Tarefa1

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1 Tarefa 1 - Cálculo numérico

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[2]:
```
Int64
```

[2]: Int64

[4]:
```
Int32
```

[4]: Int32

[6]:
```
Int16
```

[6]: Int16

[8]:
```
Int8
```

[8]: Int8

[9]:
```
Int8(100)+Int8(26)
```

[9]: 126

[10]:
```
Int8(100)+Int8(27)
```

[10]: 127

O caso abaixo nos da um overflow, resultando no resultado negativo correspondente

Como Int8 consiste em 2^8 = 256, então 256 números disponíveis, temos um intervalo de representação de -128 a 127

[11]:
```
Int8(100)+Int8(28)
```

[11]: -128

[28]:
```
Int8(2^7 - 1)
```

[28]: 127

Abaixo temos o nosso limite negativo, consistindo do -2^7, desse modo retornando -128

Caso somemos -1, teríamos um overflow, o que nos retornaria nosso maior número possivel

[31]:
```
Int8(-2^7)
```

[31]: -128

[37]:
```
Int8(-128)+Int8(-1)
```

[37]: 127

Sabemos que os números tradicionais no Julia utilizam 64bits, ou seja, nosso maior número seria 2^63 - 1 e o menor -2^63 , pois sabemos que temos 2^64 números disponíveis, sendo metade deles positivos e a outra metade negativos. Com isso em mente, temos que levar em conta o zero fazendo parte dos "positivos", então temos um intervalo que vai de de -2^63 a 2^63 - 1

Consistindo então de 0 a 2^63 -1 nos positivos e -1 a -2^63 nos negativos

[22]:
```
x = 2^63 - 1
```

[22]: 9223372036854775807

[23]:
```
x + 1
```

[23]: -9223372036854775808

Se sormarmos -1 ao menor número na representação 64bits, ele passa aser o maior número possível

[25]:
```
y = -2^63
```

[25]: -9223372036854775808

[38]:
```
y - 1
```

[38]: 9223372036854775807

[45]:
```
(BigInt(2)^63-1)^10
```

[45]: 4455508415646675013373597242420117818453694838130159772560668808816707086990958982033203334310070688731662890013605553436739351074980172000127431349940128178077122187317837794167991459381249

A partir de um ponto o programa diz que o número é grande o suficiente para considerar que é infinito, então ao invés de termos um overflow e o número ir do positivo para o negativo. Temos um "overflow" onde dizemos que o número explodiu para o infinito

[46]:
```
10.0^309
```

[46]: Inf

[47]:
```
1 / Inf
```

[47]: 0.0

[48]:
```
-1 / Inf
```

[48]: -0.0

[49]:
```
1 / -Inf
```

[49]: -0.0

Como infinito é uma representação de um conceito, então infinito menos infinito é outra representação de um conceito em que não conseguimos definir um valor.

[50]:
```
Inf - Inf # Not-a-Number
```

[50]: NaN

[51]:
```
Inf / Inf
```

[51]: NaN

[55]:
```
0 / 0
```

[55]: NaN

[53]:
```
1 / 256
```

[53]: 0.00390625

Matematicamente o x nunca é zero, porém há um limite para o quanto o computador pode armazenar, que no nosso caso é 5.0 x 10^-324

