



# Extended Capacity Warehouse Location

Analytical Decision Support Systems

Francisco Pinto  
João Soares  
João Vieira

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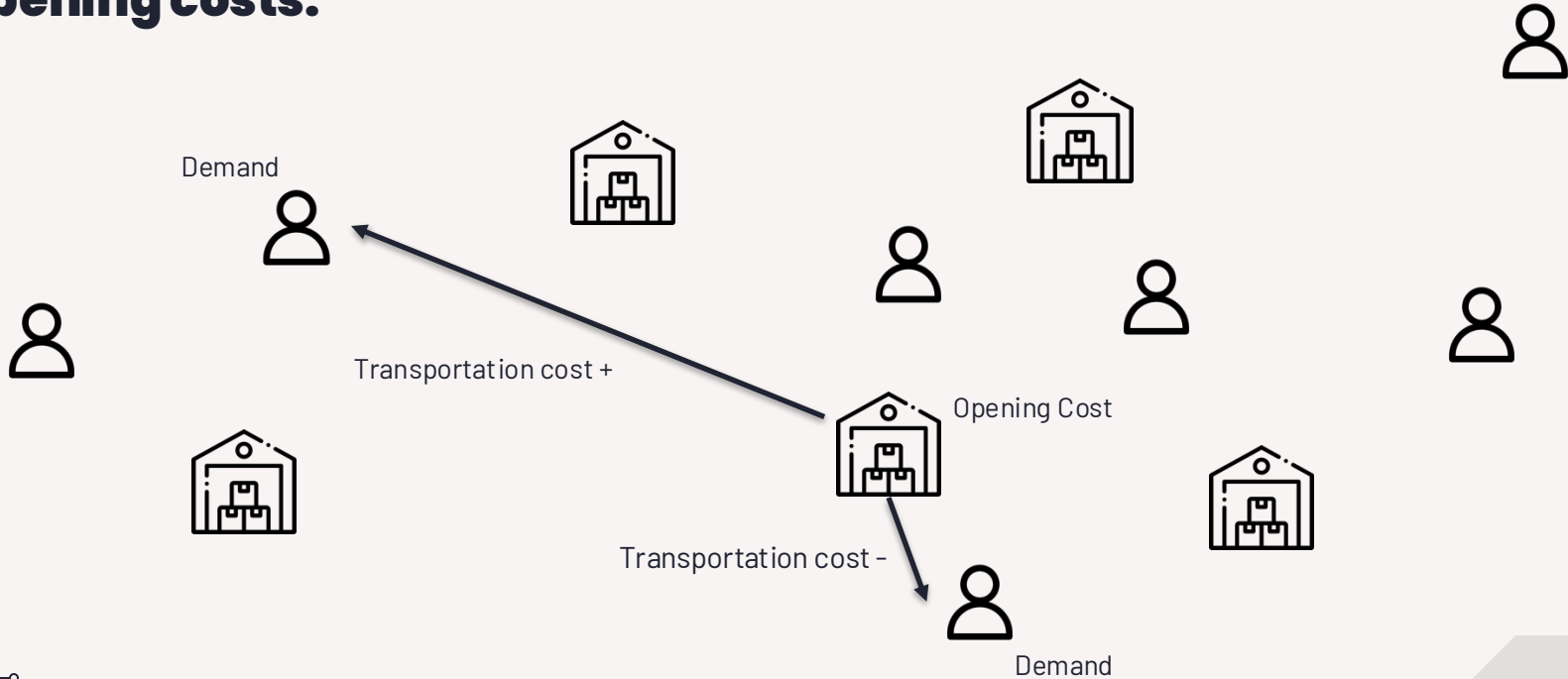
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# 1. Problem

**The problem consisted in determining which warehouses supplied each client to meet demand, taking into account transportation costs and opening costs.**



# The objective function was to minimize the total cost

## Decision Variables:

### – Binary Variables:

- $x_i \in \{0, 1\}$ : Indicates whether the warehouse  $i$  is open ( $x_i = 1$ ) or closed ( $x_i = 0$ ).
- $y_{i,j} \in \{0, 1\}$ : Indicates whether customer  $j$  is served by warehouse  $i$ .

