Fuzzy Logic and Fuzzy Cognitive Map

MATH 800 – 4

Fall 2011

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- Fuzzy Logic Introduction
  - Fuzzy Numbers
  - Fuzzy Sets
  - Fuzzy Inference System
- Examples
  - Modelling the Underground Economy in Taiwan
  - Rainfall Events Prediction
- Fuzzy Toolbox or libraries

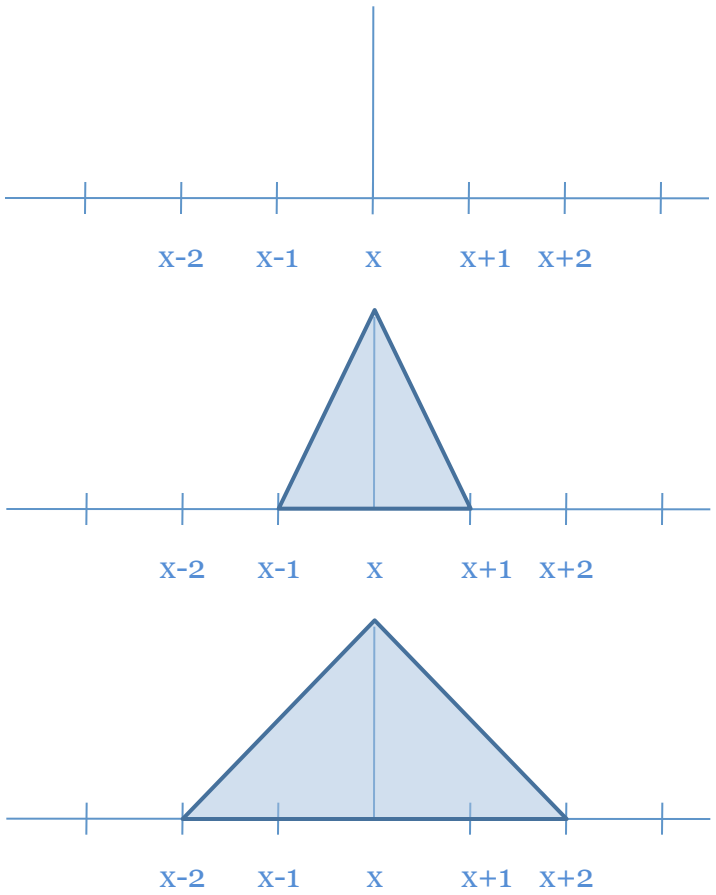
- Fuzzy Cognitive Maps
- Examples

Prof. Lotfi A. Zadeh

Prof. Bart Kosko

Fuzzy Logic Introduction

- *Fuzzy Number*
  - *Number 'x'*
  - *Near 'x'*
  - *Almost 'x'*



Fuzzy Logic Introduction

- *Fuzzy Sets*

  In a crisp set, membership or non-membership of element 'x' in set A is described by a characteristic function

  $$\mu_A(x), \text{ where } \mu_A(x) = 1 \text{ if } x \in A \text{ and } \mu_A(x) = 0 \text{ if } x \notin A.$$

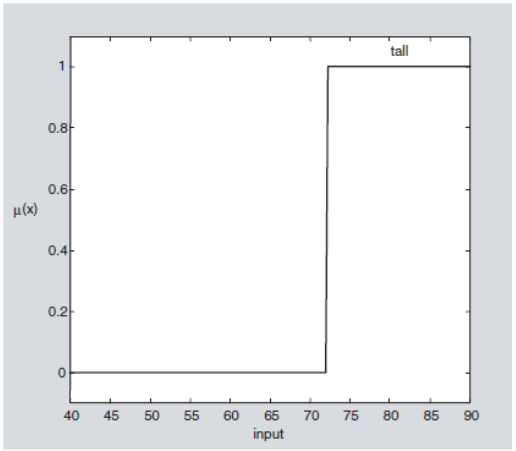
  Fuzzy set theory extends this concept by defining partial membership. A fuzzy set A on a universe of discourse U is characterized by a membership function

  $\mu_A(x)$ that takes values in the interval [0, 1].

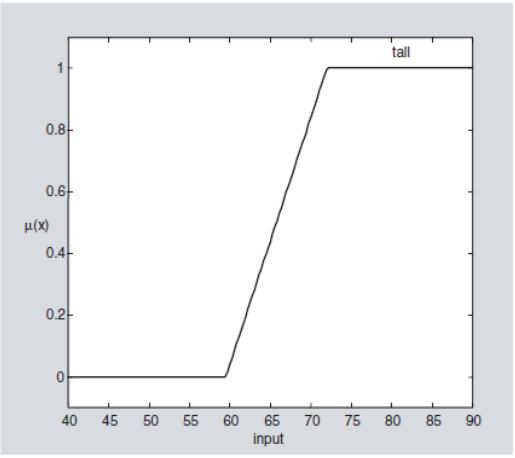
Fuzzy Logic Introduction

- *Fuzzy Sets…*

  A fuzzy set A in U may be represented as a set of ordered pairs. Each pair consists of a generic element x and its grade of membership function; that is $A = \{(x, \mu_A(x)) \mid x \in U\}$