[56]:
```
x = 1.0
while x > 0
    x = x/2
    println("x = $x")
end
```

```
x = 0.5
x = 0.25
x = 0.125
x = 0.0625
x = 0.03125
x = 0.015625
x = 0.0078125
x = 0.00390625
x = 0.001953125
x = 0.0009765625
x = 0.00048828125
x = 0.000244140625
x = 0.0001220703125
x = 6.103515625e-5
x = 3.0517578125e-5
x = 1.52587890625e-5
x = 7.62939453125e-6
x = 3.814697265625e-6
x = 1.9073486328125e-6
x = 9.5367431640625e-7
x = 4.76837158203125e-7
x = 2.384185791015625e-7
x = 1.1920928955078125e-7
x = 5.960464477539063e-8
x = 2.9802322387695312e-8
x = 1.4901161193847656e-8
x = 7.450580596923828e-9
x = 3.725290298461914e-9
x = 1.862645149230957e-9
x = 9.313225746154785e-10
x = 4.656612873077393e-10
x = 2.3283064365386963e-10
x = 1.1641532182693481e-10
x = 5.820766091346741e-11
x = 2.9103830456733704e-11
x = 1.4551915228366852e-11
x = 7.275957614183426e-12
x = 3.637978807091713e-12
x = 1.8189894035458565e-12
x = 9.094947017729282e-13
x = 4.547473508864641e-13
x = 2.2737367544323206e-13
x = 1.1368683772161603e-13
x = 5.684341886080802e-14
x = 2.842170943040401e-14
x = 1.4210854715202004e-14
x = 7.105427357601002e-15
x = 3.552713678800501e-15
```

```
x = 1.7763568394002505e-15
x = 8.881784197001252e-16
x = 4.440892098500626e-16
x = 2.220446049250313e-16
x = 1.1102230246251565e-16
x = 5.551115123125783e-17
x = 2.7755575615628914e-17
x = 1.3877787807814457e-17
x = 6.938893903907228e-18
x = 3.469446951953614e-18
x = 1.734723475976807e-18
x = 8.673617379884035e-19
x = 4.336808689942018e-19
x = 2.168404344971009e-19
x = 1.0842021724855044e-19
x = 5.421010862427522e-20
x = 2.710505431213761e-20
x = 1.3552527156068805e-20
x = 6.776263578034403e-21
x = 3.3881317890172014e-21
x = 1.6940658945086007e-21
x = 8.470329472543003e-22
x = 4.235164736271502e-22
x = 2.117582368135751e-22
x = 1.0587911840678754e-22
x = 5.293955920339377e-23
x = 2.6469779601696886e-23
x = 1.3234889800848443e-23
x = 6.617444900424222e-24
x = 3.308722450212111e-24
x = 1.6543612251060553e-24
x = 8.271806125530277e-25
x = 4.1359030627651384e-25
x = 2.0679515313825692e-25
x = 1.0339757656912846e-25
x = 5.169878828456423e-26
x = 2.5849394142282115e-26
x = 1.2924697071141057e-26
x = 6.462348535570529e-27
x = 3.2311742677852644e-27
x = 1.6155871338926322e-27
x = 8.077935669463161e-28
x = 4.0389678347315804e-28
x = 2.0194839173657902e-28
x = 1.0097419586828951e-28
x = 5.048709793414476e-29
x = 2.524354896707238e-29
x = 1.262177448353619e-29
```

```
x = 6.310887241768095e-30
x = 3.1554436208840472e-30
x = 1.5777218104420236e-30
x = 7.888609052210118e-31
x = 3.944304526105059e-31
x = 1.9721522630525295e-31
x = 9.860761315262648e-32
x = 4.930380657631324e-32
x = 2.465190328815662e-32
x = 1.232595164407831e-32
x = 6.162975822039155e-33
x = 3.0814879110195774e-33
x = 1.5407439555097887e-33
x = 7.703719777548943e-34
x = 3.851859888774472e-34
x = 1.925929944387236e-34
x = 9.62964972193618e-35
x = 4.81482486096809e-35
x = 2.407412430484045e-35
x = 1.2037062152420224e-35
x = 6.018531076210112e-36
x = 3.009265538105056e-36
x = 1.504632769052528e-36
x = 7.52316384526264e-37
x = 3.76158192263132e-37
x = 1.88079096131566e-37
x = 9.4039548065783e-38
x = 4.70197740328915e-38
x = 2.350988701644575e-38
x = 1.1754943508222875e-38
x = 5.877471754111438e-39
x = 2.938735877055719e-39
x = 1.4693679385278594e-39
x = 7.346839692639297e-40
x = 3.6734198463196485e-40
x = 1.8367099231598242e-40
x = 9.183549615799121e-41
x = 4.591774807899561e-41
x = 2.2958874039497803e-41
x = 1.1479437019748901e-41
x = 5.739718509874451e-42
x = 2.8698592549372254e-42
x = 1.4349296274686127e-42
x = 7.174648137343064e-43
x = 3.587324068671532e-43
x = 1.793662034335766e-43
