# João Antônio de Lorenzo Martins - Lista 6.

**1)** a) $\vec{u} = (1,1,1)_E$

$\|\vec{u}\| = \sqrt{1^2+1^2+1^2}$

$\|\vec{u}\| = \sqrt{3}$

b) $\vec{u} = 3\vec{i} + 4\vec{k}$

$\|\vec{u}\| = \sqrt{3^2+4^2}$

$\|\vec{u}\| = 5$

c) $\vec{u} = -\vec{i} + \vec{j}$

$\|\vec{u}\| = \sqrt{(-1)^2+1}$

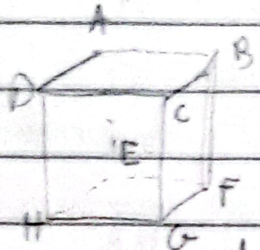
$\|\vec{u}\| = \sqrt{2}$

d) $\vec{u} = 4\vec{i} + 3\vec{j} - \vec{k}$

$\|\vec{u}\| = \sqrt{4^2+3^2+(-1)^2}$

$\|\vec{u}\| = \sqrt{26}$

**2)** a)




$\vec{e_1} = \vec{DH}$ $\quad \|\vec{e_1}\| = \|\vec{e_2}\| = \|\vec{e_3}\| = 1$ (cubo unitário)

$\vec{e_2} = \vec{DC}$ $\quad$ Todas as arestas adjacentes em um cubo são

$\vec{e_3} = \vec{DA}$ $\quad$ perpendiculares. Os vetores $\vec{e_1}, \vec{e_2}, \vec{e_3}$ compartilham o ponto D, sendo adjacentes e, portanto, perpendiculares.

Sendo $D=(0,0,0)$: $\vec{DH}=(1,0,0)$; $\vec{DC}=(0,1,0)$; $\vec{DA}=(0,0,1)$

$\vec{e_1} \cdot \vec{e_2} = 1\cdot 0 + 0 \cdot 1 + 0 \cdot 0 = 0$

$\vec{e_2} \cdot \vec{e_3} = 0 \cdot 0 + 1 \cdot 0 + 0 \cdot 1 = 0$

$\vec{e_3} \cdot \vec{e_1} = 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 1 = 0$

produto escalar de todos $= 0$

b) $\vec{u} = \vec{CD} + \vec{CB}$

$\vec{u} = -\vec{e_2} + \vec{e_3}$

$\vec{u} = -[(0,1,0)-(0,0,1)]$

$\vec{u} = -(0,1,-1)$

$\vec{u} = (0,-1,1)_E$

$\vec{v} = \vec{DC} + \vec{CB}$

$\vec{v} = \vec{e_2} + \vec{e_3}$

$\vec{v} = (0,1,1)_E$

$\vec{w} = \vec{GC}$

$\vec{w} = -\vec{DH}$

$\vec{w} = (-1,0,0)_E$

c) $\vec{f_1} = \dfrac{\vec{u}}{\|\vec{u}\|} = \left(0, \dfrac{-1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

$\vec{f_2} = \dfrac{\vec{v}}{\|\vec{v}\|} = \left(0, \dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

$\vec{f_3} = (-1,0,0)$

Seja $\dfrac{1}{\sqrt{2}} = x$

$\vec{f_1} \cdot \vec{f_2} = 0 \cdot 0 + (-x \cdot x) + x \cdot x = -x^2 + x^2 = 0$

$\vec{f_1} \cdot \vec{f_3} = 0 \cdot (-1) + (-x \cdot 0) + x \cdot 0 = 0$

$\vec{f_2} \cdot \vec{f_3} = 0 \cdot (-1) + x \cdot 0 + x \cdot 0 = 0$

· Produtos escalares $= 0$

↳ Base ortonormal

d) $M_{E\to F} = \begin{pmatrix} 0 & 0 & -1 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$ $\quad M_{F\to E} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -1 & 0 & 0 \end{pmatrix}$

a matriz M é ortogonal, pois ela transforma de uma base ortonormal para outra

e) $\vec{HB} = -\vec{DH} + \vec{DA} + \vec{DC}$

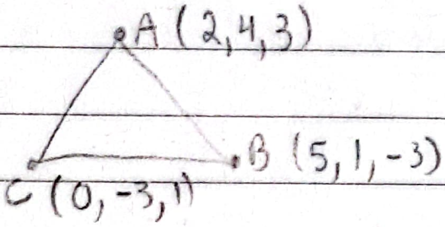
$\vec{HB} = -\vec{e_1} + \vec{e_3} + \vec{e_2}$