(a) Crisp membership function



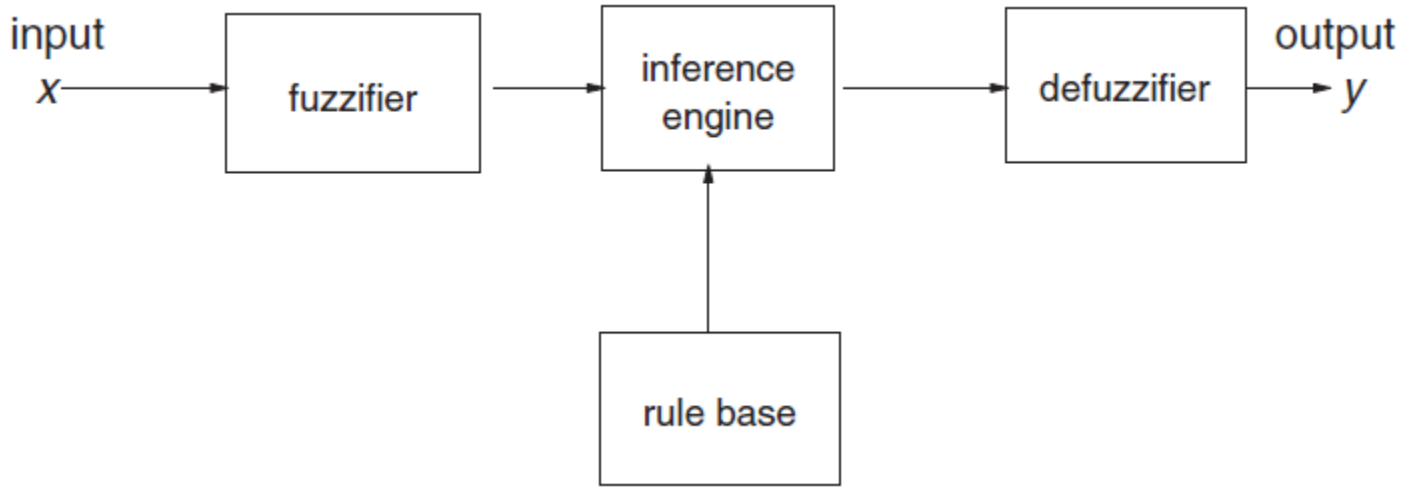
(b) Fuzzy membership function

Fuzzy Logic Introduction

- *Fuzzy Sets…*
  - *Fuzzy set operations*
    - *OR* $\mu_{A\cup B}(x) = \max\left[\mu_A(x), \mu_B(x)\right]$ $\mu_{A\cup B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x)$
    - *AND* $\mu_{A\cap B}(x) = \min\left[\mu_A(x), \mu_B(x)\right]$ $\mu_{A\cap B}(x) = \mu_A(x)\mu_B(x)$
    - *NOT* $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$

Fuzzy Logic Introduction

- *Fuzzy Inference System*



Fuzzy Logic Introduction

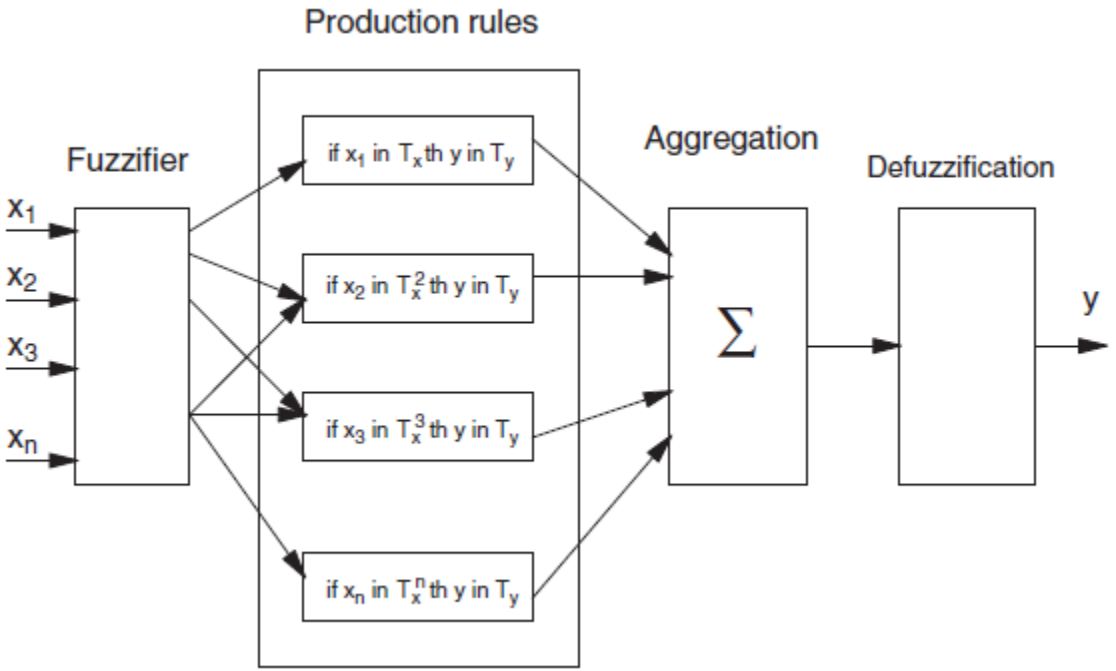
- *Fuzzy Inference System…*

Mamdani Method

- In 1975, Professor Ebrahim Mamdani of London University built one of the first fuzzy systems to control a steam engine and boiler combination. He applied a set of fuzzy rules supplied by experienced human operators.

Fuzzy Logic Introduction

- *Fuzzy Inference System…*



Fuzzy Logic Introduction

- *Fuzzy Inference System…*
  - *An example*
    - *Two inputs (x, y)*
    - *One output (z)*

    - *Rules:*

Rule1:	*If*	*x is A3*	*or*	*y is B1*	*Then*	*z is C1*
Rule2:	*If*	*x is A2*	*and*	*y is B2*	*Then*	*z is C2*
Rule3:	*If*	*x is A1*			*Then*	*z is C3*

Fuzzy Logic Introduction

- *Fuzzy Inference System…*
  - *Input x: research_funding*
  - *Input y: project_staffing*
  - *Output z: risk*

    - *Rules:*

Rule1: If research_funding is ***adequate*** *or project_staffing is* ***small*** *Then risk is* ***low***

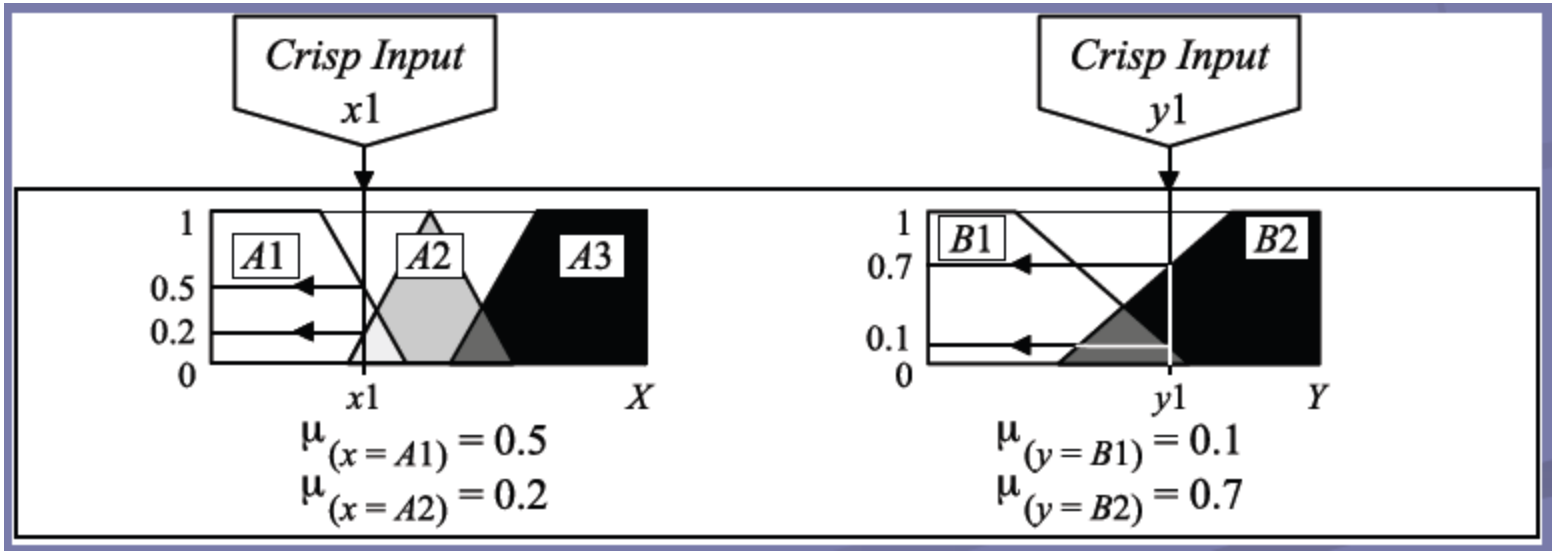
Rule2: If research_funding is ***marginal*** *and project_staffing is* ***large*** *Then risk is* ***normal***

