

Kris Untenius de Lang Martens - Lijst 1 A 7

3 3 3 3 3 3

$$\textcircled{1} \quad \text{a)} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{pmatrix} \Rightarrow \begin{array}{l} x=2 \\ y=-1 \end{array}$$

$$(x, y) = (2, -1)$$

$$\text{e)} \begin{pmatrix} 1 & 0 & 0 & -7 & 8 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & -5 & 0 \end{pmatrix} \begin{array}{l} x - 7w = 8 \\ y + 3w = 2 \\ z + w = 5 \end{array}$$

$$x = 8 + 7w$$

$$y = 2 - 3w$$

$$z = w - 5$$

$$(x, y, z, w) = (8 + 7w, 2 - 3w, w - 5, w)$$

$$\text{b)} \begin{pmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$(x, y, z, w) = (4, 3, 2, 1)$$

$$\text{f)} \begin{pmatrix} 1 & -6 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{array}{l} x_1 - 6x_2 + 3x_5 = -2 \\ x_3 + 4x_5 = 7 \\ x_4 + 5x_5 = 8 \end{array}$$

$$x_2 = \alpha \quad \text{e} \quad x_5 = \beta$$

$$\text{g)} \begin{pmatrix} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix} \begin{array}{l} x = 6 \\ y = 3 \\ z + w = 2 \end{array}$$

$$z = \alpha \quad w = 2 - z$$

$$(x, y, z, w) = (6, 3, \alpha, 2 - \alpha)$$

$$\text{d)} \begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & 2 \end{pmatrix} \begin{array}{l} x + 3z = 1 \\ y - z = 2 \end{array}$$

$$z = y - 2 \quad (z = \alpha)$$

$$x = 1 - 3\alpha \quad y = \alpha + 2$$

$$(x, y, z) = (1 - 3\alpha, \alpha + 2, \alpha)$$

$$S = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \mid \begin{array}{l} x_1 = 6\alpha - 3\beta - 2 \\ x_2 = \alpha \\ x_3 = 7 - 4\beta \\ x_4 = 8 - 5\beta \\ x_5 = \beta \end{array} \right\}$$

$$\text{2a)} \begin{cases} 3x - 4y = 1 \\ x + 3y = 9 \end{cases} \quad \begin{pmatrix} 3 & -4 & 1 \\ 1 & 3 & 9 \end{pmatrix}$$

$$\begin{cases} 5x + 8y = 34 \\ 10x + 16y = 50 \end{cases}$$

$$\begin{pmatrix} 0 & -3 & -26 \\ 1 & 3 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 9 \end{pmatrix} =$$

$$\begin{pmatrix} 5 & 8 & 34 \\ 10 & 16 & 50 \end{pmatrix} = \begin{pmatrix} 5 & 8 & 34 \\ 0 & 0 & -18 \end{pmatrix} \leftarrow$$

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$3x + 2y = -18$$

$$S = \emptyset$$

$$c) \begin{cases} x+2y=5 \\ 2x-3y=-4 \end{cases} \quad \begin{array}{l} \text{L}_2 - 2 \cdot \text{L}_1 \\ \left(\begin{array}{ccc|cc} 1 & 2 & 5 & 1 & 2 & 5 \end{array} \right) - \left(\begin{array}{ccc|cc} 0 & -7 & -14 & 0 & 1 & 2 \end{array} \right) \\ \left(\begin{array}{ccc|cc} 1 & 2 & 5 & 1 & 2 & 5 \end{array} \right) = \left(\begin{array}{ccc|cc} 2 & -6 & -4 & 1 & -3 & 2 \end{array} \right) \\ \left(\begin{array}{ccc|cc} 1 & 2 & 5 & 1 & 2 & 5 \end{array} \right) = \left(\begin{array}{ccc|cc} 1 & 3 & 1 & 1 & 3 & 1 \end{array} \right) = \left(\begin{array}{ccc|cc} 1 & 3 & 1 & 1 & 3 & 1 \end{array} \right) \\ \left(\begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 1 & 1 \end{array} \right) \quad \left(\begin{array}{ccc|cc} x & 1 & 1 & 1 & 1 & 1 \\ y & & & 2 & 2 & 2 \end{array} \right) \quad \left(\begin{array}{ccc|cc} 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right) \end{array}$$