$\vec{HB} = (-1, 1, 1)_E$

$\vec{v_F} = M^{-1}\vec{v_E}$ $\begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$\vec{v_F} = \begin{pmatrix} 0 \\ 2/\sqrt{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{2} \\ 1 \end{pmatrix} = \vec{HB}$

③ a)




$\vec{AB} = (5-2, 1-4, -3-3) = (3, -3, -6)$

$\vec{BC} = (0-5, -3-1, 1-(-3)) = (-5, -4, 4)$

$\vec{CA} = (2-0, 4-(-3), 3-1) = (2, 7, 2)$

b) $\|\vec{AB}\| = \sqrt{3^2 + (-3)^2 + (-6)^2} = \sqrt{54}$ → lados de mesma medida

$\|\vec{BC}\| = \sqrt{(-5)^2 + (-4)^2 + 4^2} = \sqrt{57}$

$\|\vec{CA}\| = \sqrt{2 \cdot 2^2 + 7^2} = \sqrt{57}$ $\Big\}$ O triângulo é isósceles em $\vec{BC}$ e $\vec{CA}$

c) Ponto médio dos lados:

$\vec{AB} = (7/2, 5/2, 0)$

$\vec{BC} = (5/2, -1, -1)$

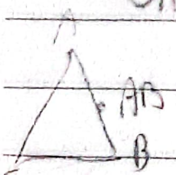
$\vec{CA} = (1, 1/2, 2)$

Mediana $= \vec{AB} - C$

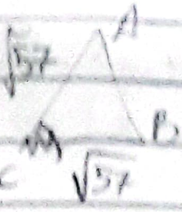
$= (7/2, 11/2, -1)$

$\frac{7 \cdot 3}{2} + \frac{11 \cdot (-3)}{2} + -6 \cdot (-1)$

$\frac{21}{2} - \frac{33}{2} + \frac{12}{2} \Rightarrow$ produto escalar $= 0$




Logo, a mediana também é mediatriz

d) 


$\vec{CB} = (5, 4, -4)$

$\vec{CA} = (2, 7, 2)$

$\vec{CB} \cdot \vec{CA} \quad 10 + 28 - 8 = 30$

$$\cos\theta = \frac{\vec{CA} \cdot \vec{CB}}{\|\vec{CA}\| \cdot \|\vec{CB}\|} = \frac{30}{\sqrt{57} \cdot \sqrt{57}} = \frac{30}{57} = \frac{10}{19}$$

$$\theta = \arccos\left(\frac{10}{19}\right)$$

e) a soma de $\vec{AB} + \vec{BC} + \vec{CA}$ deve ser zero, pois ABC é um triângulo, ou seja, um circuito fechado.

4) a) $|\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \cdot \|\vec{v}\|$

Seja $\theta$ um número $\in \mathbb{R}$:

$-1 \leq \cos\theta \leq 1$

$$-1 \leq \frac{|\vec{u} \cdot \vec{v}|}{\|\vec{u}\| \cdot \|\vec{v}\|} \leq 1$$

$-\|\vec{u}\| \cdot \|\vec{v}\| \leq |\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \cdot \|\vec{v}\|$

Se $\vec{u} = \lambda \vec{v}$, $\lambda \in \mathbb{R}$

$\cos 0 = 1$

$\to \vec{u}$

$\to \vec{v}$

$= 1$

$$\frac{|\vec{u} \cdot \vec{v}|}{\|\vec{u}\| \cdot \|\vec{v}\|} = 1$$

$|\vec{u} \cdot \vec{v}| = \|\vec{u}\| \cdot \|\vec{v}\|$

b) $\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\|^2 + 2(\vec{u} \cdot \vec{v}) + \|\vec{v}\|^2$

$|\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \cdot \|\vec{v}\| \Rightarrow \vec{u} \cdot \vec{v} \leq |\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \cdot \|\vec{v}\|$

