1 a) $r: X = \left(-5, \frac{2}{3}, 0\right) + \underbrace{\left(\frac{1}{2}, 1, 1\right)}_{\vec{v}_1}\lambda$ $s: z = 3x = 2y - 16$

$0 = 3x - z = 2y - z - 16$

$\begin{cases} 3x - z = 0 \\ 2y - z = 16 \end{cases}$ $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & -1 \\ 0 & 2 & -1 \end{vmatrix} = \vec{i}\begin{vmatrix} 0 & -1 \\ 2 & -1 \end{vmatrix} + \vec{j}\begin{vmatrix} -1 & 3 \\ -1 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix}$

$\vec{i}\cdot 2 + \vec{j}\cdot(3) + \vec{k}\cdot 6 = (2, 3, 6) = \vec{v}_2$

$\frac{|\vec{v}_1 \cdot \vec{v}_2|}{\|\vec{v}_1\|\,\|\vec{v}_2\|} = \frac{|2\cdot\frac{1}{2} + 3 + 6|}{\sqrt{\frac{1}{4}+1+1}\,\sqrt{4+9+36}} = \frac{10}{\sqrt{\frac{9}{4}}\,\sqrt{49}} = \frac{10}{\frac{3}{2}\cdot 7} = \frac{20}{21} = \cos\alpha$

$\text{sen}^2 x + \cos^2 x = 1 \Leftrightarrow \left(\frac{20}{21}\right)^2 + \text{sen}^2\alpha = 1 \quad \text{sen}^2\alpha = 1 - \frac{400}{441} = \sqrt{\frac{41}{441}} \Leftrightarrow \text{sen}\,\alpha = \frac{\sqrt{41}}{441}$

b) $r: X = (1, 1, 0) + \lambda\underbrace{(0, -1, 1)}_{\vec{v}_1}$ $s: x - y + 3 = z = 4 \Leftrightarrow \begin{cases} x - y = 1 \\ z = 4 \end{cases}$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} + \vec{j}\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -\vec{i} - \vec{j} = (1\ 1\ 0) = \vec{v}_2$

$\frac{|-1|}{\sqrt{2}\sqrt{2}} = \frac{1}{2} = \cos\alpha \qquad \text{sen}\,\alpha = \sqrt{3}/2$

c) $r: \begin{cases} x + 3z = 7 \\ y = \end{cases}$ $s: \begin{cases} x - 4y - 2z = 5 \\ y = 0 \end{cases}$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 3 \\ 0 & 1 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 0 & 3 \\ 1 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$-3\vec{i} + \vec{k} = (-3, 0, 1)$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -2 \\ 0 & 1 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 0 & -2 \\ 1 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$\vec{i}\cdot 2 + \vec{k}\cdot 1 = (2, 0, 1)$

$\cos\alpha = \frac{|(-3)2 + 1|}{\sqrt{9+1}\,\sqrt{4+1}} = \frac{|-6+1|}{\sqrt{10}\,\sqrt{5}} = \frac{5}{\sqrt{50}} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

d) $r: x = \frac{1-y}{2} = \frac{z}{3}$ $\quad s = \begin{cases} 3x + y - 5z = 0 \\ x - 2y + 3z + 1 = 0 \end{cases}$

$x = \frac{y-1}{-2} = \frac{z}{3}$

$(1, -2, 3) = \vec{v_1}$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -5 \\ 1 & -2 & 3 \end{vmatrix} = \vec{i}\begin{vmatrix} 1 & -5 \\ -2 & 3 \end{vmatrix} + \vec{j}\begin{vmatrix} -5 & 3 \\ 3 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix}$

$\vec{i}(3-10) + \vec{j}(-5-9) + \vec{k}(-6-1)$

$(-7, -14, -7) = (1, 2, 1) = \vec{v_2}$

$\cos\alpha = \frac{|1.1 + (-2)2 + 3|}{\|\vec{v_1}\| \, \|\vec{v_2}\|} = \frac{0}{\|\vec{v_1}\| \, \|\vec{v_2}\|} = 0$

(2) $r: X = (0, 2, 0) + \lambda(0\ 1\ 0) \supseteq P$ $\quad \sphericalangle(PQ, r) = 45^\circ$

