



Computação em Larga Escala

General Problems – Algorithmic analysis 3

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Summary

- *Sorting a sequence of integers*
 - *Rationale*
 - *Alternatives*
 - *Algorithm*

DETI

Sorting a sequence of integers - 1

The approach to be followed stems from the fact that it is less complex to sort a somewhat presorted sequence than a totally random one.

Thus, suppose the sequence of N values `val`, with $N = 2^k$ and $k \geq 1$, is such that both its halves, from 0 to $N/2-1$ and from $N/2$ to $N-1$, respectively, are already sorted. In order to sort the whole sequence one may use merge sorting.

Two alternatives are possible for sorting in ascending order

- both halves are sorted in ascending order (*standard merge sorting*)
- first half is sorted in ascending order and second half in descending order (*bitonic sorting*).

Another interesting feature of this approach is that the sorting of both halves is independent from one another and can be done in parallel.

Sorting a sequence of integers - 2

Standard merge sorting

Example using an 8-valued sequence

1	5	6	9	2	4	7	8	---	initial situation
1	4	6	8	2	5	7	9	---	iteration 1 (4 CAPS): first 2 values positioned
1	2	5	7	4	6	8	9	---	iteration 2 (3 CAPS): another 2 values positioned
1	2	4	6	5	7	8	9	---	iteration 3 (2 CAPS): another 2 values positioned
1	2	4	5	6	7	8	9	---	iteration 4 (1 CAPS): last 2 values positioned

```
for (m = 0; m < N/2; m++)  
  for (n = 0; (m + n) < N/2; n++)  
    CAPS (val[m+n], val[N/2+n]);
```

CAPS stands for *compare and possible swap the value*

The merge operation costs $N(N+2)/8$ CAPS, when N is even.

Sorting a sequence of integers - 3

Bitonic sorting

Example using an 8-valued sequence

4	6	8	9	7	5	2	1	---	initial situation
4	5	2	1	7	6	8	9	---	iteration 1 (4 CAPS)
2	1	4	5	7	6	8	9	---	iteration 2 (4 CAPS)
1	2	4	5	6	7	8	9	---	iteration 3 (4 CAPS)

Sorting a sequence of integers - 4

```
v = N >> 1;
nL = 1;
for (m = 0; m < k; m++)
{ n = 0;
  u = 0;
  while (n < nL)
  { for (t = 0; t < v; t++)
    CAPS (val[t+u], val[t+u+v]);
    u += (v << 1);
    n += 1;
  }
  v >>= 1;
  nL <<= 1;
}
```

CAPS stands for *compare and possible swap the value*

The merge operation costs $kN/2$ CAPS, when N is even.