$2(\vec{u} \cdot \vec{v}) = 2\|\vec{u}\| \, \|v\|$

5) a) $\cos\theta = \dfrac{|1\cdot(-2) + 0 + 2|}{\sqrt{1+1} \cdot \sqrt{4+100+4}} = \dfrac{|-2+2|}{\sqrt{2}\cdot\sqrt{108}} = 0$ $\theta = 90^\circ$

b) $\cos\theta = \dfrac{|-1+1+1|}{\sqrt{3}\cdot\sqrt{3}} = \dfrac{1}{3}$ $\theta = \arccos\left(\dfrac{1}{3}\right)$

c) $\dfrac{|3\cdot 2 + 3\cdot 1 + 1\cdot(-2)|}{\sqrt{3^2+3^2+0} \cdot \sqrt{2^2+1^2+(-2)^2}} = \dfrac{|9|}{\sqrt{18}\cdot\sqrt{9}} = \dfrac{9}{\sqrt{2}\cdot\sqrt{9}\cdot\sqrt{9}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\Rightarrow \cos\theta = \sqrt{2}/2$ $\theta = 45^\circ$

d) $\cos\theta = \dfrac{|\sqrt{3}\cdot\sqrt{3} + 1\cdot 1 + 0\cdot 2\sqrt{3}|}{\sqrt{(\sqrt{3})^2+1^2+0^2} \cdot \sqrt{(\sqrt{3})^2+1^2+(2\sqrt{3})^2}} = \dfrac{|4|}{\sqrt{4}\cdot\sqrt{3+1+4\cdot 3}} = \dfrac{4}{2\sqrt{16}}$

$\cos\theta = \dfrac{1}{2}$ $\theta = 60^\circ$

6 a) $\vec{u} = (x+1, 1, 2)$ $\quad (x+1)(x-1) + 1\cdot(-1) + 2\cdot(-2) = 0$

$\vec{v} = (x-1, -1, -2)$ $\quad x^2 - 1 + (-1) + (-4) = 0$

$x^2 - 6 = 0 \quad x = \pm\sqrt{6}$

b) $\vec{u} = (x, x, 4)$ $\quad 4x + x^2 + 4 = 0$

$\vec{v} = (4, x, 1)$ $\quad S = -4 \quad P = 4 \quad \boxed{x = -2}$

7 a) $\vec{u} = (x, y, z)$

$\vec{v} = (4, -1, 5)$

$\vec{w} = (1, -2, 3)$

$$\begin{cases} 4x - y + 5z = 0 \\ x - 2y + 3z = 0 \\ x + y + z = -1 \end{cases}$$

$2x + 2y + 2z = -2$

$$\begin{cases} 5x + 6z = -1 \\ 3x + 5z = -2 \end{cases}$$

$15x + 18z = -3$

$15x + 25z = -10$

$-7z = 7$

$z = -1$

$3x - 5 = -2$

$3x = 3 \quad x = 1$

$3 + y - 1 = -1$

$y = -3$

$\vec{u} = (3, -3, -1)$

b) $\vec{u} = (x, y, z)$

$\|\vec{u}\| = 3\sqrt{3}$

$\vec{v} = (2, 3, -1)$

$\vec{w} = (2, -4, 6)$

$$\begin{cases} \sqrt{x^2 + y^2 + z^2} = 3\sqrt{3} \\ 2x + 3y - z = 0 \\ 2x - 4y + 6z = 0 \end{cases} \Rightarrow \begin{cases} x^2 + y^2 + z^2 = 27 \\ 2x + 3y - z = 0 \\ 2x - 4y + 6z = 0 \end{cases}$$

$\to 2x + 3y - y = 0$

$2x + 2y = 0$

$x = -y$

$-7y + 7z = 0$

$z - y = 0$, logo $z = y$

$-x = y = z$

$x^2 + y^2 + z^2 = 27$

$\to 3$ ou $-3$

$$\begin{cases} \vec{u} = (3, -3, -3) \\ \vec{u}' = (-3, 3, 3) \end{cases}$$

$\vec{i} = (1, 0, 0)$

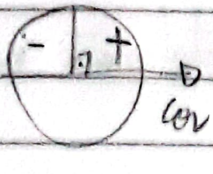
O vetor $\vec{u} = (3, -3, -3)$ forma um ângulo agudo

