costs, the wedge can be exactly interpreted as the expected distance between efficient and assigned quality. Let $\hat{\tau}$ be the estimated wedge given what the researcher observes.

Proposition 5. *Under quadratic costs, $\hat{\tau} \leq \tau(F,q)$.*

Proposition 5 shows that, in the quadratic economy, a researcher would always underestimate the expected gap between efficient and realized quality and thus would overestimate efficiency. This result follows a simple argument. In a binary setting, F consists of three variables: θ_L, θ_H, and the frequency of high types, which we call p_H^F. By observing the market share of each good, the researcher can backtrack p_H^F. Indeed, p_H^F is exactly the proportion of high-quality contracts signed. Moreover, by assuming efficiency at the top, $q_H = \theta_H$, the researcher identifies the high type. Proposition 3 guarantees both θ_H and p_H^F are estimated correctly, because in the quadratic environment, the high type is undistorted. However, issues arise when the researcher tries to estimate θ_L using q_L. By Proposition 3, q_L is smaller than the quality that would be assigned to the true low type θ_L in standard screening. To rationalize that low level of quality, the researcher would have to believe the low type is lower than θ_L. In fact, one can show their estimate of θ_L would be the threat point $\hat{\theta}_L = \theta_o(\psi)$. This mismeasurement leads to a lower value for the estimated mean $\mathbb{E}_F[\theta]$, underestimating the wedge.

Our solution to the quadratic example allows us to quantify the discrepancy between the real efficiency loss and the one estimated by the researcher, as illustrated in Figure 4. We solve our model for different levels of information costs and measure the relative level of quality degradation, given by $\frac{\tau(F,q)}{\mu}$, represented by the full curve in the figure. Then, for each k, we reproduce the estimation procedure we described above, assuming the researcher observes the equilibrium quality obtained in our model, thus leading to the estimated level of degradation given by the dashed curve. For low enough k — for example, k close to 1 —, the estimate is correct, because $\theta_o(\psi) = \theta_L$. When costs are high enough that no information is acquired, only one contract is signed in equilibrium and estimated losses are 0, but real losses are positive and often large. Crucially, even for intermediate cases, when two contracts are signed in equilibrium, the researcher could severely underestimate losses. For example, when $k \approx 1.2$, the researcher would estimate minimal losses when quality levels are in fact about 45% lower than efficient.

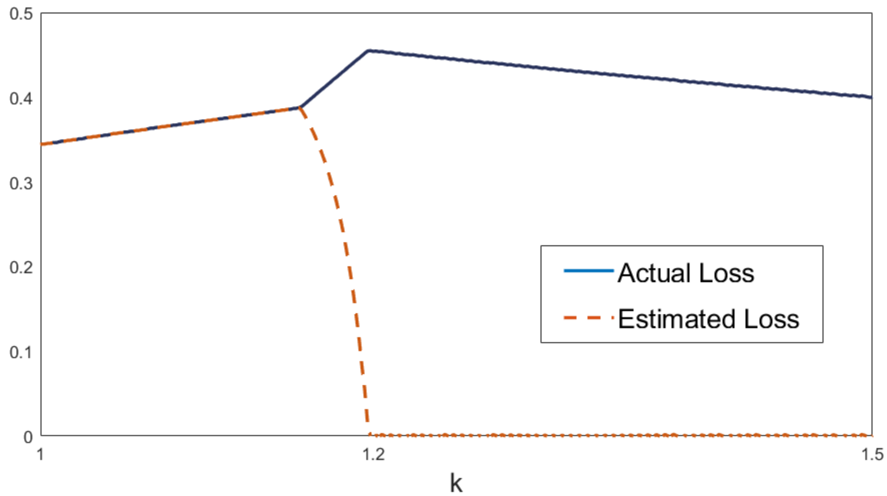


Figure 4: Underestimating Inefficiency

Notes: This figure plots real and estimated quality degradation as k varies. Degradation is measured as the percentage deviation between expected quality and the efficient one: $\frac{\tau(F,q)}{\mu}$. The picture corresponds to parameters $\{\underline{\theta}, \mu, \overline{\theta}\} = \{.2, .5, 1\}$.

4.2 Manipulating Consumer Learning

In many settings, a seller can help or obstruct consumer learning by making testing, experimenting, or having access to valuable information easier or harder. For example, insurance and mobile phone provider websites often help customers find plans that best suit their needs. In this section, we extend our model to allow the seller to manipulate information costs. Assume the buyer has a baseline level of information costs, $k_o > 0$, interpreted as the natural difficulty with which he can introspect and do research about his match value with the good. Then, we take a reduced-form approach and assume the seller can affect this level of information costs. In particular, if she wants information costs to be of level k, she can pay $\gamma w(k)$ monetary units, where $w : \mathbb{R} \to \mathbb{R}_+$ and $\gamma > 0$. $w \in \mathcal{C}^2$ is strictly convex and $w(k_o) = w'(k_o) = 0$. These costs can be seen as the price of designing a website that allows the buyer to better compare their options, or to generate false product reviews that make information acquisition more difficult (Mayzlin et al., 2014). Importantly, both helping and making information acquisition harder are costly, and γ parameterizes that cost. Let k^* be the seller-optimal level of information costs.

Proposition 6. *Let H be BMC. $\bar{\gamma}$ and $\underline{k} \leq \overline{k}$ exist such that for all $\gamma \geq \bar{\gamma}$,*[13]

1. *If $k_o \geq \overline{k}$, the seller hampers information acquisition — i.e. $k^* > k_o$*

[13]Although the results are proved for BMC costs, the intuition extends for any cost function. Note we could also solve for the same model without manipulation costs, that is, when $\gamma = 0$. In that case, because the principal's profits are U-shaped, the seller always wants to either make information freely available or prohibitively costly to acquire. Indeed, depending on assumptions about the production-cost function and parameters, either of the two options could be optimal.

2. *If $k_o \leq \underline{k}$, the seller aids information acquisition — i.e. $k^* < k_o$.*

Proposition 6 suggests firms may either have incentives to obfuscate or to make information easier to acquire, depending on acquisition costs. If the seller has a limited ability to affect the level of acquisition costs, then she would prefer to hamper consumer learning when those costs are sufficiently large, but to facilitate learning when costs are low. Here, obfuscation is profitable when the agent threatens to learn more than what the monopolist desires. For a given level of costs, preventing this threat would imply distorting production surplus and providing price discounts. On the other hand, by making acquisition harder, the seller discourages learning, achieving the same goal with smaller efficiency losses. When the agent threatens to learn less than the seller wants, facilitating learning has the same effect.

Put together, these two observations may suggest an explanation for the observed heterogeneity in how much companies try to influence consumer learning. As previously mentioned, companies offer different tools to aid buyers' selection among their products. Typically, these online tools allow for easy comparisons in some dimensions of product characteristics but not others, so they only partially help buyers learn. Additionally, learning manipulation is heterogeneous; for instance, whereas some firms help acquisition, some others make it more difficult by producing false online reviews about their products (Mayzlin et al., 2014). Our result suggests the observed heterogeneity may be related to the baseline level of information costs in the market and how expensive moving these costs is for the seller. An in-depth study of the incentives for helping or hampering learning in the presence of information-acquisition costs is an interesting area for future research.

