João Antônio de Souza Martins – Lista 1

① $A=\begin{pmatrix}1 & 0\\ 2 & 7\end{pmatrix}$ $B=\begin{pmatrix}0 & 5\\ 3 & -2\end{pmatrix}$ $C=\begin{pmatrix}-2 & 3 & -7\\ 7 & -3 & -2\end{pmatrix}$ $D=\begin{pmatrix}-3 & 2 & 0\\ 1 & 1 & 4\\ -2 & 0 & 2\end{pmatrix}$ $E=\begin{pmatrix}2 & 4 & -3\\ -1 & 0 & -4\\ -6 & 0 & -1\end{pmatrix}$

$F=\begin{pmatrix}1\\ -2\\ 0\end{pmatrix}$

a) $A + 2B$

$$\begin{pmatrix}1 & 0\\ 2 & 7\end{pmatrix} + 2\cdot\begin{pmatrix}0 & 5\\ 3 & -2\end{pmatrix}$$

$$\begin{pmatrix}1 & 0\\ 2 & 7\end{pmatrix} + \begin{pmatrix}0 & 10\\ 6 & -4\end{pmatrix}$$

$$\begin{pmatrix}1 & 10\\ 8 & 3\end{pmatrix} \;//$$

b) $A\cdot B - B\cdot A$

$$\begin{pmatrix}1 & 0\\ 2 & 7\end{pmatrix}\cdot\begin{pmatrix}0 & 5\\ 3 & -2\end{pmatrix} - \begin{pmatrix}0 & 5\\ 3 & -2\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\ 2 & 7\end{pmatrix}$$

$$\begin{pmatrix}1\cdot 0+0\cdot 3 & 1\cdot 5+0\cdot(-2)\\ 2\cdot 0+7\cdot 3 & 2\cdot 5+7\cdot(-2)\end{pmatrix} - \begin{pmatrix}0\cdot 1+5\cdot 2 & 0\cdot 0+5\cdot 7\\ 3\cdot 1+(-2)\cdot 2 & 3\cdot 0+(-2)\cdot 7\end{pmatrix}$$

$$\begin{pmatrix}0 & 5\\ 21 & -4\end{pmatrix} - \begin{pmatrix}10 & 35\\ -1 & -14\end{pmatrix} = \begin{pmatrix}-10 & -30\\ 22 & 10\end{pmatrix} \;//$$

c) $2C - D$

$$2\cdot\begin{pmatrix}-2 & 3 & 7\\ 7 & 3 & -2\end{pmatrix} - \begin{pmatrix}-3 & 2 & 0\\ 1 & 1 & 4\\ -2 & 0 & 2\end{pmatrix}$$

$2\times 3 \neq 3\times 3$

não é possível //

d) $2D^t - 3E^t$

$$2\cdot\begin{pmatrix}-3 & 2 & 0\\ 1 & 1 & 4\\ -2 & 0 & 2\end{pmatrix}^t - 3\cdot\begin{pmatrix}2 & 4 & -3\\ -1 & 0 & -4\\ -6 & 0 & -1\end{pmatrix}^t$$

$$\begin{pmatrix}-6 & 4 & 0\\ 2 & 2 & 8\\ -4 & 0 & 4\end{pmatrix}^t - \begin{pmatrix}6 & 12 & -9\\ -3 & 0 & -12\\ -18 & 0 & -3\end{pmatrix}^t$$

$$\begin{pmatrix}-6 & 2 & -4\\ 4 & 2 & 0\\ 0 & 8 & 4\end{pmatrix} - \begin{pmatrix}6 & -3 & -18\\ 12 & 0 & 0\\ -9 & -12 & -3\end{pmatrix} = \begin{pmatrix}-12 & 5 & 14\\ -8 & 2 & 0\\ 9 & 20 & 7\end{pmatrix} \;//$$

e) $D^2 + D\cdot E$

$$\begin{pmatrix}-3 & 2 & 0\\ 1 & 1 & 4\\ -2 & 0 & 2\end{pmatrix}\cdot\begin{pmatrix}-3 & 2 & 0\\ 1 & 1 & 4\\ -2 & 0 & 2\end{pmatrix} + \begin{pmatrix}-3 & 2 & 0\\ 1 & 1 & 4\\ -2 & 0 & 2\end{pmatrix}\cdot\begin{pmatrix}2 & 4 & -3\\ -1 & 0 & -4\\ -6 & 0 & -1\end{pmatrix}$$

