A Robust Fuzzy Control of A Nonlinear Magnetic Ball Suspension System

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Abstract

A methodology for the position control of a nonlinear magnetic ball suspension system based on fuzzy if-then rules is presented in this paper. With the experience of stabilizing the magnetic ball at a desired position using a PD controller, a robust complexity-reduced fuzzy controller is designed. Simulations with different initial position conditions of the ball suspension system are provided. Simulation results illustrate that the fuzzy controller is more robust than a classical PD controller in the sense of being able to stabilize a steel ball with a larger range of initial position conditions. Also, the performances of the fuzzy controller is shown to be better than a PD controller in this paper.

1 Introduction

In the past several years, a fuzzy control system has been proved to have the advantages of being robust to the variations in the system dynamics [3], and successfully utilized in the ill-defined processes [2]. Recently, applying fuzzy logic [4] to complex systems, for example, nonlinear systems [1] and unstable systems is a popular research topic. In this paper, the position control of a magnetic ball suspention system (MBSS), which is naturally unstable and nonlinear, is studied. With the experience of stabilizing the magnetic ball at a desired position using a PD controller , a robust controller is constructed based on the fuzzy if-then rules. The fuzzy logic controller developed here is complexity reduced and simply consists of seven fuzzy if-then rules. The triangular type functions are adopted for membership functions of most of the fuzzy sets, and the centroid technique is used for defuzzification. Comparisons with a classical PD controller are presented to indicate that the fuzzy controller is more robust than a PD controller in the sense of being able to stablize the steel ball with a larger range of initial position conditions. Also, the performances of the fuzzy controller is shown to be better than a PD controller in this paper.

The remainder of this paper is organized as follows. Section 2 presents the discussions of the characteristics of the nonlinear magnetic ball suspension system. The nonlinear magnetic ball suspension system with PD controller is studied in section 3. The structure of the fuzzy control system is provided in section 4. Simulation results are shown in section 5. And conclusions are presented in section 6.

2 Characteristics of the MBSS

The problem considered here is to stabilize the steel ball of the magnetic system to the desired position with different intial conditions by controlling the magnetic levitation. The magnetic ball suspension system shown in Figure 1 is an open loop system, and the dynamic equations of the magnetic ball suspension system are as follow:

Magnetic force equation:

$$f = \frac{ci^2}{y}.$$

Motion equation:

$$M\frac{d^2y}{dt^2} = Mg - f.$$

Electric circuit equation:

$$L\frac{di}{dt} = -Ri + u.$$

where

i=winding current	c=positive constant factor
y=ball position	M=ball mass
g=gravitational acceleration	R=resistance
L=winding inductance	u=voltage source

For simplicity and without loss of generality, the ball position y and winding current i are assumed to be strictly positive.

The state equations of MBSS are derived as

$$\dot{x_1} = x_2 \quad (1)$$

$$\dot{x_2} = g - \frac{cx_3^2}{mx_1} \quad (2)$$

$$\dot{x_3} = -\frac{R}{L}x_3 + \frac{u}{L} \quad (3)$$

with $x_1 = y$, $x_2 = \dot{y}$, and $x_3 = i$.

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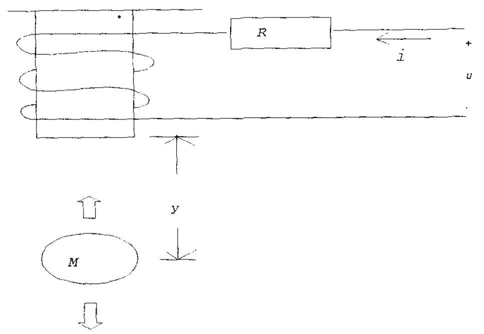

Figure 1: A Magnetic Ball Suspension System

In order to discuss of the characteristics of the MBSS, the linearization technique about the equilibrium point ($x_1 = x_{1e}, x_2 = x_{2e}$, and $x_3 = x_{3e}$) is applied to the nonlinear system MBSS. Then the vector form of the state equation for the linearized MBSS system is

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU}$$

and

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{cx_{3e}^2}{mx_{1e}^2} & 0 & \frac{-2cx_{3e}}{mx_{1e}} \\ 0 & 0 & \frac{-R}{L} \end{bmatrix}$$

Therefore, the characteristic equation of the linearized MBSS

$$\begin{aligned} |s\mathbf{I} - \mathbf{A}| &= \left|\begin{bmatrix} s & -1 & 0 \\ \frac{-cx_{3e}^2}{mx_{1e}^2} & s & \frac{2cx_{3e}}{mx_{1e}} \\ 0 & 0 & s+\frac{R}{L} \end{bmatrix}\right| \\ &= s^2(s+\frac{R}{L}) - (\frac{cx_{3e}^2}{mx_{1e}^2})(s+\frac{R}{L}) \\ &= (s+\frac{R}{L})(s^2 - \frac{cx_{3e}^2}{mx_{1e}^2}) \\ &= 0 \end{aligned} \tag{4}$$

Since

$$\frac{cx_{3e}^2}{mx_{1e}^2} > 0$$

is always true, one of the roots of s is on the right-half side of s-plane. And the linearized MBSS is unsatble. Thus the equilibrium point is unstable for the nonlinear MBSS by the theorem of the Lyapunov's linearization method.