$\vec{u} \cdot \vec{i} = 3\cdot 1 + (-3)\cdot 0 + (-3)\cdot 0 = 3$

$\vec{u}' \cdot \vec{i} = -3\cdot 1 + 3\cdot 0 + 3\cdot 0 = -3$

$\to$ se $> 90°$, $< 0$

se $< 90$, $> 0$




$$\cos\theta = \frac{|\vec{u} \cdot \vec{i}|}{\|\vec{u}\| \, \|\vec{i}\|}$$

/ /

c) $\frac{|\vec{u}\cdot\vec{v}|}{\|\vec{u}\|\,\|\vec{v}\|} = \frac{\sqrt{2}}{2} = \cos\frac{\pi}{2} \Rightarrow \frac{|\vec{u}\cdot\vec{v}|}{\sqrt{5}} \Rightarrow |\vec{u}\cdot\vec{v}| = \sqrt{5}$

$\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 + 2\vec{u}\vec{v}$

$\|\vec{u}-\vec{v}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 - 2\vec{u}\vec{v}$

$(\vec{u}+\vec{v})(\vec{u}-\vec{v}) = \vec{u}\cdot\vec{u} - \vec{v}\cdot\vec{v}$

$\|\vec{u}\|^2 - \|\vec{v}\|^2$

8) a) $1\cdot 3 + (-1)\cdot(-1) + 2\cdot 1$

$3+1+2 = 6$

$3^2 + (-1)^2 + 1^2$

$9+1+1 = 11$

$\frac{6}{11}(3,-1,1) = \left(\frac{18}{11}, \frac{-6}{11}, \frac{6}{11}\right) = proj_{\vec{u}}\vec{v}$

b) $1\cdot(-3) + 3\cdot 1 + \cancel{5\cdot 0}$

$-3+3 = 0$ → ortogonais

$(-3)^2 + 1^2 = 10$

$proj_{\vec{u}}\vec{v} = \vec{0}$

c) $(-1)\cdot(-2) + 1\cdot 1 + 1\cdot 2$

$2+1+2 = 5$

$(-2)^2 + (1)^2 + (2)^2 = 9$

$\frac{5}{9}(-2,1,2)$

$proj_{\vec{u}}\vec{v} = \left(\frac{-10}{9}, \frac{5}{9}, \frac{10}{9}\right)$

d) $1\cdot(-2) + 2\cdot(-4) + 4\cdot(-8)$

$-2-8-32 = -42$

$(-2)^2 + (-4)^2 + (-8)^2 = 4+16+64$

$84$ $-\frac{1}{2}($

$proj_{\vec{u}}\vec{v}\ (1,2,4)$

↳ $\vec{v}$ está totalmente na direção de $\vec{u}$

9) $E = (\vec{i}, \vec{j}, \vec{k})$ $\quad \vec{u} = 2\vec{i} - 2\vec{j} + \vec{k}$ $\quad \vec{v} = 3\vec{i} - 6\vec{j}$

a) $\text{proj}_{\vec{u}}\,\vec{v} = (4, -4, 2)$ $\qquad \text{proj}_{\vec{v}}\,\vec{u} = (6/5, -12/5, 0)$

$(2)(3) + (-2)(-6) + 1 \cdot 0$ $\quad \Rightarrow$

$6 + 12 = 18$

$(2)^2 + (-2)^2 + 1^2 = 9$ $\qquad (3)^2 + (-6)^2 = 45$ $\qquad \frac{18}{45} = \frac{2}{5}$

$\frac{18}{9} = 2$ $\quad 2(2, -2, 1)$ $\qquad \frac{2}{5}(3, -6, 0) = \left(\frac{6}{5}, \frac{-12}{5}, 0\right)$

$(4, -4, 2)$

b) $\vec{p}, \vec{q}$ $\quad \vec{p} + \vec{q} = \vec{v}$ $\quad \vec{p} \,//\, \vec{v}$ $\quad \vec{q} \perp \vec{u}$

