Online Appendix for

"Adverse Selection and Endogenous Information"

João Thereze *

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Contents

*Department of Economics, Princeton University — joaothereze@princeton.edu.

A Auxiliary Results

We start by proving properties of the induced information cost function, c.

Define $\mathcal{A}_c = \{(U,Q) \in \mathcal{C} : I^*_{F_o}(1-Q) \leq \mu - U\}$. We know that for any $(U,Q) \in \mathcal{A}_c$ there exists exactly one an at-most-binary $F \in \mathcal{A}$ that is compatible with it. We can then define, as in the text:

$$c(U,Q) = C(F).$$

Proposition OA 1. *c is strictly increasing in U, strictly decreasing in Q, convex in (U,Q), and strictly convex in the interior of $\mathcal{C}$.*

Proof of Proposition OA 1

Note that if F is associated with (U,Q), then:

$$I_F(x) = \begin{cases} 0 & \text{if } x \leq \frac{\mu-U}{1-Q}, \\ (1-Q)x + U - \mu & \text{if } \frac{\mu-U}{1-Q} < x \leq \frac{U}{Q} \\ x - \mu & \text{otherwise} \end{cases} \tag{1}$$

Strictly increasing in U. Fix (U,Q) and (U',Q) associated with F and F' respectively, and $U' > U$. We have:

$$I_{F'}(x) - I_F(x) = \begin{cases} 0 & \text{if } x \leq \frac{\mu-U'}{1-Q}, \\ (1-Q)x + U - \mu & \text{if } \frac{\mu-U'}{1-Q} < x \leq \frac{\mu-U}{1-Q} \\ U' - U & \text{if } \frac{\mu-U}{1-Q} < x \leq \frac{U}{Q} \\ U' - Qx & \text{if } \frac{U}{Q} < x \leq \frac{U'}{Q} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Notice the difference is non-negative everywhere, and strictly positive for at least some interval (as $U' > U$). Thus, $F \preceq_{\text{m.p.s.}} F'$ and:

$c(U',Q) = C(F') > C(F) = c(U,Q)$.

Decreasing in Q. Now let (U,Q), (U,Q') be associated with F and F', and assume $Q' > Q$. Then:

$$I_{F'}(x) - I_F(x) = \begin{cases} 0 & \text{if } x \leq \frac{\mu - U}{1-Q}, \\ (1-Q)x + U - \mu & \text{if } \frac{\mu-U}{1-Q} < x \leq \frac{\mu-U}{1-Q'} \\ (Q-Q')x & \text{if } \frac{\mu-U}{1-Q'} < x \leq \frac{U}{Q'} \\ Qx - U & \text{if } \frac{U}{Q'} < x \leq \frac{U}{Q} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The expression above is always non-positive, and it is negative at least for one interval (because $Q' > Q$). Thus $F' \preceq_{\text{m.p.s.}} F$, and we have that $c(U,Q) > c(U,Q')$.

Convex. Take any (U,Q), (U',Q') associated with F and F', respectively, and some $\lambda \in (0,1)$.

Define $F'' = \lambda F + (1-\lambda)F'$. $F'' \in \mathcal{A}$, as the mean-preserving-spread is a convex relation. Finally, define:

$$\hat{F} = F''(\mu)\delta_{\mathbb{E}_{F''}[\theta|\theta\leq\mu]} + (1 - F''(\mu))\delta_{\mathbb{E}_{F''}[\theta|\theta>\mu]}.$$

Clearly, $\hat{F} \preceq_{\text{m.p.s.}} F''$, so $\hat{F} \in \mathcal{A}$. Note that $\hat{F}$ is also binary, so there is at least one $(\hat{U},\hat{Q})$ associated with $\hat{F}$. By definition:

$$\hat{Q} = 1 - \hat{F}(\mu) = 1 - F''(\mu) = 1 - \lambda F(\mu) - (1-\lambda)F'(\mu) = \lambda Q + (1-\lambda)Q'.$$

Similarly:

$$\hat{U} = \int_\mu^1 \theta dF''(\theta) = \lambda \int_\mu^1 \theta dF(\theta) + (1-\lambda)\int_\mu^1 \theta dF'(\theta) = \lambda U + (1-\lambda)U'.$$

Thus: $(\lambda U + (1-\lambda)U', \lambda Q + (1-\lambda)Q')$ is associated with $\hat{F}$. To conclude convexity, notice:

$$\begin{aligned} \lambda c(U,Q) + (1-\lambda)c(U',Q') = \lambda C(F) + (1-\lambda)C(F') \geq C(F'') \\ \leq C(\hat{F}) = c\left(\lambda U + (1-\lambda)U', \lambda Q + (1-\lambda)Q'\right). \end{aligned} \tag{4}$$

The first inequality holds by convexity of C, and the second inequality holds by monotonicity of C. Thus, c is convex. To prove strict convexity in the interior, notice that if (U,Q) and (U',Q') are in the interior of $\mathcal{A}_c$, then $F, F' \neq \delta_\mu$. Thus, $F'' \neq \hat{F}$. Because $\hat{F} \preceq_{\text{m.p.s.}} F''$, strict monotonicity of C with informativeness implies $C(\hat{F}) < C(F'')$, and the inequality above is strict.