$s: X = (1, 2, 0) + \mu(0, 0, 1) \supseteq Q$ $\quad \sphericalangle(PQ, s) = 60^\circ$

$\frac{|\vec{PQ} \cdot \vec{v_1}|}{\|\vec{PQ}\| \, \|\vec{v_1}\|} = \frac{\sqrt{2}}{2}$; $\frac{|\vec{PQ} \cdot \vec{v_2}|}{\|\vec{PQ}\| \, \|\vec{v_2}\|} = \frac{1}{2}$; $\vec{PQ} = (1, 2-(2+\lambda), \mu) = (1, -\lambda, \mu)$

$\frac{|-\lambda|}{\sqrt{1}\sqrt{1+\lambda^2+\mu^2}} = \frac{\lambda}{\sqrt{1+\lambda^2+\mu^2}} = \frac{\sqrt{2}}{2}$; $\frac{\mu}{\sqrt{1+\lambda^2+\mu^2}} = \frac{1}{2}$

$\Rightarrow \sqrt{1+\lambda^2+\mu} = \Delta$

$\frac{\left(\frac{\lambda}{\Delta}\right)}{\left(\frac{\mu}{\Delta}\right)} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{1}{2}\right)} \Leftrightarrow \frac{\lambda}{\Delta} \cdot \frac{\Delta}{\mu} = \frac{\sqrt{2}}{2} \cdot \frac{2}{1} \Leftrightarrow \frac{\lambda}{\mu} = \sqrt{2}$ $\quad \lambda = \mu\sqrt{2}$

$|\lambda| = \sqrt{2}$

$\frac{|\mu|}{\sqrt{1 + (\mu\sqrt{2})^2 + \mu^2}} = \frac{1}{2} \Leftrightarrow \frac{\mu^2}{1+\mu^2 \cdot 2 + \mu^2} = \frac{1}{4} = 4\mu^2 = 3\mu^2 + 1$ $\quad \boxed{\mu^2 = 1}$

$|\mu| = 1$

$\mu = 1$
$\lambda = \sqrt{2}$ $\rightarrow$ $P = (0, 2+\sqrt{2}, 0)$
$Q = (1, 2, 1)$

③ a) $r: x = y - z = 0$

$\begin{cases} x=0 \\ y-z=0 \end{cases}$

$\pi: z=0$

$\hookrightarrow \vec{n} = (0,0,1)$

$\dfrac{|\vec{v}\cdot\vec{n}|}{\|\vec{v}\|\|\vec{n}\|} = \dfrac{1}{\sqrt{1}\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix} = \vec{j}\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$\vec{j}\cdot 1 + \vec{k}\cdot 1 = (0,1,1) = \vec{v}$

$\operatorname{sen}\alpha = \dfrac{\sqrt{2}}{2}$

$\alpha = \dfrac{\pi}{4}$

b) $r: -x = y = \dfrac{z-1}{2}$

$\begin{cases} x+y=0 \\ 2x+z=1 \end{cases}$

$\pi: 2x - y = 0$

$(2,-1,0)$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{vmatrix} = \vec{i} + \vec{j}\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix}$

$\vec{i} - \vec{j} + 4 = (1,-1,2)$

$\operatorname{sen}\alpha = \dfrac{|2+1|}{\sqrt{6}\sqrt{5}} = \dfrac{3}{\sqrt{30}} = \dfrac{3\sqrt{30}}{30} = \dfrac{\sqrt{30}}{10}$

$\alpha = \operatorname{arcsen}\left(\dfrac{\sqrt{30}}{10}\right)$

c) $r: X = (1,0,0) + \lambda(1,1,-2)$ $\quad \vec{v}$

$\pi: x + y - z - 1 = 0$ $\quad \vec{n}$

$\dfrac{|1+1+2|}{\sqrt{6}\sqrt{3}} = \dfrac{3}{\sqrt{18}} = \dfrac{3}{3\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

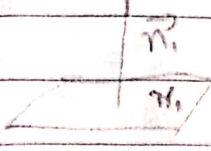
$\operatorname{sen}\dfrac{\sqrt{2}}{2} = \dfrac{\pi}{4}$