$$\begin{pmatrix}-3\cdot(-3)+2\cdot 1+0\cdot(-2) & -3\cdot 2+2\cdot 1+0\cdot 0 & -3\cdot 0+2\cdot 4+0\cdot 2\\ 1\cdot(-3)+1\cdot 1+4\cdot(-2) & 1\cdot 2+1\cdot 1+4\cdot 0 & 1\cdot 0+1\cdot 4+4\cdot 2\\ -2\cdot(-3)+0\cdot 1+2\cdot(-2) & -2\cdot 2+0\cdot 1+2\cdot 0 & -2\cdot 0+0\cdot 4+2\cdot 2\end{pmatrix} + \begin{pmatrix}-3\cdot 2+2\cdot(-1)+0\cdot 6 & 3\cdot 4+2\cdot 0+0\cdot 0 & -3\cdot(-3)+2\cdot(-4)+0\cdot(-1)\\ 1\cdot 2+1\cdot(-1)+4\cdot(-6) & 1\cdot 4+1\cdot 0+4\cdot 0 & 1\cdot(-3)+1\cdot(-4)+4\cdot(-1)\\ -2\cdot 2+0\cdot(-1)+2\cdot(-6) & -2\cdot 4+0\cdot 0+2\cdot 0 & -2\cdot(-3)+0\cdot(-4)+2\cdot(-1)\end{pmatrix}$$

$$\underbrace{\begin{pmatrix}11 & -4 & 8\\ -10 & 3 & 12\\ 10 & -4 & 4\end{pmatrix}}_{D^2} + \underbrace{\begin{pmatrix}-8 & -12 & 1\\ -23 & 4 & -11\\ -16 & -8 & 4\end{pmatrix}}_{D\cdot E} = \begin{pmatrix}3 & -16 & 9\\ -33 & 7 & 1\\ -6 & -12 & 8\end{pmatrix} \;//$$

f) $C^t \cdot A$

$\begin{pmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{pmatrix}^t \cdot \begin{pmatrix} 1 & 0 \\ 2 & 7 \end{pmatrix}$

$\underset{3\times 2}{\begin{pmatrix} -2 & 7 \\ 3 & -3 \\ -7 & -2 \end{pmatrix}} \cdot \underset{2\times 2}{\begin{pmatrix} 1 & 0 \\ 2 & 7 \end{pmatrix}}$

$\begin{pmatrix} -2\cdot 1 + 7\cdot 2 & -2\cdot 0 + 7\cdot 7 \\ 3\cdot 1 + (-3)\cdot 2 & 3\cdot 0 + (-3)\cdot 7 \\ -7\cdot 1 + (-2)\cdot 2 & -7\cdot 0 + (-2)\cdot 7 \end{pmatrix}$

$\begin{pmatrix} 12 & 49 \\ -3 & -21 \\ -11 & -14 \end{pmatrix}$ //

g) $E - A \cdot C$

$E_{3\times 3} - A_{2\times 2} \cdot C_{2\times 3}$

$E_{3\times 3} - (AC)_{2\times 3}$

Não é possível //

h) $F^t E$

$\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}^t_{3\times 1} \cdot \begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix}_{3\times 3}$

$\begin{pmatrix} 1 & -2 & 0 \end{pmatrix}_{1\times 3} \cdot \begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix}_{3\times 3}$

$a_{13} = 1\cdot 2 + (-2)\cdot(-1) + 0\cdot(-6)$

$a_{23} = 1\cdot 4 + (-2)\cdot 0 + 0\cdot 0$

$a_{33} = 1\cdot(-3) + (-2)\cdot(-4) + \cancel{0\cdot(-1)}$

$F^t \cdot E = \begin{pmatrix} 4 & 4 & 5 \end{pmatrix}$ //

i) $(BC)F$

$\begin{pmatrix} 0 & 5 \\ 3 & -2 \end{pmatrix} \cdot \begin{pmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{pmatrix} =$