■

The next result shows that, at $\kappa = 0$, there exists a unique equilibrium.

Lemma OA 1. *At $\kappa = 0$, there exists a unique equilibrium.*

Proof of Lemma OA 1

Equilibrium is defined by the equality $R_0(p) = p$, whenever this equality holds. It is clear that, $R_0(0) = \alpha\mu > 0$. We prove the function $g(p) = R(p) - p$ is decreasing, so it can cross zero at most once. For that, notice:

$$g(p) = \alpha \frac{1}{1 - F_o(p)} \int_p^1 \theta f_o(\theta) d\theta - p,$$

and consider $\hat{g}(p) = \frac{1}{1-F_o(p)} \int_p^1 \theta f_o(\theta) d\theta - p$. If $\hat{g}$ is decreasing, then so is g, since $\hat{g}$ gives more weight to the increasing term. Then, using the logconcavity of f_o, we can apply Theorem 6 in Bagnoli and Bergstrom (2005) to obtain $\hat{g}(p)$ is decreasing, and conclude the proof.

■

We conclude this section by proving that Proposition 1 in the main text generalizes for the case of mutual information costs with a uniform prior, and by providing sufficient conditions on costs and priors for demands to be rotation-ordered. *Coming soon!*

B Heterogeneous Consumers

In this section we extend the model to allow for consumer heterogeneity. Note that the baseline model has no aggregate uncertainty, because we can leverage the law of large numbers to obtain the demand curve. To keep this feature, we assume there are N groups of consumers, indexed by $i \in \{1, ..., N\}$, each of them with beliefs $\pi_i \in \Delta[0,1]$ about their valuation, ω. Additionally, each group faces an information cost $\kappa_i C_i$, satisfying the same conditions for the information cost in the homogeneous consumer case. We assume that the mass of group i in the population is τ_i, and define their initial expected valuation as $\mu_i = \mathbb{E}_{\pi_i}[\omega]$. Define $F_\infty = \sum_i \tau_i \delta_{\mu_i}$. Note that when consumers acquire no information, F_∞ is the distribution of valuations in the economy. Similar, we let $F_o = \sum_i \tau_i \pi_i$ be the distribution of valuations in the economy when consumers acquire all possible information. Following the baseline model, F_o has mean μ, and has a log-concave, continuously differentiable density f_o. The cost for the firm of serving a consumer with willingness-to-pay ω is, again, $\alpha\omega$, with $\alpha \in (0,1)$.

A group-i consumer solves the problem:

$$\max_{F_i \preceq_{\text{m.p.s.}} \pi_i} \mathbb{E}_{F_i}[\max\{\theta - p, 0\}] - \kappa_i C_i(F_i) \tag{5}$$

Let $F^p_{\kappa_i,i}$ solve the problem above, and define $\kappa = \begin{bmatrix} \kappa_1 \\ \vdots \\ \kappa_N \end{bmatrix}$. Then, demand is $D_\kappa(p) = \sum_i \tau_i \left(1 - F^p_{\kappa_i,i}(p-)\right)$, with associated inverse demand P_κ. similarly, firm's average costs at p are:

$$R_\kappa(p) = \sum_i \frac{\tau_i(1 - F^p_{\kappa_i,i}(p-))}{\sum_j \tau_j(1 - F^p_{\kappa_i,i}(p-))} E_{F^p_{\kappa_i,i}}[\theta | \theta \geq p].$$

And we let $AC_\kappa(Q) = R_\kappa(P_\kappa(Q))$. Our goal in this section is to show the following generalization of Theorem 1 and Theorem 2 from the main text.

Theorem OA 1. *Let vectors of information cost levels be $0 \leq \kappa \leq \kappa'$. Then:*

1. $D_\kappa(p) = 1 - F_\kappa(p-)$ *and* $F_\kappa \preceq_{m.p.s} F_{\kappa'}$
2. $AC_{\kappa'}(Q) \leq AC_\kappa(Q)$ *for all* $Q \in (0,1]$, *with equality at* $Q = 1$

We start with an auxiliary result. By individually applying Lemma 1 from the main text, we obtain a simplified consumer problem for each group, depending on the pair (U_i, Q_i) of expected utility and probability of purchase. Define the set $\mathcal{C}_i$ and the constraints MPC_i analogously to $\mathcal{C}$ and MPC. For simplicity, throughout this section we assume the induced cost function over (U_i, Q_i), c, is twice differentiable.

We call the aggregate expected utility U, and the aggregate level of demand Q. That is:

$$\sum_i \tau_i Q_i = Q \qquad \sum_i \tau_i U_i = U. \tag{6}$$

We say that aggregate quantities (U, Q) are consistent with consumer optimality if and only if there exists a set $\{(U_i, Q_i)\}_{i=1,\ldots,N}$, and a price p, with (U_i, Q_i) solving the group-i problem for every i given p, and such that (6) is satisfied.