These results complement the literature on obfuscation (Ellison, 2005; Ellison and Ellison, 2009; Ellison and Wolitzky, 2012; Gamp, 2016; Petrikaitė, 2018). First, in our model, the seller has incentives to manipulate consumer learning because doing so affects the credibility of the buyer's information threats. In other words, the goal of manipulation is to shift the balance of the principal-agent relationship. By contrast, in the obfuscation literature, firms manipulate consumer learning to alleviate competitive pressure (Ellison and Wolitzky, 2012), or to screen different buyers (Petrikaitė, 2018). Second, we prove that, in some circumstances, it is optimal for the seller to aid learning, whereas the work on obfuscation has extensively focused on the optimality of hiding product information. Finally, whereas most of the literature on obfuscation relies on search models, we show that a different mechanism may justify learning manipulation under costly information acquisition.

4.3 Transparency Policies

Finally, our results suggests policies that facilitate consumer learning may not benefit consumers. Transparency policies are relatively popular in markets for complex goods.[14] For example, New Hampshire provides the public with the HealthCost website, which is a price information tool for health-care costs (Brown, 2019). Importantly, this website allows consumers to compare out-of-pocket medical procedure costs across providers, taking into account personal information, as their insurance carrier and zip code. Similar tools are also available in many other states (Brown, 2019). The rationale for these policies seems to be that reducing information costs will help consumers make better decisions and, thus, increase their welfare. However, this rationale ignores equilibrium effects, which may reverse the intended effects of such intervention.

In this section, we study how equilibrium considerations affect the welfare gains of transparency policies. We assume again that agents have a baseline level of information costs k_o. We define a transparency policy as any intervention that reduces the level of information costs. The next results is a direct consequence of Proposition 4.

Corollary 1. *Let Assumption 1 hold.* $\tilde{k}$ *exists such that, for* $k_o \le \tilde{k}$*, any transparency policy harms consumers.*

Corollary 1 shows transparency policies may have unintended consequences in equilibrium: when costs are not too large, making information easier to acquire harms the consumer. Indeed, buyers make mistakes when information is costly so, for a fixed menu, consumers must be better off. However, when sellers are allowed to respond, this information friction may be beneficial to consumers. In fact, the principal steps in to prevent mistakes, providing incentives for information acquisition. She does so because she also profits from a buyer who makes fewer mistakes, since tailoring quality to agents' types increases the total surplus. When acquisition costs are low, decreasing them further has the side effect of degrading the strategic advantage of agents, and can therefore hurt them. In other words, equilibrium considerations may reverse the effect of transparency policies.

It follows from this discussion that a rationally inattentive consumer may be better off than a fully attentive one — or, equivalently, that costly information can be better than free information. This assertion relies on the fact that sellers want consumers to learn and thus are willing to incentivize them to do so for some values of information costs. When the willingness of the principal to compensate the agent for not making mistakes is high enough — that is, when costs are low — inattention increases consumer welfare. Although these consequences are derived here in the monopolistic screening framework, they seem to be

[14] See Brancaccio et al. (2017) for an example in finance, Brown (2017, 2019) for health care, and Hackethal et al. (2012) for cellular plans.

much more general: as long as sellers benefit from information acquisition and compensate agents for it in equilibrium, lower costs will not necessarily imply higher consumer surplus.

5 Discussion

Timing We studied the contracting problem when the agent acquires information *after* the principal offers the menu. This assumption is realistic: in a variety of settings, the menu of available goods is fixed, and consumers can choose to acquire information after observing it, according to their own timelines. Examples are health care and online shopping, where the available goods and terms of trade are typically readily available and easily observable. In other common environments, however, information can be acquired before the menu is offered. In particular, when observing the terms of trade depends on an action by the buyer — for example, contacting a dealer for a financial-asset quote, or reaching out to a vendor to learn about prices —, information acquisition can happen before the seller's offer is fixed. This possibility can also hold when the producer is able to make exploding or timed offers. Roesler and Szentes (2017) and Ravid et al. (2022) study bilateral trade models in which information acquisition happens before and simultaneously to the design of the mechanism, respectively. Together, their work and this paper shed light on the importance of the timing assumption in determining the outcome of mechanism design under information acquisition.

Our model complements Ravid et al. (2022) by showing our timing assumption reverses their main takeaway. The key message in Ravid et al. (2022) is that, when information is acquired simultaneously to the design of the mechanism, the buyer may be substantially better off having access to free information than being able to purchase the same information at a low cost. When buyer and seller decide at the same time, the corresponding optimal mechanism fails to induce information acquisition even when costs are arbitrarily small. As the buyer foregoes some amount of information, the authors show the price at the optimal mechanism is higher than it would be if the buyer learned more thoroughly. As a consequence, both the consumer and the producer are worse off than in the full-information equilibrium that could arise when information is free.[15] As previously discussed, we obtain the opposite result: from the point of view of the consumer, low information-acquisition costs can be strictly better than being able to acquire information for free. The reason is that the principal, acting first, chooses to compensate the agent to acquire surplus-enhancing information when doing so is inexpensive. That compensation works in the buyer's favor and may increase consumer surplus.

[15] In fact, they are worse off than under any equilibrium of the costless information economy.

Equivalence to Dynamic Acquisition. In many settings, the agent learns gradually, rather than at once. For instance, consumers deciding among health insurance plans can do various rounds of information acquisition until they decide to take one of the available offers. This dynamic acquisition process seems ubiquitous; in particular, it applies to any instance in which the available goods and their prices are kept the same over time and the buyer can decide what to learn and when to purchase in their own timeline. Therefore, the formulation of our model with a static acquisition decision may seem to leave out many possible applications.

In Online Appendix C, we use the results in Hébert and Woodford (2021) to show the equivalence between our framework and a natural dynamic model of acquisition. In particular, all the analysis in the paper remain unchanged even if the consumer continuously acquires information before deciding to stop learning and making a purchasing decision. This equivalence is described in detail in the Online Appendix. Concretely, we assume both players are infinitely patient, but the consumer incurs a fixed waiting cost per unit of time. At each point in time, the buyer faces a posterior-separable acquisition cost. The game is as follows: first, the principal sets a menu at time 0. Then, the buyer decides which information to acquire and when to stop learning and make a purchase decision. The main result in Online Appendix C shows the distribution of contract choices in this dynamic problem is the same as the one obtained in our paper. Therefore, the optimal menu in the two models is the same.

Multiple States. In the model we analyzed, the real valuation of the buyer was binary: $\vartheta \in \{\underline{\theta}, \overline{\theta}\}$. The main reason for that choice is clarity. As shown in Online Appendix D, the key results of the paper generalize naturally to a finite arbitrary number of states. In particular, under some additional conditions, both the pattern of distortions and the nonmonotonicity of the surpluses continue holding in that general setting. The characterization in that case follows the same steps as in the simpler binary model; namely, we show the appropriate conditions associated with product differentiation and the threat point are necessary and sufficient for the seller's problem.

We now briefly discuss the key technical difficulty of the general model. With two states, any posterior-separable cost can be written as a linear function of F, the distribution of posterior means. Doing so when valuations are not binary is not possible. As a consequence, to keep linearity of the acquisition-cost function, we need to write information costs as a function of the whole distribution over posterior beliefs, which is a less transparent, higher-dimension object. However, that the agent cares only about the mean of his posterior beliefs still holds. That fact simplifies the analysis of interim behavior: once information is acquired, rents and quality levels are still functions of posterior types. We can then apply methods that mirror the ones developed for the simpler problem to tackle the less tractable, general model.