④ $r: X = A + \vec{v}\cdot\alpha$ $\quad \vec{v} = (a,b,c)$

$\pi_1: x+y+z=0$

$\angle(r, \pi_2) = 45°$, $\pi_2 = x - y = 0$

$\vec{n_2} = (1,-1,0)$



$\vec{n_1}\cdot\vec{v} = 0$

$a+b+c=0$

$a^2+b^2+c^2=1 \Rightarrow$ vetor unitário

$\dfrac{|a-b|}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{2}} = \dfrac{\sqrt{2}}{2} = \dfrac{|a-b|}{\sqrt{a^2+b^2+c^2}} = 1 \Leftrightarrow |a-b| = 1$

$\begin{cases} a+b+c=0 \\ a^2+b^2+c^2=1 \\ |a-b|=1 \end{cases}$

$c = -b - a = -b - b - 1 = -2b - 1$

$\Rightarrow a = b + 1$

$(b-1)^2 + b^2 + (-2b-1)^2 = 0$

$(b+1)^2 + b^2 + (2b+1)^2 = 1$

$b^2 + 2b + 1 + b^2 + 4b^2 + 4b + 1 = 1$; $b = \frac{-6 \pm 2\sqrt{3}}{12} = \frac{-3 \pm \sqrt{3}}{6}$

$6b^2 + 6b + 1 = 0$

$b = \frac{-6 \pm \sqrt{36-24}}{12}$

$b = \frac{-6 \pm \sqrt{12}}{12}$

Vetor unitário $= \left(\frac{3+\sqrt{3}}{6}, \frac{-3+\sqrt{3}}{6}, \frac{-\sqrt{3}}{6}\right)$

$a = 1 + \left(\frac{-3+\sqrt{3}}{6}\right) = \frac{6-3+\sqrt{3}}{6} = \frac{3+\sqrt{3}}{6}$

$c = -2\left(\frac{-3+\sqrt{3}}{6}\right) - \frac{6}{6} = \frac{+6-2\sqrt{3}-6}{6} = \frac{-\sqrt{3}}{6}$

5) a) $\pi_1: 2x + y - z - 1 = 0$ $\quad \pi_2: x - y + 3z - 10 = 0$

$\vec{n}_1 = (2, 1, -1)$ $\quad \vec{n}_2 = (1, -1, 3)$

$\cos\theta = \frac{|2 - 1 - 3|}{\sqrt{6}\sqrt{11}} = \frac{|-2|}{\sqrt{66}} = \frac{2\sqrt{66}}{66} = \frac{\sqrt{66}}{33}$ $\quad \theta = \arccos\left(\frac{\sqrt{66}}{33}\right)$

b) $\pi_1: X = (1,0,0) + t(1,0,1) + u(-1,0,0)$ $\quad \pi_2: x + y + z = 0$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 0 & \vec{j} & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = \vec{j}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \vec{j}$ $\quad \vec{n}_2 = (1, 1, 1)$

$\vec{n}_1 = (0, 1, 0)$

$\cos\theta = \frac{|1|}{\sqrt{3}\sqrt{1}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\quad \theta = \arccos\left(\frac{\sqrt{3}}{3}\right)$

c) $\pi_1: Y = (1,0,0)t + u(1,1,1)$ $\quad \pi_2: X = (1,0,0) + t(-1,2,0) + u(0,1,0)$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \vec{j}\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $\quad \begin{vmatrix} 0 & 0 & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \vec{k}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \vec{k} = (0\ 0\ 1)$

$-\vec{j} + k = (0\ -1\ 1)$

$\frac{|1|}{\sqrt{2}\sqrt{1}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \theta = \frac{\pi}{4}$

/ /

D S T Q Q S S

⑥ $\pi_1: 2x - y + z = 0$ $\quad \pi_2 \Rightarrow P \supset \pi_2, P = (1,2,3)$ $\quad \pi_2 \perp \vec{v}, \vec{v} = (1,-2,1)$

$\angle(\pi_1, \pi_2) =$ $\quad$ inútil

$n_1 = (2,-1,1) \quad n_2 = (1,-2,1) \Rightarrow \dfrac{|2+2+1|}{\sqrt{6}\sqrt{6}} = \dfrac{5}{6} \quad \theta = \arccos\left(\dfrac{5}{6}\right)$ //