Finally, define the aggregate cost function, which encompasses the total cost of information in the society:

$$\overline{c}(U, Q, \kappa_1, \ldots, \kappa_N) = \min_{\{(U_i, Q_i) \in \mathcal{C}_i\}_{i=1,\ldots,N}} \left\{ \sum_i \tau_i \kappa_i c_i(U_i, Q_i) : \text{MPC}_i \text{ for all } i, (6) \right\},$$

with the convention that $\overline{C} = \infty$ if (U, Q) cannot be achieved.

We start by showing the key result that allows for extending Theorem 2 to this environment.

Proposition OA 2. *(U, Q) is consistent with consumer optimality if and only if there exists p such that it solves:*

$$\max_{(U,Q)} \{U - pQ - \overline{c}(U, Q, \kappa_1, \ldots, \kappa_N)\} \tag{7}$$

Moreover, $\overline{c}$ is increasing in U, decreasing in Q and supermodular in (U, κ_i) for any $i = 1, ..., N$.

Proof of Proposition OA 2.

Consistency implies optimization. Assume (U, Q) is consistent and let $\{(U_i, Q_i)\}$ and p be as in the definition. We then have:

$$\overline{U} - p\overline{Q} - \sum_i \tau_i \kappa_i c_i(U_i, Q_i) = \sum_i \tau_i \left[U_i - pQ_i - k_i c_i(U_i, Q_i)\right]$$
$$= \sum_i \tau_i \left[\max_{(\tilde{U}_i, \tilde{Q}_i) \in \mathcal{C}_i \cap \mathrm{MPC}_i} \left\{\tilde{U}_i - p\tilde{Q}_i - \kappa_i c_i(\tilde{U}_i, \tilde{Q}_i)\right\}\right] \geq \tilde{U} - p\tilde{Q} - \sum_i \tau_i k_i c_i(\tilde{U}_i, \tilde{Q}_i),$$

for all $\tilde{U}, \tilde{Q}$ compatible with individual optimization. The first equality is by definition of consistency, and the second equality is from (U_i, Q_i) solving group-i's problem. The inequality holds because the sum of the maximum is larger than the maximum of the sum. This proves that consistency implies that (U, Q) solves problem 7.

Optimization implies consistency. Let (U, Q) solve problem 7 for price p. Then, let $\{(U_i, Q_i)\}$ be such that:

$$\sum_i \tau_i \kappa_i c_i(U_i, Q_i) = \overline{c}(U, Q).$$

For a contradiction, assume that group i strictly prefers $(U_i', Q_i') \in \mathcal{C}_i \cap \mathrm{MPC}_i$ to (U_i, Q_i). Then consider the pair (U', Q') such that:

$$\sum_{j \neq i} \tau_j U_j + \tau_i U_i' = U' \qquad\qquad \sum_{j \neq i} \tau_j Q_j + \tau_i Q_i' = Q'.$$

We have:

$$\begin{aligned} U - pQ - \overline{c}(U, Q) = \sum_j \tau_j \left\{U_j - pQ_j - \kappa_j c_j(U_j, Q_j)\right\} < \\ \sum_{j \neq i} \tau_j \left\{U_j - pQ_j - \kappa_j c_j(U_j, Q_j)\right\} + \tau_i \left\{U_i' - pQ_i' - \kappa_i c_i(U_i', Q_i')\right\} \leq U' - pQ' - \overline{c}(U', Q'), \end{aligned} \tag{8}$$

where the first inequality follows because group i strictly benefits from (U_i', Q_i'), and the last inequality follows because the minimal cost $\overline{c}$ can only be smaller than the cost obtained by that specific distribution of U_j and Q_j. The argument above leads to a contradiction with (U, Q) solving 7, because (U', Q') is obviously feasible. We conclude that the optimization implies consistency.

$\bar{c}$ is increasing in U and decreasing in Q. We prove the result for U. The proof for Q is analogous. Fix Q and let $U > U'$, both admissible. Assume $\{(U_i, Q_i)\}$ are such that:

$$\sum_i \tau_i \kappa_i c_i(U_i, Q_i) = \bar{c}(U, Q).$$

Then, consider the following procedure: choose $i = 1$ and set

$$U_1' = \max\left\{U_1 + \frac{U' - U}{\tau_1}, \mu_1 Q_1\right\}.$$

Compute $U^1 = \tau_1 U_1' + \sum_{i>1} \tau_i U_i$. If $U^1 > U'$, repeat the procedure with $i = 2$, defining

$$U_2' = \max\left\{U_2 + \frac{U' - U^1}{\tau 2}, \mu_2 Q_2\right\}.$$

Proceeding iteratively, it will eventually be the case that $U^i = U'$ — otherwise, $U' < \sum_i \tau_i \mu_i Q_i$, and was not admissible to begin with. Additionally, each U_i' is individually admissible: it satisfies $\mathcal{C}$ by the requirement that $U_i' \geq \mu_i Q_i$, and it satisfies MPC$_i$ because $U_i' < U_i$. Assume the procedure stops at $i = j$. We then have:

$$\bar{c}(U, Q) > \sum_{i \leq j} \tau_i \kappa_i c_i(U_i', Q_i) + \sum_{i > j} \kappa_i c_i(U_i, Q_i) \geq \bar{c}(U', Q).$$

where the inequality comes from each c_i being an increasing function of the first argument and $U_i' < U_i$.