Appendix: Auxiliary Results and Proofs

We start proving an auxiliary result. The information acquisition problem can be written in terms of rents as:

$$\begin{aligned} \max_{G\in\Delta(\Theta)} \quad & \mathbb{E}_G\left[U(\theta)-kH(\theta)\right] \\ \text{s.t.} \quad & \mathbb{E}_G[\theta]=\mu \end{aligned} \tag{1}$$

Lemma 3. *Problem 1 has a solution. $F\in\Delta(\Theta)$ solves it if and only if it satisfies BC and there exists $\psi\in\mathbb{R}$ such that:*[16]

$$\operatorname{supp} F \subseteq \arg\max_{v\in\Theta}\{U(v)-kH(v)-\psi v\} \tag{2}$$

Proof of Lemma 3

Recall that it is without loss of generality to assume (q,t) is bounded. We start by proving existence of a solution. We then proceed to show necessity and sufficiency of condition 2.

Existence. Because contracts are bounded and Θ is compact, interim rents U are bounded by definition. Thus, $U-kH$ is a bounded function and the objective function is trivially continuous with respect to F in the weak topology. Additionally, because Θ is compact, $\Delta(\Theta)$ inherits compactness in the topology of weak convergence, by an application of Prokhorov's theorem. Finally, the set of F satisfying BC is closed under weak convergence, implying that the constraint set is a closed subset of a compact space, being itself compact. As a consequence, under the topology of weak convergence, problem 1 is one of maximizing a continuous function over a compact set and, therefore, has a solution.

Necessity. That BC is necessary is trivial, as it is a constraint in the problem. For 2, Start by defining the Lagrangian:

$$L(F,\psi)=\mathbb{E}_F\left[U(\theta)-kH(\theta)-\psi\theta\right]$$

Notice that, as $\mu\in(\underline{\theta},\overline{\theta})$, it is an interior point of the set $\{y\in\mathbb{R}:\mathbb{E}_F[\theta]=y \quad \text{for some} \quad F\in\Delta(\Theta)\}$. Given that, Luenberger (1997), Chapter 8, Problem 7 proves that if F solves 1, then there exists ψ such that $F\in\arg\max_{G\in\Delta(\Theta)} L(G,\psi)$. We now prove that this implies 2.

[16] This result is a consequence of the Lagrangian Lemma in Caplin et al. (2017). We adapt it to our framework and provide a short proof.

Define $\chi \equiv \arg\max_{\theta\in\Theta}\{U(\theta) - kH(\theta) - \psi\theta\}$. Assume, so as to find a contradiction, that $v \in \operatorname{supp} F$ exists such that $v \notin \chi$. It is immediate that F cannot put positive weight outside of χ. Then, assume v is a continuity point of F. That implies there is a neighborhood of v, N_1, such that $N_1 \in \operatorname{supp} F$. However, because $U - kH$ is continuous, there is another neighborhood of v, N_2, such that for all $x \in N_2$, $x \notin \chi$. Then, F puts positive weight on $N = N_1 \cap N_2$ with $N \cap \chi = \emptyset$, which is a contradiction. Thus, 2 is necessary.

Sufficiency. Assume F satisfies BC and 2 for ψ. Because it satisfies 2, it clearly maximizes $L(G,\psi) = \mathbb{E}_G[U(\theta) - kH(\theta) - \psi\theta]$. Define the auxiliary Lagrangian $\tilde{L}$ as $\tilde{L}(G,\lambda) = \mathbb{E}_G[U(\theta) - kH(\theta) - \lambda\cdot(1,-1)\theta]$, for $\lambda \in \mathbb{R}^2$.

Because F maximizes L, it must also maximize $\tilde{L}$ when $\lambda\cdot(1,-1) = \psi$. Take $\lambda > 0$ such that this is the case which is, of course, always possible. Luenberger (1997), Chapter 8.4, Theorem 1 shows that if F maximizes $\tilde{L}$ it also solves:

$$\begin{aligned} \max_{G\in\Delta(\Theta)} \quad & \mathbb{E}_G[U(\theta) - kH(\theta)] \\ \text{s.t.} \quad & \mathbb{E}_G[\theta] \le \mathbb{E}_F[\theta] \\ & -\mathbb{E}_G[\theta] \le -\mathbb{E}_F[\theta] \end{aligned}$$

Because F satisfies BC, $\mathbb{E}_F[\theta] = \mu$. Therefore, the problem above is equivalent to the acquisition problem.

■

Proof of Lemma 1

By the characterization of IC, U is differentiable almost everywhere, except at discontinuities of q and $U'(\theta) = q(\theta)$. If q is continuous at $\theta \in \operatorname{supp} F \cap (\underline{\theta}, \overline{\theta})$, then first order condition is necessary and implies:

$$q(\theta) - kH'(\theta) = \psi$$

We want to prove that, indeed, q is continuous in $\operatorname{supp} F \cap (\underline{\theta}, \overline{\theta})$, so the equality above holds. Assume, to obtain a contradiction, that this is not the case, so there is θ in that set such that q is discontinuous. By IC, q is monotonic, so we must have:

$$\lim_{z\uparrow\theta} q(z) < \lim_{z\downarrow\theta} q(z)$$

We start proving that θ is not a discontinuity point of F.

F **is not discontinuous at** θ**.** Assume that is the case. Then, pick a small $\varepsilon > 0$. Denote $\theta_+ \equiv \theta + \varepsilon$ and $\theta_- \equiv \theta - \varepsilon$. Define $\tilde{F}$ such that:

$$\tilde{F}(v) = \begin{cases} F(v) & , \text{ if } v < \theta_- \\ F(v) + \frac{dF(\theta)}{2}) & , \text{ if } v \in [\theta_-, \theta) \\ F(v) - \frac{dF(\theta)}{2}) & , \text{ if } v \in [\theta, \theta_+) \\ F(v) & , \text{ if } v \geq \theta_+ \end{cases}$$

$\tilde{F}$ clearly satisfies Bayesian consistency. Now consider:

$$\begin{aligned} &\mathbb{E}_{\tilde{F}}[U - kH] - \mathbb{E}_F[U - kH] = \\ \Big(U(\theta_-) - H(\theta_-)\Big)\frac{dF(\theta)}{2} + \Big(U(\theta_+) - kH(\theta_+)\Big)\frac{dF(\theta)}{2} - \Big(U(\theta) - kH(\theta)\Big)dF(\theta) \\ =&\left(\int_\theta^{\theta_+} (q(v) - kH'(v))dv - \int_{\theta_-}^{\theta} (q(v) - kH'(v)dv\right)\frac{dF(\theta)}{2} \\ \geq&\left(\lim_{v\downarrow\theta} q(v) - \lim_{v\uparrow\theta} q(v)\right)\varepsilon\frac{dF(\theta)}{2} + k(H'(\theta_-) - H'(\theta_+))\varepsilon\frac{dF(\theta)}{2} \geq 0 \end{aligned}$$

where the last inequality holds for small enough ε, as the term in the first parentheses is bounded away from zero, whereas the term in the second parentheses goes to zero as ε approaches zero. This implies that $\tilde{F}$ increases the value of the objective function of the agent, which is a contradiction with optimality of F, so θ cannot be a point of discontinuity of F.