Rule3: If research_funding is ***inadequate*** *Then risk is* ***high***

Fuzzy Logic Introduction

- *Fuzzy Inference System…*

  Step 1: Fuzzification



Fuzzy Logic Introduction

- *Fuzzy Inference System…*

  Step 2: Rule Evaluation

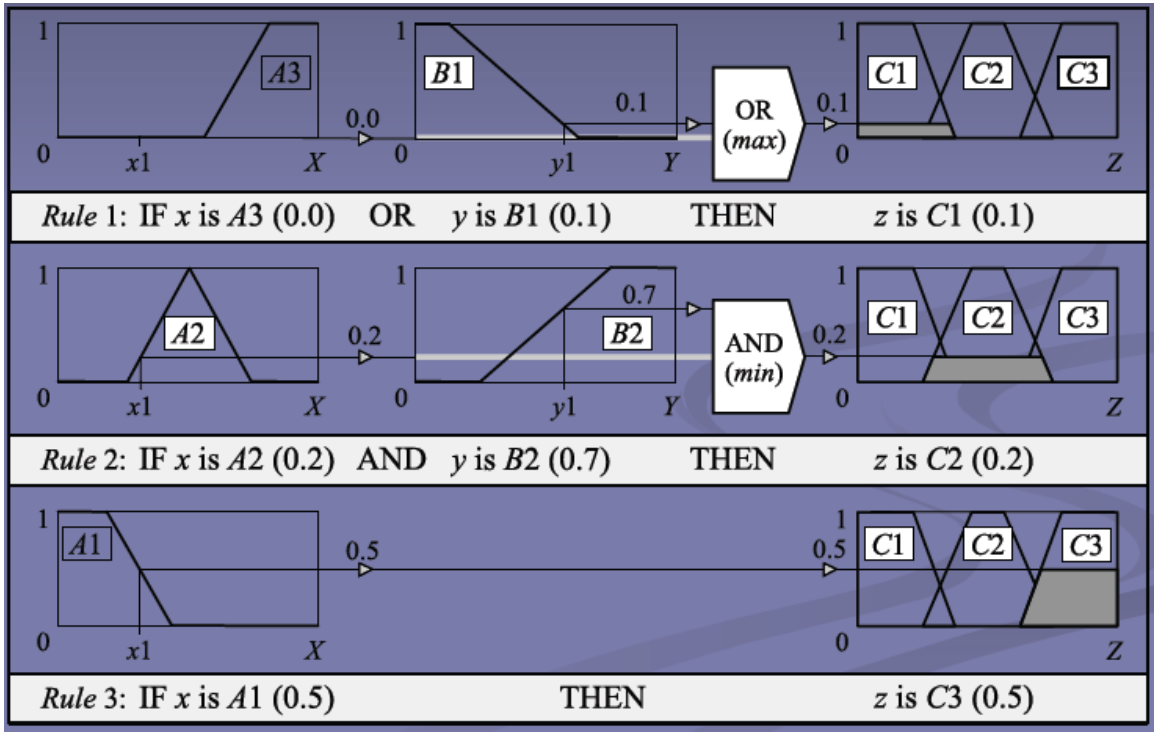
Antecedent → Consequent

$\mu_{(x=A1)} = 0.5$, $\mu_{(x=A2)} = 0.2$, $\mu_{(y=B1)} = 0.1$ and $\mu_{(y=B2)} = 0.7$

Fuzzy Logic Introduction

- *Fuzzy Inference System…*

  Step 2: Rule Evaluation…

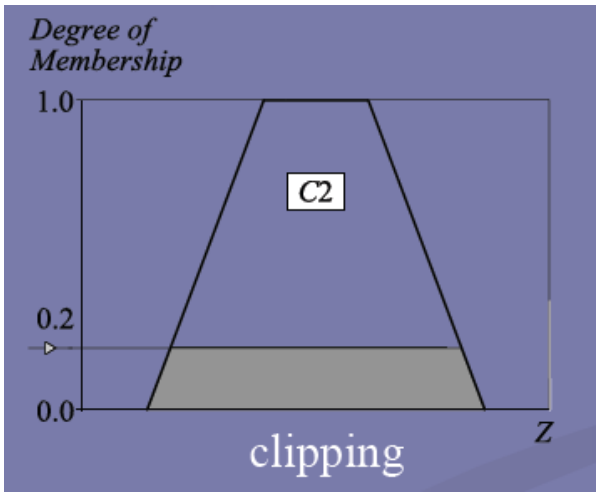


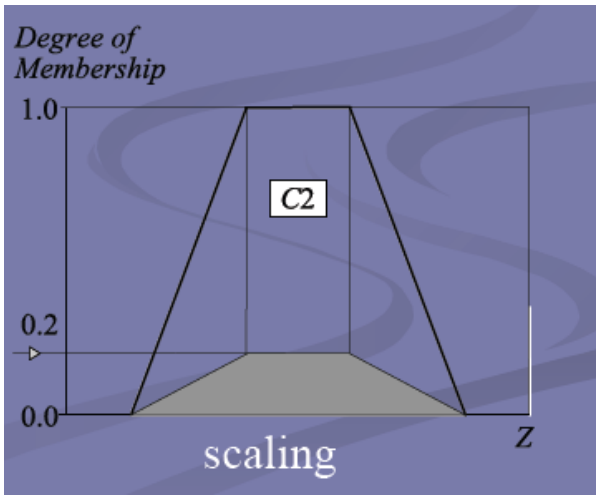
Fuzzy Logic Introduction

- *Fuzzy Inference System…*

  Step 2: Rule Evaluation…

  The result of the antecedent evaluation can be applied to the membership function of the consequent in two different ways:

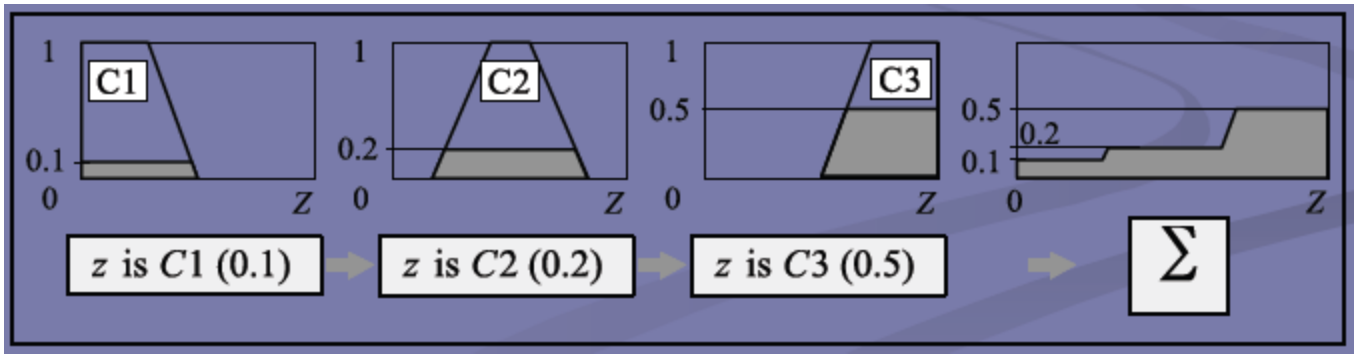




Fuzzy Logic Introduction

- *Fuzzy Inference System…*

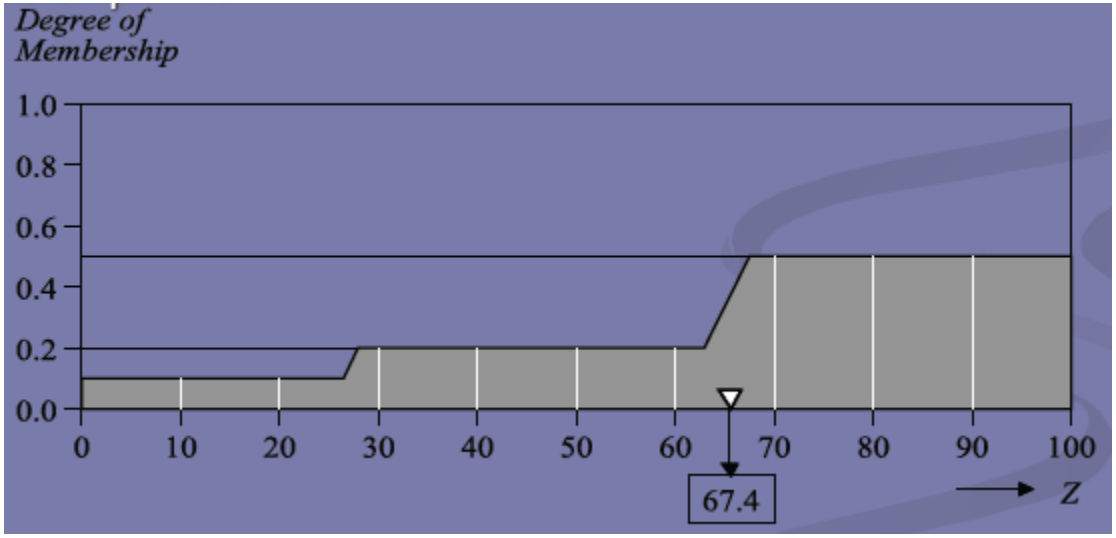
  Step 3: Rule Evaluation



Fuzzy Logic Introduction

- *Fuzzy Inference System…*

  Step 4: Defuzzification



$$COG = \frac{\int_a^b \mu_A(x)\,x\,dx}{\int_a^b \mu_A(x)\,dx}$$

$$COG = \frac{(0+10+20)\times 0.1 + (30+40+50+60)\times 0.2 + (70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$

Example 1:

- A Fuzzy Logic Approach to Modeling the Underground Economy in Taiwan

Inputs:

- Tax Rate (TR)
- Degree of government regulations (REG)

Output

- The size of Underground Economy (UE)

Example 1…

T.H.-K. Yu et al. / Physica A 362 (2006) 471–479

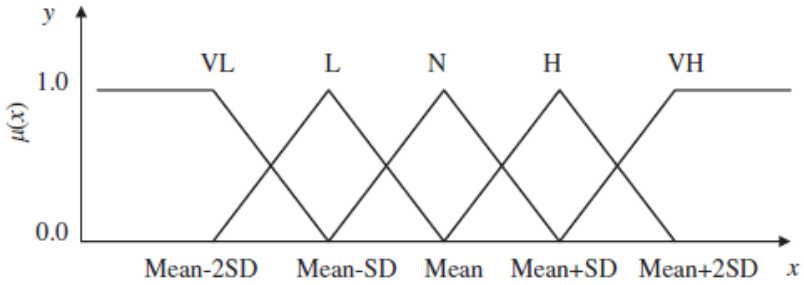


Fig. 1. Membership functions.

$$\text{index of } UE = \frac{\Sigma(\mu_{UE} y_i)}{\Sigma \mu_{UE}}$$

T.H.-K. Yu et al. / Physica A 362 (2006) 471–479

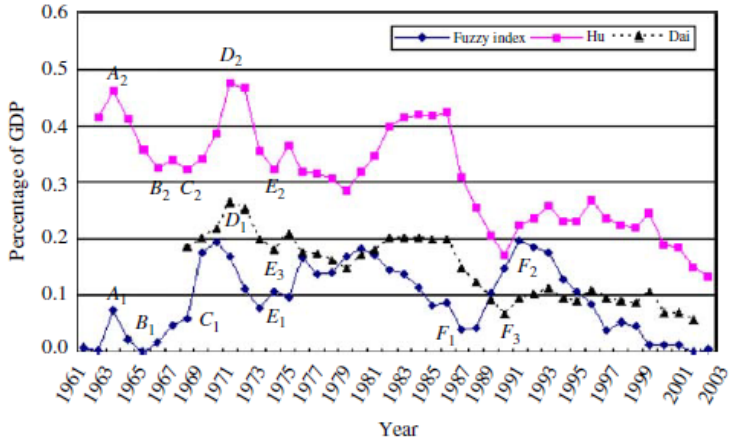


Fig. 2. Comparison of sizes of the underground economy.

If REG = ***VH*** and TR = ***VH*** Then UE = ***VB***

Example 2:

- Rainfall events prediction using rule-based fuzzy inference system

Inputs:

- Relative humidity
- Total cloud cover
- Wind direction
- Temperature and
- Surface pressure

Output

- Rainfall events

S. Asklany et al. / Atmospheric Research 101 (2011) 228–236

Fig. 1. Mersa Matruh and Cairo cities on the Egypt map.