### – Integer Variables:

- $\text{amountServed}_{i,j} \geq 0$ : Amount of goods transported from warehouse  $i$  to customer  $j$ .

## Objective Function:

$$\text{Minimize } \sum_{i=1}^{nW} \text{fixedCost}_i \cdot x_i + \sum_{i=1}^{nW} \sum_{j=1}^{nC} \text{transportCost}_{ij} \cdot \text{amountServed}_{ij}$$

## Constraints:

1. All Customers must be assigned to warehouses
2. All Customers demand must be satisfied
3. Warehouse capacity can not be surpassed
4. The amount served to a customer from all warehouses must not exceed demand
5.  $x, y$  must be 0,1 and amountServed larger or equal to zero



# The decision variables and objective function of MIP and CP were the same, but the constraint had some changes



## Decision Variables:

- **Binary Variables:**
  - $x_i \in \{0, 1\}$ : Indicates whether the warehouse  $i$  is open (1) or closed (0).
  - $y_{ij} \in \{0, 1\}$ : Indicates whether the client  $j$  is served by the warehouse  $i$  (1) or not (0).
- **Integer Variables:**
  - $\text{amountServed}_{ij} \in [0, 15000]$ : Quantity of units supplied from warehouse  $i$  to client  $j$ .

## Objective Function:

$$\text{Minimize} \quad \sum_{i=1}^{nW} \text{fixedCost}_i \cdot x_i + \sum_{i=1}^{nW} \sum_{j=1}^{nC} \text{transportCost}_{ij} \cdot \text{amountServed}_{ij}$$

## Constraints:

1. All Customers must be assigned to warehouses
2. All Customers demand must be satisfied
3. Warehouse capacity can not be surpassed
4. The amount served to a customer from all warehouses must not exceed demand
5. A Customer can only be assigned to an open warehouse
6. If a Customer is assigned to a warehouse, the warehouse must supply at least 1 unit

### 3. Extended Restrictions

## Three Restrictions were added in order to add more complexity to the problem

#### Constraints:

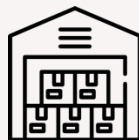
1. A warehouse if open must serve at least **80% of its capacity**



2. Certain pairs of competitive customers **cannot be served by the same warehouse**



3. Some warehouses if open **force other warehouse to open**



#### Objectives with Constraints:

- To **stress the MIP solvers** and observe the impact of the adding complexity
- To create conditions for a possible **better performance of the CP solvers** and observe the impact of the adding complexity

## 4. Analysis of Results

**To test the performance of the solvers of IBM (CPLEX), we utilized the file cap44 with a time limit of 10 minutes**

Metric	MIP (IBM Solver)	CP (IBM Solver)
Execution Time (seconds)	0.64	600.17
Optimal Result	1235500,45	1586247,50
Gap	0%	98.35%

- The MIP solver **successfully** computed the **optimal solution in 0.64 seconds** while the CP was **not able to achieve a solution**.
- Although adding the restrictions **worsen the performance of both solvers**, MIP was **significantly more affected** when in comparison with the base model, particularly for **higher numbers of instances**.