F **is not continuous at** θ**.** If that was the case, there would be a neighborhood N of θ such that $N \subset \operatorname{supp} F$. Let $\varepsilon > 0$ and $\theta_- = \theta - \varepsilon$, $\theta_+ = \theta + \varepsilon$, such that $\theta_-, \theta_+ \in N$. Notice that ε can be taken so that q is continuous at both θ_- and θ_+ - as q is increasing, it has at most countable discontinuities. Continuity of q at $\theta_-, \theta_+ \in \operatorname{supp} F$ implies, as shown before:

$$q(\theta_+) - kH'(\theta_+) = \psi = q(\theta_-) - kH'(\theta_-)$$

which can be reorganized as:

$$q(\theta_+) - q(\theta_-) = kH'(\theta_+) - kH'(\theta_-)$$

By taking ε sufficiently small, the right hand side can be made as close to zero as one desires - as H' is continuous, whereas the right hand side is bounded below by $\lim_{v\downarrow\theta} q(v) - \lim_{v\uparrow\theta} q(v)$, providing the desired contradiction.

As a consequence of the last two paragraphs, we obtained a contradiction with q discontinuous in

$\operatorname{supp} F \cap (\underline{\theta}, \overline{\theta})$. Then, and if θ is in that set, $q(\theta) - H'(\theta) = \psi$. We finish the proof by showing the result for $\overline{\theta} \in \operatorname{supp} F$. The result for $\underline{\theta}$ is symmetric.

If $\overline{\theta} \in \operatorname{supp} F$. Assume $q(\overline{\theta}) - kH'(\overline{\theta}) < \psi$. Consider $\varepsilon > 0$:

$$\begin{aligned} U(\overline{\theta}) - kH(\overline{\theta}) - \psi\overline{\theta} - (U(\overline{\theta}-\varepsilon) - kH(\overline{\theta}-\varepsilon) - \psi(\overline{\theta}-\varepsilon)) = \\ \int_{\overline{\theta}-\varepsilon}^{\overline{\theta}} (q(z) - kH'(z))dz - \psi\varepsilon \\ \leq (q(\overline{\theta}) - KH'(\overline{\theta}-\epsilon) - \psi)\varepsilon \end{aligned}$$

Where the last inequality comes from monotonicty of q and convexity of H. By continuity of H' and the assumption that $q(\overline{\theta}) - H'(\overline{\theta}) < \psi$, for sufficiently small ε the last expression must become smaller than zero, finding a contradiction with 2. Therefore, $q(\overline{\theta}) - H'(\overline{\theta}) \geq \psi$. A similar argument establishes the result for $\underline{\theta}$, so necessity of M is concluded. ■

Proof of Lemma 2

Throughout this proof, we say that a menu is feasible for the principal if it satisfies IC and IA. Notice that 2 implies:

$$U(\theta) - kH(\theta) - \psi\theta \geq -kH(\theta_o(\psi)) - \psi\theta_o(\psi) \quad \text{for all} \quad \theta \in \operatorname{supp} F \tag{3}$$

Fix F and let a feasible (U, q) and ψ satisfy 3 with strict inequality. Consider the alternative menu $(\tilde{U}, \tilde{q})$ with $\tilde{U}(\theta) = \max\{U(\theta) - \varepsilon, 0\}$, for some $\varepsilon > 0$ such that $(\tilde{U}, q)$ still satisfies 3, and $\tilde{q}(\theta) = q(\theta)\mathbb{1}_{\tilde{U}(\theta)>0}$. Such an ε exists, because we assumed 3 held with strict inequality. Next, we show this menu is feasible. We finish the proof by showing that it is also more profitable.

Feasibility. First, it satisfies IA. To see that, recall by Lemma 3 that IA is equivalent to 2. Then, because (U, q) is feasible, 2 holds for it:

$$\operatorname{supp} F \subseteq \arg\max_{v \in \Theta} \{U(v) - kH(v) - \psi v\}$$

However, $\tilde{U}(v) - kH(v) - \psi v = \max\{U(v) - kH(v) - \psi v - \epsilon, -kH(v) - \psi v\}$, which is a monotonic transformation of $U(v) - kH(v) - \psi v$. Thus:

$$\arg\max_{v \in \Theta} \{U(v) - kH(v) - \psi v\} = \arg\max_{v \in \Theta} \{\tilde{U}(v) - kH(v) - \psi v\}$$

and IA holds for the new menu $(\tilde{U}, q)$.

For IC, notice that $\tilde{q}$ is increasing and satisfies individual rationality by definition. Finally:

$$\tilde{U}(\theta) = \max\{0, U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} q(v)dv - \varepsilon\} = \max\{0, U(\underline{\theta}) - \varepsilon\} + \int_{\underline{\theta}}^{\theta} q(v)\mathbb{1}_{\tilde{U}(v)>0}dv$$

so the envelope condition also holds and $(\tilde{U},\tilde{q})$ satisfy IC. Thus, it is feasible.

Profitability. We finally show $(\tilde{U},\tilde{q})$ is more profitable than (U,q). Notice that $\tilde{U} < U$. We prove that, in the support of F, $q = \tilde{q}$. To see that, take $\theta \in \operatorname{supp} F$. Because $(\tilde{U},\tilde{q})$ is feasible, we have, by reordering 3:

$$\tilde{U}(\theta) \geq kH(\theta) - kH(\theta_o(\psi)) + \psi(\theta - \theta_o(\psi))$$
$$\geq kH(\theta) - kH(\theta_o(\psi)) - kH'(\theta_o(\psi))(\theta - \theta_o(\psi)) > 0$$

where the first inequality comes from reordering 3, the second from the definition of $\theta_o(\psi)$ and the third from strict convexity of H. Then, $\tilde{U}(\theta) > 0$, implying $\tilde{q}(\theta) = q(\theta)$. Because $\tilde{U} < U$ and, in the support of F, $\tilde{q} = q$, $(\tilde{U},\tilde{q})$ must be more profitable than (U,q). Thus, (U,q) cannot be optimal, because $(\tilde{U},\tilde{q})$ is also feasible. Therefore, for any optimal menu, 3 must hold with equality: that is, TP holds, as we wanted to prove.

■

Lemma 4 (Binary is Sufficient). *Assume the principal's problem has a solution. Then, there are $F,(U,q)$ solving it such that $|\operatorname{supp} F| \leq 2$ and $|q(\Theta)/\{0\}| \leq 2$.*

Proof of Lemma 4

By Lemma 3, the problem of the principal can be written as:

$$\max_{G,(U,q),\psi} \{\mathbb{E}_G[\theta q(\theta) - c(q(\theta)) - U(\theta)] : IC,\ BC \quad \text{and} \quad 2\}$$

where we have rewritten profits in the usual surplus minus rents format. Fix any solution to this problem, $(U,q),F,\psi$. First, we show it is sufficient to focus on binary distributions. Then we show we can restrict the menu accordingly.

Binary distribution. For fixed $(U,q),\psi$, we show the principal is at least as well off with a binary distribution. Consider the simplex $\Delta(\operatorname{supp} F)$. By Winkler (1988), the problem of maximizing profits in this simplex subject to BC is solved by an at-most binary distribution, call it $\tilde{F}$. Because $\tilde{F}$ solves this problem, it is at least as profitable as F for the principal. We now show it is feasible under the principal's problem.

Notice that IC does not depend on F, so $(U,q),\psi,\tilde{F}$ satisfy IC. Additionally, because $\operatorname{supp}\tilde{F} \subset \operatorname{supp} F$, then $(U,q),\psi,\tilde{F}$ satisfy 2. Thus, $\tilde{F}$ is feasible. That implies that focusing on at-most-binary distributions is sufficient.