$\begin{pmatrix} 0\cdot(-2) + 5\cdot 7 & 0\cdot 3 + 5\cdot(-3) & 0\cdot(-7) + 5\cdot(-2) \\ 3\cdot(-2) + (-2)\cdot 7 & 3\cdot 3 + (-2)\cdot(-3) & 3(-7) + (-2)\cdot(-2) \end{pmatrix}$

$\to \underset{2\times 3}{\begin{pmatrix} 35 & -15 & -10 \\ -20 & 15 & -17 \end{pmatrix}} \cdot \underset{3\times 1}{\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}} = \begin{pmatrix} 35\cdot 1 + (-15)\cdot(-2) + (-10)\cdot 0 \\ -20\cdot 1 + 15\cdot(-2) + (-17)\cdot 0 \end{pmatrix} = \begin{pmatrix} 65 \\ -50 \end{pmatrix}$ //

---

(2) a) $A_{2\times 3} B_{3\times 4}$
$C_{2\times 4}$
BA indefinido

b) $A_{4\times 1} B_{1\times 2}$
$C_{4\times 2}$, BA indefinido

c) $A_{1\times 2} B_{3\times 1}$
C indefinido
BA definido

d) $A_{5\times 2} B_{2\times 3}$
$C_{5\times 3}$, BA indefinido

e) $A_{4\times 4} B_{3\times 3}$
C indefinido
BA indefinido

f) $A_{4\times 2} B_{2\times 4}$
$C_{4\times 4}$, BA definido

g) $A_{2\times 1} B_{1\times 3}$
$C_{2\times 3}$
BA indefinido

h) $A_{2\times 2} B_{2\times 2}$
$C_{2\times 2}$, BA definido

---

(3) a) $\underset{2\times 3}{a_{ij}} = 3i - 2j$ $\quad A = \begin{pmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \end{pmatrix}$ //

$\begin{pmatrix} 3.1-2.1 & 3.1-2.2 & 3.1-2.3 \\ 3.2-2.1 & 3.2-2.2 & 3.2-2.3 \end{pmatrix}$

b) $b_{ij\,3\times 3} \begin{cases} 3i+j, & \text{se } i=j \\ i^2-j, & \text{se } i\neq j \end{cases}$

$\begin{pmatrix} 3\cdot 1+1 & 1-2 & 1-3 \\ 2^2-1 & 3\cdot 2+2 & 2^2-3 \\ 3^2-1 & 3^2-2 & 3\cdot 3+3 \end{pmatrix}$

$B = \begin{pmatrix} 4 & -1 & -2 \\ 3 & 8 & 1 \\ 8 & 7 & 12 \end{pmatrix}$ //

c) $c_{ij}\ 1\times4 = j^{i}$

$C = \begin{pmatrix} 1^1 & 2^1 & 3^1 & 4^1 \end{pmatrix}$

$C = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}$ //

d) $d_{ij}\ 4\times4 = \begin{cases} i^2+j^2, & \text{se } i=j \\ 2ij, & \text{se } i\neq j \end{cases}$

$$\begin{pmatrix} 1^2+1^2 & 2\cdot1\cdot2 & 2\cdot1\cdot3 & 2\cdot1\cdot4 \\ 2\cdot2\cdot1 & 2^2+2^2 & 2\cdot2\cdot3 & 2\cdot2\cdot4 \\ 2\cdot3\cdot1 & 2\cdot3\cdot2 & 3^2+3^2 & 2\cdot3\cdot4 \\ 2\cdot4\cdot1 & 2\cdot4\cdot2 & 2\cdot4\cdot3 & 4^2+4^2 \end{pmatrix}$$

$$D = \begin{bmatrix} 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 6 & 12 & 18 & 24 \\ 8 & 16 & 24 & 32 \end{bmatrix}$$ //