Example 2…

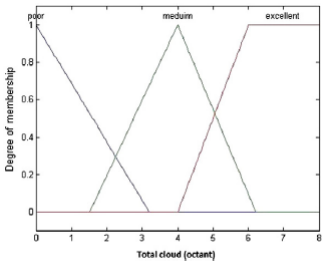


Fig. 3. Membership functions associated with the total cloud are referred t as "poor", "medium" and "excellent."

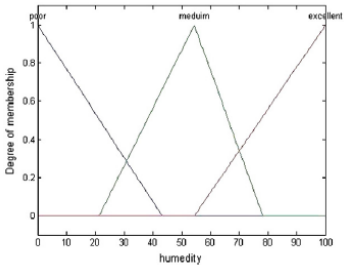
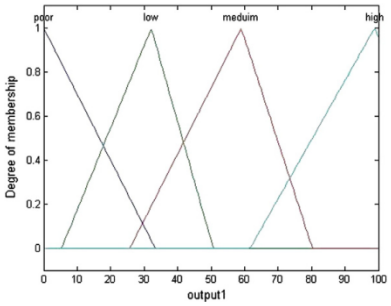


Fig. 2. Membership functions associated with the relative humidity are referred to as "poor", "medium" and "excellent."

…



b. IF humidity is poor AND IF total cloud is poor AND IF wind direction is poor AND IF pressure is high AND IF temperature is poor THEN rain percentage is low.

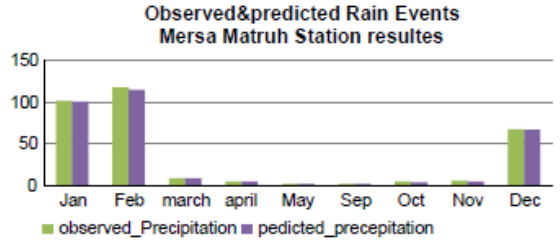


Fig. 8. FIS Output compared with observed rain events for Mersa Matruh station.

Table 1
Twenty years tested rainfall events for Cairo airport station (HECA).

Month	No. of rain events	No. of success forecasts
Jan.	388	301
Feb.	337	316
March	285	279
April	116	100
May	38	32
June	21	12
July	41	32
Aug.	47	45
Sep.	42	37
Oct	70	62
Nov	134	105
Dec	313	275

Toolboxes and Libraries for FL

Fuzzy Logic Toolbox for MATLAB:
http://www.mathworks.com/products/fuzzylogic/index.html

Fuzzy Logic package for Java (jFuzzyLogic)
http://jfuzzylogic.sourceforge.net/html/index.html

Fuzzy Logic libraries for C++ (JFuzzyQt)
http://sourceforge.net/projects/jfuzzyqt/

Q....Q...Q ???

Q: What is fuzzy logic and why do critics call it "the cocaine of science?"

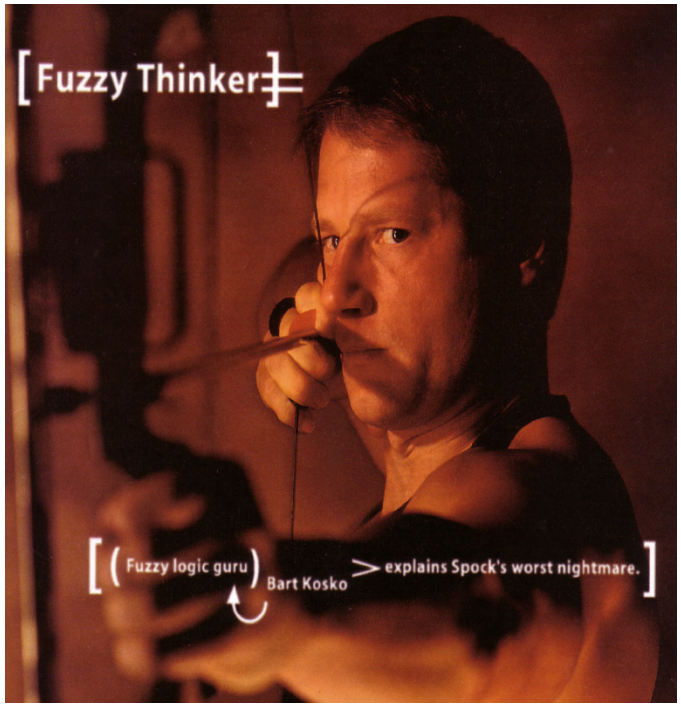
Kosko:

Fuzzy logic is a way of ***doing science without math***.

It's a new branch of machine intelligence that tries to make ***computers think the way people think*** and not the other way around.

You don't ***write equations for how to wash clothes***. Instead you load a chip with ***vague rules*** like "***if the wash water is dirty, add more soap***," and "***if very dirty, add a lot more.***"

You can never get the ***science right to more than a few decimal places.*** That's one reason we ***find chaos when we look at things up close***.



http://sipi.usc.edu/~kosko/index.html

Fuzzy Logic… so far

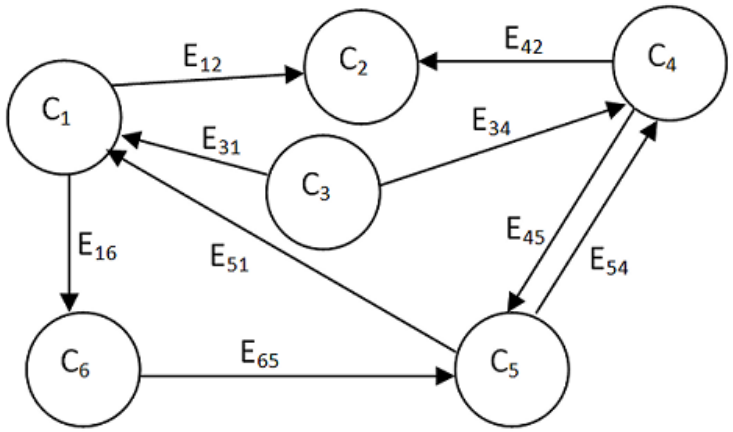
- Over 53,000 papers listed in the INSPEC database
- More than 15,000 in the Math Science Net database.
- Fuzzy-logic-related patents:
  - Over 4800 in Japan
  - 1500 + in the United States.

Fuzzy Cognitive Map

- *Introduction*
  - *Fuzzy Virtual worlds:*
    - *"Virtual worlds show how actors relate to one another … Events cause one another to some degree…"*
    - *"Fuzzy cognitive maps (FCMs) show how causal concepts affect one another to some degree… Causal concepts in a virtual worlds include* ***events***, ***values***, ***moods***, ***trends***, *or* ***goals****…"*

Fuzzy Cognitive Map

- *Introduction…*
  - *Basic structure of FCM*



- Each node in FCM represents a concept.
- Each arc (C_i, C_j) is directed as well as weighted, and represents causal link between concepts, showing how concept C_i causes concept C_j.