$\bar{c}$ is supermodular in (U, κ_i). For this part of the proof, we use the differentiability assumption. We prove supermodularity by proving $\bar{c}_{U,\kappa_i} \geq 0$.

Because $\bar{c}$ is defined by minimization, we can write its Lagrangian as:

$$\sum_i \tau_i \left\{\kappa_i c_i(U_i, Q_i) - \psi(U_i - U) - \eta(Q_i - Q) - \phi_i\left(I^*_{\pi_i}(1 - Q_i) + U - i - \mu_i\right)\right\}$$

By the standard envelope theorem, we have that $\bar{c}_U = \psi$. Thus, to show supermodularity, all we have to show is that ψ increases with κ_i. That is what we do next.

For fixed vector $\kappa_1, ..., \kappa_N$, and fixed (U, Q), we reorder the group indexes such that MPC$_i$ does not bind for $i = 1, ..., n$, and it binds otherwise. If $n = N$, it does not bind for any group of consumers. Let $\{U_i, Q_i\}$ be such that:

$$\sum_i \tau_i \kappa_i c_i(U_i, Q_i) = \bar{c}(U, Q).$$

Minimizing the Lagrangian from $\bar{c}$, the following conditions must hold, for some ψ, η and all $j \leq n$:

$$\kappa_j c_{j,U} = \psi \tag{9}$$

$$\kappa_j c_{j,Q} = \eta, \tag{10}$$

and

$$\kappa_i c_{j,U} \pi_j^{-1}(1-Q_j) + \kappa_i c_{j,Q} = \psi \pi_j^{-1}(1-Q_j) + \eta, \tag{11}$$

for $n < j \le N$. Additionally, for $n < j \le N$, MPC$_j$ must bind: $I^*_{\pi_j}(1-Q_j) = \mu_j - U_j$. Finally, equation 6 must hold.

We want to obtain $\frac{d\psi}{d\kappa_i}$ — which we simply denote by $d\psi$ henceforth. By total differentiation of 9 and 10, we obtain, for each j:

$$\begin{bmatrix} dU_j \\ dQ_j \end{bmatrix} = \frac{H_j^{-1}}{\kappa_j} \cdot \begin{bmatrix} d\psi - c_{i,U}\mathbb{1}_{j=i} \\ d\eta - c_{i,Q}\mathbb{1}_{j=i} \end{bmatrix}, \tag{12}$$

where H_j is the hessian matrix of c_j at (U_j, Q_j). Proceeding in the same way with 11:

$$dQ_j = \frac{\pi_j^{-1}(1-Q_j)\left(d\psi - c_{i,U}\mathbb{1}_{j=i}\right) + \left(d\eta - c_{i,Q}\mathbb{1}_{j=i}\right)}{\left(v^\top H^j v - (\kappa_j c_{j,u} - \psi)\frac{1}{\pi_j'(1-Q_j)}\right)}, \tag{13}$$

where $v = \begin{bmatrix} \pi_i^{-1}(1-Q_j) \\ 1 \end{bmatrix}$. By second-order conditions, the term in the denominator is positive. Besides, by the fact that, for $n < j \le N$, MPC$_j$ holds, we have:

$$dU_j = \pi^{-1}(1-Q_j)dQ_j \tag{14}$$

Putting 9 and 10 together:

$$\begin{bmatrix} dU_j \\ dQ_j \end{bmatrix} = \frac{1}{\left(v^\top H^j v - (\kappa_j c_{j,u} - \psi)\frac{1}{\pi_j'(1-Q_j)}\right)} \begin{bmatrix} \pi_j^{-1\,2}(1-Q_j) & \pi_j^{-1}(1-Q_j) \\ \pi_j^{-1}(1-Q_j) & 1 \end{bmatrix} \cdot \begin{bmatrix} d\psi - c_{i,U}\mathbb{1}_{j=i} \\ d\eta - c_{i,Q}\mathbb{1}_{j=i} \end{bmatrix} \tag{15}$$

Notice that, by 6, $\sum_j dQ_j = \sum_j dU_j = 0$. Aggregating over 15 and 12:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = A \cdot \begin{bmatrix} d\psi - c_{i,U}\mathbb{1}_{j=i} \\ d\eta - c_{i,Q}\mathbb{1}_{j=i} \end{bmatrix}, \tag{16}$$

where A is the sum of positive definite matrices, from the conditions in 12, and positive semidefinite matrices from the conditions in 12, and is, therefore, positive definite. We can then conclude:

$$d\psi = c_{i,U} > 0.$$

We have then proved that $\bar{c}$ is supermodular. ■

Proof of Theorem OA 1

Demand Rotations. Let V_{i,κ_i} be the value function for group i's problem. Under the smoothness of c, V_{i,κ_i} is continuous on $[0,1]$ and differentiable. When $p=0$, it is clear that one obtains no information and purchases the good with probability one, achieving the highest possible utility, $V_{i,\kappa_i}(0)=\mu$. Similarly, at $p=1$ it is dominant not to acquire any information and purchase the good with probability 0, achieving $V_{i,\kappa_i}(1)=0$.