## 4. Analysis of Results

**Results revealed that higher number of instances led to an overall negative impact on the solvers performance**

- The files used differed in the **number of clients:**

Cap44 - 16

Cap92 - 25

Cap123/124 - 50

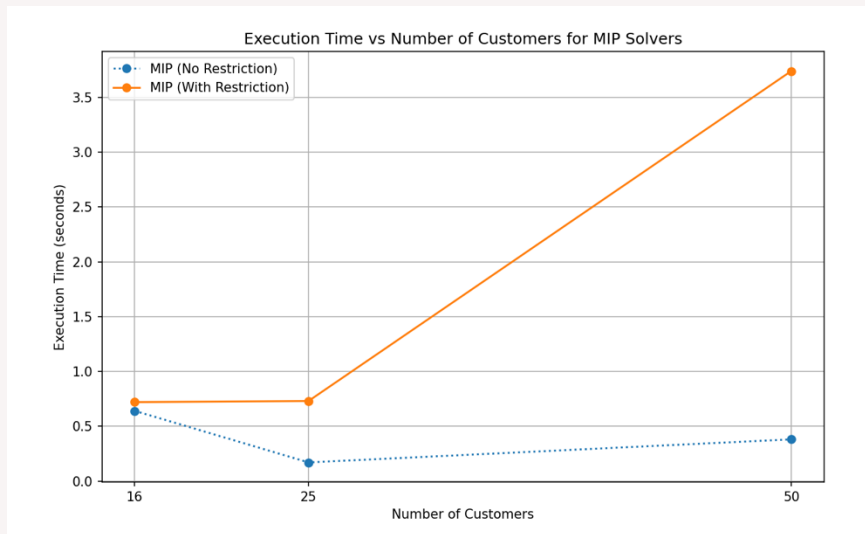
**MIP**

Higher  
**Execution  
Times**

**CP**

Higher  
**Optimality Gaps**

Slower **Branch  
Speed**





## 4. Analysis of Results

**After 10 minutes, the improvements to performance became marginal**

Time Limit	2 minutes	5 minutes	10 minutes	30 minutes	60 minutes
Time (seconds)	120.14s	300.15s	600.37s	1800.09s	3600.06s
Best Solution Found	1820190.39	1542333.29	1452414.71	1430627.43	1429537.83
Gap	98.56%	98.31%	98.20%	98.17%	98.17%
N <sup>o</sup> Branches	2187509	10217278	21947969	56112605	95797503
N <sup>o</sup> Fails	529694	2398765	5577895	16290684	31055462
Memory Usage	168 MB	226.6 MB	283.1 mb	380.9 MB	474.9 MB
Search Sp. (br. /s)	18213.6	34045.0	36559.5	31172.7	26610.2

## 4. Analysis of Results

**Altering search strategys had mixed results in the performance of the model , but the Auto (default setting) had the best overall performance**

Search Strategy	DepthFirst	Restart	Multipoint	IterativeDiving	Neighborhood	Auto (default)
Time (seconds)	600.17s	600.18s	600.17s	600.21s	600.40s	600.37s
Best Solution Found	Not Found	1615092.60	1849364.34	1592999.19	Not Found	1452414.71
Gap	-	98.38%	100%	98.36%	-	98.20%
Nº Branches	6323802	38101122	16739693	19203337	176087	21947969
Nº Fails	3159049	8142085	6325995	4497714	-	5577895
Memory Usage	66.4 MB	234.3 MB	340.4 MB	261.4 MB	175.5 MB	283.1 MB
Search Sp. (br. /s)	10539.2	63486.4	27898.4	31996.0	293.5	36559.5

Failed to find  
Optimal  
Solution

High Memory  
Usage and  
worst Solution

Failed to find  
solution and  
low Search  
Speed

Best Overall

High Search  
Speed

2nd best  
solution

## 4. Analysis of Results

**Log verbosity refers to how much information the program displays while it's running**

Verbosity Level	Quiet	Terse	Normal (default)	Verbose
Time (seconds)	-	600.36s	600.37s	600.42s
Lower Bound	-	26142.21	26142.21	26142.21
Best Solution Found	1485744.28	1451732.90	1452414.71	1452506.40
Gap	-	98.20%	98.20%	98.20%
Nº Branches	-	23138180	21947969	21493557
Nº Fails	-	5910487	5577895	5444298
Memory Usage (MB)	-	283.7	283.1	282.4
Search Speed (br. /s)	-	38542.9	36559.5	35799.9

Higher Verbosity Setting:

- **Slower Search Speed**
- **Worst Solution Found**

## Comparison of OR-Tools and Dcplex

### Base Results

MIP OR-Tools solver: CBC

CP OR-Tools solver: CP-SAT solver

- MIP-based solvers and OR-tools CP achieve **optimal solutions**
- Docplex MIP implementation shows **superior speed**
- Cplex CP struggles with execution time and solution quality