$\vec{p} = (a, b, c)$ $\Rightarrow$ $(2, -2, 1)\lambda = (2\lambda, -2\lambda, \lambda)$

$\vec{q} = (x, y, z)$

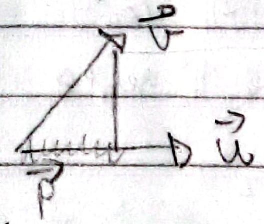
$$\Rightarrow 2x - 2y + z = 0 \qquad \begin{cases} x + \lambda \cdot 2 = 3 \\ y + \lambda(-2) = -6 \\ z + \lambda \cdot 1 = 0 \end{cases} \qquad \begin{aligned} z &= -\lambda \\ y &= -6 + 2\lambda \\ x &= 3 - 2\lambda \end{aligned}$$

$2 \cdot (3 - 2\lambda) - 2 \cdot (-6 + 2\lambda) - \lambda = 0$ $\qquad z = -2$ $\qquad \vec{p} = (4, -4, 2)$

$6 - 4\lambda + 12 - 4\lambda - \lambda = 0$ $\qquad y = -2$ $\qquad \vec{q} = (-1, -2, -2)$

$18 - 9\lambda = 0$ $\qquad x = -1$

$\lambda = \frac{-18}{-9} = 2$

c) 


$q \perp u$

$q = \vec{v} - \vec{p}$ $\rightarrow$ tira a componente horizontal $\Rightarrow = \text{proj}_{\vec{u}}\,\vec{v}$

$\Rightarrow A_{\square} = \|\vec{u}\| \cdot \|\vec{q}\|$

$= \sqrt{9} \cdot \sqrt{(-1)^2 + (-2)^2 + (-2)^2}$

$= \sqrt{9} \cdot \sqrt{9} = 9$ //

10 a) $\vec{u}=(3,3)$ $\vec{u}\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 3 & 3 & 0\\ 5 & 4 & 0\end{vmatrix}$ $\vec{k}\begin{vmatrix}3 & 3\\ 5 & 4\end{vmatrix}$ $=-3\vec{k}$

$\vec{v}=(5,4)$ $12-15$

$\|\vec{u}\times\vec{v}\|=3$

b) $\vec{u}=(7,0,-5)$ $\vec{u}\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 7 & 0 & -5\\ 1 & 2 & -1\end{vmatrix}$ $=\vec{i}(-(-10))-\vec{j}[-7-(-5)]+k[14]=10\vec{i}+2\vec{j}+14\vec{k}$

$\vec{v}=(1,2,-1)$

$\|\vec{u}\times\vec{v}\|=\sqrt{100+4+196}=\sqrt{300}=\sqrt{3}\cdot\sqrt{100}=10\sqrt{3}$

c) $\vec{u}=(1,-3,1)$ $\vec{u}\times v=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & -3 & 1\\ 1 & 1 & 4\end{vmatrix}$ $\vec{i}(-12-1)-j(4-1)+k(1-(-3))$

$\vec{v}=(1,1,4)$ $=-13i-3j+4k$

$\sqrt{13^2+3^2+4^2}=\sqrt{194}$

$\|\vec{u}\times\vec{v}\|=\sqrt{194}$

d) $\vec{u}=(2\ 1\ 2)$ $\vec{u}=\vec{v}.2$ $\vec{u}\times\vec{v}=0$

$\vec{v}=(4\ 2\ 4)$ $\|\vec{u}\times\vec{v}\|=0$

11 a) $\vec{u}\cdot\vec{v}=\|\vec{u}\|\,\|\vec{v}\|\cos\theta$ $\xrightarrow[\text{quadrado}]{\text{elevar ao}}$ $(\vec{u}\cdot\vec{v})^2=\|\vec{u}\|^2\|\vec{v}\|^2\cos^2\theta$ (1)

$\|\vec{u}\times\vec{v}\|=\|\vec{u}\|\,\|\vec{v}\|\operatorname{sen}\theta$ $\quad$ $\|\vec{u}\times\vec{v}\|^2=\|\vec{u}\|^2\|\vec{v}\|^2\operatorname{sen}^2\theta$ (2)

(1)+(2) $\Rightarrow\|\vec{u}\times\vec{v}\|^2+(\vec{u}\cdot\vec{v})^2=(\operatorname{sen}^2\theta+\cos^2\theta)\|\vec{u}\|^2\|\vec{v}\|^2$

$\Rightarrow\|\vec{u}\times v\|^2+(\vec{u}\cdot\vec{v})^2=\|\vec{u}\|^2\|\vec{v}\|^2$ Q.E.D.