By the envelope theorem:

$$V_{i,\kappa_i}(p) = V_{i,\kappa_i(0)} - \int_0^p \left(1 - F^{\nu}_{i,\kappa_i}(\nu)\right) d\nu \tag{17}$$

By convexity of V_{i,κ_i}, it must be that $F^n u_{i,\kappa_i}(\nu)$ is a decreasing function of ν. By the argument of no information acquired for $p \in \{0,1\}$, $D_\kappa(0)=1$ and $D_\kappa(1)=0$. Smoothness of information costs implies $F^n u_{i,\kappa_i}(\nu)$ is continuous. Now, $D_\kappa(p)=\sum_i \tau_i(1-F^p_{i,\kappa_i}(p)$, so D_κ is a complementary CDF.

For the second assertion, note that, because at $p=0$ no information is acquired, $V_{i,\kappa_i}(0)=V_{i,\kappa_i'}(0)$. Similarly, $V_{i,\kappa_i}=V_{i,\kappa_i'}(1)=0$. Then, applying 17, we have for $p\in[0,1]$:

$$0 \leq \sum_i \tau_i V_{i,\kappa_i'}(p) - \sum_i \tau_i V_{i,\kappa_i}(p) = \int_0^p D_\kappa(\nu)d\nu - \int_0^p D_{\kappa'}(\nu)d\nu$$

where the inequality stems from the problems being identical except for a higher information cost for each group. Because D_κ is an inverse CDF, these inequalities imply that $1-D_{\kappa'}$ second-order stochastically dominates $1-D_\kappa$. The equality of value functions at $p\in\{0,1\}$ further implies the mean-preserving contraction relation.

Cost Rotations. Fix some $Q\in[0,1]$. Let (U,Q) and (U',Q') be the aggregate quantities for (κ,p) and (κ',p') respectively. Consider the vector $k^1=\{k_1',k_2,...,k_N\}$, and let (U^1,Q) be compatible with κ^1. By Proposition OA 2, there exists some p^1 such that (U^1,Q) solves 7. Because of the supermodularity of $\bar{c}$ in Proposition OA 2, that optimization is submodular in (k_1,U), and, by Topki's Lemma, $U^1 \leq U$.

Now, for $m>1$, define (U^m,Q) as the aggregate quantity compatible with $\kappa^m=\{\kappa_1',\kappa_2',...,\kappa_m',\kappa_{m+1},...,\kappa_N\}$.

Then, by the same argument, defining: $\kappa^{m+1} = \{\kappa'_1, \kappa'_2, ..., \kappa'_{m+1}, \kappa_{m+2}, ..., \kappa_N\}$, we obtain $U^{m+1} \leq U^m$. Because $U^N = U'$, we have $U' \leq U$.

Finally, notice that, for any vector κ,

$$AC_\kappa(Q) = \frac{\sum_i \tau_i U_i}{Q} = \frac{U}{Q},$$

to conclude the result. ■

C General Firms' Costs

In this section we extend the baseline model to accommodate more general firms' costs. That requires a restriction on the set of information cost functions. Formally, we allow firms' costs to be any increasing and continuous function $\alpha : [0,1] \to [0,1]$. Because consumers' utilities are still linear, they need only to keep track of their distribution of posterior means. However, the analysis now depends on the distribution of posterior means held by consumers, which we denote by $\tau \in \delta\delta[0,1]$, because firms' costs are no longer linear in consumers' posterior beliefs. Thus, we need to define information costs appropriately. Note that Proposition 2 in the main text is already proved to encompass this case. Thus, the goal of this section is to generalize Theorem 1 and Theorem 2.

To make the problem tractable, we start by discretizing the state space. Let $\Omega = \{\omega_1, ..., \omega_N\} \subset [0,1]$ be this discretized space, and we will denote $\pi \in \operatorname{supp} \tau$ as a probability vector over Ω with $\pi = \begin{bmatrix} \pi_1 \\ \vdots \\ p_N \end{bmatrix}$, $\sum_i \pi_i = 1$, and $\pi_i \geq 0$. Prior information is given by $F_o \in \Delta\Omega$ with $f_o = \begin{bmatrix} f_{o,1} \\ \vdots \\ f_{o,N} \end{bmatrix}$. Our key assumption about information costs is:

Assumption OA 1. *The information cost function $I : \Delta\Delta\Omega \to \mathbb{R}_+$ satisfies:*

1. ***Posterior-separability:*** *$I(\tau) = \mathbb{E}_\tau[H(\pi)]$, with H bounded, strictly convex, and $H(f_o) = 0$;*
2. ***State-separability:*** *$H(\pi) = \sum_i (h(\pi_i) - h(f_{o,i}))$;*
3. ***Smoothness:*** *h is twice continuously differentiable.*

The crucial addition over what is assumed in the literature is state-separability. Note that both quadratic costs and mutual information satisfy these assumptions. Consumers solve:

$$\max_{\tau\in\Delta\Delta[0,1]} \mathbb{E}_\tau\left[\max\{\mathbb{E}_\pi[\omega]-p,0\}-\kappa H(\pi)\right].$$

Let τ_κ^p solve the consumer's problem for price p. Then, demand satisfies $D_\kappa(p)=\tau(\{\pi:\mathbb{E}_\pi[\omega]\geq p\})$. Inverse demand P_κ is defined in the usual way. Finally, average costs can be written as:

$$R_\kappa(p)=\mathbb{E}_\tau\left[\mathbb{E}_\pi[\omega]\middle|\mathbb{E}_\pi[\omega]\geq\pi\right].$$

As usual, $AC_\kappa(Q)=R_\kappa(P_\kappa(Q))$.

