Extended Capacity Warehouse Location

Analytical Decision Support Systems

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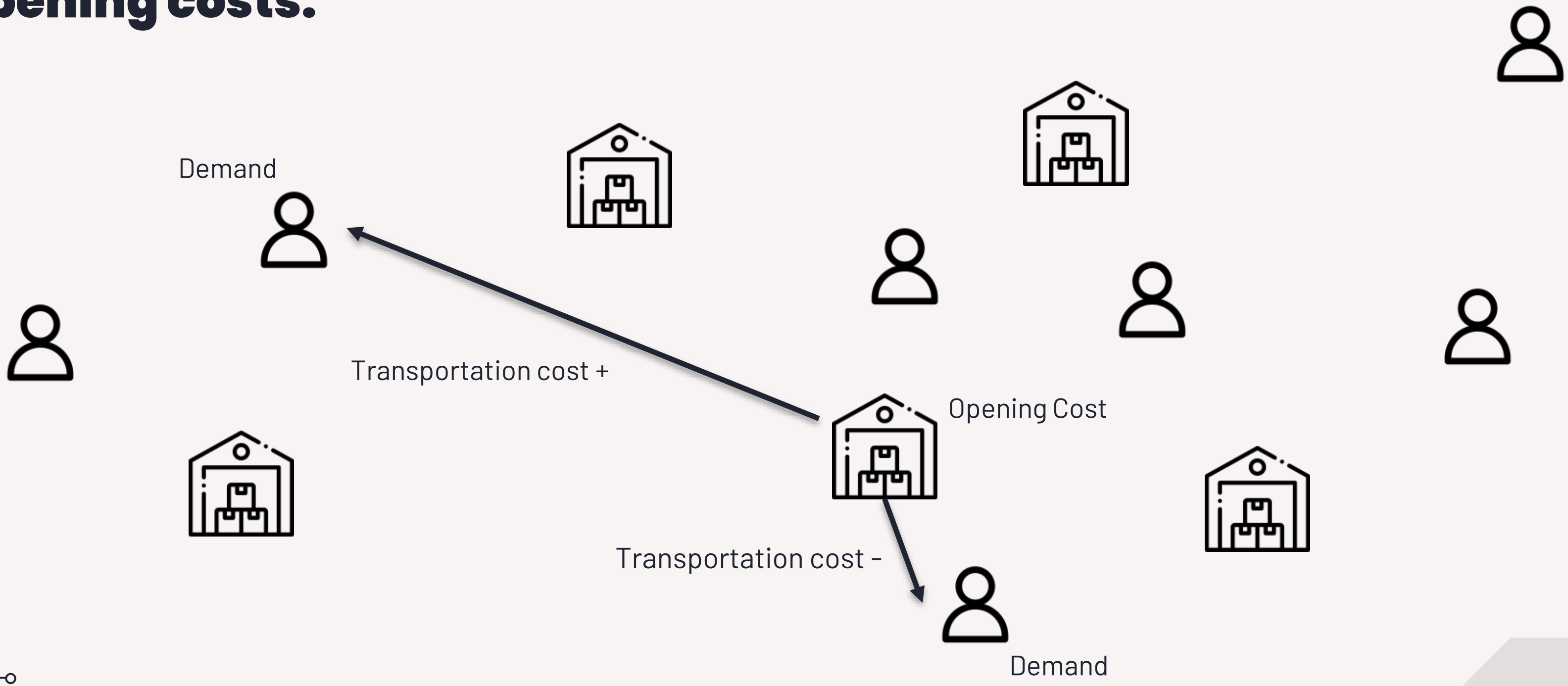
1. Problem

The problem consisted in determining which warehouses supplied each client to meet demand, taking into account transportation costs and opening costs.



The objective function was to minimize the total cost

Decision Variables:

- Binary Variables:
  - $x_i \in \{0,1\}$: Indicates whether the warehouse i is open ($x_i = 1$) or closed ($x_i = 0$).
  - $y_{i,j} \in \{0,1\}$: Indicates whether customer j is served by warehouse i.
- Integer Variables:
  - $\text{amountServed}_{i,j} \geq 0$: Amount of goods transported from warehouse i to customer j.