b) $\begin{cases} x+2y-z=2 \\ 2x-y+3z=9 \\ 3x+3y-2z=3 \end{cases}$

$$d) \begin{cases} 3x+2y-5z=8 \\ 2x-4y-2z=-4 \\ x-2y-3z=-4 \end{cases} \quad \begin{array}{l} \left(\begin{array}{ccc|cc} 3 & 2 & -5 & 8 & 1 & -2 & -3 & -4 \end{array} \right) \quad \left(\begin{array}{ccc|cc} 1 & 2 & -1 & 2 & 1 & 2 & -1 & 2 \end{array} \right) \\ \left(\begin{array}{ccc|cc} 2 & -4 & -2 & -4 & 2 & -1 & 3 & 9 \end{array} \right) = \left(\begin{array}{ccc|cc} -1 & 2 & -1 & 2 & 0 & -5 & 5 & 5 \end{array} \right) \\ \left(\begin{array}{ccc|cc} 1 & -2 & -3 & -4 & 3 & 2 & -5 & 8 \end{array} \right) \quad \left(\begin{array}{ccc|cc} 1 & 2 & -1 & 2 & 1 & 0 & 1 & 4 \end{array} \right) \\ \left(\begin{array}{ccc|cc} 1 & -2 & -3 & -4 & 1 & -2 & -3 & -4 \end{array} \right) \quad \left(\begin{array}{ccc|cc} 0 & -1 & 1 & 1 & 0 & 1 & 1 & 1 \end{array} \right) \\ \left(\begin{array}{ccc|cc} 0 & 0 & -2 & -2 & 3 & 2 & -5 & 8 \end{array} \right) = \left(\begin{array}{ccc|cc} 0 & -3 & 1 & 3 & 0 & 0 & 2 & 6 \end{array} \right) \\ \left(\begin{array}{ccc|cc} 3 & 2 & -5 & 8 & 0 & 0 & 1 & 1 \end{array} \right) \quad \left(\begin{array}{ccc|cc} 1 & 0 & 1 & 4 & 1 & 0 & 0 & 1 \end{array} \right) \\ \left(\begin{array}{ccc|cc} 1 & -2 & -3 & -4 & 1 & -2 & -3 & -4 \end{array} \right) \quad \left(\begin{array}{ccc|cc} 0 & -1 & 1 & 1 & 0 & -1 & 0 & -2 \end{array} \right) \\ \left(\begin{array}{ccc|cc} 0 & -4 & 4 & -4 & 0 & -1 & 1 & -1 \end{array} \right) = \left(\begin{array}{ccc|cc} 0 & 0 & 1 & 3 & 0 & 0 & 1 & 3 \end{array} \right) \\ \left(\begin{array}{ccc|cc} 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{array} \right) \end{array}$$

$S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}$

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$$S = \left\{ \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \right\}$$



$$g) \begin{cases} x + 3z = -8 \\ 2x - 4y = -4 \\ 3x - 2y - 5z = 26 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 3 & -8 \\ 2 & -4 & 0 & -4 \\ 3 & -2 & -5 & 26 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 3 & -8 \\ 1 & -2 & 0 & -2 \\ 3 & -2 & -5 & 26 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 3 & -8 \\ 1 & -2 & 0 & -2 \\ 3 & -2 & -5 & 26 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 3 & -8 \\ 0 & 2 & 3 & -6 \\ 0 & -2 & -4 & 50 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 3 & -8 \\ 0 & 2 & 3 & -6 \\ 0 & 0 & -1 & 44 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 3 & -8 \\ 0 & 2 & 3 & -6 \\ 0 & 0 & -1 & 44 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 2 & 0 & 6 \\ 0 & 0 & -1 & 4 \end{array} \right) \rightarrow \left(\begin{array}{c|c} x & 4 \\ y & 3 \\ z & -4 \end{array} \right)$$

