João Antônio de Souza Martins - Lista 2

1) a) $\begin{vmatrix} 1 & 2 \\ -4 & 3 \end{vmatrix}$ $1 \cdot (-1)^2 \cdot 3 + 2 \cdot (-1)^3 \cdot (-4)$

$3 + 8$ $\quad det = 11$

b) $\begin{vmatrix} \sqrt{2} & 3\sqrt{6} \\ 2 & \sqrt{3} \end{vmatrix}$ $\sqrt{2} \cdot (-1)^2 \cdot \sqrt{3} + 3\sqrt{6} \cdot (-1)^3 \cdot 2$ $\quad det = -5\sqrt{6}$

$\sqrt{6} + (-6\sqrt{6})$

c) $\begin{vmatrix} \pi & 0 & 2 \\ 5 & -1 & 1 \\ 1 & 0 & 0 \end{vmatrix}$ $1 \cdot (-1)^4 \cdot \begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} + 0 \cdot (-1)^5 \cdot \begin{vmatrix} & \\ & \end{vmatrix} + 0 \cdot (-1)^6 \cdot \begin{vmatrix} & \\ & \end{vmatrix}$

$\cancel{0 \cdot (-1)^2 \cdot 1} + 2 \cdot (-1)^3 \cdot -1$ $\quad det = 2$

d) $\begin{vmatrix} -2 & 1 & -1 \\ 1 & 5 & 4 \\ -3 & 4 & 2 \end{vmatrix}$ $-2 \cdot \cancel{(-1)^2} \cdot \begin{vmatrix} 5 & 4 \\ 4 & 2 \end{vmatrix} + 1 \cdot (-1)^3 \begin{vmatrix} 1 & 4 \\ -3 & 2 \end{vmatrix} + (-1) \cdot \cancel{(-1)^4} \cdot \begin{vmatrix} 1 & 5 \\ -3 & 4 \end{vmatrix}$

$-2 \cdot \begin{vmatrix} 5 & 4 \\ 4 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 4 \\ -3 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 5 \\ -3 & 4 \end{vmatrix}$

$-2 \cdot (5 \cdot \cancel{(-1)^2} \cdot 2 + 4 \cdot (-1)^3 \cdot 4) - (1 \cdot \cancel{(-1)^2} \cdot 2 + 4 \cdot (-1)^3 \cdot (-3)) - (1 \cdot \cancel{(-1)^2} \cdot 4 + 5 \cdot (-1)^3 \cdot (-3))$

$-2 \cdot (-6) - 14 - 19 \Rightarrow 12 - 14 - 19$ $\quad det = -21$

e) $\begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & 5 \\ 2 & 4 & 2 \end{vmatrix}$ $\cancel{0 \cdot (-1)^2 \begin{vmatrix} & \\ & \end{vmatrix}} + 2 \cdot (-1)^3 \cdot \begin{vmatrix} 1 & 5 \\ 2 & 2 \end{vmatrix} + \cancel{0 \cdot (-1)^4 \cdot \begin{vmatrix} & \\ & \end{vmatrix}}$

$-2 \cdot \begin{vmatrix} 1 & 5 \\ 2 & 2 \end{vmatrix}$ $\quad 1 \cdot \cancel{(-1)^2} \cdot 2 + 5 \cdot (-1)^3 \cdot 2$ $\quad det = 16$

$2 + -10 \quad -8 \cdot (-2) \Rightarrow$

f) $\begin{vmatrix} 3 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix}$ $3 \cdot \cancel{(-1)^2} \cdot \begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} + 0 + 0 + 0$

$3 \cdot \left(\cancel{1 \cdot (-1)^2} \cdot \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} + 0 + \cancel{1 \cdot (-1)^4} \begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix} \right)$

$det = -6$

$\cancel{1 \cdot (-1)^2} \cdot (-1) + 0 + -1 \cdot \cancel{(-1)^2} \cdot 1 + 0$

$-1 + (-1)$

$3 \cdot (-2) = -6$

g) $\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 5 & 1 & 2 & 5 & 3 \\ 7 & 2 & \sqrt{5} & 0 & 0 \\ 10 & -3 & 6 & 1 & 0 \\ -2 & -3 & 0 & 0 & 0 \end{vmatrix}$ $1 \cdot (-1)^2 \cdot \begin{vmatrix} 1 & 2 & 5 & 3 \\ 2 & \sqrt{5} & 0 & 0 \\ -3 & 6 & 1 & 0 \\ -3 & 0 & 0 & 0 \end{vmatrix} \Rightarrow -3 \cdot (-1)^5 \cdot \begin{vmatrix} 2 & 5 & 3 \\ \sqrt{5} & 0 & 0 \\ 6 & 1 & 0 \end{vmatrix} \Rightarrow 3 (\sqrt{5} \cdot (-1)^3 \begin{vmatrix} 5 & 3 \\ 1 & 0 \end{vmatrix})$