3 the MBSS with PD Controller

From the discussion in section 2, the open loop MBSS is an unstable system. In this section, the stability of the MBSS with a PD controller is studied. The close loop MBSS with a controller is shown in Figure 2. The output of the PD controller, which is the voltage source u, is defined as

$$u = k_p e + k_d \dot{e}$$

where e is the error of the system ($e = r - y$) and both of the k_p, k_d are constants. Then the state equations of the close loop MBSS with a step input r are

$$\dot{x_1} = x_2 \tag{5}$$

$$\dot{x_2} = g - \frac{cx_3^2}{mx_1} \tag{6}$$

$$\dot{x_3} = -\frac{R}{L}x_3 + \frac{k_p e + k_d \dot{e}}{L} \tag{7}$$

$$= -\frac{R}{L}x_3 + \frac{k_p r}{L} - \frac{k_p x_1}{L} - \frac{k_d x_2}{L} \tag{8}$$

with $x_1 = y$, $x_2 = \dot{y}$, and $x_3 = i$.

As the procedure in section 2, the MBSS with the PD controller is linearized about the equilibrium point and the characteristic equation is

$$\begin{aligned} |s\mathbf{I} - \mathbf{A}_{\mathbf{pd}}| &= \left|\begin{bmatrix} s & -1 & 0 \\ \frac{-cx_{3e}^2}{mx_{1e}^2} & s & \frac{2cx_{3e}}{mx_{1e}} \\ \frac{k_p}{L} & \frac{k_d}{L} & s+\frac{R}{L} \end{bmatrix}\right| \\ &= s^3 + \frac{R}{L}s^2 - (\frac{cx_{3e}^2}{mx_{1e}^2} + \frac{2cx_{3e}}{Lmx_{1e}}k_d)s - \frac{2ck_p x_{3e}}{Lmx_{1e}} \\ &\quad - \frac{cx_{3e}^2 R}{mx_{1e}^2 L} \\ &= 0 \end{aligned} \tag{9}$$

In order to make the closed loop MBSS with PD controller stable, the roots of the characteristic equation must lie on the left half of the s-plane. To test this, we can adopt the Routh-Hurwitz criterion[6].

$$\begin{array}{lcc} s^3 & 1 & -(\frac{cx_{3e}^2}{mx_{1e}^2} + \frac{2cx_{3e}}{Lmx_{1e}}k_d) \\ s^2 & \frac{R}{L} & -\frac{2ck_p x_{3e}}{Lmx_{1e}} - \frac{cx_{3e}^2 R}{mx_{1e}^2 L} \\ s^1 & -(\frac{cx_{3e}^2}{mx_{1e}^2} + \frac{2cx_{3e}}{Lmx_{1e}}k_d) + (\frac{2ck_p x_{3e}}{Rmx_{1e}} + \frac{cx_{3e}^2}{mx_{1e}^2}) & 0 \\ s^0 & -\frac{2ck_p x_{3e}}{Lmx_{1e}} - \frac{cx_{3e}^2 R}{mx_{1e}^2 L} & \end{array}$$

For the system to be stable, the elements in the first column must have the same sign. And the following constraints must be satisfied.

$$\begin{cases} k_d < \frac{L}{R}k_p \\ k_p < \frac{-x_{3e}R}{2x_{1e}} \end{cases}$$

It is easy to design a PD controller with parameters k_p and k_d selected to satisfy two requirements above. Therefore, the linearized closed loop MBSS with the PD controller can be stabilized. And from the theorem of the Lyapunov's linearization method, the