$$i) \begin{cases} x - 3y + 4z - w = 2 \\ 2x - y + 3z - 2w = 19 \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & -3 & 4 & -1 & 2 \\ 2 & -1 & 3 & -2 & 19 \end{array} \right) \rightarrow \left(\begin{array}{cccc|c} 1 & -3 & 4 & -1 & 2 \\ 0 & 5 & -5 & 0 & 15 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & -3 & 4 & -1 & 2 \\ 0 & 1 & -1 & 0 & 3 \end{array} \right) \rightarrow \left(\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 11 \\ 0 & 1 & -1 & 0 & 3 \end{array} \right)$$

$$\begin{aligned} x - z - w &= 11 & z &= \alpha \\ y - z &= 3 & w &= \beta \end{aligned}$$

$$S = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 11 + \alpha + \beta \\ 3 + \alpha \\ \alpha \\ \beta \end{pmatrix} \right\}$$

$$b) \begin{cases} x + 2y + 3z = 10 \\ 3x + 4y + 6z = 23 \\ 2x + 2y + 3z = 13 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 3 & 4 & 6 & 23 \\ 2 & 2 & 3 & 13 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 2 & 4 & 6 & 20 \\ 3 & 4 & 6 & 23 \\ 2 & 2 & 3 & 13 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 1 & 0 & 0 & 3 \\ 2 & 2 & 3 & 13 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 2 & 3 & 7 \\ 2 & 2 & 3 & 13 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 2 & 3 & 7 \\ 2 & 0 & 0 & 6 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 2 & 3 & 7 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$S = \left\{ (x, y, z) = \left(3, \frac{7 - 3\alpha}{2}, \alpha \right) \right\}$$

$$\begin{array}{l}
 \textcircled{3} \text{ a) } \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{array} \right) \equiv \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & 1 \\ 0 & -2 & 1 & 0 & -5 \end{array} \right) \equiv \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & 1 \\ 0 & 0 & -7 & 0 & -7 \end{array} \right) \equiv \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{array} \right) \\
 \left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 & 1 \end{array} \right) \equiv \left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{array} \right) \quad x+w=1 \quad w=\alpha \quad \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1-\alpha \\ 2 \\ 1 \\ \alpha \end{pmatrix}
 \end{array}$$

$$\begin{array}{l}
 \text{b) } \left(\begin{array}{cccc|c} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{array} \right) \equiv \left(\begin{array}{cccc|c} 0 & 1 & 1 & -2 & 1 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{array} \right) \equiv \left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & -1 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{array} \right) \equiv \left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & -1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & -2 & 1 \end{array} \right) \\
 \sim \left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{array} \right)
 \end{array}$$

$$\begin{array}{l}
 \left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{array} \right) \equiv \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & 1 \end{array} \right) \quad x+z=0 \quad \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1-\alpha \\ 2+\alpha \\ \alpha-1 \\ \alpha \end{pmatrix} \\
 y-w=2 \\
 -z+w=1 \quad w=\alpha \\
 -z=1-\alpha
 \end{array}$$

$$\begin{array}{l}
 \text{c) } \left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & 3 & 0 \end{array} \right) \equiv \left(\begin{array}{cccc|c} 0 & 1 & 2 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 \end{array} \right) \equiv \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right) \equiv \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right) \equiv \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \\
 (x, y, z) = (0, 0, 0)
 \end{array}$$

$$\begin{array}{l}
 \textcircled{4} \quad \begin{cases} x - 2y + z = 1 \\ 2x - 5y + z = -2 \\ 3x - 7y + 2z = -1 \end{cases} \quad \text{a) } \begin{cases} x - 2y + z = 2 \\ 2x - 5y + z = -1 \\ 3x - 7y + 2z = 2 \end{cases} \quad \text{b) } \begin{cases} 1 & -2 & 1 & | & 1 & | & 2 \\ 2 & -5 & 1 & | & -2 & | & -1 \\ 3 & -7 & 2 & | & -1 & | & 2 \end{cases}
 \end{array}$$