$3\sqrt{5} \left(- \begin{vmatrix} 5 & 3 \\ 1 & 0 \end{vmatrix} \right)$

$-3\sqrt{5} \cdot \begin{vmatrix} 5 & 3 \\ 1 & 0 \end{vmatrix}$

$1 \cdot (-1)^3 \cdot 3 + 0$

(-3)

$-3\sqrt{5} \cdot (-3)$ $det = 9\sqrt{5}$ //

h) $\begin{vmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 \end{vmatrix}$ $3 \cdot (-1)^2 \cdot \begin{vmatrix} 0 & 0 & 0 & -2 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 \end{vmatrix} \Rightarrow 3 \cdot (-2) \cdot (-1)^5 \begin{vmatrix} 0 & 0 & -2 \\ 2 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \Rightarrow 6 \cdot 1 \cdot (-1)^5 \cdot \begin{vmatrix} 0 & -2 \\ 2 & 0 \end{vmatrix}$

$6 \cdot 2 \cdot (-1)^3 \cdot (-2)$

$det = 24$ //

② a) $det(A+B)$

$\begin{pmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{pmatrix} + \begin{pmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{pmatrix} = \begin{pmatrix} 7 & -2 & 14 \\ 3 & 2 & 10 \\ 4 & -8 & 2 \end{pmatrix} \Rightarrow \begin{vmatrix} 7 & -2 & 14 \\ 3 & 2 & 10 \\ 4 & -8 & 2 \end{vmatrix} = 2 \begin{vmatrix} 7 & -2 & 14 \\ 3 & 2 & 10 \\ 2 & -4 & 1 \end{vmatrix} = 4 \begin{vmatrix} 7 & -1 & 14 \\ 3 & 1 & 10 \\ 2 & -2 & 1 \end{vmatrix}$

$4 \cdot \begin{vmatrix} 7 & -1 & 14 \\ 3 & 1 & 10 \\ 2 & -2 & 1 \end{vmatrix} \begin{matrix} 7 & -1 \\ 3 & 1 \\ 2 & -2 \end{matrix}$ $-97 - (-115)$

$115 - 97$ $det = 72$ //

$18 \cdot 4$

$(28 - 140 - 3)$ $(7 - 20 - 84)$ 72

b) $det(AB)$

$\begin{pmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{pmatrix} \begin{pmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{pmatrix} = \begin{pmatrix} 3 \cdot 4 + (-5) \cdot (-1) + 7 \cdot 3 & 3 \cdot 3 + 0 + 7 & 3 \cdot 7 - 10 - 28 \\ 16 - 2 + 24 & 12 + 0 + 8 & 28 + 4 - 32 \\ 4 + 9 + 18 & 3 + 0 + 6 & 7 + (-18) - 24 \end{pmatrix} = \begin{pmatrix} 38 & 16 & -17 \\ 38 & 20 & 0 \\ 31 & 9 & -35 \end{pmatrix}$

$$\begin{vmatrix} 38 & 16 & -17 \\ 38 & 20 & 0 \\ 31 & 9 & -35 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & -4 & -17 \\ 38 & 20 & 0 \\ 31 & 9 & 35 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & -4 & -17 \\ 19 & 10 & 0 \\ 31 & 9 & -35 \end{vmatrix} \cdot 2$$

$l_1 \leftarrow l_1 - l_2$

$$\begin{vmatrix} 0 & 4 & -17 \\ 19 & 10 & 0 \\ 31 & 9 & -35 \end{vmatrix}\begin{matrix} 0 & -4 \\ 14 & 10 \\ 31 & 9 \end{matrix}$$

$det = 2(-2907 - [-2610])$

$det = -594$ //

$-17 \cdot 310 + 0 + 1402 \cdot 19 \quad 0 + 0 + 171 \cdot (-17)$

$-5270 + 2660 \quad -2907$

$-2610 \quad -2907$

c) $det(B^t A^t)$ $\quad |X| = |X^t|$

$B^t \cdot A^t = (AB)^t$

$$\begin{pmatrix} 38 & 16 & -17 \\ 38 & 20 & 0 \\ 31 & 9 & -35 \end{pmatrix}^t = \begin{matrix} 38 & 38 & 31 \\ 16 & 26 & 9 \\ -17 & 0 & -35 \end{matrix}$$