⑦ a) $r: x - 1 = 2y = z$ $\quad A = (1,1,0) \quad B = (0,1,1)$

$$\begin{cases} x - z = 1 \\ 2y - z = 0 \end{cases}$$

$z = 0$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1 \\ 0 & 2 & -1 \end{vmatrix} = \vec{i}\begin{vmatrix} 0 & -1 \\ 2 & -1 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & -1 \\ 0 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}$$

Ponto $(1\ 0\ 0)$

$\vec{i} \cdot 2 + \vec{j} + 2\vec{k} = (2,1,2)$

$r: X = (1,0,0) + (2,1,2)\alpha \quad X = (1+2\alpha, \alpha, 2\alpha)$

$dist(X, A) = dist(X, B)$

$(2\alpha)^2 + (\alpha-1)^2 + (2\alpha)^2 = \sqrt{(1+2\alpha)^2 + (\alpha-1)^2 + (2\alpha-1)^2}$

$4\alpha^2 + 4\alpha^2 = 4\alpha^2 + 2\alpha + 1 + 4\alpha^2 - 2\alpha + 1$ $\quad \boxed{2 = 0}$ //

Não há ponto que equidiste de A e B

b) $r: X = (0,0,4) + \lambda(4,2,-3)$ $\quad A = (2,2,5) \quad B = (0,0,1)$

$X = (4\lambda, 2\lambda, 4 - 3\lambda)$

$dist(X, A) = dist(X, B)$

$\left[(4\lambda-2)^2 + (2\lambda-2)^2 + (-3\lambda-1)^2\right]^{1/2} = \left[(4\lambda)^2 + (2\lambda)^2 + (3-3\lambda)^2\right]^{1/2}$

$16\lambda^2 - 16\lambda + 4 + 4\lambda^2 - 8\lambda + 4 + 9\lambda^2 + 6\lambda + 1 = 16\lambda^2 + 4\lambda^2 + 9 - 18\lambda + 9\lambda^2$

$-16\lambda - 8\lambda + 7 + 6\lambda + 1 = 9 - 18\lambda$

$0 = 0$

Todos os pontos de r equidistam de A e B

/ /

c) $X=(2+\lambda, 3+\lambda, -3+\lambda)$ $A=(1,1,0)$ $B=(2,2,4)$

$[dist(X,A)]^2 = [dist(X,B)]^2$

$(1+\lambda)^2+(2+\lambda)^2+(-3+\lambda)^2 = \lambda^2 + (1+\lambda)^2 + (\lambda-7)^2$

$\lambda^2 + 4\lambda + 4 + \lambda^2 - 6\lambda + 9 = \lambda^2 + \lambda^2 - 14\lambda + 49$

$-2\lambda + 14\lambda = 49 - 9 - 4 \iff 12\lambda = 36$ $\lambda = 3$

O ponto que equidista de A e B é $(5,6,0)$

8) a) $P=(-2,0,1)$ $r: X=(1,-2,0)+(3,2,1)\lambda$

$\vec{PA} = (1,-2,0)-(-2,0,1) = (3,-2,-1)$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & 1 \\ -3 & 2 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} + \vec{j}\begin{vmatrix} 1 & 3 \\ 1 & -3 \end{vmatrix} + \vec{k}\begin{vmatrix} 3 & 2 \\ -3 & 2 \end{vmatrix}$

$-6\vec{j} + 12k = (0,-6,12)$

$\dfrac{\|\vec{PA} \times \vec{v}\|}{\|\vec{v}\|} = \dfrac{\sqrt{36+144}}{\sqrt{14}}$

$\dfrac{\sqrt{180}}{\sqrt{14}} = \sqrt{\dfrac{180}{14}} = \sqrt{\dfrac{90}{7}} = \dfrac{\sqrt{90 \cdot 7}}{7} = \boxed{\dfrac{3\sqrt{70}}{7} = \text{distância}}$

b) $P=(1,-1,4)$ $r: \dfrac{x-2}{4} = \dfrac{y}{-3} = \dfrac{1-z}{2} \rightarrow \dfrac{z-1}{-2}$

$\vec{PA} = (2,0,1)-(1,-1,4) = (1,1,-3)$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -3 \\ 4 & -3 & -2 \end{vmatrix} = \vec{i}\begin{vmatrix} 1 & -3 \\ -3 & -2 \end{vmatrix} + \vec{j}\begin{vmatrix} -3 & 1 \\ -2 & 4 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 1 \\ 4 & -3 \end{vmatrix}$