$$\begin{array}{l}
 \left(\begin{array}{cccc|c} 1 & -2 & 1 & 1 & 2 \\ 0 & -1 & -4 & -5 \\ 0 & -1 & -4 & -4 \end{array} \right) \equiv \left(\begin{array}{cccc|c} 1 & -2 & 1 & 1 & 2 \\ 0 & 1 & 1 & 4 & 5 \\ 0 & 1 & 1 & 4 & 4 \end{array} \right) \equiv \left(\begin{array}{cccc|c} 1 & -2 & 1 & 1 & 2 \\ 0 & 1 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right) \quad S = \emptyset \quad y+z=4 \quad z=\alpha \\
 b \Rightarrow \quad a \Rightarrow x-2y+z=1 \\
 x = 8-2\alpha - \alpha + 1 \\
 = 9-3\alpha
 \end{array}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9-3\alpha \\ 4-\alpha \\ \alpha \end{pmatrix}$$

$$\textcircled{5} \text{ a) } A = \begin{pmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & 4 \end{pmatrix} \quad D = 0 \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (A + 4I_3)X = 0$$

$$\left(\begin{array}{ccc|cc} 1 & 0 & 5 & 4 & 0 \\ 1 & 1 & 1 & 0 & 4 \\ 0 & 1 & 4 & 0 & 4 \end{array} \right) \xrightarrow{\begin{array}{l} R_1 - R_3 \\ R_2 - R_3 \\ R_3 \cdot 4 \end{array}} \left(\begin{array}{ccc|cc} 1 & 0 & 5 & 4 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 16 \end{array} \right) \quad \left\{ \begin{array}{l} 5x + 5z = 0 \\ x + y + z = 0 \\ 4z = 0 \end{array} \right. \quad \left\{ \begin{array}{l} 5x + 5z = 0 \\ x + y + z = 0 \\ y = 0 \end{array} \right.$$

$$X = \begin{pmatrix} -z \\ 0 \\ z \end{pmatrix} \quad X = z \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \quad \boxed{X = -z}$$

$$\text{b) } AX = 2X$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 5 & x \\ 1 & 1 & 1 & y \\ 0 & 1 & 4 & z \end{array} \right) \xrightarrow{\begin{array}{l} R_1 - R_3 \\ R_2 - R_3 \\ R_3 \cdot 2 \end{array}} \left(\begin{array}{ccc|c} 1 & 0 & 5 & x \\ 0 & 1 & 1 & y \\ 0 & 0 & 4 & z \end{array} \right) \quad \left\{ \begin{array}{l} x + 5z = 2x \\ x + y + z = 2y \\ y - 4z = 2z \end{array} \right. \quad \left\{ \begin{array}{l} -x + 5z = 0 \\ x - y + z = 0 \\ y - 6z = 0 \end{array} \right. \quad \left. \begin{array}{l} i \\ ii \\ iii \end{array} \right.$$

$$\text{i} + \text{iii} = x - 5z = 0 \Rightarrow x = 5z \quad \text{i} - \text{ii} = -y + 6z = 0 \Rightarrow y = 6z \quad X = \begin{pmatrix} 5z \\ 6z \\ z \end{pmatrix} = z \begin{pmatrix} 5 \\ 6 \\ 1 \end{pmatrix}$$

$$\textcircled{6} \text{ a) } \begin{cases} x + y + z = 2 \\ 2x + 3y + 2z = 5 \\ 2x + 3y + (a^2 - 1)z = a+1 \end{cases} \quad \left| \begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & a^2-1 & a+1 \end{array} \right. \quad \begin{array}{l} a^2 - 1 \neq 2 \\ a^2 \neq 3 \\ a \neq \pm \sqrt{3} \end{array} \quad \text{SPD}$$

$$\left| \begin{array}{ccc|c} 2 & 1 & 1 & 2 \\ 5 & 3 & 2 & 5 \\ a+1 & 3 & a^2-1 & a+1 \end{array} \right. \quad \left| \begin{array}{ccc|c} 2 & 1 & 2 & a^2+2 \\ 5 & 3 & 5 & a^2-a-1 \\ a+1 & 3 & a^2+2 & 1-4 \cdot (-1) \end{array} \right. \quad \frac{1 \pm \sqrt{5}}{2} \quad D_X$$

$$\left| \begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 2 & 3 & 5 & 2 \\ 2 & 3 & a+1 & 2+a+1 \end{array} \right. \quad \left| \begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & a+1 & 0 \end{array} \right. \quad \begin{array}{l} \text{SPD: } a \in \mathbb{R} - \{-\sqrt{3}, \sqrt{3}\} \\ \text{SPI: nenhum } a \\ \text{SI: } a \in \{-\sqrt{3}, \sqrt{3}\} \end{array}$$

$\frac{1 \pm \sqrt{5}}{2}$ não dá para zerar!