x = 8.96831017167883e-44
x = 4.484155085839415e-44
```

```
x = 2.2420775429197073e-44
x = 1.1210387714598537e-44
x = 5.605193857299268e-45
x = 2.802596928649634e-45
x = 1.401298464324817e-45
x = 7.006492321624085e-46
x = 3.503246160812043e-46
x = 1.7516230804060213e-46
x = 8.758115402030107e-47
x = 4.3790577010150533e-47
x = 2.1895288505075267e-47
x = 1.0947644252537633e-47
x = 5.473822126268817e-48
x = 2.7369110631344083e-48
x = 1.3684555315672042e-48
x = 6.842277657836021e-49
x = 3.4211388289180104e-49
x = 1.7105694144590052e-49
x = 8.552847072295026e-50
x = 4.276423536147513e-50
x = 2.1382117680737565e-50
x = 1.0691058840368783e-50
x = 5.345529420184391e-51
x = 2.6727647100921956e-51
x = 1.3363823550460978e-51
x = 6.681911775230489e-52
x = 3.3409558876152446e-52
x = 1.6704779438076223e-52
x = 8.352389719038111e-53
x = 4.176194859519056e-53
x = 2.088097429759528e-53
x = 1.044048714879764e-53
x = 5.22024357439882e-54
x = 2.61012178719941e-54
x = 1.305060893599705e-54
x = 6.525304467998525e-55
x = 3.2626522339992623e-55
x = 1.6313261169996311e-55
x = 8.156630584998156e-56
x = 4.078315292499078e-56
x = 2.039157646249539e-56
x = 1.0195788231247695e-56
x = 5.0978941156238473e-57
x = 2.5489470578119236e-57
x = 1.2744735289059618e-57
x = 6.372367644529809e-58
x = 3.1861838222649046e-58
x = 1.5930919111324523e-58
```

```
x = 7.965459555662261e-59
x = 3.982729777831131e-59
x = 1.9913648889155653e-59
x = 9.956824444577827e-60
x = 4.9784122222889134e-60
x = 2.4892061111444567e-60
x = 1.2446030555722283e-60
x = 6.223015277861142e-61
x = 3.111507638930571e-61
x = 1.5557538194652854e-61
x = 7.778769097326427e-62
x = 3.8893845486632136e-62
x = 1.9446922743316068e-62
x = 9.723461371658034e-63
x = 4.861730685829017e-63
x = 2.4308653429145085e-63
x = 1.2154326714572542e-63
x = 6.077163357286271e-64
x = 3.0385816786431356e-64
x = 1.5192908393215678e-64
x = 7.596454196607839e-65
x = 3.7982270983039195e-65
x = 1.8991135491519597e-65
x = 9.495567745759799e-66
x = 4.7477838728798994e-66
x = 2.3738919364399497e-66
x = 1.1869459682199748e-66
x = 5.934729841099874e-67
x = 2.967364920549937e-67
x = 1.4836824602749686e-67
x = 7.418412301374843e-68
x = 3.7092061506874214e-68
x = 1.8546030753437107e-68
x = 9.273015376718553e-69
x = 4.636507688359277e-69
x = 2.3182538441796384e-69
x = 1.1591269220898192e-69
x = 5.795634610449096e-70
x = 2.897817305224548e-70
x = 1.448908652612274e-70
x = 7.24454326306137e-71
x = 3.622271631530685e-71
x = 1.8111358157653425e-71
x = 9.055679078826712e-72
x = 4.527839539413356e-72
x = 2.263919769706678e-72
x = 1.131959884853339e-72
x = 5.659799424266695e-73
```

```
x = 2.8298997121333476e-73
x = 1.4149498560666738e-73
x = 7.074749280333369e-74
x = 3.5373746401666845e-74
x = 1.7686873200833423e-74
x = 8.843436600416711e-75
x = 4.421718300208356e-75
x = 2.210859150104178e-75
x = 1.105429575052089e-75
x = 5.527147875260445e-76
x = 2.7635739376302223e-76
x = 1.3817869688151111e-76
x = 6.908934844075556e-77
x = 3.454467422037778e-77
x = 1.727233711018889e-77
x = 8.636168555094445e-78
x = 4.3180842775472223e-78
x = 2.1590421387736112e-78
x = 1.0795210693868056e-78
x = 5.397605346934028e-79
x = 2.698802673467014e-79
x = 1.349401336733507e-79
x = 6.747006683667535e-80
x = 3.3735033418337674e-80
x = 1.6867516709168837e-80
x = 8.433758354584419e-81
x = 4.2168791772922093e-81
x = 2.1084395886461046e-81
x = 1.0542197943230523e-81
x = 5.271098971615262e-82
x = 2.635549485807631e-82
x = 1.3177747429038154e-82
x = 6.588873714519077e-83
x = 3.2944368572595385e-83
x = 1.6472184286297693e-83
x = 8.236092143148846e-84
x = 4.118046071574423e-84
x = 2.0590230357872116e-84
x = 1.0295115178936058e-84