Fuzzy Cognitive Map

- *Introduction…*
  - *Basic structure of FCM…*

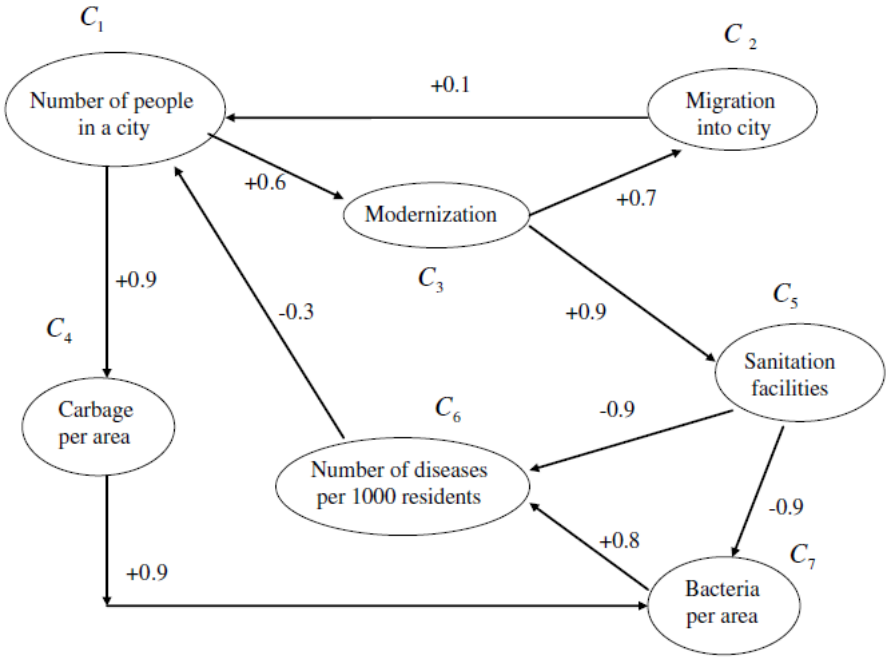
$$W = \begin{cases} w_{ij} > 0; \textit{expresses positive causality} \\ w_{ij} = 0; \textit{expresses no causality} \\ w_{ij} < 0; \textit{expresses negative causality} \end{cases}$$

excitatory

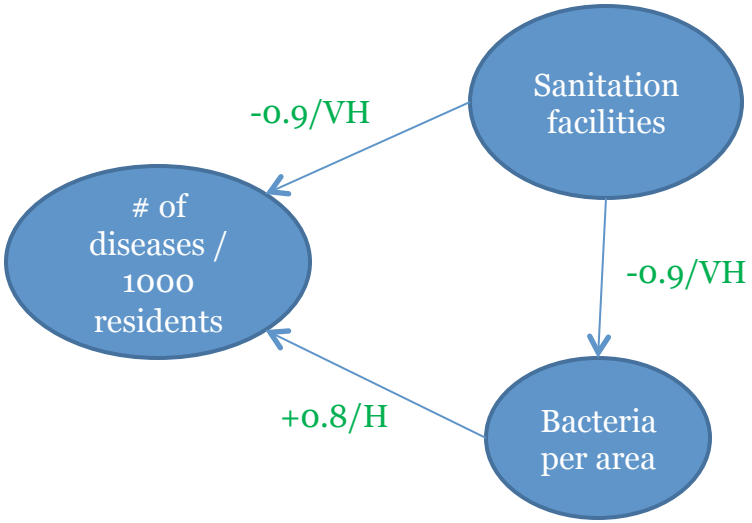
inhibitory

Fuzzy Cognitive Map

- *Introduction…*
  - *Basic structure of FCM…*



A civil engineering FCM



Fuzzy Cognitive Map

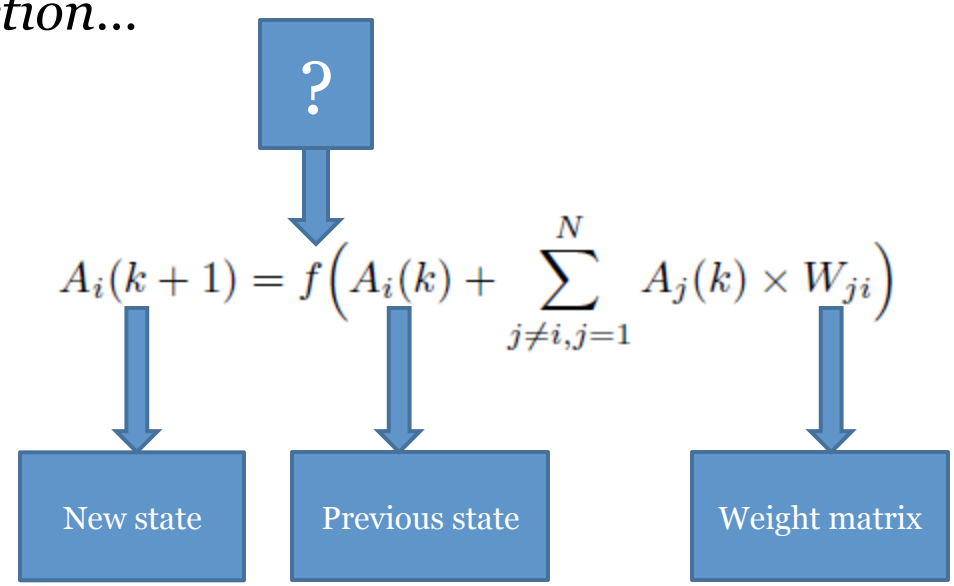
- *Introduction…*
  - *Adjacency matrix*

$W=$

	C1	C2	C3	…
C1	0	VH	VL	.
C2	H	0	0	.
C3	VL	H	0	.
…	.	.	.	.

Fuzzy Cognitive Map

- *Introduction…*



Fuzzy Cognitive Map

- *Introduction…*
  - *Transfer function of FCM*

(a) $$f_{\text{sign}}(x) = \begin{cases} 1, & x > 0, \\ 0, & x \leqslant 0. \end{cases}$$

(b) $$f_{\text{tri}}(x) = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0. \end{cases}$$

(c) $$f(x) = \tanh(x) \text{ or } f(x) = \frac{e^{2x} - 1}{e^{2x} + 1}.$$

Fuzzy Cognitive Map

- *FCM Inference Algorithm*

Step 1: Definition of the initial vector **A** that corresponds to the elements-concepts identified by experts' suggestions and available knowledge.