Binary menus. Now start with a solution $(U,q),\psi,F$ with F at-most-binary. Define the following alternative menu:

$$\tilde{U}(\theta) = \max\left\{0, \max_{v\in\operatorname{supp} F}\{U(v)+(\theta-v)q(v)\}\right\}$$

Let $v(\theta) \in \operatorname{supp} F$ be the largest argmax of the maximization problem in the parentheses above. We define $\tilde{q}(\theta) = q(v(\theta))\mathbb{1}_{\tilde{U}(\theta)>0}$. Notice that $(\tilde{U},\tilde{q})$ was constructed to satisfy IC. It is also easy to see it coincides with (U,q) in $\operatorname{supp} F$, so it satisfies 2. Now, this new menu produces exactly the same profits as (U,q), but notice that $q(\Theta) \subset q(\operatorname{supp} F) \cup \{0\}$, proving the result, since $\operatorname{supp} F$ is at-most-binary.

■

Proof of Proposition 1

Recall the problem of the principal can be written as:

$$\max_{G,(U,q)} \{\mathbb{E}_G[\theta q(\theta) - c(q(\theta)) - U(\theta)] : IC \quad \text{and} \quad IA\} \tag{4}$$

where we have rewritten profits in the usual surplus minus rents format. We call the value of this problem L^*. Let the value of problem P be L^P. We start showing that $L^* \leq L^P$. Then, we prove that the converse holds by constructing a transformation that allows us to express the solution to one of the problems as a solution to the other.

First, $L^* \leq L^P$. Take a solution of 4, $\{F,(U,q)\}$. By Lemma 4, assume without loss of generality that F is at-most-binary and $|q(\Theta)/\{0\}| \leq 2$. Because the solution must satisfy IA, by Lemma 3, it also satisfies 2 for some ψ, and F is Bayesian consistent. Additionally, by Lemma 1, IA and IC imply M, and by Lemma 2, optimality in the principal's problem 4 implies TP.

Just as in the text, identify $\operatorname{supp} F = \{\theta_L, \theta_H\}$, with $\theta_L \leq \mu \leq \theta_H$, and then define $q_i \equiv q(\theta_i)$ and $U_i \equiv U(\theta_i)$, $i \in \{L,H\}$. Then, $\left\{F, \{U_i, q_i\}_{i\in\{L,H\}}, \psi\right\}$ is feasible in P, because TP and M depend only on the points in the support of F. Additionally, it is easy to see that profits under $\left\{F, \{U_i, q_i\}_{i\in\{L,H\}}, \psi\right\}$ in P are the same as L^*. Because L^P is the maximum profits obtained at P, $L^* \leq L^P$.

Next, we prove that $L^P \leq L^*$. Take a feasible element of P, $\left\{F, \{U_i, q_i\}_{i\in\{L,H\}}, \psi\right\}$. We show we can construct from that a feasible element of 4, $\{F,(U,q)\}$ that preserves the profits of the principal. The proof proceeds in two steps.

Step 1: Extension to a menu that satisfies IC. For $\theta_i \in \operatorname{supp} F$, Let $q(\theta_i) \equiv q_i$ and $U(\theta_i) \equiv U_i$, so these functions are defined in $\operatorname{supp} F$. We now extend them. Define, for $\theta \notin \operatorname{supp} F$:

$$U(\theta) = \max\{0, \max_{v \in \operatorname{supp} F}\{U(v) + (\theta - v)q(v)\}\}$$

Let $v(\theta)$ be the largest argmax of the maximization problem in the parentheses above. We define $\tilde{q}(\theta) = q(v(\theta))\mathbb{1}_{\tilde{U}(\theta)>0}$. (U,q) is clearly individually rational. We prove that is it also incentive compatible. Start with $\theta, \theta' \in \operatorname{supp} F$. We have:

$$\begin{aligned} U(\theta) - U(\theta') - (\theta - \theta')q(\theta') &= \\ U(\theta') + kH(\theta) - kH(\theta') + \psi(\theta - \theta') - U(\theta') - (\theta - \theta')q(\theta') &\geq \\ kH(\theta) - kH(\theta') + \psi(\theta - \theta') - (\theta - \theta')(\psi + kH'(\theta')) &\geq 0 \end{aligned}$$

where the first equality is a consequence of TP, the first inequality cancels repeated terms and applies M, and the second inequality cancel the new repeated terms and uses convexity of H. This implies incentive compatibility holds in $\operatorname{supp} F$. It is easy to see that U was constructed so that it satisfied incentive compatibility outside of the support, so (U,q) satisfy IC, finishing step 1.

Step 2. $F,(U,q)$ **satisfy IA** We start by proving that $g(\theta) \equiv U(\theta) - kH(\theta) - \psi\theta$ is concave by parts. Notice that, $q(\Theta) \subset \{q_o \equiv 0, q_L, q_H\}$. q is non-decreasing, because (U,q) satisfies IC, so we can define the intervals $I_i = \{\theta \in \Theta : q(\theta) = q_i\}$, for $i \in \{o, L, H\}$. Now, $g|_{I_i}$ is differentiable and we have:

$$g'|_{I_i}(\theta) = q_i - H'(\theta) - \psi$$

which is decreasing, so $g|_{I_i}$ is concave for each $i \in \{o, L, H\}$. Then, by M and the definition of $\theta_o \equiv \theta_o(\psi)$, for each i, $\theta_i \in \arg\max_{v \in I_i}\{g(v)\}$. That implies:

$$\arg\max_{v \in \Theta}\{g(v)\} \subseteq \max_{i \in \{o,L,H\}}\{g(\theta_i)\}$$

But by TP, $g(\theta_i)$ is a constant across i's. Because $\operatorname{supp} F = \{\theta_L, \theta_H\}$ we have:

$$\operatorname{supp} F \subseteq \arg\max_{v \in \Theta} g(\theta)$$

which is exactly 2 which, by Lemma 3 implies IA, when summed with the fact that F satisfies BC.

We proved that $\{F,(U,q)\}$ is feasible under 4. But because the menu coincides with $\{U_i,q_i\}_{i\in\{L,H\}}$ in the support of F, it obtains the same profit in 4 as in P. Because this was done for an arbitrary feasible element of P we have: $L^P \leq L^*$.

We have then established $L^P = L^*$ and constructed a mapping between solutions to the problems, proving that they are equivalent.

Existence of Solution. As previously argued, P is an optimization in 7 variables. Recall that we identify F with its support: $\{\theta_L,\theta_H\}$, $\theta_L \leq \mu \leq \theta_H$. Notice that $p_L^F = 1-p_H^F$, and $p_H^F = \frac{\mu-\theta_L}{\theta_H-\theta_L}$, for $\theta_H > \theta_L$. Given M, For $\theta_H = \theta_L = \mu$, we have $q_H = q_L$ and $U_H = U_L$. Because of that, p_H^F is immaterial in that case, so we define $p_H^F = 0$. By inspection, the restriction of the objective function to the constraint set is continuous in $\left\{\{\theta_i,U_i,q_i\}_{i\in\{L,H\}},\psi\right\}$. We proceed to prove the constraint set is compact.

M, TP and BC are clearly closed. As previously argued, q_i,U_i are bounded, and $\theta_i \in \Theta$ are also bounded, for $i\in\{L,H\}$. Then, we just need to prove that ψ is bounded. By M, we have, for $\theta_L < \theta_H$:

$$q_H - q_L \geq kH'(\theta_H) - kH'(\theta_L)$$

Notice that this implies $H'(\theta_i)$, $i\in\{L,H\}$ is uniformly bounded. To see that, assume there is a sequence of feasible choices, with θ_i^n, and $H'(\theta_H^n)\geq n$ unbounded. Then, because q_i are bounded, we have that $H'(\theta_L^n)$ also grows unboundedly, so that the right hand side of the inequality above remains bounded. But this is impossible because $\theta_L^n \leq \mu$ implies $kH'(\theta_L^n) \leq kH'(\mu) < \infty$. A similar argument holds if we assume θ_L^n is unbounded below. Then, using M again we obtain:

$$q_L - kH'(\theta_L) \leq \psi \leq q_H - kH'(\theta_H)$$

which guarantees that ψ is bounded. Thus, P is a problem of maximizing a continuous function over a compact set and, therefore, it has a solution.