x = 5.147557589468029e-85
x = 2.5737787947340145e-85
x = 1.2868893973670072e-85
x = 6.434446986835036e-86
x = 3.217223493417518e-86
x = 1.608611746708759e-86
x = 8.043058733543795e-87
x = 4.021529366771898e-87
x = 2.010764683385949e-87
```

```
x = 1.0053823416929744e-87
x = 5.026911708464872e-88
x = 2.513455854232436e-88
x = 1.256727927116218e-88
x = 6.28363963558109e-89
x = 3.141819817790545e-89
x = 1.5709099088952725e-89
x = 7.854549544476363e-90
x = 3.9272747722381812e-90
x = 1.9636373861190906e-90
x = 9.818186930595453e-91
x = 4.909093465297727e-91
x = 2.4545467326488633e-91
x = 1.2272733663244316e-91
x = 6.136366831622158e-92
x = 3.068183415811079e-92
x = 1.5340917079055395e-92
x = 7.670458539527698e-93
x = 3.835229269763849e-93
x = 1.9176146348819244e-93
x = 9.588073174409622e-94
x = 4.794036587204811e-94
x = 2.3970182936024055e-94
x = 1.1985091468012028e-94
x = 5.992545734006014e-95
x = 2.996272867003007e-95
x = 1.4981364335015035e-95
x = 7.490682167507517e-96
x = 3.745341083753759e-96
x = 1.8726705418768793e-96
x = 9.363352709384397e-97
x = 4.6816763546921983e-97
x = 2.3408381773460992e-97
x = 1.1704190886730496e-97
x = 5.852095443365248e-98
x = 2.926047721682624e-98
x = 1.463023860841312e-98
x = 7.31511930420656e-99
x = 3.65755965210328e-99
x = 1.82877982605164e-99
x = 9.1438991302582e-100
x = 4.5719495651291e-100
x = 2.28597478256455e-100
x = 1.142987391282275e-100
x = 5.714936956411375e-101
x = 2.8574684782056875e-101
x = 1.4287342391028437e-101
x = 7.143671195514219e-102
```

```
x = 3.5718355977571093e-102
x = 1.7859177988785547e-102
x = 8.929588994392773e-103
x = 4.464794497196387e-103
x = 2.2323972485981933e-103
x = 1.1161986242990967e-103
x = 5.5809931214954833e-104
x = 2.7904965607477417e-104
x = 1.3952482803738708e-104
x = 6.976241401869354e-105
x = 3.488120700934677e-105
x = 1.7440603504673385e-105
x = 8.720301752336693e-106
x = 4.3601508761683463e-106
x = 2.1800754380841732e-106
x = 1.0900377190420866e-106
x = 5.450188595210433e-107
x = 2.7250942976052165e-107
x = 1.3625471488026082e-107
x = 6.812735744013041e-108
x = 3.4063678720065206e-108
x = 1.7031839360032603e-108
x = 8.515919680016301e-109
x = 4.257959840008151e-109
x = 2.1289799200040754e-109
x = 1.0644899600020377e-109
x = 5.3224498000101884e-110
x = 2.6612249000050942e-110
x = 1.3306124500025471e-110
x = 6.653062250012736e-111
x = 3.326531125006368e-111
x = 1.663265562503184e-111
x = 8.31632781251592e-112
x = 4.15816390625796e-112
x = 2.07908195312898e-112
x = 1.03954097656449e-112
x = 5.19770488282245e-113
x = 2.598852441411225e-113
x = 1.2994262207056124e-113
x = 6.497131103528062e-114
x = 3.248565551764031e-114
x = 1.6242827758820155e-114
x = 8.121413879410078e-115
x = 4.060706939705039e-115
x = 2.0303534698525194e-115
x = 1.0151767349262597e-115
x = 5.075883674631299e-116
x = 2.5379418373156492e-116
```

```
x = 1.2689709186578246e-116
x = 6.344854593289123e-117
x = 3.1724272966445615e-117
x = 1.5862136483222808e-117
x = 7.931068241611404e-118
x = 3.965534120805702e-118
x = 1.982767060402851e-118
x = 9.913835302014255e-119
x = 4.9569176510071274e-119
x = 2.4784588255035637e-119
x = 1.2392294127517818e-119
x = 6.196147063758909e-120
x = 3.0980735318794546e-120
x = 1.5490367659397273e-120
x = 7.745183829698637e-121
x = 3.8725919148493183e-121
x = 1.9362959574246591e-121
x = 9.681479787123296e-122
x = 4.840739893561648e-122
x = 2.420369946780824e-122
x = 1.210184973390412e-122
x = 6.05092486695206e-123
x = 3.02546243347603e-123
x = 1.512731216738015e-123
x = 7.563656083690075e-124
x = 3.7818280418450374e-124
x = 1.8909140209225187e-124
x = 9.454570104612593e-125
x = 4.727285052306297e-125