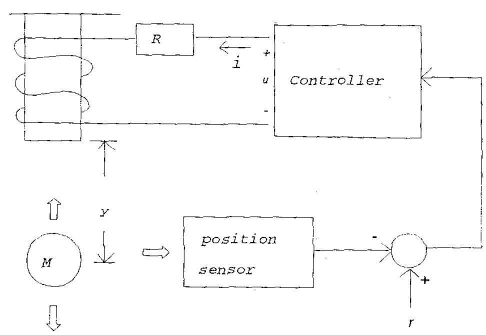

Figure 2: A Close Loop Magnetic Ball Suspension System

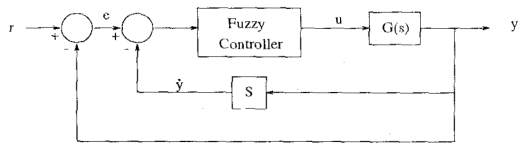

Figure 3: Block Diagram of The Fuzzy Control System

equilibrium point is asymptotically stable for the actural nonlinear closed loop MBSS with the PD controller. However, the linearized model for a nonlinear system can only be applied to the close neighborhood of the equilibrium point. The stability of the nonlinear system with an initial condition far away from the equilibrium point is not guarantied by the Lyapunov's linearization method. Since a larger stable region is desired for the MBSS, to apply fuzzy techniques to the design of a controller for MBSS in next section is a reasonable approach.

4 Design of A Robust Fuzzy Controller

As discussed in section 2, the MBSS is naturally nonlinear and unstable. It is known from section 3 that the PD controller can stabilize the MBSS in the neighborhood of the equilibrium point. Moreover, that the complexity of the fuzzy controller is decreased by reducing the number of fuzzy if then rules is a common sense. And the number of fuzzy if then rules decreases as the number of the input variables decreased. Based on these information, a robust complexity-reduced fuzzy controller with a structure of combination of region-wise PD-like controllers is designed.

The block diagram of the fuzzy control system for the magnetic ball suspension system (MBSS) is shown in Figure 3, where the r, y are the reference input position and the output position, respectively. In order to have the complexity of the fuzzy control system reduced, the input x to the fuzzy controller is taken to be the weighted difference of e and $\dot{y}$, that is,

$$x = e - k_d \dot{y}. \tag{10}$$

And then the fuzzy controller is simply based on only seven fuzzy if then rules.

The universes of discourse of the input x and output u are fuzzily partitioned into seven fuzzy sets which are "negative big (nb)", "negative medium (nm)""negative small (ns)", "zero (ze)", "positive small (ps)", "positive medium (pm)", and "positive big (pb)". The membership functions of these fuzzy sets for the input x and output u are defined by triangular shape functions in Figure 4 and Figure 5. And the seven fuzzy if then rules are designed as in Table 1. One example of the fuzzy if then rules is given as,

R1: "If x is zero, then u is zero."

With the partition method in Figure 4 and Figure 5, the triangular type membership functions, and the centroid defuzzification techbique, the output u of the fuzzy controller with the input x is defined as [5]

$$u = \frac{\sum_{l=1}^{n} p_l m_l(x)}{\sum_{l=1}^{n} m_l(x)}.$$

where $p_l's$ are the points at which the membership value is maximum for the corresponding fuzzy sets of the output variable, n denote the number of fuzzy if-then rules, and $m_l's$ represent the membership values.

x						
nb	nm	ns	ze	ps	pm	pb
pb	pm	ps	ze	ns	nm	nb

Table 1: The fuzzy control rules.

Also note that the denominator of the output u is always equal to one. Therefore, u is simplified to be

$$u = \sum_{l=1}^{n} p_l m_l(x). \tag{11}$$

As shown in Figure 4, the universe of discourse of the input variable x can be generally grouped into two different type of regions (I and II). When the input variable is region (II), only one of the fuzzy sets is involved, and the membership value $m_{i+1}(x)$ is equal to a constant value 1. Consequently, when the input variable x is in region (II), the output of the fuzzy controller, u, is equal to p_{i+1}, which is also a constant and represents a step input with the amplitude p_{i+1}. Note that the output $y(t)$ of the plant does not affect the input of the plant in the region (II), that is, the system behaves like a open loop system in this region.

For each input variable in region (I), there are two of the fuzzy sets involved, and the membership values $m_i(x)$ and $m_{i+1}(x)$ are calculated as

$$m_i(x) = \frac{b_i + a_i - x}{a_i} \quad \text{and} \quad m_{i+1}(x) = \frac{-b_i + x}{a_i}$$

Substitute the membership values $m_i(x)$ and $m_{i+1}(x)$ into Eq. 11 , the output of the fuzzy controller

$$\begin{aligned} u &= p_i(b_i + a_i - x)/a_i + p_{i+1}(-b_i + x)/a_i \\ &= [p_i + b_i(p_i - p_{i+1})/a_i] + (p_{i+1} - p_i)x/a_i \end{aligned} \tag{12}$$

After substituting Eq. 10 into Eq. 12, we have

$$u = \gamma_i + \beta_i(e - k_d \dot{y}) \tag{13}$$

where γ_i and β_i are both constants in the region (I), and

$$\gamma_i = p_i + b_i(p_i - p_{i+1})/a_i \quad and \quad \beta_i = (p_{i+1} - p_i)/a_i$$

Thus, the fuzzy controller can be defined as a combination of two open loop controllers and six region-wise PD-like controllers.