4) $A = \begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 2 \\ 1 & 4 & 5 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -7 \end{pmatrix}$

a) $(BA)_{23} = B_{l_2} \cdot A_{c_3}$

$2\cdot1 + (-1)\cdot2 + 4\cdot5 =$

20 //

b) $(AB)_{23} = A_{l_2} \cdot B_{c_3}$

$-2\cdot3 + (-3)\cdot4 + 2\cdot(-7)$

-32 //

c) $(B^2)_{31} = B_{l_3} \cdot B_{c_1}$

$-3\cdot1 + (-1)\cdot2 + (-7)\cdot(-3)$

16 //

d) $tr(A) = 1 + (-3) + 5$

$tr(A) = 3$ //

e) $tr(B^t) = tr\,B = 1 + (-1) + -7 = -7$ //

f) $tr(A-B) = tr(A) - tr(B)$

$3 - (-7) = 10$ //

g) $tr(AB) = (AB)_{11} + (AB)_{22} + (AB)_{33} = 2 + 1 - 16 = -13$ //

$(AB)_{11} = 1\cdot1 + 2\cdot2 + 1\cdot(-3) = 2$

$(AB)_{22} = -2\cdot0 + (-3)(-1) + 2(-1) = 1$

$(AB)_{33} = 1\cdot3 + 4\cdot4 + 5\cdot(-7) = -16$

5) $A = \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$ $C = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$

a) $2X + A = 3B + C$

$2X + \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} = 3\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$

$2X = \begin{pmatrix} 6 & 3 \\ 12 & 9 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix}$

$2X = \begin{pmatrix} 7 & -2 \\ 11 & 3 \end{pmatrix}$

$\frac{2X}{2} = \begin{pmatrix} 7 & -2 \\ 11 & 3 \end{pmatrix} \cdot \frac{1}{2}$

$X = \begin{pmatrix} 3,5 & -1 \\ 5,5 & 1,5 \end{pmatrix}$ //

b) $Y + A = \frac{1}{2}(B-C)^T$

$$Y + \begin{pmatrix}-1 & 7\\ 2 & 6\end{pmatrix} = \frac{1}{2}\left[\begin{pmatrix}2 & 1\\ 4 & 3\end{pmatrix}^T - \begin{pmatrix}0 & 2\\ 1 & 0\end{pmatrix}^T\right]$$

$$Y + \begin{pmatrix}-1 & 7\\ 2 & 6\end{pmatrix} = \left[\begin{pmatrix}2 & 4\\ 1 & 3\end{pmatrix} - \begin{pmatrix}0 & 1\\ 2 & 0\end{pmatrix}\right] \cdot \frac{1}{2}$$

$$Y + \begin{pmatrix}-1 & 7\\ 2 & 6\end{pmatrix} = \begin{pmatrix}2 & 3\\ -1 & 3\end{pmatrix} \cdot \frac{1}{2}$$

$$Y + \begin{pmatrix}-1 & 7\\ 2 & 6\end{pmatrix} = \begin{pmatrix}1 & 1,5\\ -0,5 & 1,5\end{pmatrix}$$

$$Y = \begin{pmatrix}1 & 1,5\\ -0,5 & 1,5\end{pmatrix} - \begin{pmatrix}-1 & 7\\ 2 & 6\end{pmatrix}$$

$$Y = \begin{pmatrix}2 & -5,5\\ -2,5 & -4,5\end{pmatrix} //$$

c) $3X + A = B - X$

$$3X + \begin{pmatrix}-1 & 7\\ 2 & 6\end{pmatrix} = \begin{pmatrix}2 & 1\\ 4 & 3\end{pmatrix} - X$$

$$4X = \begin{pmatrix}2 & 1\\ 4 & 3\end{pmatrix} - \begin{pmatrix}-1 & 7\\ 2 & 6\end{pmatrix}$$

$$4X = \begin{pmatrix}3 & -6\\ 2 & -3\end{pmatrix}$$

$$X = \begin{pmatrix}3/4 & -3/2\\ 1/2 & -3/4\end{pmatrix} //$$

d) $\begin{cases} X + Y = 3A \\ X - Y = 2B + C \end{cases}$ (+)

$$2X = 3A + 2B + C$$

$$2X = 3\begin{pmatrix}-1 & 7\\ 2 & 6\end{pmatrix} + 2\begin{pmatrix}2 & 1\\ 4 & 3\end{pmatrix} + \begin{pmatrix}0 & 2\\ 1 & 0\end{pmatrix}$$