$\cos^2x+\operatorname{sen}^2x=1$

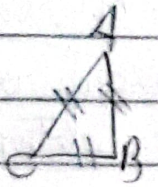
b) $\|\vec{u} \times \vec{v}\|$ , $\vec{u} \cdot \vec{v} = 3$ , $\|\vec{u}\| = 1$ , $\|\vec{v}\| = 5$.

$\|\vec{u} \times \vec{v}\|^2 + (\vec{u} \cdot \vec{v})^2 = \|\vec{u}\|^2 \|\vec{v}\|^2$

$\|\vec{u} \times \vec{v}\|^2 + 3^2 = 1^2 \cdot 5^2$

$\|\vec{u} \times \vec{v}\|^2 = 16$

$\|\vec{u} \times \vec{v}\| = \sqrt{16}$. $\|\vec{u} \times \vec{v}\| = 4$

c)  lado $l$ Produto vetorial $= \|\vec{AB}\| \cdot \|\vec{AC}\| \cdot \text{sen } 60 = l^2 \cdot \frac{\sqrt{3}}{2}$

12) a) $\begin{cases} \vec{x} \cdot (2\vec{i} + 3\vec{j} + 4\vec{k}) = 9 \\ \vec{x} \times (-\vec{i} + \vec{j} - \vec{k}) = -2\vec{i} + 2\vec{k} \end{cases}$ $x = a\vec{i} + b\vec{j} + c\vec{k}$

$\begin{cases} 2a + 3b + 4c = 9 \\ 2a + 2b = 4 \end{cases}$

$b + 4c = 5$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ -1 & 1 & -1 \end{vmatrix}$

$\vec{i}(-b-c) - \vec{j}(-a-(-c)) + \vec{k}(a-(-b))$

$= -\vec{i}(b+c) - \vec{j}(-a+c) + \vec{k}(a+b)$

$= -2\vec{i} + 2\vec{k}$

$\begin{cases} b+c = 2 \\ c = a \\ a+b = 2 \end{cases}$ $a + b = 2$

$\begin{cases} a+b = 2 \\ 4a + b = 5 \end{cases}$ $3a = 3$

$a = 1$ $b = 1$ $c = 1$ $\vec{x} = (1, 1, 1)$

b) $\begin{cases} x \times (1,0,1) = 2(1,1,-1) \\ \|\vec{x}\| = \sqrt{6} \end{cases}$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ 1 & 0 & 1 \end{vmatrix}$

$\vec{i}(b) - \vec{j}(a-c) + \vec{k}(-b)$

$= \vec{i} \cdot b + j(c-a) + \vec{k}(-b)$

$\begin{cases} b = 2 \\ c - a = 2 \\ -b = -2 \end{cases}$ $\begin{cases} b = 2 \\ a - c = -2 \end{cases}$

$a^2 + b^2 + c^2 = \sqrt{6}$

$a^2 + b^2 + c^2 = 6$

$\begin{cases} a^2 + c^2 = 2 \\ a - c = -2 \end{cases}$ $\Rightarrow c^2 + (c-2)^2 = 2$

$c^2 + c^2 - 2 \cdot 2 \cdot c + 4 = 2$

$a = c - 2$ $2c^2 - 4c + 4 = 2$

$2c^2 - 4c + 2 = 0$ $\vec{x} = (-1, 2, 1)$

$c^2 - 2c + 1 = 0$

$S = 2$ 1 e 1

$P = 1$

$c = 1 \Rightarrow a = -1$

c) $\|\vec{x}\| = \sqrt{3}$ $\quad \vec{x} \perp \vec{u}$ $\quad \vec{x} \perp \vec{v}$ $\quad \cos\theta = \frac{|\vec{x}\cdot\vec{j}|}{\|\vec{x}\|\|\vec{j}\|} < 0$