For region i of type (I), the state equations of the closed loop MBSS with a PD-like controller are

$$\dot{x}_1 = x_2 \tag{14}$$

$$\dot{x}_2 = g - \frac{c x_3^2}{m x_1} \tag{15}$$

$$\begin{aligned} \dot{x}_3 &= -\frac{R}{L}x_3 + \frac{k_p e + k_d \dot{e}}{L} &(16)\\ &= -\frac{R}{L}x_3 + \frac{\beta_i r + \gamma_i}{L} - \frac{\beta_i x_1}{L} - \frac{\beta_i k_d x_2}{L} &(17) \end{aligned}$$

with $x_1 = y$, $x_2 = \dot{y}$, and $x_3 = i$.

The MBSS with the PD-like controller is linearized about the equilibrium point with the same procedures in section 2 and the characteristic equation is

$$
\begin{aligned}
\left|s\mathbf{I}-\mathbf{A_{pd}}\right| &= \left|\begin{bmatrix} s & -1 & 0 \\ \frac{-cx_{3e}^2}{mx_{1e}^2} & s & \frac{2cx_{3e}}{mx_{1e}} \\ \frac{\beta_i}{L} & \frac{\beta_i k_d}{L} & s+\frac{R}{L} \end{bmatrix}\right| \\
&= s^3+\frac{R}{L}s^2-\left(\frac{cx_{3e}^2}{mx_{1e}^2}+\frac{2cx_{3e}}{Lmx_{1e}}\beta_i k_d\right)s-\frac{2c\beta_i x_{3e}}{Lmx_{1e}} \\
&\quad -\frac{cx_{3e}^2 R}{mx_{1e}^2 L} \\
&= 0 \qquad (18)
\end{aligned}
$$

For the closed loop MBSS with a PD-like controller in region i to be stable, the roots of the characteristic equation must lie on the left half of the s-plane.

$$
\begin{array}{lcc}
s^3 & 1 & -\left(\frac{cx_{3e}^2}{mx_{1e}^2}+\frac{2cx_{3e}}{Lmx_{1e}}\beta_i k_d\right) \\
s^2 & \frac{R}{L} & -\frac{2c\beta_i x_{3e}}{Lmx_{1e}}-\frac{cx_{3e}^2 R}{mx_{1e}^2 L} \\
s^1 & -\left(\frac{cx_{3e}^2}{mx_{1e}^2}+\frac{2cx_{3e}}{Lmx_{1e}}\beta_i k_d\right)+\left(\frac{2c\beta_i x_{3e}}{Rmx_{1e}}+\frac{cx_{3e}^2}{mx_{1e}^2}\right) & 0 \\
s^0 & -\frac{2c\beta_i x_{3e}}{Lmx_{1e}}-\frac{cx_{3e}^2 R}{mx_{1e}^2 L} &
\end{array}
$$

By the Routh-Hurwitz criterion, the elements in the first column must have the same sign in order for the system to be stable. Therefore, the following constraints must be satisfied.

$$
\begin{cases} k_d > \frac{L}{R} \\ \beta_i < \frac{-x_{3e}R}{2x_{1e}} \end{cases}.
$$

It is easy to select parameters β_i and k_d to satisfy two requirements above. Therefore, the linearized closed loop MBSS with the PD-like controller in region i can also be stabilized. And from the theorem of the Lyapunov's linearization method, the equilibrium point is asymptotically stable for the actural nonlinear closed loop MBSS with the PD-like controller. Note that since the winding current and position of the magnetic ball are assumed to be strictly positive, k_d is greater than a constant $\frac{L}{R}$ for all the regions. Thus, k_d can be selected as a constant for all the regions.

As we can see that the structure of the fuzzy controller is a combination of two open loop controllers and six region-wise PD-like controllers. Therefore, the fuzzy controller is expected to be more flexible than the classical PD controller. The comparisons of simulations results of the magenetic ball suspension system with the fuzzy controller and a PD controller in the following section justify that fuzzy controller is more flexible and robust than the classical controller in the sense of being able to stabilize the steel ball with a larger range of initial position conditions.