■

Proof of Proposition 2

We start by constructing a Lagrangian for the principal's problem. For that, we add two auxiliary variables to the problem. Write

$$q_i = kH'(\theta_i) + \psi + a_i$$

By M, $a_L \leq 0$, with equality for all $\theta_L > \underline{\theta}$ and $a_H \geq 0$, with equality for all $\theta_H < \overline{\theta}$. With that definition, we can eliminate M and plug q in the profit function. Additionally, recall that $\theta_o(\psi) = \arg\max_{v\in\Theta}\{-kH(v) - \psi v\} \leq \theta_L$, as $q_L \geq 0$. Using that, we solve TP for U_i to obtain:

$$U_i = kH(\theta_i) + \psi\theta + \max_{v\in\Theta}\{-kH(v) - \psi v\}$$

This then allows us to eliminate the constraint TP and to, again, rewrite the profit function plugging this equation for U_i. In order for the problems to be equivalent, then, we need to impose two types of constraints on a_i. In particular, $a_L \leq 0$, $a_H \geq 0$, to which we associate multipliers α_i, and $a_L(\theta_L - \underline{\theta}) = 0$ and $a_H(\overline{\theta} - \theta_H) = 0$, to which we associate β_i. For shortness, we encode $\vartheta_L = \underline{\theta}$ and $\vartheta_H = \overline{\theta}$. Finally, recall that $q_i \geq 0$ is also a constraint, that we associate with multiplier y_i. We can then write the following Lagrangian for P, plugging in the equation above for U_i:

$$\begin{aligned}\mathcal{L}(\{\theta_i, a_i, \alpha_i, \beta_i, y_i\}_{i\in\{L,H\}}, \psi) = \sum_{i\in\{L,H\}} p_i^F \Big\{ & \theta_i\,(kH'(\theta_i) + \psi + a_i) - c\,(kH'(\theta_i) + \psi + a_i) \\ & - \Big(kH(\theta_i) + \psi\theta_i + \max_{v\in\Theta}\{-kH(v) - \psi v\}\Big) \\ & - \alpha_i a_i - \beta_i a_i(\theta_i - \vartheta_i) - y_i\,(kH'(\theta_i) + \psi + a_i)\Big\}\end{aligned} \tag{5}$$

Notice that, by M, $q_H \geq q_L$, so $y_H = 0$, and we omit it whenever convenient. Start with first order conditions for ψ:

$$[\psi]: \quad \mu - \theta_o(\psi) = \sum_{i\in\{L,H\}} p_i^F [\theta_i - c'(q_i) - y_i] \tag{6}$$

Then, for a_i:

$$[a_L]: \quad \theta_L - c'(q_L) \geq \alpha_L + \beta_L(\theta_L - \underline{\theta}) + y_L \tag{7}$$

with equality if $a_L < 0$, and:

$$[a_H]: \quad \theta_H - c'(q_H) \leq \alpha_H + \beta_H(\theta_H - \overline{\theta}) \tag{8}$$

with equality if $a_H > 0$.

We prove, from now, that q_L is underprovided.

q_L is underprovided — $a_L = 0$. If $q_L = 0$, this is obvious. So we prove it for $q_L > 0$ — thus, $y_L = 0$. First, if $\theta_L < \theta_H$, equation 6 and strict convexity of c imply:

$$c'(q_L) < \sum_i p_i^F c'(q_i) = \theta_o(\psi) \leq \theta_L$$

By contrast, if $\theta_L = \mu$, by 6:

$$c'(kH'(\mu) + \psi) = \theta_o(\psi)$$

By a simple application of Topkis' lemma, $\theta_o(\psi)$ is nonincreasing with ψ. Define: $g(\psi) = c'(kH'(\mu) + \psi) - \theta_o(\psi)$, which is then strictly increasing with ψ. Additionally, $g(-kH'(\mu)) = -\mu < 0$, and g is unbounded — because $\theta_o(\psi) \leq \theta_L$. Thus, there is a unique $\psi > -kH'(\mu)$ with $g(\psi) = 0$. Because $\psi > -kH'(\mu)$, $\theta_o(\psi) < \mu$ and we have:

$$c'(q) = \theta_o(\psi) < \mu$$

as we wanted to prove. We finish by arguing that this implies $a_L = 0$. Assume $a_L < 0$. Because we just proved that q_L is underprovided, we can increase a_L, which would increase q_L, improving profits, while not violating any constraint. Then, it must be that $a_L = 0$.

First Order Conditions for θ_i. Now, define $L_i \equiv \theta_i q_i - c(q_i) - (kH(\theta_i) + \psi\theta + \max_{v\in\Theta}\{-kH(v) - \psi v\})$. Also, let $p^F \equiv p_H^F$. For $\theta_H > \theta_L$:

$$\frac{dp^F}{d\theta_H} = -\frac{p^F}{\theta_H - \theta_L} \qquad \text{and} \qquad \frac{dp^F}{d\theta_L} = -\frac{(1-p^F)}{\theta_H - \theta_L}$$

We start by focusing on the case in which $\theta_i \neq \mu$, $i \in \{L, H\}$, so $\theta_H > \theta_L$. Take first order conditions of $\mathcal{L}$ with respect to θ_i to obtain:

$$[\theta_H]: \qquad -\frac{p^F}{\theta_H - \theta_L}(L_H - L_L) + p^F\left(a_H + (\theta_H - c'(q_H))\, kH''(\theta_H)\right) - p^F \beta_H a_H \geq 0 \tag{9}$$

with equality if $\theta_H < \overline{\theta}$, and:

$$[\theta_L]: \qquad -\frac{(1-p^F)}{\theta_H - \theta_L}(L_H - L_L) + (1-p^F)(\theta_L - c'(q_L) - y_L)\, kH''(\theta_L) \leq 0 \tag{10}$$

with equality if $\theta_L > \underline{\theta}$.

We now prove the main results of the proposition.

High Quality is underprovided. From the first order conditions for θ_i:

$$(1-\beta_H)a_H + (\theta_H - c'(q_H))kH''(\theta_H) \geq (\theta_L - c'(q_L) - y_L)kH''(\theta_L) \tag{11}$$

with equality if $\underline{\theta} < \theta_L < \theta_H < \overline{\theta}$.

We proceed by analysing two cases. Start with $a_H = 0$. Then, the inequality above becomes:

$$(\theta_L - c'(q_L) - y_L)kH''(\theta_L) \leq (\theta_H - c'(q_H))kH''(\theta_H)$$

Because the left hand side of 6 is positive and H is strictly convex, we have that at least one of the sides of inequality above is positive. Thus $c'(q_H) < \theta_H$.

Now, assume $a_H > 0$. In this case, notice that it must be $\theta_H = \overline{\theta}$ and $\alpha_H = 0$. Then, by 8, $\theta_H = c'(q_H)$, and we are done.