Objective Function:

$$\text{Minimize} \quad \sum_{i=1}^{nW} \text{fixedCost}_i \cdot x_i + \sum_{i=1}^{nW} \sum_{j=1}^{nC} \text{transportCost}_{ij} \cdot \text{amountServed}_{ij}$$

Constraints:

1. All Customers must be assigned to warehouses
2. All Customers demand must be satisfied
3. Warehouse capacity can not be surpassed
4. The amount served to a customer from all warehouses must not exceed demand
5. x, y must be 0,1 and amountServed larger or equal to zero

The decision variables and objective function of MIP and CP were the same, but the constrainst had some changes

Decision Variables:

- **Binary Variables:**
  - $x_i \in \{0,1\}$: Indicates whether the warehouse i is open (1) or closed (0).
  - $y_{ij} \in \{0,1\}$: Indicates whether the client j is served by the warehouse i (1) or not (0).
- **Integer Variables:**
  - $\text{amountServed}_{ij} \in [0, 15000]$: Quantity of units supplied from warehouse i to client j.

Objective Function:

$$\text{Minimize} \quad \sum_{i=1}^{nW} \text{fixedCost}_i \cdot x_i + \sum_{i=1}^{nW} \sum_{j=1}^{nC} \text{transportCost}_{ij} \cdot \text{amountServed}_{ij}$$

Constraints:

1. All Customers must be assigned to warehouses
2. All Customers demand must be satisfied
3. Warehouse capacity can not be surpassed
4. The amount served to a customer from all warehouses must not exceed demand
5. A Customer can only be assigned to an open warehouse
6. If a Customer is assigned to a warehouse, the warehouse must supply at least 1 unit

3. Extended Restrictions

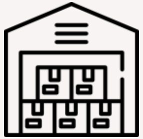
Three Restrictions were added in order to add more complexity to the problem

Constraints:

1. A warehouse if open must serve at least **80% of its capacity**

2. Certain pairs of competitive customers **cannot be served by the same warehouse**

3. Some warehouses if open **force other warehouse to open**



Objectives with Constraints:

- To **stress the MIP solvers** and observe the impact of the adding complexity
- To create conditions for a possible **better performance of the CP solvers** and observe the impact of the adding complexity

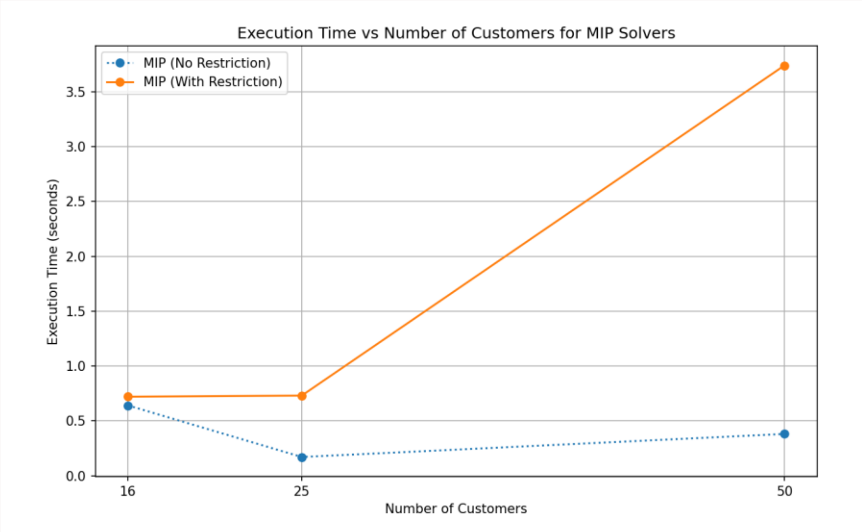
To test the performance of the solvers of IBM (CPLEX), we utilized the file cap44 with a time limit of 10 minutes

Metric	LP (IBM Solver)	CP (IBM Solver)
Execution Time (seconds)	0.64	600.17
Optimal Result	1235500,45	1586247,50
Gap	0%	98.35%

- The MIP solver **successfully** computed the **optimal solution in 0.64 seconds** while the CP was **not able to achieve a solution**.
- Although adding the restrictions **worsen the performance of both solvers**, MIP was **significantly more affected** when in comparison with the base model, particularly for **higher numbers of instances**.