11

0 1 2 3 4 5 6 7

b) $\begin{cases} x + 2y - 3z = 4 \\ 3x - y + 5z = 2 \\ 4x + y + (a^2 - 14)z = a + 2 \end{cases}$

$$\begin{array}{c|ccccc} & 1 & 2 & -3 & 4 & \\ \text{D} = & \left| \begin{array}{cccc} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2 - 14 & a + 2 \end{array} \right| & = & \left| \begin{array}{cccc} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & 14 & -10 \end{array} \right| & \neq 0 \quad a^2 - 14 \neq \pm 4 \end{array}$$

d) $a=4$

$$\left(\begin{array}{cccc} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & 2 & 6 \end{array} \right) \left(\begin{array}{cccc} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & 14 & -10 \end{array} \right) \xrightarrow{\text{(SP)}} a=4$$

$$l_3 + l_3 - 4 \cdot l_1$$

$$l_3 + l_2 - 3 \cdot l_1$$

d) $a=-4$

$$\left(\begin{array}{cccc} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & 2 & -2 \end{array} \right) \left(\begin{array}{cccc} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & 14 & -18 \end{array} \right) \xrightarrow{\text{(SI)}} a=-4$$

$$l_2 + l_2 - 3l_1$$

$$l_3 + l_3 - 4l_1$$

• SPD: $a \in \mathbb{R} - \{-4, 4\}$ • SPI: $a=4$ • SI: $a=-4$

⑦ $A = \begin{pmatrix} 2 & -2 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & \frac{1}{2} & 0 \\ 3 & 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 & \frac{1}{2} & 1 \\ 3 & 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 12 & 0 & \frac{3}{2} & 3 \\ 12 & 4 & 0 & 4 \end{pmatrix}$

$$\begin{pmatrix} 4 & 0 & \frac{1}{2} & 1 \\ 0 & 4 & -\frac{3}{2} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \frac{1}{8} & \frac{1}{4} \\ 0 & 1 & \frac{3}{8} & \frac{1}{4} \end{pmatrix}$$

11

b) $B = \begin{pmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & -2 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & -2 & 0 & -1 & 0 & 0 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 4 & 2 & 0 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 2 & -1 & 2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -2 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 3 & -8 & 0 & 4 & 0 & 0 \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 & 3 & 2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 8 & 0 & -1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 8 & 0 & -1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 8 & 0 & -1 & 2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 8 & -8 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -8 & 1 & -2 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 8 & 0 & 0 & 3 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 8 & 0 & -1 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & -\frac{1}{8} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 8 & -1 & 2 & -6 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{pmatrix}$$

c) $C = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 1 & -3 \\ 1 & 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 1 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 & 5 \\ 0 & 1 & -1 & 3 \end{pmatrix}$

d) $D = \begin{pmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 1 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 & 1 & 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 1 & 0 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 3 & 2 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 3 & 2 & -1 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & -1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 1 & \frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

e) $\begin{pmatrix} 2 & -1 & 0 & 1 \\ 0 & 2 & 0 & -1 \\ 1 & 0 & 2 & 3 \\ 0 & 0 & -2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 & 3 & 0 & 0 & 0 \end{pmatrix}$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 0 & -1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 & 3 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 & -1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & -2 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -2 & 0 & 0 & 0 & 1 \end{pmatrix}$$



$$\left| \begin{array}{r} 10230010 \\ 020-10100 \\ 2-1011000 \\ 00-200001 \end{array} \right| \left| \begin{array}{r} 10030011 \\ 020+0100 \\ 00-200001 \\ 2-1011000 \end{array} \right| \left| \begin{array}{r} 10030011 \\ 020-10100 \\ 00-200001 \\ 0-16510-2-2 \end{array} \right|$$