$\therefore$

$det = -594$ //

$\hookrightarrow det = -594$

d) $det(3A - 2C + B)$

$$3\begin{pmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{pmatrix} + \begin{vmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 & -1 \\ 6 & 9 & -2 \\ 8 & 12 & -3 \end{vmatrix}$$

$$\begin{pmatrix} 9 & -15 & 21 \\ 12 & 6 & 24 \\ 3 & -27 & 18 \end{pmatrix} + \begin{vmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{vmatrix} - \begin{vmatrix} 4 & 6 & -2 \\ 12 & 18 & -4 \\ 16 & 24 & -6 \end{vmatrix} = \begin{vmatrix} 9 & -18 & 30 \\ -1 & -12 & 30 \\ -10 & -50 & 20 \end{vmatrix} \Rightarrow 10\begin{vmatrix} 9 & -18 & 3 \\ -1 & -12 & 3 \\ -10 & -50 & 2 \end{vmatrix}$$

$$10 \cdot 3\begin{vmatrix} 3 & -6 & 1 \\ -1 & -12 & 3 \\ -10 & -50 & 2 \end{vmatrix} = 30 \cdot 2\begin{vmatrix} 3 & -6 & 1 \\ -1 & -12 & 3 \\ -5 & -25 & 1 \end{vmatrix} = 60\begin{vmatrix} 3 & -6 & 1 \\ -1 & -12 & 3 \\ -8 & -19 & 0 \end{vmatrix}\begin{matrix} 3 & -6 \\ -1 & -12 \\ -8 & -19 \end{matrix}$$

$60 \cdot (163 - (-75))$

$det = 14280$ //

$12 \cdot 8 + 9 \cdot (-19) \quad 18 \cdot 8 + 19$

e) $\det(AC^T)$

$$\underset{A}{\begin{pmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{pmatrix}} \cdot \underset{C^T}{\begin{pmatrix} 2 & 6 & 8 \\ 3 & 9 & 12 \\ -1 & -2 & -3 \end{pmatrix}} \Rightarrow 2\begin{vmatrix} 3 & -5 & 7 \\ 2 & 1 & 4 \\ 1 & -9 & 6 \end{vmatrix}$$

$\det(A \cdot C^T) = \det A \cdot \det C^T$

$\det A \cdot 0$

$\therefore \det(A \cdot C^T) = 0$

$$|C^T| = 2\begin{vmatrix} 1 & 3 & 4 \\ 3 & 9 & 12 \\ -1 & -2 & -3 \end{vmatrix} \quad 2 \cdot 3\begin{vmatrix} 1 & 3 & 4 \\ 1 & 3 & 4 \\ -1 & -2 & -3 \end{vmatrix} \to \det = 0$$

③ $A_{4\times4}$; $\det(A) = -2$

a) $\det(A^T) = \det A$
$= -2$

c) $\det(A^7)$
$\boxed{\det(A)}^7$
$(-2)^7 = -128$

d) $\det(A^{-1}) = \dfrac{1}{\det A}$
$= -\dfrac{1}{2}$

b) $\det(6A)$
$6^4 \cdot \det A$
$6^4 \cdot -2 = -2592$

④ $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -3$

a) $\begin{vmatrix} a & b & c \\ d & e & f \\ 4g & 4h & 4i \end{vmatrix} = 4\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4 \cdot (-3)$
-12

b) $\begin{vmatrix} a & b & -2c \\ 3d & 3e & -6f \\ g & h & -2i \end{vmatrix} = 3\begin{vmatrix} a & b & -2c \\ d & e & -2f \\ g & h & -2i \end{vmatrix} = -6\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$
$-6 \cdot (-3)$
18

c) $\begin{vmatrix} -a & -b & -c \\ g & h & i \\ -d & -e & -f \end{vmatrix} \Rightarrow (-1)^2 \begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix}$

$-\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$

d) $\begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix} = -\begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix} = (-1)^2 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$
-3

Digitalizado com CamScanner

e) $\begin{vmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{vmatrix} = \begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g & h & i \end{vmatrix} + \begin{vmatrix} a & b & c \\ a & b & c \\ g & h & i \end{vmatrix} \Rightarrow 2\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \quad 2\cdot|-3| = -6$

$\det = 0$

f) $\begin{vmatrix} ka+a & kb+b & ck+c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} ka & kb & kc \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = k\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -3k-3$