Method	cap44	cap92	cap123	cap124
<b>Optimal Reference Value</b>	1 235 500,45	855 733,50	895 302,33	946 051,33
<b>MIP - OR-Tools</b>	<b>Optimal</b>	<b>Optimal</b>	<b>Optimal</b>	<b>Optimal</b>
Execution Time (s)	0,400	0,410	1,430	3,030
<b>CP - OR-Tools</b>	<b>Optimal</b>	<b>Optimal</b>	<b>Optimal</b>	<b>Optimal</b>
Execution Time (s)	0,359	0,325	3,452	4,593
<b>MIP - cplex</b>	<b>Optimal</b>	<b>Optimal</b>	<b>Optimal</b>	<b>Optimal</b>
Execution Time (s)	0,640	0,170	0,380	0,380
<b>CP - cplex</b>	<i>Suboptimal</i>	<i>Suboptimal</i>	<i>Suboptimal</i>	<i>Suboptimal</i>
Difference (%)	28,4%	7,1%	28,0%	17,7%
Execution Time (s)	600,130	600,080	600,190	600,150

## Comparison of OR-Tools and D0cplex

### Results with added Restrictions

Method	cap44	cap92	cap123	cap124
<b>Optimal Achieved Value</b>	1 327 373,35	1 080 811,69	1 095 811,69	1 118 311,69
<b>MIP - OR-Tools</b>	<b>Optimal</b>	<b>Optimal</b>	<b>Optimal</b>	<b>Optimal</b>
Execution Time (s)	1,440	22,700	429,280	291,230
<b>CP - OR-Tools</b>	<b>Optimal</b>	<b>Optimal</b>	<b>Optimal</b>	<b>Optimal</b>
Execution Time (s)	3,639	14,49	112,01	92,87
<b>MIP - cplex</b>	<b>Optimal</b>	<b>Optimal</b>	<b>Optimal</b>	<b>Optimal</b>
Execution Time (s)	0,720	0,730	3,740	3,440
<b>CP - cplex</b>	<i>Suboptimal</i>	<i>Suboptimal</i>	<i>Suboptimal</i>	<i>Suboptimal</i>
Difference (%)	9,42%	27,18%	77,15%	38,56%
Execution Time (s)	600,340	600,130	600,360	600,170

- **CP OR-Tools outperforms MIP OR-Tools** in several cases, benefiting from its hybrid approach.
- D0cplex MIP maintains **optimal** solutions with **fastest computation times**
- CP CPLEX consistently **underperforms** across all test scenarios

# Comparison of OR-Tools and DOcplex

## Key Solver Performance Findings

### CPLEX MIP: Superior Performance

- Why: Advanced simplex methods optimized for large-scale linear problems

### OR-Tools CP: Surprising Strength

Outperformed OR-Tools MIP with the new restrictions

- Why: Combinatorial Handling, Efficient Algorithms, Reduced Overhead

### CPLEX CP: Underperformer- Poor performance across all test cases

- Why: Less mature CP implementation compared to its MIP capabilities

### OR-Tools MIP Solver: Unexpectedly Poor Performance

- Why: The MIP solver (CBC), exhibited subpar performance, especially for larger datasets.



## 6. Conclusions and Future Work

### Conclusion

1. **CPLEX MIP model exceled** compared to the rest while **CPLEX CP struggled**
2. OR-Tools CP performed **unexpectedly well**. On the other hand, OR-Tools MIP performed **unexpectedly bad**

### Future Work

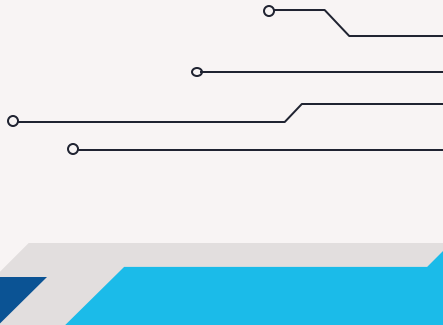
1. Explore the model **OR-Tools** in more detail.
2. Test with **more instances**
3. Try different and more diverse **constraitns**



# THANKS!

Do you have any questions?