Theorem OA 2. *Let Assumption OA 1 hold, and information levels be $0\leq\kappa\leq\kappa'$. Then:*

1. $D_\kappa(p)=1-F_\kappa(p-)$ *and* $F_\kappa\preceq_{m.p.s}F_{\kappa'}$
2. $AC_{\kappa'}(Q)\leq AC_\kappa(Q)$ *for all* $Q\in(0,1]$, *with equality at* $Q=1$

To prove this result, we depend on a key comparative statics result. We say τ is binary if $|\operatorname{supp}\tau|=2$. In that case, we say $(Q,\theta_H)\in\mathcal{A}(\tau)$ if $\operatorname{supp}\tau=\{\pi^L,\pi^H\}$, $\theta_i=\sum_i\omega_i\pi^i$, for $i\in\{L,H\}$ and $\theta_H>\theta_L$, and $Q=\tau_H$. Note that the consumer does not need anything other information about τ other than (Q,θ_H). Because of that, the key variable in this section is the cheapest distribution over posteriors τ that induces a given pair (Q,θ_H). Formally:

$$\tau(Q,\theta_H)\in\arg\min_\tau\{I(\tau):(Q,\theta_H)\in\mathcal{A}(\tau)\ \text{ and }\ \mathbb{E}_\tau[\pi]=f_o\} \tag{18}$$

Finally, for $f,g\in\Delta\Omega$, let $f\preceq_1 g$ denote that g first order stochastically dominates f.

Proposition OA 3. *Problem 18 has a unique and at-most binary solution. Let (Q,θ_H) and (Q,θ_H') be such that $\theta_H>\theta_H'$. Then, $\pi^H\preceq_1\pi_H'$.*

Proof of Proposition OA 3

Existence, Uniqueness and at-most Binary. Existence is trivial. Because H is strictly convex, I is strictly increasing in the mean-preserving spread order in τ. Thus, following a similar argument to the proof of Lemma 1 in the main appendix proves the optimal τ is unique and at-most binary.

Rewriting the problem. Let's start by rewriting the optimization problem in 18 as:

$$
\begin{aligned}
\min_{\pi^H} \quad & QH(\pi^H) + (1-Q)H\left(\frac{f_o - Q\pi^H}{1-Q}\right): \\
\text{s.t.} \quad & \sum_i \pi_i^H = 1 && \text{(1a)} \\
& \sum_i \omega_i \pi_i^H = \theta_H && \text{(1b)} \\
& 0 \le \pi_i^H \le \frac{f_{o,i}}{Q} \text{ for all } i = 1, \ldots, N. && \text{(1c)}
\end{aligned}
$$

To construct this objective, we have simply used the fact that τ can be substituted by Q or $1-Q$, solved for π^L in the Bayesian consistency constraint, rewritten as: $f_o = Q\pi^H + (1-Q)\pi^L$. (1a) constraints π^H to integrate to one, (1b) makes sure the average of π^H is θ_H, and (1c) translates the requirement that both π^H and π^L are non-negative.

Showing First-Order Stochastic Dominance. Let γ be a multiplier associated with (1a), λ associated with (1b), ν_i, β_i associated to the non-positivity constraint and the upper bound constraint, respectively. Then, first-order conditions imply:

$$
Q\left(h'(\pi_i^H) - h'\left(\frac{f_{o,i} - Q\pi_i^H}{1-Q}\right)\right) = \gamma + \lambda\omega_i + \nu_i - \beta_i
$$

Because H is convex, h is convex and, thus h' is increasing. As a consequence, the left hand side above is increasing in π_i^H. Thus, we can define $\ell(\cdot)$, increasing, to obtain:

$$
\pi_i^H = \ell(\gamma + \lambda\omega_i + \nu_i - \beta_i)
$$

We can then write:

$$
\pi_i^H = \begin{cases} 0 & \text{if } \ell(\gamma + \lambda\omega_i) < 0, \\ \ell(\gamma + \lambda\omega_i) & \text{if } 0 \le \ell(\gamma + \lambda\omega_i) \le \frac{f_{o,i}}{1-Q}, \\ \frac{f_{o,i}}{1-Q} & \text{otherwise.} \end{cases} \tag{19}
$$

We finish by proving this implies first order stochastic dominance. Let (γ', λ') be associated with the solution to (Q, θ_H'), and (γ, λ) to the solution to (Q, θ_H), with $\theta_H' > \theta_H$. We let π^H and π'^H be defined by the optimization, $\ell_i = \ell(\gamma + \lambda\omega_i)$ and $\ell_i' = \ell(\gamma' + \lambda'\omega_i)$. Because ℓ is increasing:

$$\ell_i > \ell'_i \iff \gamma + \lambda\omega_i > \gamma' + \lambda'\omega_i.$$

The remainder of the proof consists of ruling out cases for (γ, λ) and (γ', λ').