$-3(k+1)$

5) $A = \begin{bmatrix} 10 & 8 & 40 & -2 \\ 4 & 6 & 20 & 4 \\ -5 & -7 & -30 & 1 \\ 3 & -6 & -30 & 12 \end{bmatrix}$ $\det = 10\begin{vmatrix} 10 & 8 & 4 & -2 \\ 4 & 6 & 2 & -4 \\ -5 & -7 & -3 & 1 \\ 3 & -6 & -3 & 12 \end{vmatrix} = 10\cdot 2^2 \begin{vmatrix} 5 & 4 & 2 & -1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 1 \\ 3 & -6 & -3 & 12 \end{vmatrix} = 40\cdot 3\begin{vmatrix} 5 & 4 & 2 & -1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 1 \\ 1 & -2 & -1 & 4 \end{vmatrix}$

$120\begin{vmatrix} 5 & 4 & 2 & -1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 1 \\ 1 & -2 & -1 & 4 \end{vmatrix} = 120\begin{vmatrix} 0 & -3 & -1 & 0 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 1 \\ 1 & -2 & -1 & 4 \end{vmatrix} = 120\begin{vmatrix} 2 & 0 & 0 & -2 \\ 2 & 3 & 1 & -2 \\ -5 & 7 & -3 & 1 \\ 1 & -2 & -1 & 4 \end{vmatrix} = 120\cdot(-1)^2\begin{vmatrix} 2 & 0 & 0 & -2 \\ 2 & 3 & 1 & -2 \\ 5 & 7 & 3 & -1 \\ -1 & 2 & 1 & -4 \end{vmatrix}$

$l_1 \leftarrow l_1 + l_3 \quad l_1 \leftarrow l_1 + l_2$

$-120\begin{vmatrix} 2 & 0 & 0 & 2 \\ 2 & 3 & 1 & 2 \\ 5 & 7 & 3 & 1 \\ -1 & 2 & 1 & 4 \end{vmatrix} = -120\begin{vmatrix} 2 & 0 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 5 & 7 & 3 & -4 \\ -1 & 2 & 1 & 5 \end{vmatrix} \Rightarrow 2\cdot(-1)^2\begin{vmatrix} 3 & 1 & 0 \\ 7 & 3 & -4 \\ 2 & 1 & 5 \end{vmatrix} + 0 + 0 + 0 \quad -240\begin{vmatrix} 3 & 1 & 0 \\ 7 & 3 & -4 \\ -1 & 0 & 5 \end{vmatrix}$

$C_4 \leftarrow C_4 - C_1 \quad l_3 \leftarrow l_3 - l_1$

$\begin{vmatrix} 3 & 1 & 0 \\ 7 & 3 & -4 \\ -1 & 0 & 5 \end{vmatrix}\begin{matrix} 3 & 1 \\ 7 & 3 \\ -1 & 0 \end{matrix}$ $49-35$ $-240\cdot 14 =$

14 -3360

$4+0+35 \quad 45+4+0$

6 a) $\begin{vmatrix} 4 & 6 & x \\ 7 & 4 & 2x \\ 5 & 2 & -x \end{vmatrix} = -128$ $\quad 2x\begin{vmatrix} 4 & 3 & 1 \\ 7 & 2 & 2 \\ 5 & 1 & -1 \end{vmatrix} = -128$ $\quad x\begin{vmatrix} 9 & 4 & 0 \\ 7 & 2 & 2 \\ 5 & 1 & -1 \end{vmatrix} = -64$ $\quad x\begin{vmatrix} 9 & 4 & 0 \\ 17 & 4 & 0 \\ 5 & 1 & -1 \end{vmatrix} = -64$

$l_1 \leftarrow l_1 + l_3 \qquad l_2 \leftarrow l_2 + 2 \cdot l_3 \qquad l_2 \leftarrow l_2 - l_1$

$x\begin{vmatrix} 9 & 4 & 0 \\ 8 & 0 & 0 \\ 5 & 1 & -1 \end{vmatrix} = -64$ $\qquad \begin{vmatrix} 9 & 4 & 0 \\ 8 & 0 & 0 \\ 5 & 1 & -1 \end{vmatrix}\begin{matrix} 9 & 4 \\ 8 & 0 \\ 5 & 1 \end{matrix}$ $\qquad x \cdot 32 = -64$