Step 2: Multiply the initial vector **A** with the matrix ***W*** defined by experts

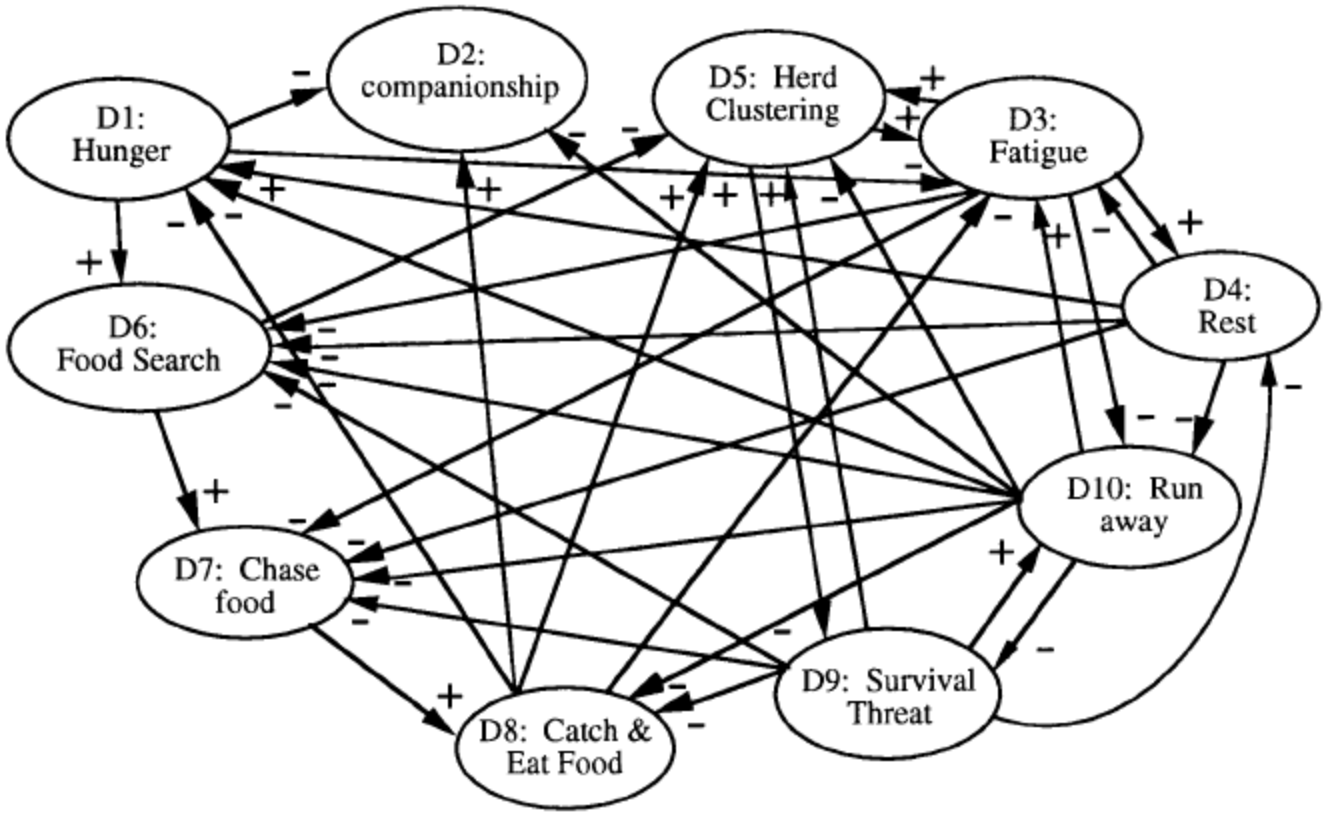
Step 3: The resultant vector **A** at time step *k* is updated using function threshold '*f* '.

Step 4: This new vector is considered as an initial vector in the next iteration.

Step 5: Steps 2–4 are repeated until *epsilon* (where *epsilon* is a residual, describing the minimum error difference among the subsequent concepts)