$\vec{u} = (-3, 0, 3)$ $\quad \vec{v} = (2, -2, 0)$

$\cos\theta = \frac{b}{\sqrt{3}}$ $\quad b < 0$

$E = (\vec{i}, \vec{j}, \vec{k})$ $\quad |\vec{x}\cdot\vec{j}| = a\cdot 0 + b\cdot 1 + c\cdot 0$

$|\vec{x}\cdot\vec{j}| = b$

$\begin{cases} -3a + 3c = 0 \\ 2a - 2b = 0 \end{cases}$ $\quad \sqrt{a^2+b^2+c^2} = \sqrt{3}$ $\quad a = \pm 1 \to$ como os valores precisam ser negativos para o ângulo ser obtuso,

$a = b \quad a = c$ $\quad a^2+b^2+c^2 = 3$ $\quad a=b=c=-1$

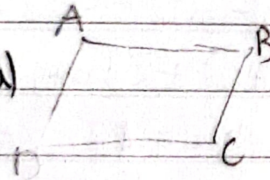
$\therefore a < 0$ $\quad 2a^2 + c^2 = 3$

$b < 0$ $\quad \cancel{3}a^2 = \cancel{3}$

$c < 0$ $\quad a^2 = 1$

$a = \pm\sqrt{1}$ $\quad \vec{x} = (-1, -1, -1)$

13) a)



$\vec{AB} = (1, 1, -1)$ $\quad D = (5, 3, 3)$

$A = (3, 2, -1)$

$\vec{AD} = D - A$ $\quad \|\vec{AB} \times \vec{AD}\| \Rightarrow$

$\vec{AD} = (5,3,3) - (3,2,-1)$

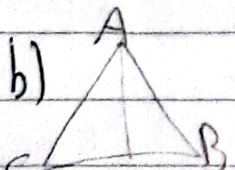
$\vec{AD} = (2, 1, 4)$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 2 & 1 & 4 \end{vmatrix}$ $\quad \vec{i}(4 - -1) + $ $= \vec{j}(4 - (-2)) +$ $\vec{k}(1 - 2)$

$5\vec{i} - 6\vec{j} - \vec{k} = (5, -6, -1)$

$\sqrt{5^2 + (-6)^2 + (-1)^2} = \sqrt{25+36+1} = \sqrt{62}$

b)



$\vec{AB} = (-1, 1, 0)$ $\quad \|\vec{AB} \times \vec{AC}\| \Rightarrow$

$\vec{AC} = (0, 1, 3)$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$ $\quad 3i - j(-3) + \vec{k}(-1)$ $= 3\vec{i} + 3\vec{j} - \vec{k}$

$\sqrt{3^2+3^2+1^2} = \frac{\sqrt{19}}{2}$

$A_\Delta = \frac{\sqrt{19}}{2}$

$\frac{\sqrt{19}}{\cancel{2}} = \frac{\|\vec{BC}\|\cdot h}{\cancel{2}}$ $\quad \vec{BC} = \vec{BA} + \vec{AC}$

$\vec{BC} = (1, -1, 0) + (0, 1, 3)$ $\quad \Rightarrow \sqrt{19} = \sqrt{10}\cdot h$

$\vec{BC} = (1, 0, 3)$ $\quad \|\vec{BC}\| = \sqrt{1^2+3^2} = \sqrt{10}$ $\quad h = \sqrt{\frac{19}{10}} = \sqrt{1{,}9}$

14 a) $\vec{u} = (x_1, y_1, z_1)$ , $\vec{v}(x_2, y_2, z_2)$ , $\vec{z}(x_3, y_3, z_3)$

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = (u \times v) \cdot w = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \quad \vec{i}(y_2 z_3 - z_2 y_3) + \vec{j}(-x_2 z_3 + z_2 x_3) + \vec{k}(x_2 y_3 - y_2 x_3)$$