Aggregate distortions. We can rewrite 6 as:

$$\sum_{i\in\{L,H\}} p_i^F c'(q_i) = \theta_o(\psi) - p_L^F y_L \leq \max\{\theta_L, p_H^F \theta_H\} = \sum_{i\in\{L,H\}} p_i^F c'(q_i^s)$$

where the first inequality comes from $q_L \geq 0$ and the definition of $\theta_o(\psi)$, and the last equality by the known pure screening solution. To see that the inequality in the middle holds, consider the following two cases. First, if $q_L > 0$, $y_L = 0$ and the inequality is true by $\theta_o(\psi) \leq \theta_L$. Now, if $y_L \neq 0$, we have, by 6:

$$\tau(F,q) = p_H^F c'(q_H) \leq p_H^F \theta_H$$

using the result that q_H is underprovided. That proves the aggregate distortions result. We next prove these two results when $\theta_i = \mu$ for some i.

No Acquisition: $\operatorname{supp} F = \{\mu\}$**.** We proved before that:

$$c'(q) = \theta_o(\psi) < \mu$$

This shows that both the underprovision and the aggregate distortion results hold strictly in this case.

k large enough. Recall that, q_i, $i \in \{L,H\}$ is bounded — uniformly on k by strict convexity of c. Just as in the proof of existence in Proposition 1, this implies that $\{kH'(\theta_i)\}_{i\in\{L,H\}}$ is uniformly bounded on k. Thus, for large enough k, $kH'(\theta_H) < kH'(\overline{\theta})$, as the term in the right grows unbounded. Thus, because we proved that $c'(q_H) = \theta_H$ only when $\theta_H = \overline{\theta}$, we have that, for high enough k, $c'(q_H) < \theta_H$.

Notice that, with the same argument as above, we can conclude that, for high enough k, $\theta_L, \theta_H \in (\underline{\theta}, \overline{\theta})$. Recall that 6 can be written as

$$\sum_i p_i^F c'(kH'(\theta_i) + \psi) = \theta_o(\psi) - p_L^F y_L$$

If $q_L = 0$, because $\overline{\theta} > \theta_H$, $c'(q_H) < \theta_H$ so $\sum_i p_i^F c'(kH'(\theta_i)+\psi) < p_H^F \theta_H$. If, on the other hand, $q_L > 0$, $y_L = 0$ and because $\theta_L > \underline{\theta}$, we have $\theta_L > \theta_o(\psi)$. Applying both of these arguments to the equation above, we get:

$$\sum_i p_i^F c'(q_i) < \max\{\theta_L, p_H^F \theta_H\} = \sum_i p_i^F c'(q_i^s)$$

H is UMC. If that is the case, then for $k > 0$ it is clear that $\theta_i \in (\underline{\theta}, \overline{\theta})$ for $i \in \{L, H\}$. Thus the same argument holds as for when k is large enough.

■

Proof of Proposition 4

Let $Q(k)$ be the value function of P at k. We can apply the envelope theorem to 5 to conclude that the derivative of the profit function satisfies:

$$Q'(k) = -\sum_{i \in \{L,H\}} p_i^F \left\{ H(\theta_i^k) - H(\theta_o(\psi^k)) - (\theta_i^k - c'(q_i^k) - y_i^k) H'(\theta_i^k) \right\} \tag{12}$$

where superscript k denotes that the variable solves P for k. We start the proof showing that profits are decreasing for small k and increasing for large k. We finally prove that profits are quasiconvex, which established the U-shape. Then we move on to consumer surplus.

Profits decrease for small k. When $k = 0$, the solution to P is full information and the optimal contract is the second-best contract for that information: $\operatorname{supp} F^0 = \operatorname{supp} \overline{F} = \{\underline{\theta}, \overline{\theta}\}$, $q^0 = q^s$. Notice that this implies $\theta_o(\psi^0) = \theta_L$. We know $c'(q_H^0) = \overline{\theta}$. Applying this in 6, we have

$$\theta_L - c'(q_L^0) - y_L = \frac{p_H^{\overline{F}}}{p_L^{\overline{F}}} (\overline{\theta} - \underline{\theta})$$

Plugging these into 12:

$$Q'(0) = -p_H^F \{H(\overline{\theta}) - H(\underline{\theta}) - H'(\underline{\theta})(\overline{\theta} - \underline{\theta})\} < 0$$

where the inequality is due to strong convexity of H. Then, we obtain the intended result.

Profits increase for large k. In Lemma OA 1, we prove that there is $\overline{k}$ such that, for $k \geq \overline{k}$ $\operatorname{supp} F^k = \{\mu\}$. It is direct that $\theta_o(\psi^k) < \mu$ for any finite k. Thus, using 6:

$$Q'(k) = -\{H(\mu) - H(\theta_o(\psi^k)) - H'(\mu)(\mu - \theta_o(\psi^k)\} > 0$$

for $k \geq \overline{k}$, where we used, again, strict convexity of H.

Profits are quasiconvex. For $k > 0$, define an auxiliary variable $\tilde{\psi} = \frac{\psi}{k}$. With that, we can rewrite TP in a different form:

$$U_i = k\{H(\theta_i) + \tilde{\psi} + \max_{v \in \Theta}\{-H(v) - \tilde{\psi} v\}\}, \quad \text{for all } \ i \in \{L, H\} \tag{13}$$

Additionally, we add variables a_i to write M as

$$q_i - kH'(\theta_i) - \psi - a_i \mathbb{1}_{\theta_i = \vartheta_i} = 0, \quad \text{for all } \ i \in \{L, H\} \tag{14}$$

By inspection, it should be clear that the following problem is equivalent to P:

$$\max_{F, \{U_i, q_i, a_i\}_{i \in \{L,H\}}, \tilde{\psi}} \Big\{ \sum_{i \in \{L,H\}} p_i^F \left[\theta_i q_i - c(q_i) - U_i\right] : 14, \ \ 13, \ \ \mathbb{1}_{a_L > 0} \leq 0, \mathbb{1}_{a_H < 0} \leq 0 \quad \text{and} \quad BC \Big\}$$

Here is the rationale for this new version of the problem: we added two auxiliary variables that only have a role when a state is fully revealed. These variables make the former inequalities in M into equations, at a cost of an additional constraint. The constraints are such that $a_L \leq 0 \leq a_H$, but we encode them into slightly more cumbersome notation for a reason that becomes apparent in the next paragraph. Additionally, we use the fact that ψ is not profit-relevant, to do a change of variables.

For brevity, define $X \equiv \{F, \{U_i, q_i, a_i\}_{i \in \{L,H\}}, \tilde{\psi}\}$. Notice that we can write the constraint set as a vector inequality: $g(X, k) \leq 0$. g is clearly continuous in k. It is also concave in k, as all constraints are affine in k. The addition of $\{a_i\}$ and of the inequality constraints in the above form guarantee that there is always a solution X^* for this reformulated problem such that $g(X^*, p) = 0$. To see that, notice that choosing $a_i = 0$ if $\theta_i \notin \{\underline{\theta}, \overline{\theta}\}$ gives equality in all constraints in that case. In the opposite case, equalities are guaranteed by definition.

We can then apply Theorem 3.1, part (b) in Xu (2001) to obtain that the value function of the problem above is quasiconvex.

Consumer Surplus increases for small k. First, at $k = 0$ there is full information. If $q_L^s = 0$ for full information, we have that consumer surplus is zero at $k = 0$. Because consumer surplus is always positive at positive k, we have our result. By usual arguments, $q_L^s = 0$ if and only if

$$\underline{\theta} - \frac{p^{\overline{F}}}{1 - p^{\overline{F}}}(\overline{\theta} - \underline{\theta}) \geq 0$$

which can be rewritten as a function of the prior mean to obtain condition 1 in Assumption 1.