We start ruling out $(\gamma', \lambda') > (\gamma, \lambda)$ or $(\gamma, \lambda) > (\gamma', \lambda')$. Assume $(\gamma', \lambda') > (\gamma, \lambda)$. But that implies $\ell_i < \ell'_i$ for all i and thus $\pi_i^H \leq \pi_i'^H$ for all i. If they are equal, we have a contradiction with $\theta_H < \theta'_H$. Otherwise, we must have:

$$1 = \sum_i \pi_i^H < \sum_i \pi_i'^H = 1,$$

which is a contradiction. We can rule out $(\gamma, \lambda) > (\gamma', \lambda')$ with the symmetric argument.

We now rule out $\gamma' > \gamma$, $\lambda' < \lambda$. Assume that was the case. Because ℓ is increasing, there exists exactly 1 $\hat{\omega}$ such that $\ell_i < \ell'_i$ for all $\omega < \hat{\omega}$. Notice this implies:

$$\pi_i^H < \pi_i'^H \implies \omega < \hat{\omega}.$$

Because π^H and π'^H are probabilities, the equation above implies $\pi'^H \preceq_1 \pi^H$. But that contradicts $\theta'_H > \theta_H$.

Thus, it must be the case that $\gamma' \leq \gamma$, $\lambda' \geq \lambda$, with at least one of the inequalities strict. But by the argument above, this implies $\pi^H \preceq_1 \pi'^H$, as we wanted to prove. ■.

Proof of Theorem OA 2

Demand Rotations. For the first part, define, for any $F \in \Delta[0,1]$:

$$C(F) = \min_{\tau} \{I(\tau) : \mathbb{E}_\tau[\pi] = f_o, F(\theta) = \tau(\{\pi : \mathbb{E}_\pi[\omega] \leq \theta\})\}$$

C is clearly well-defined, and the minimum is achieved. By Berge's Maximum theorem, $C(F)$ is lower-semicontinuous. Because I is strictly increasing in informativeness, C is naturally strictly increasing in informativeness. Additionally, let $F = \delta_\mu$. Then, if τ is in the constraint set, $\tau = \delta_{f_o}$. Thus, $C(\delta_\mu) = I(\delta_{f_o}) = H(f_o) = 0$.

Finally, let $F, F' \in \Delta[0,1]$ and $\lambda \in (0,1)$, and τ and τ' attain the cost minimization for F and F' respectively. Then:

$$\begin{aligned}\lambda C(F)+(1-\lambda)C(F') = \lambda I(\tau)+(1-\lambda)I(\tau') &= I\left(\lambda\tau+(1-\lambda)\tau'\right)\\ &\geq C\left(\lambda F+(1-\lambda)F'\right),\end{aligned} \tag{20}$$

where the second equality uses linearity of I and the inequality follows from the fact that the constraint set in the minimization above is convex. Thus, C is a convex function.

We have then concluded that C satisfies all the conditions in the main text. It is straightforward to see that the consumers' problem is equivalent to:

$$\max_{F\in\mathcal{A}} \mathbb{E}_F[\max\{\theta-p,0\}]-\kappa C(F).$$

Thus, the proof of Theorem 1 carries over to this problem.

Cost Rotations. Following the proof of Theorem 2, we obtain that if (U,Q) solves the consumer's problem for some (p,κ) and (U',Q) solves the consumer's problem for (p',κ'), with $\kappa'>\kappa$, then $U'\leq U$. Let τ and τ' be the respective distributions over posteriors that solve the cost minimization problem. We have $\operatorname{supp}\tau=\{\pi^L,\pi^H\}$, and $\operatorname{supp}\tau'=\{\pi'^L,\pi'^H\}$.

By definition:

$$\sum_i \omega_i\pi^H=\frac{U}{Q}\geq\frac{U'}{Q}=\sum_i\omega_i\pi'^H.$$

Therefore, by Proposition OA 3, $\pi'^H\preceq_1\pi^H$. To conclude, note that:

$$AC_\kappa(Q)=\mathbb{E}_{\pi^H}[\alpha(\omega)]\leq\mathbb{E}_{\pi'^H}[\alpha(\omega)]=AC_{\kappa'}(Q), \tag{21}$$

where the inequality follows from $\pi'^H\preceq_1\pi^H$ and α strictly increasing. We have thus proved the cost rotation result. ■

D Monopoly

Section Coming Soon!

E The Highest Equilibrium

In this appendix we show the highest equilibrium of the trading game — that is, the equilibrium with the highest price — is the only one that is not trivial. Formally, we prove it is the sole equilibrium that can be attained by standard undercutting arguments. We thus propose a refinement that allows us to formalize this notion. Intuitively, we discretize the decision set of firms, show that there the equilibrium is unique, and then show the limit of such equilibria is the highest equilibrium.