$\vec{i}(-2-9) + \vec{j}(-12+2) + \vec{k}(-3-4)$

$\vec{i}(-11) + \vec{j}(-10) + \vec{k}(-7) = (-11,-10,-7)$

$\dfrac{\|\vec{PA} \times \vec{v}\|}{\|\vec{v}\|} = \dfrac{\sqrt{121+100+49}}{\sqrt{16+9+4}}$

$= \dfrac{\sqrt{270}}{\sqrt{29}}$

$= \dfrac{3\sqrt{30}}{\sqrt{29}}$

$\boxed{\text{distância} = \dfrac{3\sqrt{870}}{29}}$

c) $P = (0, -1, 0)$ $\qquad r: x = 2y - 3 = 2z - 1$

$\vec{PA} = (1, 3, 1)$

$$\begin{cases} 2y - x = 3 & x = 0 \quad (0, \frac{3}{2}, \frac{1}{2}) \\ 2z - x = 1 & x = 1 \quad (1, 2, 1) \end{cases}$$

$r: X = (1, 2, 1) + (1, \frac{1}{2}, \frac{1}{2})\alpha \qquad v = \frac{1}{2}\vec{v}$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} + \vec{j}\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 2\vec{i} + \vec{j} + (-5)k$$

$(2, 1, -5)$

$$\frac{\|\vec{PA} \times \vec{v}\|}{\|\vec{v}\|} = \frac{\sqrt{4+1+25}}{\sqrt{6}} = \frac{\sqrt{30}}{\sqrt{6}} = \sqrt{5} = \text{distância}$$

9) $\pi_1: x + y = 2 \qquad \pi_2 = x = y + z \qquad s: x = y = z + 1 \Rightarrow X = (0, 0, -1) + (1, 1, 1)\lambda$ $\quad v$

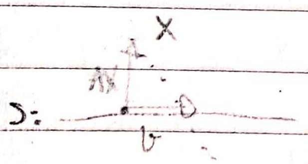
Interseção π_1 e π_2:

$$\begin{cases} x + y = 2 & x = 0 \quad (0, 2, -2) = P_1 \\ x - y - z = 0 & y = 0 \quad (2, 0, 2) = P_2 \end{cases}$$

$\vec{P_1P_2} = (2, -2, 4)$

$X = (2, 0, 2) + (1, -1, 2)\lambda$

$X = (2 + \lambda, -\lambda, 2 + 2\lambda)$



$A - X = (-2 - \lambda, \lambda, -3 - 2\lambda)$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2-\lambda & \lambda & -3-2\lambda \\ 1 & 1 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} \lambda & -3-2\lambda \\ 1 & 1 \end{vmatrix} + \vec{j}\begin{vmatrix} -3-2\lambda & -2-\lambda \\ 1 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} -2-\lambda & \lambda \\ 1 & 1 \end{vmatrix}$$

$\vec{i}(\lambda + 3 + 2\lambda) + \vec{j}(-3 - 2\lambda + 2 + \lambda) + \vec{k}(-2 - \lambda - \lambda)$

$\vec{i}(3\lambda + 3) + j(-\lambda - 1) + \vec{k}(-2 - 2\lambda)$

$$\frac{\sqrt{(3\lambda+3)^2 + (\lambda+1)^2 + (2\lambda+2)^2}}{\sqrt{3}} = \frac{\sqrt{14}}{\sqrt{3}} \iff (3(\lambda+1))^2 + (\lambda+1)^2 + (2(\lambda+1))^2$$

$9(\lambda+1)^2 + (\lambda+1)^2 + 4(\lambda+1)^2$

$\sqrt{14(\lambda+1)^2} = \sqrt{14}$

$\sqrt{14} \cdot \sqrt{(\lambda+1)^2} = \sqrt{14}$

$|\lambda + 1| = 1$

$\lambda = 0$

$\lambda = -2$

Para $\lambda = 0 \Rightarrow (2, 0, 2)$

Para $\lambda = -2 \Rightarrow (0, 2, -2)$

10 a) $P=(1,3,4)$ $\quad \pi: x=(1,0,0)+\lambda\underset{\vec{r}}{(1,0,0)}+\mu\underset{\vec{u}}{(-1,0,3)}$