Results revealed that higher number of instances led to an overall negative impact on the solvers performance

- The files used differed in the **number of clients:**
  Cap44 – 16
  Cap92 - 25
  Cap123/124 - 50



MIP

Higher **Execution Times**

CP

Higher **Optimality Gaps**

Slower **Branch Speed**

Altering search strategys had mixed results in the performance of the model , but the Auto (default setting) had the best overall performance

Search Strategy	DepthFirst	Restart	Multipoint	IterativeDiving	Neighborhood	Auto (default)
Time (seconds)	600.17s	600.18s	600.17s	600.21s	600.40s	600.37s
Best Solution Found	Not Found	1615092.60	1849364.34	1592999.19	Not Found	1452414.71
Gap	-	98.38%	100%	98.36%	-	98.20%
Nº Branches	6323802	38101122	16739693	19203337	176087	21947969
Nº Fails	3159049	8142085	6325995	4497714	-	5577895
Memory Usage	66.4 MB	234.3 MB	340.4 MB	261.4 MB	175.5 MB	283.1 MB
Search Sp. (br. /s)	10539.2	63486.4	27898.4	31996.0	293.5	36559.5

Failed to find Optimal Solution

High Search Speed

High Memory Usage and worst Solution

2nd best solution

Failed to find solution and low Search Speed

Best Overall

Log verbosity refers to how much information the program displays while it's running

Verbosity Level	Quiet	Terse	Normal (default)	Verbose
Time (seconds)	-	600.36s	600.37s	600.42s
Lower Bound	-	26142.21	26142.21	26142.21
Best Solution Found	1485744.28	1451732.90	1452414.71	1452506.40
Gap	-	98.20%	98.20%	98.20%
Nº Branches	-	23138180	21947969	21493557
Nº Fails	-	5910487	5577895	5444298
Memory Usage (MB)	-	283.7	283.1	282.4
Search Speed (br. /s)	-	38542.9	36559.5	35799.9

Higher Verbosity Setting:

- **Slower Search Speed**
- **Worst Solution Foun**

Comparison of OR-Tools and DOcplex

Base Results

- MIP-based solvers and OR-tools CP achieve **optimal solutions**
- Docplex MIP implementation shows **superior speed**
- Cplex CP struggles with execution time and solution quality

Method	**cap44**	**cap92**	**cap123**	**cap124**
Optimal Reference Value	1 235 500,45	855 733,50	895 302,33	946 051,33
LP - OR-Tools	**Optimal**	**Optimal**	**Optimal**	**Optimal**
Execution Time (s)	0,400	0,410	1,430	3,030
CP - OR-Tools	**Optimal**	**Optimal**	**Optimal**	**Optimal**
Execution Time (s)	0,359	0,325	3,452	4,593
LP - Cplex	**Optimal**	**Optimal**	**Optimal**	**Optimal**
Execution Time (s)	0,640	0,170	0,380	0,380
CP - Cplex	*Suboptimal*	*Suboptimal*	*Suboptimal*	*Suboptimal*
Difference (%)	28,4%	7,1%	28,0%	17,7%
Execution Time (s)	600,130	600,080	600,190	600,150

Comparison of OR-Tools and DOcplex

Results with added Restrictions

Method	**cap44**	**cap92**	**cap123**	**cap124**
Optimal Achieved Value	1 327 373,35	1 080 811,69	1 095 811,69	1 118 311,69
LP - OR-Tools	**Optimal**	**Optimal**	**Optimal**	**Optimal**
Execution Time (s)	1,440	22,700	429,280	291,230
CP - OR-Tools	**Optimal**	**Optimal**	**Optimal**	**Optimal**
Execution Time (s)	3,639	14,49	112,01	92,87
LP - Cplex	**Optimal**	**Optimal**	**Optimal**	**Optimal**
Execution Time (s)	0,720	0,730	3,740	3,440
CP - Cplex	*Suboptimal*	*Suboptimal*	*Suboptimal*	*Suboptimal*
Difference (%)	9,42%	27,18%	77,15%	38,56%
Execution Time (s)	600,340	600,130	600,360	600,170

- **CP OR-Tools outperforms MIP OR-Tools** in several cases, benefiting from its hybrid approach.
- DOcplex MIP maintains **optimal** solutions with **fastest computation times**
- CP CPLEX consistently **underperforms** across all test scenarios

Comparison of OR-Tools and DOcplex

Key Solver Performance Findings

CPLEX MIP: Superior Performance

- Why: Advanced simplex methods optimized for large-scale linear problems

OR-Tools CP: Surprising Strength

Outperformed OR-Tools MIP with the new restrictions

- **Why:** Combinatorial Handling, Efficient Algorithms, Reduced Overhead

CPLEX CP: Underperformer- Poor performance across all test cases

- Why: Less mature CP implementation compared to its MIP capabilities

6. Conclusions and Future Work

Conclusion

1. **CPLEX MIP model exceled** compared to the rest while **CPLEX CP struggled**

2. OR-Tools CP performed **unexpectedly well**. On the other hand, OR-Tools MIP performed **unexpectedly bad**

Future Work

1. Explore the model **OR-Tools** in more detail.

2. Test with **more instances**

3. Try different and more diverse **constraitns**

THANKS!