$\Rightarrow$ chamando cada um dos termos de, respectivamente, $T_1, T_2, T_3$

$\vec{v} \times \vec{w} = (T_1, T_2, T_3)$

$\vec{u} \cdot (T_1, T_2, T_3) \Rightarrow x_1 T_1 + x_2 T_2 + x_3 T_3$

$$x_1 |det_1| + x_2 \left[-|det_2|\right] + x_3 |det_3|$$

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} //$$

Seguindo a mesma lógica para: $(\vec{u} \times v) \cdot \vec{w}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = \underbrace{\vec{i}\,|det_1|}_{D_1} + \underbrace{\vec{j}\left[-|det_2|\right]}_{D_2} + \underbrace{\vec{k}\,|det_3|}_{D_3}$$

$$(D_1, D_2, D_3) \cdot \vec{w} = x_3 D_1 + y_3 D_2 + z_3 D_3 = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

usando essa linha para expandir dessa vez, geramos os mesmos resultados,

14) b) $[\vec{u}, \vec{v}, \vec{w}] = 12$

$$\begin{vmatrix} 1 & 3 & 2 \\ 0 & 1 & -2 \\ -1 & 2 & 0 \end{vmatrix} = -1[3(-2)-2] - 2[-2] = -(-6-2) + 4 \quad 8+4=12$$

$[\vec{u}, \vec{w}, \vec{v}]$ = permuta linhas; $*(-1) = -12$

$[v, 2\vec{u}, \vec{u}$ = 2 permuta linhas + 2 · l; $*(-1)^2$ 2· $= 12 \cdot 2 \cdot \cancel{1}^2 = 24$

$[\vec{u}, 3\vec{v} - 2\vec{u}, \vec{w} + 3\vec{u}] = 36$

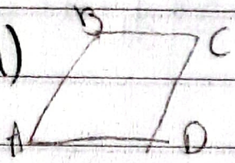
$\hookrightarrow [\vec{u}, 3\vec{v}, \vec{w} + 3\vec{u}] + [\vec{u}, -2\vec{u}, \vec{w} + 3\vec{u}]$ ▷ * multipla "

$3[\vec{u}, \vec{v}, \vec{w} + 3\vec{u}]$

$3[\vec{u}, \vec{v}, \vec{w}] + \cancel{3[\vec{u}, \vec{v}, 3\vec{u}]}$

$3 \cdot 12 = 36$

15) a)




$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 0 & 3 & 3 \end{vmatrix} = \vec{i}[-3] - j[3-0] + k[-3] = -3i - 3\vec{j} - 3\vec{k} \quad \sqrt{27}$$

$\vec{AB} = \vec{AF} + \vec{EA}$

$\vec{AB} = \vec{AF} + \vec{EB} + \vec{BA}$

$\vec{AB} = \vec{AF} - \vec{BE} - \vec{AB}$

$\vec{AB} = \vec{AF} - (\vec{BE} + \vec{AB})$

$\vec{AB} = \vec{AF} - (3, 2, 3)$

$= (3, 5, 6) - (3, 2, 3) = (0\ 3\ 3)$

$\|\vec{AB} \times \vec{AD}\| = \sqrt{27} = 3\sqrt{3}$

AE

$D(3, 2, 3)$

b) a área do paralelepípedo será dada por $[\vec{AB}, \vec{AD}, \vec{AE}]$

$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 3 & 3 \\ 3 & 2 & 3 \end{vmatrix} = (3 \cdot 3 - 3 \cdot 2) + (-3 \cdot 3) = 3 - 9 = -6$$

$\to$ volume é positivo, então volume $= 6$

/ /

c) Altura em relação à face ABCD ⇒ $h \cdot ABCD = \text{volume}$

$h = \text{volume}/ABCD$

$h = \frac{6}{\sqrt{27}} = \frac{6}{3\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

d) Volume Tetraedro EABD $= \frac{1}{3} Ab \cdot h = \frac{1}{3} \cdot \frac{\sqrt{27}}{2} \cdot \frac{6}{\sqrt{27}} = \frac{6}{6} = 1$

$Ab = \frac{ABCD}{2}$ ou $\frac{1}{6} \cdot$ volume do paralelepípedo

e) volume EABD $= \frac{1}{3} Ab \cdot h = \frac{1}{3} \frac{\sqrt{27}}{2} \cdot h = 1 \Rightarrow h = \frac{6}{\sqrt{27}} = \frac{6\sqrt{27}}{27}$

$\frac{6 \cdot 3\sqrt{3}}{27} = \frac{2\sqrt{3}}{3}$