x = 2.3636425261531484e-125
x = 1.1818212630765742e-125
x = 5.909106315382871e-126
x = 2.9545531576914354e-126
x = 1.4772765788457177e-126
x = 7.386382894228589e-127
x = 3.6931914471142943e-127
x = 1.8465957235571472e-127
x = 9.232978617785736e-128
x = 4.616489308892868e-128
x = 2.308244654446434e-128
x = 1.154122327223217e-128
x = 5.770611636116085e-129
x = 2.8853058180580424e-129
x = 1.4426529090290212e-129
x = 7.213264545145106e-130
x = 3.606632272572553e-130
x = 1.8033161362862765e-130
x = 9.016580681431383e-131
```

```
x = 4.5082903407156913e-131
x = 2.2541451703578456e-131
x = 1.1270725851789228e-131
x = 5.635362925894614e-132
x = 2.817681462947307e-132
x = 1.4088407314736535e-132
x = 7.044203657368268e-133
x = 3.522101828684134e-133
x = 1.761050914342067e-133
x = 8.805254571710335e-134
x = 4.4026272858551673e-134
x = 2.2013136429275836e-134
x = 1.1006568214637918e-134
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x = 2.7516420536594796e-135
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x = 3.2066693760143564e-291
x = 1.6033346880071782e-291
x = 8.016673440035891e-292
x = 4.008336720017946e-292
x = 2.004168360008973e-292
x = 1.0020841800044864e-292
x = 5.010420900022432e-293
x = 2.505210450011216e-293
x = 1.252605225005608e-293
x = 6.26302612502804e-294
x = 3.13151306251402e-294
x = 1.56575653125701e-294
x = 7.82878265628505e-295
x = 3.914391328142525e-295
x = 1.9571956640712625e-295
x = 9.785978320356312e-296
x = 4.892989160178156e-296
x = 2.446494580089078e-296
x = 1.223247290044539e-296
x = 6.116236450222695e-297
x = 3.0581182251113476e-297
x = 1.5290591125556738e-297
x = 7.645295562778369e-298
x = 3.8226477813891845e-298
x = 1.9113238906945923e-298
x = 9.556619453472961e-299
x = 4.778309726736481e-299
x = 2.3891548633682403e-299
x = 1.1945774316841202e-299
x = 5.972887158420601e-300
x = 2.9864435792103004e-300
x = 1.4932217896051502e-300
x = 7.466108948025751e-301
x = 3.7330544740128755e-301
x = 1.8665272370064378e-301
x = 9.332636185032189e-302
x = 4.6663180925160944e-302
x = 2.3331590462580472e-302
x = 1.1665795231290236e-302
x = 5.832897615645118e-303
x = 2.916448807822559e-303
x = 1.4582244039112795e-303
x = 7.291122019556398e-304
x = 3.645561009778199e-304
```

```
x = 1.8227805048890994e-304
x = 9.113902524445497e-305
x = 4.5569512622227484e-305
x = 2.2784756311113742e-305
x = 1.1392378155556871e-305
x = 5.696189077778436e-306
x = 2.848094538889218e-306
x = 1.424047269444609e-306
x = 7.120236347223045e-307
x = 3.5601181736115222e-307
x = 1.7800590868057611e-307
x = 8.900295434028806e-308
x = 4.450147717014403e-308
x = 2.2250738585072014e-308
x = 1.1125369292536007e-308
x = 5.562684646268003e-309
x = 2.781342323134e-309
x = 1.390671161567e-309
x = 6.953355807835e-310
x = 3.4766779039175e-310
x = 1.73833895195875e-310
x = 8.691694759794e-311
x = 4.345847379897e-311
x = 2.1729236899484e-311
x = 1.086461844974e-311
x = 5.43230922487e-312
x = 2.716154612436e-312
x = 1.35807730622e-312
x = 6.7903865311e-313
x = 3.39519326554e-313
x = 1.69759663277e-313
x = 8.487983164e-314
x = 4.243991582e-314
x = 2.121995791e-314
x = 1.0609978955e-314
x = 5.304989477e-315
x = 2.65249474e-315
x = 1.32624737e-315
x = 6.63123685e-316
x = 3.3156184e-316
x = 1.6578092e-316
x = 8.289046e-317
x = 4.144523e-317
x = 2.0722615e-317
x = 1.036131e-317
x = 5.180654e-318
x = 2.590327e-318
x = 1.295163e-318
```

```
x = 6.4758e-319
x = 3.2379e-319
x = 1.61895e-319
x = 8.095e-320
x = 4.0474e-320
x = 2.0237e-320
x = 1.012e-320
x = 5.06e-321
x = 2.53e-321
x = 1.265e-321
x = 6.3e-322
x = 3.16e-322
x = 1.6e-322
x = 8.0e-323
x = 4.0e-323
x = 2.0e-323
x = 1.0e-323
x = 5.0e-324
x = 0.0
```