Do you have any questions?

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MIP Mathematical Constraints

1. **Assignment of Customers to Warehouses:** Each customer must be served by at least one warehouse:

$$\sum_{i=1}^{nW} y_{ij} \geq 1 \quad \forall j = 1, \ldots, nC$$

2. **Demand Satisfaction:** The total amount served to each customer must meet their demand:

$$\sum_{i=1}^{nW} \text{amountServed}_{ij} = \text{demand}_j \quad \forall j = 1, \ldots, nC$$

3. **Capacity Constraints:** The total amount served by a warehouse cannot exceed its capacity if it is open:

$$\sum_{j=1}^{nC} \text{amountServed}_{ij} \leq \text{capacity}_i \cdot x_i \quad \forall i = 1, \ldots, nW$$

4. **Linking Transportation and Assignment:** The amount served to a customer from all the warehouses cannot exceed the customer demand:

$$\text{amountServed}_{i,j} \leq \text{demand}_j \cdot y_{i,j} \quad \forall i \in \text{Warehouses}, \forall j \in \text{Customers}$$

$$y_{i,j} \leq x_i \quad \forall i \in \text{Warehouses}, \forall j \in \text{Customers}$$

5. **Non-Negativity and Binary Constraints:**

$$x_i, y_{i,j} \in \{0, 1\}, \quad \text{amountServed}_{i,j} \geq 0$$

CP Mathematical Constraints

1. **Assignment of Customers to Warehouses:** Each customer must be served by at least one warehouse.

$$\sum_{i=1}^{nW} y_{ij} \geq 1 \quad \forall j = 1, \ldots, nC$$

2. **Demand Satisfaction:** The total quantity served to each client must equal their demand.

$$\sum_{i=1}^{nW} \text{amountServed}_{ij} = \text{demand}_j \quad \forall j = 1, \ldots, nC$$

3. **Capacity Constraints:** The total quantity served by each warehouse cannot exceed its capacity.

$$\sum_{j=1}^{nC} \text{amountServed}_{ij} \leq \text{capacity}_i \cdot x_i \quad \forall i = 1, \ldots, nW$$

4. **Consistency Between Quantity and Assignment:** The quantity supplied from warehouse i to client j must be zero if client j is not assigned to warehouse i.

$$\text{amountServed}_{ij} \leq \text{demand}_j \cdot y_{ij} \quad \forall i = 1, \ldots, nW,\ j = 1, \ldots, nC$$

5. **Assignment to Open Warehouses:** A client can only be assigned to an open warehouse.

$$y_{ij} \leq x_i \quad \forall i = 1, \ldots, nW,\ j = 1, \ldots, nC$$

6. **Binding Variables:** If a client is assigned to a warehouse, the warehouse must supply at least one unit.

$$\text{amountServed}_{ij} \geq y_{ij} \quad \forall i = 1, \ldots, nW,\ j = 1, \ldots, nC$$

CPLEX performance results of MIP and CP models for different number of instances

Table 3. Comparison of LP Solvers with and without Restrictions Across Datasets

Dataset	Metric	LP (No Restriction)	LP (With Restriction)
cap44	Execution Time (seconds)	0.64	0.72
	Solution Found	1235500.45	1327496.93
cap92	Execution Time (seconds)	0.17	0.73
	Solution Found	855733.5	1080811.69
cap123	Execution Time (seconds)	0.38	3.74
	Solution Found	895302.32	1095811.69
cap124	Execution Time (seconds)	0.38	3.44
	Solution Found	946051.33	1118345.66

Table 4. Comparison of CP Solvers with and without Restrictions Across Datasets

Dataset	Metric	CP (No Restriction)	CP (With Restriction)
cap44	Execution Time (seconds)	600.17	600.37
	Solution Found	1586247.5	1452414.71
	Optimality Gap (%)	98.35%	98.20%
	Search Sp. (br. /s)	30962.3	36559.5
cap92	Execution Time (seconds)	600.14	600.19
	Solution Found	916489.13	1374624.01
	Optimality Gap (%)	98.39%	98.87%
	Search Sp. (br. /s)	23945.0	20403.8
cap123	Execution Time (seconds)	600.28	600.46
	Solution Found	1145804.16	1941252.85
	Optimality Gap (%)	98.47%	99.10%
	Search Sp. (br. /s)	14505.6	13330.3
cap124	Execution Time (seconds)	600.26	600.25
	Solution Found	1113363.93	1549509.59
	Optimality Gap (%)	97.75%	98.39%
	Search Sp. (br. /s)	12256.9	11319.7