$x = -2$ //

$0 - (-32) \qquad det = 32$

b) $\begin{vmatrix} 3 & 5 & 7 \\ 2x & x & 3x \\ 4 & 6 & 7 \end{vmatrix} = 39$ $\quad x\begin{vmatrix} 3 & 5 & 7 \\ 2 & 1 & 3 \\ 4 & 6 & 7 \end{vmatrix} = 39$ $\quad x\begin{vmatrix} 3 & 5 & 7 \\ 2 & 1 & 3 \\ 1 & 1 & 0 \end{vmatrix} = 39$ $\quad x\begin{vmatrix} 3 & 5 & 7 \\ 1 & 0 & 3 \\ 1 & 1 & 0 \end{vmatrix} = 39$

$l_3 \leftarrow l_3 - l_1 \qquad l_2 \leftarrow l_2 - l_3$

$\begin{vmatrix} 3 & 5 & 7 \\ 1 & 0 & 3 \\ 1 & 1 & 0 \end{vmatrix}\begin{matrix} 3 & 5 \\ 1 & 0 \\ 1 & 1 \end{matrix}$ $\qquad 15 + 7 - 9$

$13 \qquad x \cdot 13 = 39$

$x = 3$ //

$9 \qquad (15 + 7)$

b) $\begin{vmatrix} x+3 & x+1 & x+4 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix} = -7$ $\quad \begin{vmatrix} x & x & x \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix} + \begin{vmatrix} 3 & 1 & 4 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix} = -7$ $\quad \underbrace{x\begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix}}_{det\ A} + \underbrace{\begin{vmatrix} 3 & 1 & 4 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix}}_{det\ B} = -7$

$det\ A = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & 3 \\ 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 4 & 5 & 3 \\ 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 4 & 0 & 3 \\ 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{vmatrix}\begin{matrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{matrix}$ $\quad 0 - 1 \quad -1$

$l_3 \leftarrow l_3 - 2l_2 \quad l_1 \leftarrow l_1 - l_3 \quad l_2 \leftarrow l_2 - 5l_1 \quad l_2 \leftarrow l_2 - 3l_3 \qquad 1$

$det\ B = \begin{vmatrix} 3 & 1 & 4 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 4 \\ 4 & 5 & 3 \\ 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 4 \\ 1 & 5 & 3 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 0 \\ 1 & 5 & 3 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 0 \\ 1 & 5 & 0 \\ 0 & 0 & 1 \end{vmatrix}\begin{matrix} -1 & 1 \\ 1 & 5 \\ 0 & 0 \end{matrix}$

$1 \quad -5 \quad -6$

$l_3 \leftarrow l_3 - 2l_2 \quad c_1 \leftarrow c_1 - c_3 \quad l_1 \leftarrow l_1 - 4l_3 \quad l_2 \leftarrow l_2 - 3l_3$

$-x - 6 = -7$

$-x = -1$

$x = 1$ //

d) $\begin{pmatrix} x & x+2 \\ 1 & x \end{pmatrix}$ singular → det = 0

$\begin{vmatrix} x & x+2 \\ 1 & x \end{vmatrix}$ $x^2 - x - 2 = 0$ $S=1$ $P=-2$ $x' = 2$ $x'' = -1$

$-x-2$ x^2

$X = 2$ ou $X = -1$ //

e) $\begin{pmatrix} x-4 & 0 & 3 \\ 2 & 0 & x-9 \\ 0 & 3 & 0 \end{pmatrix}$ det $\neq 0$

$\begin{vmatrix} x-4 & 0 & 3 \\ 2 & 0 & x-9 \\ 0 & 3 & 0 \end{vmatrix} \begin{matrix} x-4 & 0 \\ 2 & 0 \\ 0 & 3 \end{matrix}$

$3(x-4)(x-9)$ 18

$18 - 3(x-4)(x-9) \neq 0$

$3(x-4)(x-9) \neq 18$

$(x-4)(x-9) \neq 6$

$x^2 - 9x - 4x + 36 \neq 6$

$x^2 - 13x + 30 = 0$

$S = 13$ $P = 30$ 10 e 3

X pode assumir qualquer valor, exceto 3 e 10 //

7) a) $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $A^{-1} = \dfrac{1}{ad-bc} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ //

Condição de Existência: $ad \neq bc$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$ad - bc \neq 0$

cofatores $\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$ adjunta $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ det A $ad - bc$

b) $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 4 & 7 \\ 1 & 2 \end{pmatrix}$

- $A^{-1} = \dfrac{1}{3 \cdot 2 - 1 \cdot 5} \cdot \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} = \dfrac{1}{1} \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$ //
- $B^{-1} = \dfrac{1}{4 \cdot 2 - 7 \cdot 1} \begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix} = \dfrac{1}{1} \begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix} = \begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix}$ //
- $(AB)^{-1} = B^{-1} A^{-1} = \begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 4+35 & -2-21 \\ -2-20 & 1+12 \end{pmatrix} = \begin{pmatrix} 39 & -23 \\ -22 & 13 \end{pmatrix}$ //