Francisco Pinto  
João Soares  
João Vieira





# MIP Mathematical Constraints

1. **Assignment of Customers to Warehouses:** Each customer must be served by at least one warehouse:

$$\sum_{i=1}^{nW} y_{ij} \geq 1 \quad \forall j = 1, \dots, nC$$

2. **Demand Satisfaction:** The total amount served to each customer must meet their demand:

$$\sum_{i=1}^{nW} \text{amountServed}_{ij} = \text{demand}_j \quad \forall j = 1, \dots, nC$$

3. **Capacity Constraints:** The total amount served by a warehouse cannot exceed its capacity if it is open:

$$\sum_{j=1}^{nC} \text{amountServed}_{ij} \leq \text{capacity}_i \cdot x_i \quad \forall i = 1, \dots, nW$$

4. **Linking Transportation and Assignment:** The amount served to a customer from all the warehouses cannot exceed the customer demand:

$$\text{amountServed}_{i,j} \leq \text{demand}_j \cdot y_{i,j} \quad \forall i \in \text{Warehouses}, \forall j \in \text{Customers}$$

$$y_{i,j} \leq x_i \quad \forall i \in \text{Warehouses}, \forall j \in \text{Customers}$$

5. **Non-Negativity and Binary Constraints:**

$$x_i, y_{i,j} \in \{0, 1\}, \quad \text{amountServed}_{i,j} \geq 0$$

# CP Mathematical Constraints

1. **Assignment of Customers to Warehouses:** Each customer must be served by at least one warehouse.

$$\sum_{i=1}^{nW} y_{ij} \geq 1 \quad \forall j = 1, \dots, nC$$

2. **Demand Satisfaction:** The total quantity served to each client must equal their demand.

$$\sum_{i=1}^{nW} \text{amountServed}_{ij} = \text{demand}_j \quad \forall j = 1, \dots, nC$$

3. **Capacity Constraints:** The total quantity served by each warehouse cannot exceed its capacity.

$$\sum_{j=1}^{nC} \text{amountServed}_{ij} \leq \text{capacity}_i \cdot x_i \quad \forall i = 1, \dots, nW$$

4. **Consistency Between Quantity and Assignment:** The quantity supplied from warehouse  $i$  to client  $j$  must be zero if client  $j$  is not assigned to warehouse  $i$ .

$$\text{amountServed}_{ij} \leq \text{demand}_j \cdot y_{ij} \quad \forall i = 1, \dots, nW, j = 1, \dots, nC$$

5. **Assignment to Open Warehouses:** A client can only be assigned to an open warehouse.

$$y_{ij} \leq x_i \quad \forall i = 1, \dots, nW, j = 1, \dots, nC$$

6. **Binding Variables:** If a client is assigned to a warehouse, the warehouse must supply at least one unit.

$$\text{amountServed}_{ij} \geq y_{ij} \quad \forall i = 1, \dots, nW, j = 1, \dots, nC$$

## CPLEX performance results of MIP and CP models for different number of instances

Dataset	Metric	MIP (No Restriction)	MIP (With Restriction)
cap44	Execution Time (seconds)	0.64	0.72
	Solution Found	1235500.45	1327496.93
cap92	Execution Time (seconds)	0.17	0.73
	Solution Found	855733.5	1080811.69
cap123	Execution Time (seconds)	0.38	3.74
	Solution Found	895302.32	1095811.69
cap124	Execution Time (seconds)	0.38	3.44
	Solution Found	946051.33	1118345.66

**Table 4.** Comparison of CP Solvers with and without Restrictions Across Datasets

Dataset	Metric	CP (No Restriction)	CP (With Restriction)
cap44	Execution Time (seconds)	600.17	600.37
	Solution Found	1586247.5	1452414.71
	Optimality Gap (%)	98.35%	98.20%
	Search Sp. (br. /s)	30962.3	36559.5
cap92	Execution Time (seconds)	600.14	600.19
	Solution Found	916489.13	1374624.01
	Optimality Gap (%)	98.39%	98.87%
	Search Sp. (br. /s)	23945.0	20403.8
cap123	Execution Time (seconds)	600.28	600.46
	Solution Found	1145804.16	1941252.85
	Optimality Gap (%)	98.47%	99.10%
	Search Sp. (br. /s)	14505.6	13330.3
cap124	Execution Time (seconds)	600.26	600.25
	Solution Found	1113363.93	1549509.59
	Optimality Gap (%)	97.75%	98.39%
	Search Sp. (br. /s)	12256.9	11319.7