Let our trading game, Γ, be defined by a prior, F_o; information costs; C, an action set for strategy sets of consumers: $A_c = \mathcal{A} \times \Delta[0,1]^{[0,1]}$, where $(F,a) \in A_c$ implies F is an information structure, and a is a function of signal realizations in $[0,1]$ to a distribution over purchasing (1) or not purchasing (0); finally, the action set of firms is $A_f = [0,1]$, any price between 0 and 1.[1] That is, $\Gamma = \left\{F_o, C, A_c, A_f\right\}$. Recall that $\mathcal{E}$ is the set of SPNE price-quantity pairs of Γ, where (p^e, Q^e) is the equilibrium pair with the highest prices. Let $\mathcal{E}_p$ denote the set of equilibrium prices.

For each integer $N > 0$, we consider the sequence of games $\Gamma^N = \left\{F_o, C, A_c, A_f^N\right\}$, where $A_f^N = \bar{\mathcal{E}^p} \cap \{\frac{m}{N}\}_{m\in[N]}$, with $[N] = \{0,1,...,N\}$. In words, we are considering a grid of price choices that excludes equilibrium prices. Let $\mathcal{E}^N$ be the set of Subgame Perfect Nash Equilibria of this game.

Proposition OA 4. *Fix a base game Γ. For N large enough, $\mathcal{E}^N$ is a singleton. Moreover, let (p^N, Q^N) be the largest equilibrium prices and quantities of this game. Then $(p^N, Q^N) \to (p^e, Q^e)$.*

Proof of Proposition OA 4.

Note that R_κ is still defined in the same way as in the original text, so the only change is in the first stage, where firms choose prices.

First, notice that any equilibrium in which at least one firm makes positive profits must be symmetric. Otherwise, the firm with the highest prices have incentives to deviate to the price of the other firm and share their customer base, making positive profits.

Second, some firm must make profits in equilibrium. To see that, assume the firms make zero profits at price p. That implies $p^e < p$. Indeed, if $p \leq p^e$ and generates zero profits, it is an equilibrium price, and it is thus excluded from the grid. However, if $p^e < p$, and N is large enough, there exists $p' \in A_f^N \cap (p^e, p)$ such that one firm can deviate to p', attract all consumers, and make positive profits. Thus, it is impossible firms make zero profits in equilibrium. This and the previous paragraph proves that, for large enough N, equilibrium is symmetric and both firms make positive profits.

Then, consider a price posted by both firms (p, Q), with $p \in A_f^N$. Assume $p - \frac{1}{N}$ is such that $R_\kappa\left(p - \frac{1}{N}\right) < p - \frac{1}{N}$. One of the firms must be making less than half of the profits, regardless of how the customer base is shared. Then, for high enough N, and by continuity of R_κ and D_κ:

[1] Evidently, allowing firms to charge other prices does not change anything.

$$\frac{1}{2}D_\kappa(p)(p - R_\kappa(p)) < D_\kappa\left(p - \frac{1}{N}\right)\left(p - \frac{1}{N} - R_\kappa\left(p - \frac{1}{N}\right)\right)$$

So that if p is an equilibrium price, then $p - \frac{1}{N}$ does not generate profits. Define $S = \{p \in A_f^N : p - \frac{1}{N} \in A_f^N, R_\kappa\left(p - \frac{1}{N}\right) \geq p - \frac{1}{N}\}$.

Let $\lfloor p^e \rfloor^N = \min\{p \in A_f^N : p \geq p^e\}$. We conclude by proving this is the unique equilibrium price for high enough N. First, notice $\lfloor p^e \rfloor^N \in S$, because either $\lfloor p^e \rfloor^N - \frac{1}{N} < p^e$, and thus cannot have positive profits, or $\lfloor p^e \rfloor^N - \frac{1}{N} < p^e \neq A_f^N$. Similarly, for $p < p^e$, profits are strictly negative, so they cannot be equilibrium. Now, we just need to prove any $p > \lfloor p^e \rfloor^N$ with $p \in S$ can be ruled out. If $p^e = \max S$, we are done. So assume that is not the case. Let $p^N = \min\{p \in S : p > \lfloor p^e \rfloor\}$, and define $\pi^N = \max_{p \in [\lfloor p^e \rfloor^N, p^N) \cap A_f^N} D_\kappa(p)(p - R_\kappa(p))$. Notice that π^N is bounded away from zero, as it converges to the highest profits between p^e and the next point of zero profits.

Now, consider $p \in S$, with $p > \lfloor p^e \rfloor^N$. Because $p - \frac{1}{N} < 0$, and profits are continuous, when N is sufficiently large, the profits obtained at p are close to zero, whereas π_N is bounded away from zero. Moreover, it is clear that $p^N < p$. Thus, deviating for p^N will attract all consumers and increase profits. Thus, for large enough N, p cannot be an equilibrium. We then conclude $\lfloor p^e \rfloor^N$ is the only equilibrium candidate.

To prove $\lfloor p^e \rfloor^N$ is an equilibrium, notice that any deviation to a lower p in the grid generates negative profits, and to a higher p loses all customer. This proves that, for large enough N, $\lfloor p^e \rfloor^N$ is the unique equilibrium. It is trivial to see that $\lfloor p^e \rfloor^N \to p^e$, concluding the proof. ■

References

Bagnoli, M., and Bergstrom, T. (2005). Log-concave probability and its applications. *Economic Theory*, *26*(2), 445–469.