8a) $A=\begin{pmatrix}2 & -2\\ 3 & 1\end{pmatrix}$ cofatora $=\begin{pmatrix}(-1)^2\cdot 1 & (-1)^3\cdot 3\\ (-1)^3\cdot(-2) & (-1)^4\cdot 2\end{pmatrix}=\begin{pmatrix}1 & -3\\ 2 & 2\end{pmatrix}$ //

b) $B=\begin{pmatrix}2 & -2 & 0\\ 1 & 2 & 1\\ 0 & 1 & -1\end{pmatrix}$ cofatora $=\begin{pmatrix}-3 & 1 & 1\\ -2 & -2 & -2\\ -2 & -2 & 6\end{pmatrix}$

$a_{11}=\cancel{(-1)^2}\cdot\begin{vmatrix}2 & 1\\ 1 & -1\end{vmatrix}=-2-1=-3$ $\quad a_{21}=(-1)^3\begin{vmatrix}-2 & 0\\ 1 & -1\end{vmatrix}=-1\cdot 2=-2$

$a_{12}=(-1)^3\cdot\begin{vmatrix}1 & 1\\ 0 & -1\end{vmatrix}=(-1)^3\cdot(-1)=1$ $\quad a_{22}=\cancel{(-1)^4}\begin{vmatrix}2 & 0\\ 0 & -1\end{vmatrix}=-2$

$a_{13}=\cancel{(-1)^4}\begin{vmatrix}1 & 2\\ 0 & 1\end{vmatrix}=1$ $\quad a_{23}=(-1)^5\begin{vmatrix}2 & -2\\ 0 & 1\end{vmatrix}=-1\cdot 2=-2$

$a_{31}=\cancel{(-1)^4}\begin{vmatrix}-2 & 0\\ 2 & 1\end{vmatrix}=-2$ $\quad a_{33}=\cancel{(-1)^6}=\begin{vmatrix}2 & -2\\ 1 & 2\end{vmatrix}=4-(-2)=6$

$a_{32}=(-1)^5\begin{vmatrix}2 & 0\\ 1 & 1\end{vmatrix}=-2$

9a) $A=\begin{pmatrix}2 & -2\\ 3 & 1\end{pmatrix}\Rightarrow cof(A)=\begin{pmatrix}\cancel{(-1)^2}\cdot 1 & (-1)^3\cdot 3\\ (-1)^3\cdot(-2) & \cancel{(-1)^4}\cdot 2\end{pmatrix}=\begin{pmatrix}1 & -3\\ 2 & 2\end{pmatrix}\Rightarrow adj(A)=\begin{pmatrix}1 & 2\\ -3 & 2\end{pmatrix}$

$-6 \quad det=8 \quad 2$

$A^{-1}=\frac{1}{8}\cdot\begin{pmatrix}1 & 2\\ -3 & 2\end{pmatrix}=\begin{pmatrix}\frac{1}{8} & \frac{1}{4}\\ -3/8 & \frac{1}{4}\end{pmatrix}$ //

b) $B=\begin{pmatrix}2 & -2 & 0\\ 1 & 2 & 1\\ 0 & 1 & -1\end{pmatrix}$

$C_{13}=\cancel{(-1)^4}\begin{vmatrix}1 & 2\\ 0 & 1\end{vmatrix}=1$ $\quad C_{23}=(-1)^5\begin{vmatrix}2 & -2\\ 0 & 1\end{vmatrix}=-2$

$C_{21}=(-1)^3\begin{vmatrix}-2 & 0\\ 1 & -1\end{vmatrix}=-2$ $\quad C_{31}=\cancel{(-1)^4}\begin{vmatrix}-2 & 0\\ 2 & 1\end{vmatrix}=-2$

$C_{11}=\cancel{(-1)^2}\begin{vmatrix}2 & 1\\ 1 & -1\end{vmatrix}=-3$

$C_{22}=\cancel{(-1)^4}\begin{vmatrix}2 & 0\\ 0 & -1\end{vmatrix}=-2$ $\quad C_{32}=(-1)^5\begin{vmatrix}2 & 0\\ 1 & 1\end{vmatrix}=-2$