$\vec{AP}=(1,3,4)-(1,0,0)=(0,3,4)$

$$\frac{|\vec{AP}\cdot(\vec{r}\times\vec{u})|}{\|\vec{r}\times\vec{u}\|} \qquad \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ -1 & 0 & 3 \end{vmatrix} = \vec{i}\begin{vmatrix} 0 & 0 \\ 0 & 3 \end{vmatrix} + \vec{j}\begin{vmatrix} 0 & 1 \\ 3 & -1 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix}$$

$\vec{j}\cdot -3 = (0,-3,0)$

$\vec{AP}\cdot(\vec{r}\times\vec{u}) = 0+(-3)\cdot 3+0 = |-9| = 9$

$\frac{9}{\sqrt{0^2+(-3)^2+0^2}} = \frac{9}{\sqrt{9}} = 3 \Rightarrow$ distância

b) $P=(0,0,-6)$ $\quad \pi: x-2y-2z-6=0$ $\quad z=0 \Rightarrow A=(0,0,-3)$

$\vec{PA}=(0,0,-3)-(0,0,-6)=(0,0,3)$ $\quad \vec{n}=(1,-2,-2)$

$\frac{|\vec{PA}\cdot\vec{n}|}{\|\vec{n}\|} = \frac{|0+0+(-6)|}{\sqrt{1+4+4}} = \frac{6}{\sqrt{9}} = \frac{6}{3} = 2 \Rightarrow$ distância //

c) $P=(1,1,1)$ $\quad \pi: 2x-y+2z-3=0$ $\quad z=0 \Rightarrow 2x-y=3$

$\vec{PA}=(2,1,0)-(1,1,1)$ $\quad \vec{n}=(2,-1,2)$ $\quad A=(2,1,0)$ $\quad x=2;\ y=1$

$\vec{PA}=(1,0,-1)$

$\frac{|\vec{PA}\cdot\vec{n}|}{\sqrt{4+1+4}} = \frac{|2+0-2|}{3} = \frac{0}{3} = 0$ $\qquad 2-1+2=3 \Leftarrow \boxed{4-1=3}$

$\Rightarrow$ Distância zero, $P \in \pi$.

(11) $r: x = 2 - y = y + z$ $\quad \pi: x - 2y - z = 1 \rightarrow (0,0,-1) = P_3$

$\begin{cases} -x + y + z = 0 \\ -y - x = -2 \end{cases}$ $\quad \vec{n} = (1,-2,-1)$

$P_1 = (0,2,-2)$ $\quad \overrightarrow{P_1P_2} = (2,0,2) - (0,2,-2)$

$P_2 = (2,0,2)$ $\quad = (2,-2,4)$ $\Rightarrow r: X = (2,0,2) + (2,-2,4)\lambda$

$X = (2 + 2\lambda, -2\lambda, 2 + 4\lambda)$

$\overrightarrow{P_3X}\ (2 + 2\lambda, -2\lambda, 2 + 4\lambda) - (0,0,-1) = (2 + 2\lambda, -2\lambda, 3 + 4\lambda)$

$\dfrac{|\overrightarrow{P_3X} \cdot \vec{n}|}{\sqrt{1+4+1}} = \dfrac{|\overrightarrow{P_3X} \cdot \vec{n}|}{\sqrt{6}} = \sqrt{6} = \overrightarrow{P_3X} \cdot \vec{n} = 6$

$\Rightarrow \overrightarrow{P_3X} \cdot \vec{n} = |2 + 2\lambda + 4\lambda - 3 - 4\lambda| = 6 \Leftrightarrow |2\lambda - 1| = 6 \rightarrow 2\lambda - 1 = 6$

$2\lambda - 1 = -6$

$\lambda \in \{-5/2, 7/2\}$ $\quad X_1 = (2 + (-\frac{5}{2}) \cdot 2, -2 \cdot (-\frac{5}{2}), 2 + 4(-\frac{5}{2})) = (-3, 5, -8)$

$X_2 = (2 + 2 \cdot \frac{7}{2}, -2 \cdot \frac{7}{2}, 2 + 4 \cdot \frac{7}{2}) = (9, -7, 16)$