$$\left| \begin{array}{r} 10030011 \\ 020-10100 \\ 00-200001 \\ 0-20-102-4 \end{array} \right| \left| \begin{array}{r} 10030011 \\ 020-10100 \\ 00-200001 \\ 000-1121-44 \end{array} \right|$$

$$\left| \begin{array}{r} 110033001111 \\ 0220-1101100 \\ 00-200001 \\ 000-1121-44 \end{array} \right| \left| \begin{array}{r} 11000063-1-1 \\ 02200-21044 \\ 00-207001 \\ 000+121-44 \end{array} \right| \left| \begin{array}{r} 63-1-1 \\ -153311 \\ 000-12 \\ 21-141411 \end{array} \right|$$

(8) $\begin{cases} c + 2s + 3b = 26 \\ 2c + 5s + 6b = 60 \\ 2c + 3s + 4b = 40 \end{cases}$

 $c + 16 + 6 = 26 \quad c = 4$
 $2c + 5s + 6b = 60 \quad \Rightarrow s = 8 \quad b = 2$
 $2c + 3s + 4b = 40 \quad 2s + 2b = 20 \quad 8 + 5 = 10$

R: Os valores unitários da calça, shorts e blusa são, respectivamente, R\$ 4,00, R\$ 8,00, R\$ 2,00.

(9) $5s + 2c + 6b = 2200 \quad 5 \cdot 2b + 2 \cdot 3b + 6b = 2200$

$c = 3b$

$22b = 2200$

$c = b + s \quad s = -b + c \quad (s = 2b) \quad b = 100 \quad c = 300 \quad s = 200$

R: Foram vendidos 100 sanduíches, 300 cachorros e 200 banana-split.

(10) $\begin{cases} 40t + 30s + 10p = 7000 \\ 20t + 40s + 30p = 6000 \\ 10t + 20s + 40p = 5000 \end{cases}$

$40t + 30s + 10p = 7000$

$20t + 40s + 30p = 6000 \quad \Rightarrow \quad 40t + 80s + 60p = 12000 \quad 100p = 8000$

$10t + 20s + 40p = 5000 \quad \Rightarrow \quad 40t + 80s + 160p = 20000 \quad p = 80$

$t + 2s + 4p = 500$

$t + 40 + 320 = 500$

$t = 140$

$50s + 50p = 5000$

$s + p = 100$

$s = 20$

$\left\{ \begin{array}{l} \text{Torta} = \text{R\$ 140} \\ \text{Salada} = \text{R\$ 20} \end{array} \right.$

$\left\{ \begin{array}{l} \text{Pizza} = \text{R\$ 80} \end{array} \right.$

$$\begin{array}{l}
 \text{11) } \begin{array}{l} 2A + 3B + C = 8420 \\ A + 2B + 2C = 7940 \\ 4A + 3B = 8110 \end{array} \quad \begin{array}{l} 4A + 6B + 2C = 18840 \\ A + 2B + 2C = 7940 \\ 3A + 4B = 8900 \end{array} \quad \begin{array}{l} 5 \\ - \\ 3840 \\ \hline 26700 \\ 5740 \end{array} \\
 \begin{array}{l} A = \frac{8110 - 3B}{4} \\ B = \frac{8110 - 4A}{3} \\ B = 1610 \end{array} \quad \begin{array}{l} 3A + 4 \cdot \left(\frac{8110 - 4A}{3} \right) = 8900 \\ 9A + 3240 - 16A = -7A + 32440 = 8900 \\ -7A = 26700 - 32440 \\ 7A = 5740 \quad A = 820 // \end{array} \\
 \begin{array}{l} 1640 + 3000 + 1830 = 8420 - C \\ C = 8420 - 1640 - 4830 \\ C = 1950 \end{array} \quad \begin{array}{r} 28420 \\ -1640 \\ \hline 56780 \\ -4830 \\ \hline 1950 \end{array}
 \end{array}$$

R: As distâncias das pistas A, B e C, em metros, são respectivamente, 820, 1610, 1950.