$C_{12}=(-1)^3\begin{vmatrix}1 & 1\\ 0 & -1\end{vmatrix}=1$

$C_{33}=\cancel{(-1)^6}\begin{vmatrix}2 & -2\\ 1 & 2\end{vmatrix}=6$

$\operatorname{cof} B = \begin{pmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & 6 \end{pmatrix}$ $\operatorname{adj} B = \begin{pmatrix} -3 & -2 & -2 \\ 1 & -2 & -2 \\ 1 & -2 & 6 \end{pmatrix}$ $= \frac{1}{8} \begin{pmatrix} -3 & -2 & -2 \\ 1 & -2 & -2 \\ 1 & -2 & 6 \end{pmatrix}$

$\begin{vmatrix} 2 & 2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{vmatrix} = 2 \begin{vmatrix} 1 & -1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{vmatrix} = 2 \begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 0 \\ 0 & 1 & -1 \end{vmatrix} \begin{matrix} 1 & -1 \\ 1 & 3 \\ 0 & 1 \end{matrix}$

-3

$\det = -8$

$B^{-1} = \begin{pmatrix} 3/8 & \frac{1}{4} & \frac{1}{4} \\ -1/8 & \frac{1}{4} & \frac{1}{4} \\ -1/8 & \frac{1}{4} & -3/4 \end{pmatrix}$ //

c) $C = \begin{pmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}$ $C_{11} = (-1)^2 \cdot \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = 1$ $C_{13} = (-1)^4 \cdot \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2$

$C_{12} = (-1)^3 \cdot \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = -1$ $C_{21} = (-1)^3 = \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} = 1$

$C_{22} = (-1)^4 \cdot \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$ $C_{23} = (-1)^5 \cdot \begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix} = -1$ $C_{31} = (-1)^4 \cdot \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix}$ 1

$C_{32} = (-1)^5 \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix} = 2$ $C_{33} = (-1)^6 \begin{vmatrix} 0 & -1 \\ 2 & 0 \end{vmatrix} = 2$ $\frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 2 & -1 & 2 \end{pmatrix}$

$\operatorname{cof} C = \begin{pmatrix} 1 & -1 & 2 \\ 1 & -1 & -1 \\ 1 & 2 & 2 \end{pmatrix}$ $\operatorname{adj} = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 2 \\ 2 & -1 & 2 \end{pmatrix}$ $\begin{vmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix} \begin{matrix} 0 & -1 \\ 2 & 0 \\ 1 & 1 \end{matrix}$ $C^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & 2/3 \\ 2/3 & -\frac{1}{3} & 2/3 \end{pmatrix}$ //

$\det = 1 + 2 = 3$

d) $D = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 3 \\ 0 & 0 & -1 & 0 \end{pmatrix} \Rightarrow \begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 3 \\ 0 & 0 & -1 & 0 \end{vmatrix}$ $1 \cdot (-1)^4 \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \\ 0 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix}$

$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{matrix}$

$- (-1)$ $\det = +1$

$$C_{11} = (-1)^2 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 2 \\ 0 & -1 \end{matrix} = 3$$

$- -3$

$$C_{12} = (-1)^3 \begin{vmatrix} 0 & 0 & 0 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = 0$$

$$C_{13} = (-1)^4 \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$$C_{14} = (-1)^5 \begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & -1 \end{vmatrix} \begin{matrix} 0 & 1 \\ 2 & 0 \\ 0 & 0 \end{matrix} = -2$$

$- - -2$

$$C_{21} = (-1)^3 \begin{vmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} \begin{matrix} 0 & 0 \\ 0 & 2 \\ 0 & -1 \end{matrix} = 0$$

$$C_{22} = (-1)^4 \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} \begin{matrix} 1 & 0 \\ 2 & 2 \\ 0 & -1 \end{matrix} = 1$$

$- -3 \qquad -2$

$$C_{23} = (-1)^5 \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$$C_{24} = (-1)^6 \begin{vmatrix} 1 & 0 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & -1 \end{vmatrix} = 0$$

$$C_{31} = (-1)^4 \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \begin{matrix} 0 & 0 \\ 1 & 0 \\ 0 & -1 \end{matrix} = -1$$

-1

$$C_{32} = (-1)^5 \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = 0$$

$$C_{33} = (-1)^6 \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$$C_{34} = (-1)^7 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix} = 1$$

-1

$$C_{41} = (-1)^5 \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 2 & 3 \end{vmatrix} \begin{matrix} 0 & 0 \\ 1 & 0 \\ 0 & 2 \end{matrix} = -2$$

2

$$C_{42} = (-1)^6 \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 2 & 2 & 3 \end{vmatrix} = 0$$

$$C_{43} = (-1)^7 \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{matrix} = -1$$