[68]:
```julia
import Pkg; Pkg.add("Plots")
using Plots
gr(size=(400,300))
```

```
  Resolving package versions…
 No Changes to `~/.julia/environments/v1.6/Project.toml`
 No Changes to `~/.julia/environments/v1.6/Manifest.toml`
```

[68]:
```julia
Plots.GRBackend()
```

Aqui eu vou pegar o número zero, que é meu x. Vou pegar um vetor de números X de todos que eu passar, onde o primeiro é zero E vou pegar um vetor de distâncias D entre números Então eu quero saber, a partir de um número, qual o próximo número posso guardar na máquina?

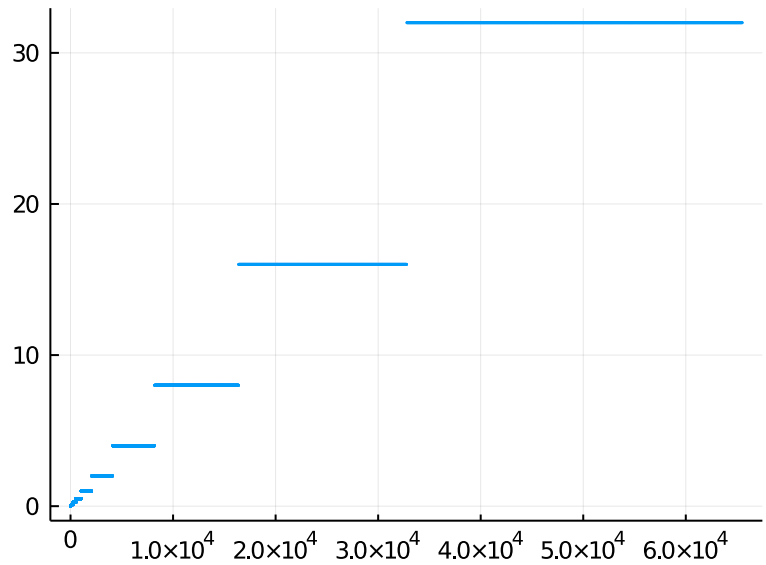
[69]:
```julia
x = Float16(0.0)
X = Float16[0.0]
D = Float16[0.0]
while x < Inf
    y = nextfloat(x) #aqui eu pego o próximo float
    d = y -x #calculo a distância entre esses dois
    x = y
    push!(X,x) #guardo o valor do x atual
    push!(D,d) #guardo a distância que estou do número atual
end
```

Aqui eu vou plotar pontos X em relação a distância D entre um número e o anterior Sabemos que a distância entre dois números inteiros consecutivos é sempre um. Já no

ponto flutuante, não é o caso. Conforme vamos aumentando, a mantissa vai aumentando também, porém há momentos em que a mantissa não consegue aumentar e eu sou obrigado a mudar a base Quando temos um número grande o suficiente e queremos fazer ele menos um, a máquina não consegue, não é possível escrever. Temos então que a precisão não é infinita

[71]:
```
scatter(X, D, ms=1, m=(stroke(0)), leg=false)
```

[71]:



[73]:
```
length(D)
```

[73]: 31745

[63]:
```
0.1 + 0.2
```

[63]: 0.30000000000000004

Sabemos que isto está a nível de hardware, ou seja, ele não consegue armazenar com precisão os valores e os mesmos não são tratados via software

[64]:
```
0.3
```

[64]: 0.3

Esse número tem a ver com a precisão do hardware

[65]:
```
0.1 + 0.2 - 0.3
```

[65]: 5.551115123125783e-17

A precisão da máquina é definida como sendo o menor valor para o qual 1 + epsilon > 1 Sempre temos para qualquer x real, existe um x' na máquina tal que a distância entre esses dois é limitada por esse cara relativamente, ou seja |x - x'| <= epsilon

[75]:
```
e = 1.0
```

[75]: 1.0

Temos que enquanto 1 mais um número real positivo for maior que 1, que seria sempre verdade dividimos ele sempre por 2.

Quando acabar eu vou multiplicar ele por 2 e eu vou ter o menor valor que ainda faz diferença

[76]:
```
while 1 + e > 1
    e = e / 2
end
e = 2e
```

[76]: 2.220446049250313e-16

Eps é uma função do Julia que nos diz quem é o epsilon da máquina. Sabemos também que o epsilon depende do número de bits, então para o float32 é um valor diferentee float16 também

[77]:
```
eps(Float64)
```

[77]: 2.220446049250313e-16

[78]:
```
eps(Float32)
```

[78]: 1.1920929f-7

[79]:
```
eps(Float16)
```

[79]: Float16(0.000977)

[81]:
```
e
```

[81]: 2.220446049250313e-16

[82]:
```
1 + 1e-16
```

[82]: 1.0

A soma aqui nos casos abaixo deixa de ser associativa

[86]:
```
(1+ 1e-16) + 1e-16
```

[86]: 1.0

[87]:
```
1+ (1e-16 + 1e-16)
```

[87]: 1.0000000000000002

$$\frac{1}{2\sqrt{a}} = \frac{d}{dx}\left(\sqrt{x}\right)|_{x=a} = \lim_{h\to 0}\frac{\sqrt{a+h}-\sqrt{a}}{h} \approx \frac{\sqrt{a+h}-\sqrt{a}}{h} = \frac{1}{\sqrt{a+h}+\sqrt{a}}$$

***

Quando eu aproximo por um h finito, eu faço um erro e quando eu aproximo o

$$\frac{\sqrt{a+h}-\sqrt{a}}{h}$$

no hardware, eu faço outro erro e consequentemente esses dois erros são conflitantes