Henceforth, we prove the result holds for $q_L^s > 0$. Under Assumption 1, H is BMC. In this case, by Lemma OA 2 in the Online Appendix, there is always a $\underline{k} > 0$ such that $\operatorname{supp} \overline{F} = \operatorname{supp} F^k = \{\underline{\theta}, \overline{\theta}\}$ and $q^k = q^s$ for that distribution, for all $k \leq \underline{k}$. Let $W(k)$ and $CS(k)$ denote the welfare and consumer surplus obtained at the optimal solution, respectively. Then, notice that because the distribution and optimal quality do not change in this interval, welfare changes only to the extent that acquisition costs increase. We can then calculate the derivative of consumer surplus at zero using the fact that profits, Q are welfare minus consumer surplus:

$$\begin{aligned} CS'(0) = W'(0) - Q'(0) &= -\sum_{i \in \{L,H\}} p_i^{\overline{F}}[H(\theta_i)] + p_H^{\overline{F}}\{H(\overline{\theta}) - H(\underline{\theta}) - H'(\underline{\theta})(\overline{\theta} - \underline{\theta})\} \\ &= -H(\underline{\theta}) - p_H^{\overline{F}} H'(\underline{\theta})(\overline{\theta} - \underline{\theta}) > 0 \end{aligned}$$

where the last inequality comes from strong convexity of H and the fact that $H(\mu) = 0$.

Consumer Surplus decreases for high k We know from Lemma OA 1 in the Online Appendix that there is $\overline{k} > 0$ such that $\operatorname{supp} F^k = \{\mu\}$ for all $k \geq \overline{k}$. Additionally, for k sufficiently high, we have $\theta_o^k \equiv \theta_o(\psi^k) > \underline{\theta}$. Then, by 6.

$$c'\left(kH'(\mu) - kH'(\theta_o^k)\right) = \theta_o^k$$

Differentiation then provides:

$$\frac{d\theta_o^k}{dk} = \frac{c''(q^k)\frac{q^k}{k}}{1 + c''(q^k)kH''(\theta_o^k)}$$

We can apply that in the derivative of $CS(k)$ to obtain:

$$CS'(k) = \frac{CS}{k}(k) - \frac{H''(\theta_o^k)c''(q^k)q^k}{1 + c''(q^k)kH''(\theta_o^k)}(\mu - \theta_o^k)$$

By the mean value theorem, there is $m_k \in [\theta_o^k, \mu]$ such that $\frac{CS}{k}(k) = H''(m_k)\frac{(\mu - \theta_o^k)^2}{2}$. We then have that the derivative above can be rewritten as:

$$CS' = \left(H''(m_k)\frac{(\mu-\theta_o^k)}{2} - \frac{H''(\theta_o^k)c''(q^k)q^k}{1+c''(q^k)kH''(\theta_o^k)}\right)(\mu-\theta_o^k)$$

The first term in parentheses converges to zero, but not the second, which means that as k is large enough the whole derivative is negative.

■

Proof of Proposition 5

Recall that the distribution F to be estimated is composed of three elements: $\operatorname{supp} F = \{\theta_L, \theta_H\}$ and $1 - F(\theta_L) = p_H^F$. Denote the mean of F by μ. As argued in the text, p_H^F is correctly identified from the distribution of signed contracts and θ_H by the high quality efficiency, following Proposition 3. So we focus on the estimation of θ_L. Call this estimator $\hat{\theta}_L$. That value can be estimated using the second best contract formula. In particular:

$$q_L = \hat{\theta}_L - \frac{p_H^F}{1-p_H^F}(\theta_H - \hat{\theta}_L)$$

Notice, however, that, under costly information acquisition, $\mathbb{E}_F[q] = \theta_o$. Assuming that q_L solves the second order problem, we obtain:

$$\hat{\theta}_L = E_F[q] = \theta_o \leq \hat{\theta}_L$$

Therefore, the estimated mean of the distribution F is $\hat{\mu} = p_H^F\theta_H + (1-p_H^F)\hat{\theta}_L \leq p_H^F\theta_H + (1-p_H^F)\theta_L = \mu$. Because the expected quality is known, the estimated wedge must be weakly smaller than the real wedge. That is:

$$\hat{\tau}(F,q) = \hat{\mu} - p_H^F q_H - (1-p_H^F)q_L \leq \mu - p_H^F q_H - (1-p_H^F)q_L = \tau(F,q)$$

■

Proof of Proposition 6

Start recalling, by Lemma OA 1, that there is $\overline{k}$ high enough with $\operatorname{supp} F^k = \{\mu\}$ for all $k > \overline{k}$. Assume $k_o > \overline{k}$. By the proof of Proposition 4, the derivative of profits in that region is:

$$Q'(k) = -\{H(\mu) - H(\theta_o(\psi^k)) - H'(\mu)(\mu - \theta_o(\psi^k))\} > 0$$

By the proof of Proposition 2, $\theta_o(\psi^k)$ is increasing in k for $k \geq \overline{k}$, so the profit function is concave. Consider then the problem:

$$\max_k Q(k) - \gamma N(k) \tag{15}$$

If we constraint $k \geq k_o$, this problem is well defined: it is a (strictly) convex optimization. Because $N'(k_o) = 0$, N is strictly convex and $Q'(k) \to 0$ as $k \to \infty$, there is always a solution $k_\gamma > k_o$ that solves this problem.

Now, assume the solution to the unconstrained problem is $k' < k_o$. Because the profits are U-shaped, there exists a unique $\hat{k}$ such that $Q'(\hat{k}) = 0$. Additionally, $\hat{k} < k_o$. Because N is decreasing for $k < k_o$ and Q is increasing for $k > \hat{k}$, we have that $Q(k) - \gamma N(k) < Q(k_o)$ for $k \in [\hat{k}, k_o]$. Thus, it must be that $k' \leq \hat{k}$. Define $\overline{\gamma} = \frac{{}_k Q(k) - Q(k_o)}{N(\hat{k})}$ and notice that, for $\gamma > \overline{\gamma}$:

$$Q(k') - \gamma N(k') < \max_k |Q(k)| - \gamma N(\hat{k}) < Q(k_o) < Q(k_\gamma) - \gamma N(k_\gamma)$$

where the first inequality comes from $k' \leq \hat{k} \leq k_o$ and the fact that N is strictly decreasing for $k < k_o$. The second inequality comes from the definition of $\overline{\gamma}$ and $\gamma > \overline{\gamma}$, and the third inequality from the fact that k_γ is an optimal solution to the right of k_o. Thus, we proved that for $\gamma > \overline{\gamma}$, the optimal solution for the principal is $k^* = k_\gamma > k_o$, obfuscating information.

For the second part, recall that, for BMC costs, there is $\underline{k} > 0$ small enough such that information is not acquired. By the proof of Proposition 4, we know that profits are linear in $k \in [0, \underline{k}]$. Let $k_o \leq \underline{k}$, and consider the problem in 15. First, consider that problem constraint to $k \leq k_o$. That is a strictly convex problem and, because $N'(k_o) = 0$, there exists a solution $k_\gamma < k_o$ to that problem.

Now, assume the solution to the unconstrained problem is $k' > k_o$. Because $Q(k)$ is decreasing at k_o and $N(k)$ is increasing, we know again that $k' > \hat{k}$. Using the same definition of $\overline{\gamma}$ as before, we obtain the same chain of inequalities and the proof is finished.

■

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