(12) a) $r: X = (2,1,0) + \lambda(1,-1,1)$ $\quad s: x + y + z = 2x - y - 1 = 0$

$s: X = (0,-1,1) + (1,2,-3)\lambda$ $\quad \begin{cases} x + y + z = 0 \\ 2x - y = 1 \end{cases}$ $\quad x = 0 \Rightarrow (0,-1,1)$

$\overrightarrow{X_rX_s} = (0,-1,1) - (2,1,0) = (-2,-2,1)$ $\quad x = 1 \Rightarrow (1,1,-2)$

$\begin{vmatrix} 1 & -1 & 1 \\ 1 & 2 & -3 \\ -2 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 2 & -3 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & -3 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ -2 & -2 \end{vmatrix}$ $\quad \dfrac{|\overrightarrow{X_rX_s} \cdot (\vec{v} \times \vec{u})|}{\|\vec{v} \times \vec{u}\|}$

$(2-6) + (1-6) + (-2+4) = -4 - 5 + 2 = -7$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 1 \\ 1 & 2 & -3 \end{vmatrix} = \vec{i}\begin{vmatrix} -1 & 1 \\ 2 & -3 \end{vmatrix} + \vec{j}\begin{vmatrix} 1 & 1 \\ -3 & 1 \end{vmatrix} \vec{k}\begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix}$ $\quad \dfrac{|-7|}{\sqrt{1+16+9}} = \dfrac{7}{\sqrt{26}}$

$\vec{i}(3-2) + \vec{j}(1+3) + \vec{k}(2+1)$

$\text{distância} = \dfrac{7\sqrt{26}}{26}$

b) $r: \frac{x+4}{3} = \frac{y}{4} = \frac{z+5}{-2}$ $\quad s: X = (21, -5, 2) + \lambda(6, -4, -1)$

$\frac{\overrightarrow{X_rX_s} \cdot (\vec{v} \times \vec{u})}{\|\vec{v} \times \vec{u}\|}$

$r: X = (-4, 0, -5) + (3, 4, -2)\mu$

$\overrightarrow{X_rX_s} = (25, -5, 7)$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & -4 & -1 \\ 3 & 4 & -2 \end{vmatrix} = \vec{i}\begin{vmatrix} -4 & -1 \\ 4 & -2 \end{vmatrix} + \vec{j}\begin{vmatrix} -1 & 6 \\ -2 & 3 \end{vmatrix} + \vec{k}\begin{vmatrix} 6 & -4 \\ 3 & 4 \end{vmatrix}$

$\vec{i}(+8+4) + \vec{j}(-3+12) + \vec{k}(24+12) = (12, 9, 36)$

$\frac{|25 \cdot 12 + -5 \cdot 9 + 7(-36)|}{\sqrt{12^2 + 9^2 + 36}} = \frac{|-507|}{\sqrt{1521}} = \frac{|-507|}{39} = \boxed{13} \Rightarrow$ Distância

c) $r: \frac{x-1}{-2} = 2y = z$ $\quad s: X = (0,0,2) + \lambda(-4, 1, 2) \to \vec{v}$

$r: X = (1, 0, 0) + (-2, \frac{1}{2}, 1)\mu \to u$

$r: \frac{x-1}{-2} = \frac{y}{1/2} = z$ $\quad \vec{v} = \vec{u} \cdot 2 \Rightarrow$ paralelas

$\overrightarrow{X_sX_r} = (1, 0, -2)$ $\quad \frac{\|\overrightarrow{X_sX_r} \times (4, 1, 2)\|}{\|(4, 1, 2)\|}$

r

s

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -2 \\ -4 & 1 & 2 \end{vmatrix} = \vec{i}\begin{vmatrix} 0 & -2 \\ 1 & 2 \end{vmatrix} + \vec{j}\begin{vmatrix} -2 & 1 \\ 2 & -4 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 0 \\ -4 & 1 \end{vmatrix}$

$\vec{i}(-2) + \vec{j} \cdot 6 + \vec{k} \cdot 1 = (2, 6, 1)$

$\frac{\sqrt{4+36+1}}{\sqrt{16+1+4}} = \frac{\sqrt{41}}{\sqrt{21}} = \boxed{\frac{\sqrt{861}}{21}}$ Distância

/ /

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13 a) $r: X=(1,9,4)+\lambda\underset{\vec{v}}{(3,3,3)}$ $\pi: X=(5,7,9)+\lambda(1,0,0)+u(0,1,0)$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = k$$

$\vec{n}=(0,0,1)$

$\vec{n}\cdot\vec{v}\neq 0$, reta fura o plano

distância $=0$.