[120]:
```
h = 10.0^(-14)
a = 1.0
1 / 2 / sqrt(a)
```

[120]: 0.5

[121]:
```
(sqrt(a + h) - sqrt(a)) / h
```

[121]: 0.4884981308350689

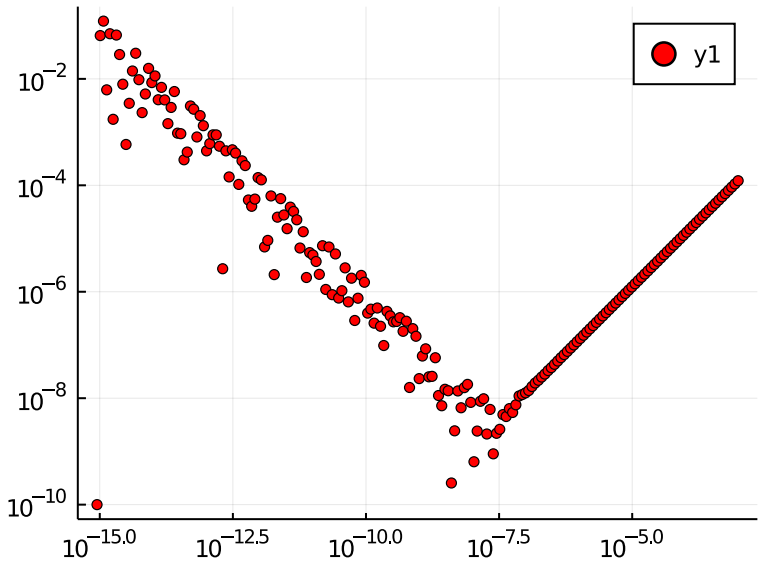
[124]:
```
using Plots
gr(size=(400,300))
plot(rand(3),rand(3))
nothing
```

Aqui temos para cada um dos valores de h, calcula a aproximação, calcula o erro e faz um gráfico desse erro

[135]:
```
FUNC(h) = max(abs((sqrt(1 + h) - 1) / h - 0.5), 1e-10)
#E(h) = max(abs(1 / (sqrt(1 + h) + 1) - 0.5), 1e-16)
valores_h = [2.0^p for p = range(-50, -10, length=200)]

plot(xaxis=:log, yaxis=:log)
scatter!(valores_h, FUNC.(valores_h), c=:red, ms=3)
#scatter!(valores_h, E.(valores_h), c=:blue, ms=3)
```

[135]:



A precisão da máquina nos diz o máximo de distância do número real, mas não quer dizer que eu sempre vou estar a uma distância epsilon do número real

[131]:
```
setprecision(2^14)
BigFloat(1) / 10 + BigFloat(2) / 10
```

[131]:
```
0.300000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
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000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000000000000000000000000000000000000000000000
```

```
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
0000000000000000000000000000000000000000000000000000000000000000000000000000000000
```