Fuzzy Cognitive Map

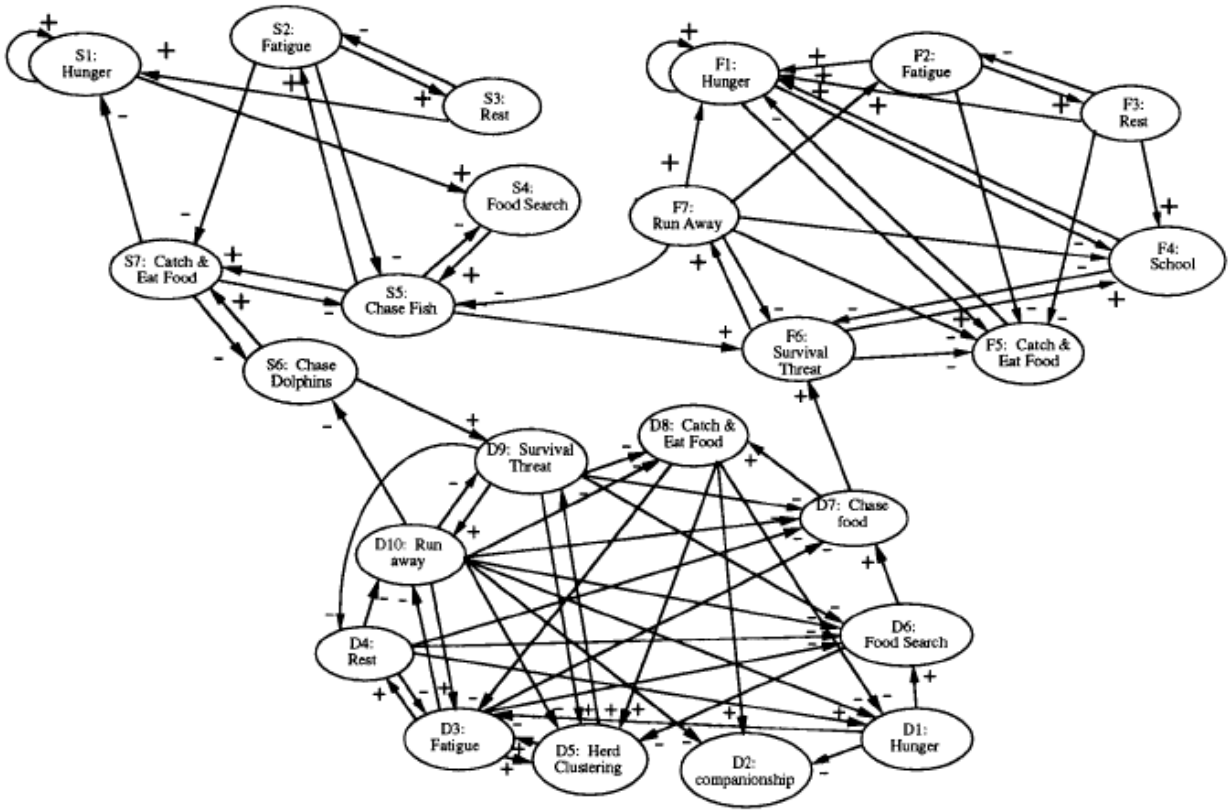
- *Example 1:*



Trivalent FCM for the control of a dolphin actor in virtual world

Fuzzy Cognitive Map

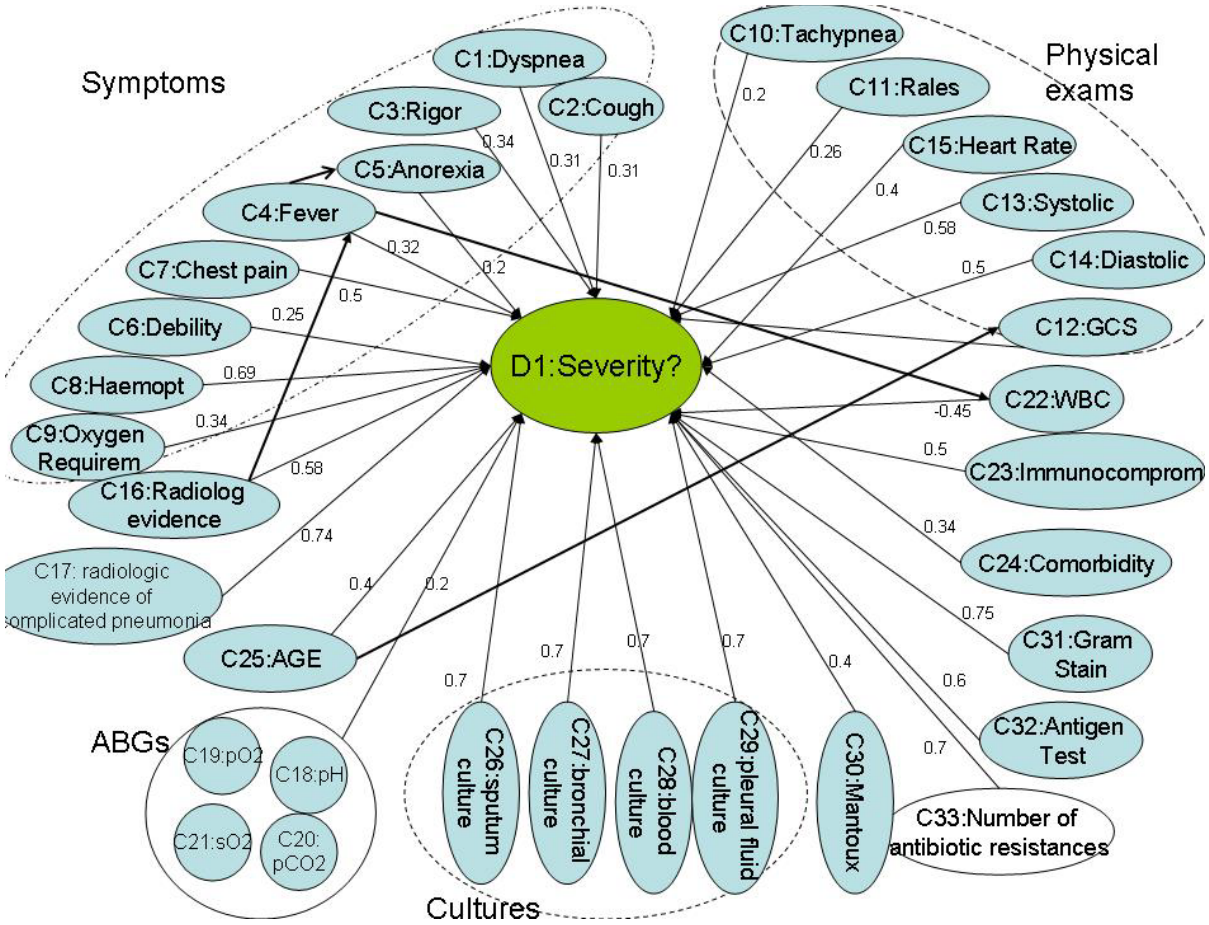
- *Example 2:*



FCM for dolphin, fish and sharks in virtual world

Fuzzy Cognitive Map

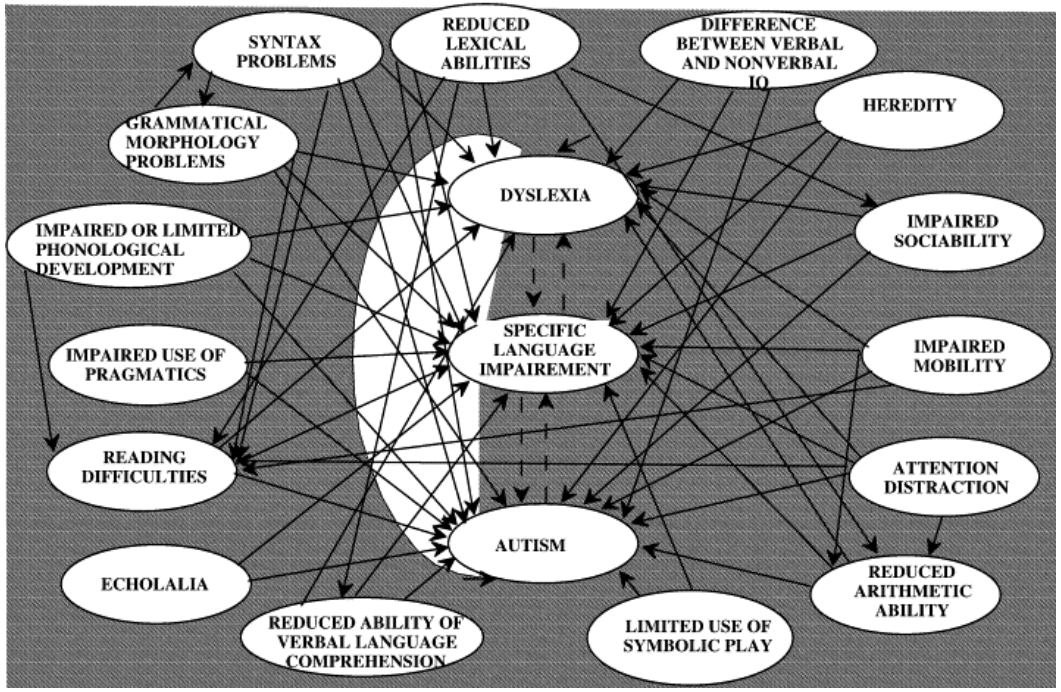
- *Example 3:*



FCM model for predicting the severity index of pulmonary infection

Fuzzy Cognitive Map

- *Example 4:*



FCM differential diagnosis model of SLI from dyslexia and autism

FCM?