$$2X = \begin{pmatrix}-3 & 21\\ 6 & 18\end{pmatrix} + \begin{pmatrix}4 & 2\\ 8 & 6\end{pmatrix} + \begin{pmatrix}0 & 2\\ 1 & 0\end{pmatrix}$$

$$2X = \begin{pmatrix}1 & 25\\ 15 & 24\end{pmatrix} \quad X = \begin{pmatrix}0,5 & 12,5\\ 7,5 & 12\end{pmatrix} //$$

$$\begin{pmatrix}0,5 & 12,5\\ 7,5 & 12\end{pmatrix} + Y = \begin{pmatrix}-3 & 21\\ 6 & 18\end{pmatrix}$$

$$Y = \begin{pmatrix}-3 & 21\\ 6 & 18\end{pmatrix} - \begin{pmatrix}0,5 & 12,5\\ 7,5 & 12\end{pmatrix}$$

$$Y = \begin{pmatrix}-3,5 & 8,5\\ -1,5 & 6\end{pmatrix} //$$

6) $A = \begin{pmatrix}1 & 1/x\\ x & 1\end{pmatrix}$

$$A^2 = 2A$$

$$\underbrace{\begin{pmatrix}1 & \frac{1}{x}\\ x & 1\end{pmatrix} \cdot \begin{pmatrix}1 & \frac{1}{x}\\ x & 1\end{pmatrix}} = \begin{pmatrix}2 & 2/x\\ 2x & 2\end{pmatrix}$$

$$(A^2)_{11} = 1 \cdot 1 + \frac{1}{x} \cdot x = 2$$

$$(A^2)_{12} = 1 \cdot \frac{1}{x} + \frac{1}{x} \cdot 1 = 2/x$$

$$(A^2)_{21} = x \cdot 1 + 1 \cdot x = 2x$$

$$(A^2)_{22} = x \cdot \frac{1}{x} + 1 \cdot 1 = 2$$

$\longrightarrow$

$(A^2)_{11} = 2 \quad (A^2)_{12} = 2/x$

$(A^2)_{21} = 2x \quad (A^2)_{22} = 2$

$$\begin{pmatrix} 2 & 2/x \\ 2x & 2 \end{pmatrix} = \begin{pmatrix} 2 & 2/x \\ 2x & 2 \end{pmatrix} \checkmark$$

$$(A^3) = \begin{pmatrix} 2 & 2/x \\ 2x & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1/x \\ x & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2\cdot1 + \frac{2}{x}\cdot x & 2\cdot1 + 2\cdot\frac{1}{x} \\ 2x\cdot1 + 2x & 2x\cdot\frac{1}{x} + 2\cdot1 \end{pmatrix} = \begin{pmatrix} 8 & 8/x \\ 8x & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 4/x \\ 4x & 4 \end{pmatrix}$$

$$(A^4) = \begin{pmatrix} 4 & \frac{4}{x} \\ 4x & 4 \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4\cdot1 + \frac{4}{x}\cdot x & 4\cdot\frac{1}{x} + \frac{4}{x}\cdot1 \\ 4x\cdot1 + 4\cdot x & 4x\cdot\frac{1}{x} + 4 \end{pmatrix}$$

$$\to A^n = \begin{pmatrix} 2^{n-1} & \frac{2^{n-1}}{x} \\ 2^{n-1}\cdot x & 2^{n-1} \end{pmatrix}$$

$$A^n = 2^{n-1} \cdot A \ //$$

⑦ $\{X = AB \quad Y = AC\}$

a) $A(B+C)$ * distributiva

$AB + AC$

$X + Y \ //$

c) $C^t A^t$

$(A\cdot C)^t$

Y^t

d) $(ABA)C$ → propriedade associativa

$(AB)(AC)$

$XY \ //$

b) $B^t A^t$

$(A\cdot B)^t$

$X^t \ //$ ⇒ (!) produto de duas matrizes transpostas é igual a transposta do produto dessas matrizes em ordem inversa.