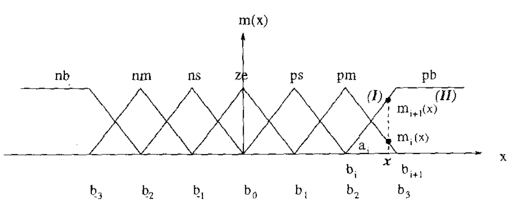

Figure 4: Membership functions of input variables

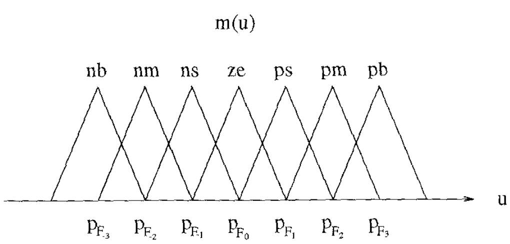

Figure 5: Membership functions of output variables

5 Simulation results

The classical PD controller used here to be compared with the fuzzy controller has the transfer function $C(s)$ defined as

$$C(s) = Kp + Kds$$

where $Kp = 2$ and $Kd = .4$. And the fuzzy controller designed here has the parameters $b_i's$ and $p_i's$ selected as

$$[b_{-3}\ b_{-2}\ b_{-1}\ b_0\ b_1\ b_2\ b_3] = [-.0445\ -.0422\ -.04\ 0\ .04\ .0422\ .0445]$$

$$[p_{-3}\ p_{-2}\ p_{-1}\ p_0\ p_1\ p_2\ p_3] = [-6.86\ -3.43\ -1.37\ 0\ 1.37\ 3.43\ 6.86]$$

Simulations of stabilizing the steel ball of MBSS at the desired position $r = .1$ with different initial position conditions are provided. Simulation results indicate that if only the variations of initial position are considered and other initial conditions are set to be zero, the steel ball of MBSS controlled by PD controller can be stabilized at r with the initial position range $y_{0PD} = [.033\ \ .63]$ and the steel ball of MBSS can be stabilized at r with a larger initial position range $y_{0F} = [.01\ \ .79]$ when the fuzzy controller is used. Figure 6 and Figure 7 show that the steel ball can be stabilized at r only with the fuzzy controller when the initial positions are out of the range y_{0PD}. Figure 8 illustrates that the performances of MBSS with the fuzzy controller is better than the performances of the MBSS with the PD controller in the sense of short settle time.

6 Conclusions

A robust controller based on the fuzzy if-then rules for the position control of a magnetic ball suspension system is presented in this paper. Human experience is utilized to construct the fuzzy logic controller. The fuzzy logic controller is shown to be more robust than a classical PD controller. Also, the performances of the fuzzy controller is also shown to be better than the performances of the PD controller. The success of the fuzzy logic application on the control of a nonlinear, naturally unstable magnetic ball suspension system in this paper encourages future studies in more complex nonlinear fuzzy control systems.

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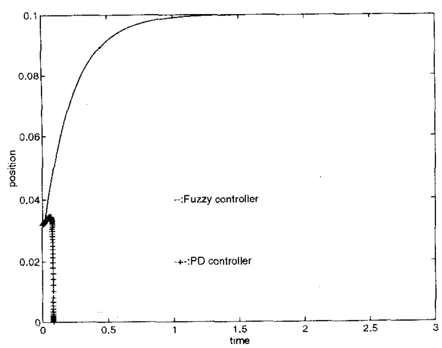

Figure 6: Comparion of performances of the fuzzy controller and the PD controller with initial position, .032.

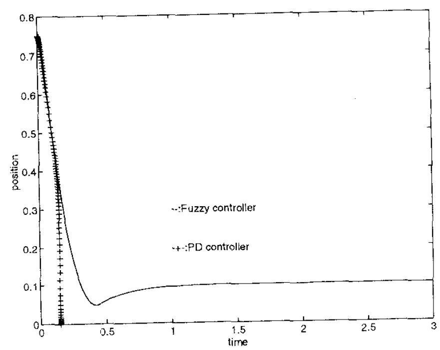

Figure 7: Comparion of performances of the fuzzy controller and the PD controller with initial position, .75.

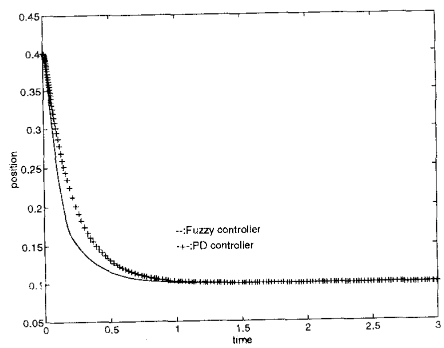

Figure 8: Comparion of performances of the fuzzy controller and the PD controller with initial position, .4.