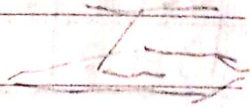
b) $r: x-y+z=0=2x+y-z-3$ $\pi=y-z=4$

$$\begin{cases} x-y+z=0 \\ 2x+y-z=3 \end{cases}$$

$z+1=y$

$y=z+1$

$\vec{n}=(0,1,-1)$



$3x=3$ $x=1$ $X_1=(1,1,0)$ $X_2=(1,2,1)$ $r: X=(1,1,0)+(0,1,1)\lambda$

$$D=\frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}=\frac{|0+1\cdot1+0-4|}{\sqrt{1^2+1^2}}=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}$$

c) $r: x=y-1=z+3$ $\vec{v}$ $\pi: 2x+y-3z-10=0$

$r: X=(0,1,-3)+(1,1,1)\lambda$ $\vec{n}=(2,1,-3)$

$\vec{n}\cdot\vec{v}=0$, reta e plano são paralelos.

$$D=\frac{|2\cdot0+1\cdot1+(-3)^2+(-10)|}{\sqrt{2^2+1^2+(3)^2}}=\frac{10-10}{\sqrt{12}}=0$$

r está em π
$\Rightarrow$ distância $=0$.

14 a) $\pi: 2x-y+2z+0=0$ $\pi_2=4x-2y+4z-21=0$

$2(2,-1,2)=(4,-2,4)\Rightarrow$ paralelos

$$\frac{|0-21|}{\sqrt{4^2+2^2+4^2}}=\frac{21}{\sqrt{36}}=\frac{21}{6}=\boxed{\frac{7}{2}}\Rightarrow \text{distância}$$

b) $\pi_1 = 2x + 2y + 2z = 5$ $\pi_2 : X = (2,1,2) + \lambda(-1,0,3) + \mu(1,1,0)$

$$\begin{vmatrix} x-2 & y-1 & z-2 \\ -1 & 0 & 3 \\ 1 & 1 & 0 \end{vmatrix}$$

$(x-2)(-3) + (y-1)(3) + (z-2)(-1) = 0$

$= -3x + 6 + 3y - 3 + 2 - z = 0$

$-3x + 3y - z + 5 = 0$

$(3,-3,1) = \alpha(2,2,2) \Rightarrow \nexists$

não paralelos, distância $= 0$

c) $\pi : x + y + z = 0$ $\pi_2 : 2x + y + z + 2 = 0$

$(1,1,1) = \alpha(2,1,1)$, $\alpha \nexists$ $\Rightarrow$ não paralelos distância $= 0$

15) $\pi : \begin{cases} r: x + z = 5 = y + 4 \\ s: X = (4,1,1) + \lambda(4,2,-3) \end{cases}$

$r: \begin{cases} x + z = 5 \\ y = 1 \end{cases}$ $X_1 = (0,1,5)$

$X_2 = (5,1,0)$

$\overrightarrow{X_1X_2} = (5,0,-5)$

$r: (0,1,5) + \mu(1,0,-1)$

$$\begin{vmatrix} x & y-1 & z-5 \\ 1 & 0 & -1 \\ 4 & 2 & -3 \end{vmatrix} = x\begin{vmatrix} 0 & -1 \\ 2 & -3 \end{vmatrix} + (y-1)\begin{vmatrix} -1 & 1 \\ -3 & 4 \end{vmatrix} + (z-5)\begin{vmatrix} 1 & 0 \\ 4 & 2 \end{vmatrix}$$

$2x + (y-1)(-1) + (z-5)2 = 2x - y + 2z + 1 - 10 = 0$

$\pi = 2x - y - 2z - 9 = 0$

$\dfrac{|-9-d|}{\sqrt{9}} = 2 \Leftrightarrow |-9-d| = 6 \rightarrow -9-d = 6$ $d = -15$

$9 + d = 6$ $d = -3$

Os planos que distam dois de π são:

$\pi_2 = 2x - y - 2z - 3 = 0$

$\pi_3 = 2x - y - 2z - 15 = 0$