⑧ a) $A = \begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$

★ $A = A^t$

$$\begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix} = \begin{pmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{pmatrix}$$

$\cdot\ x+2 = 2x-3$

$\boxed{5 = x} \ //$

$$A = \begin{pmatrix} 4 & 7 \\ 7 & 6 \end{pmatrix} //$$

b) $B = \begin{pmatrix} 0 & -4 & 2 \\ x & 0 & 1-z \\ y & 2z & 0 \end{pmatrix}$

★ $B^t = -B$

$$\begin{pmatrix} 0 & x & y \\ -4 & 0 & 2z \\ 2 & 1-z & 0 \end{pmatrix} = \begin{pmatrix} 0 & 4 & -2 \\ -x & 0 & z-1 \\ y & -2z & 0 \end{pmatrix} \quad \begin{cases} x = 4 \\ y = -2 \\ z = -1 \end{cases} //$$

$1 - z = -2z \quad z = -1$

$1 = -z$

9)

$$3\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2t \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+t & 3 \end{pmatrix} \Rightarrow \begin{pmatrix} 3x & 3y \\ 3z & 3t \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2t \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+t & 3 \end{pmatrix}$$

- $3x = x + 4$; $2x = 4$; $x = 2$
- $3y = 6 + x + y$; $2y = 6 + 2$; $y = 4$
- $3t = 2t + 3$; $t = 3$
- $3z = -1 + z + t$; $2z = -1 + 3$; $z = 1$

$$\boxed{S = \{(x,y,z,t) \mid (x,y,z,t) = (2,4,3,1)\}}$$

---

10)

a) $\begin{pmatrix} \cos\theta & \text{sen}\,\theta \\ -\text{sen}\,\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\text{sen}\,\theta \\ \text{sen}\,\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & -\text{sen}\,\theta \\ \text{sen}\,\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & \text{sen}\,\theta \\ -\text{sen}\,\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

- $\begin{pmatrix} \cos\theta\cdot\cos\theta + \text{sen}\,\theta\cdot\text{sen}\,\theta & \cos\theta\cdot -\text{sen}\,\theta + \text{sen}\,\theta\cos\theta \\ -\text{sen}\,\theta\cdot\cos\theta + \cos\theta\cdot\text{sen}\,\theta & -\text{sen}\,\theta\cdot(-\text{sen}\,\theta) + \cos\theta\cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2 = (R\theta \cdot R\theta^t)$

- $\begin{pmatrix} \cos\theta\cdot\cos\theta + (-\text{sen}\,\theta)\cdot(-\text{sen}\,\theta) & \cos\theta\cdot\text{sen}\,\theta + -\text{sen}\,\theta\cos\theta \\ \text{sen}\,\theta\cdot\cos\theta + \cos\theta\cdot(-\text{sen}\,\theta) & \text{sen}\,\theta\cdot\text{sen}\,\theta + \cos\theta\cdot\cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2 = (R\theta^t \cdot R\theta)$

$\text{sen}^2\theta + \cos^2\theta = 1$

b) $AA^t = A^tA = I_n$

$$\begin{pmatrix} 1 & 0 & x \\ 0 & \frac{1}{\sqrt2} & y \\ 0 & 1/\sqrt2 & z \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ x & y & z \end{pmatrix} = \begin{pmatrix} 1+0+x^2 & 0+0+xy & xz \\ xy & \frac12 + y^2 & \frac12 + yz \\ xz & \frac12 + xz & \frac12 + z^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$x^2 + 1 = 1$

$x = 0$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ 0 & y & z \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt2} & y \\ 0 & \frac{1}{\sqrt2} & z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{y}{\sqrt2} + \frac{z}{\sqrt2} \\ 0 & y\cdot\frac{1}{\sqrt2} + z\cdot\frac{1}{\sqrt2} & y^2 + z^2 \end{pmatrix}$$

$\frac{y}{\sqrt2} + \frac{z}{\sqrt2} = 0 \Rightarrow y = -z$

$y^2 + z^2 = 1$

$y^2 + (-y)^2 = 1$

$2y^2 = 1 \quad y = \pm\sqrt{\frac12}$

$$S = \{(x,y,z) \mid x = 0,\ y \in \{1/\sqrt2 ; -1/\sqrt2\},\ z = -y\}$$