$-2 \qquad +3$

$$C_{44} = (-1)^8 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{matrix} = 2$$

2

$$\mathrm{cof}(D) = \begin{pmatrix} 3 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -2 & 0 & -1 & 2 \end{pmatrix}$$

$$\mathrm{adj}(D) = \begin{vmatrix} 3 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 0 & 1 & -2 \end{vmatrix}$$

$*\ \det = 1 \to \mathrm{adj}(D) = D^{-1}$

$$D^{-1} = \begin{pmatrix} 3 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 0 & 1 & -2 \end{pmatrix}$$

Digitalizado com CamScanner

10 a) $2A^{*} = C - XB$

$XB + 2A^{*} = C - XB + XB$

$XB + 2A^{*} = C$

$XB + 2A^{*} - 2A^{*} = C - 2A^{*}$

$XB = C - 2A^{*}$

$XB \cdot B^{-1} = (C - 2A^{*}) \cdot B^{-1}$

$X I_3 = (C - 2A^{*}) B^{-1}$

$X = (C - 2A^{*}) B^{-1}$ //

B precisa ser invertível, ou seja $\det(B) \neq 0$

b) $X = \left[\begin{pmatrix} 4 & 4 & 0 \\ 0 & 8 & -2 \\ -2 & 0 & 6 \end{pmatrix} - 2 \begin{pmatrix} 1 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & -1 & 2 \end{pmatrix}^{*} \right] \cdot \begin{pmatrix} 3 & -2 & 6 \\ 2 & -1 & 5 \\ 1 & 0 & 3 \end{pmatrix}^{-1}$

$\underbrace{\left(\begin{pmatrix} 4 & 4 & 0 \\ 0 & 8 & -2 \\ -2 & 0 & 6 \end{pmatrix} - \begin{pmatrix} 2 & 0 & -2 \\ 4 & 6 & 0 \\ 0 & -2 & 4 \end{pmatrix}^{*} \right)}$ $\cdot \begin{pmatrix} 3 & -2 & 6 \\ 2 & -1 & 5 \\ 1 & 0 & 3 \end{pmatrix}^{-1}$

$\begin{matrix} 3 & -2 & 6 & 3 & -2 \\ 2 & -1 & 5 & 2 & -1 \\ 1 & 0 & 3 & 1 & 0 \end{matrix}$

$-4 \quad -3 - 2 \quad -5 - (-4) = -1$

$\begin{pmatrix} 4 & 4 & 0 \\ 0 & 8 & -2 \\ -2 & 0 & 6 \end{pmatrix} - \begin{pmatrix} 2 & 4 & 0 \\ 0 & 6 & -2 \\ -2 & 0 & 4 \end{pmatrix}$

$= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

$C_{11} = (-1)^2 \begin{vmatrix} -1 & 5 \\ 0 & 3 \end{vmatrix} = -3$ $\quad C_{21} = (-1)^3 \begin{vmatrix} -2 & 6 \\ 0 & 3 \end{vmatrix} = 6$

$C_{12} = (-1)^3 \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = -1$ $\quad C_{22} = (-1)^4 \begin{vmatrix} 3 & 6 \\ 1 & 3 \end{vmatrix} = 3$

$C_{13} = (-1)^4 \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = 1$ $\quad C_{23} = (-1)^5 \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = -2$

$C_{31} = (-1)^4 \begin{vmatrix} -2 & 6 \\ -1 & 5 \end{vmatrix} = -4$ $\quad -6 \quad -10$ $\quad C_{32} = (-1)^5 \begin{vmatrix} 3 & 6 \\ 2 & 5 \end{vmatrix} = -3$ $\quad C_{33} = (-1)^6 \begin{vmatrix} 3 & -2 \\ 2 & -1 \end{vmatrix} = 1$

$\text{cof} \begin{pmatrix} -3 & -1 & 1 \\ 6 & 3 & -2 \\ -4 & -3 & 1 \end{pmatrix} \Rightarrow \text{adj} \begin{pmatrix} -3 & 6 & -4 \\ -1 & 3 & -3 \\ 1 & -2 & 1 \end{pmatrix}$

$B^{-1} = \begin{pmatrix} -3 & 6 & -4 \\ -1 & 3 & -3 \\ 1 & -2 & 1 \end{pmatrix} \cdot \frac{1}{-1} = \begin{pmatrix} 3 & -6 & 4 \\ 1 & -3 & 3 \\ -1 & 2 & -1 \end{pmatrix}$