[133]:
```
BigFloat(pi)
```

[133]:
```
3.14159265358979323846264338327950288419716939937510582097494459230781640628620
9986280348253421170679821480865132823066470938446095505822317253594081284811174
5028410270193852110555964462294895493038196442881097566593344612847564823378678
3165271201909145648566923460348610454326648213393607260249141273724587006606315
5881748815209209628292540917153643678925903600113305305488204665213841469519415
1160943305727036575959195309218611738193261179310511854807446237996274956735188
57527248912279381830119491298336733624406566430860213949463952247371907021798609
43702770539217176293176752384674818467669405132000568127145263560827785771342757
78960917363717872146844090122495343014654958537105079227968925892354201995611212
90219608640344181598136297747713099605187072113499999983729780499510597317328160
96318595024459455346908302642522308253344685035261931188171010003137838752886587
53320838142061717766914730359825349042875546873115956286388235378759375195778185
77805321712268066130019278766111959092164201989380952572010654858632788659361533
81827968230301952035301852968995773622599413891249721775283479131515574857242454
15069595082953311686172785588907509838175463746493931925506040092770167113900984
88240128583616035637076601047101819429555961989467678374494482553797747268471040
47534646208046684259069491293313677028989152104752162056966024058038150193511253
38243003558764024749647326391419927260426992279678235478163600934172164121992458
63150302861829745557067498385054945885869269956909272107975093029553211653449872
02755960236480665499119881834797753566369807426542527862551818417574672890977772
79380008164706001614524919217321721477235014144197356854816136115735255213347574
18494684385233239073941433345477624168625189835694855620992192221842725502542568
87671790494601653466804988627232791786085784383827967976681454100953883786360950
68006422512520511739298489608412848862694560424196528502221066118630674427862203
91949450471237137869609563643719172874677646575739624138908658326459958133904780
27590099465764078951269468398352595709825822620522489407726719478268482601476990
90264013639443745530506820349625245174939965143142980919065925093722169646151570
98583874105978859597729754989301617539284681382686838689427741559918559252459539
59431049972524680845987273644695848653836736222626099124608051243884390451244136
54976278079771569143599770012961608944169486855584840635342207222582848864815845
60285060168427394522674676788952521385225499546667278239864565961163548862305774
56498035593634568174324112515076069479451096596094025228879710893145669136867228
74894056010150330861792868092087476091782493858900971490967598526136554978189312
97848216829989487226588048575640142704775551323796414515237462343645428584447952
65867821051141354735739523113427166102135969536231442952484937187110145765403590
27993440374200731057853906219838744780847848968332144571386875194350643021845319
10484810053706146806749192781911979399520614196634287544406437451237181921799983
91015919561814675142691239748940907186494231961567945208095146550225231603881930
14209376213785595663893778708303906979207734672218256259966150142150306803844773
45492026054146659252014974428507325186660021324340881907104863317346496514539057
96268561005508106658796998163574736384052571459102897064140110971206280439039759
51567715770042033786993600723055876317635942187312514712053292819182618612586732
15791984148488291644706095752706957220917567116722910981690915280173506712748583
22287183520935396572512108357915136988209144421006751033467110314126711136990865
85163983150197016515116851714376576183515565088490998985998238734552833163550764
79185
```

```
3589322618548963213293308985706420467525907091548141654985946163718027098199430 9
9244889575712828905923233260972997120844335732654893823911932597463667305836041 4
2813883032038249037589852437441702913276561809377344403070746921120191302033038 0
1976211011004492932151608424448596376698389522868478312355265821314495768572624 3
3441893039686426243410773226978028073189154411010446823252716201052652272111660 3
9666557309254711055785376346682065310989652691862056476931257058635662018558100 7
2936065987648611791045334885034611365768675324944166803962657978771855608455296 5
4126654085306143444318586769751456614068007002378776591344017127494704205622305 3
8994561314071127000407854733269939081454664645880797270826683063432858785698305 2
3580893306575740679545716377525420211495576158140025012622859413021647155097925 9
2309907965473761255176567513575178296664547791745011299614890304639947132962107 3
4043751895735961458901938971311179042978285647503203198691514028708085990480109 4
1214722131794764777262241425485454033215718530614228813758504306332175182979866 2
2371721591607716692547487389866549494501146540628433663937900397692656721463853 0
6736096571209180763832716641627488880078692560290228472104031721186082041900042 2
9661711963779213375751149595015660496318629472654736425230817703675159067350235 0
7283540567040386743513622224771589150495309844489333 11
```

Definição de quantos bits de precisão queremos, mas nesse caso aqui pode se tornar muito lento

[134]:
```
setprecision(2^15)
sqrt(big"2")^2 - 2
```

[134]:
```
-1.4129671931155272885554804375717436907474887820545013118188285159380758593320 3
2541582207048707354961538816795326572752765551515170858120315735745353919733923 0
2893161539938044155880023867028163736517680099263563259094136619942839235731249 3
9011100936585695252104670102074324065723752579793187037740715378506499325042883 4
4936412635847919085238688545762903317954433719595951711260478891553341348402289 0
0308644780889748639074023270783554907425258226077677444597315120888798685677129 2
7719260427374096235800947615456972191898019790584265475419779119597263166360697 4
0150953600409184327035854001506454507389757458001773498175860266023892382922836 2
1602979033604230942158459997792959016829670392975028831099449779666139080696269 6
8919155829501912396447204075559995839278832256462168921725383430108451942863901 2
7667355183440938215851124461670687366825299229804293245382144204205604487730488 8
3856603760935874080161032819017159436540079378686702589561871946669632009887533 7
6746935349988903290282055939637548939681710222630064711787291304593309139879772 5
6785219410196560937567310602571208817349659110332183424692570225652112549432512 2
4736380066138223851534082530658745440081387021974608591431334366867777331864285 2
4960997101509070389949236950878270485812467178459017860251892525223256473473145 6
6378527464343528757147893978239921008478384602411480156788182926828554938610799 8
9053708040296969536693263053036396999523257194700185705460427003001042801734873 2
1394657843045622879171180798500961612382204837798971201394022800518456861909309 2
1253626443239878080336209730282769876349912705578687150328250784716344284952144 3
0502832475748928522419547981860386047596309225206416357642178331754524995595693 0
3770799409055976213615287909590115447750250645980493289156857605097627613819126 6
5590093045830893502332156924304726034141487151652199530243918012198569987903966 3
```

1941943989483770370076448788243928527978493939481652598990108110809076102928843 3
6673399411740549542469131649655000814212119208839155882163529473912410306226850 3
8827829605350035848499311138367050712083778489219092037647031291682276912156977 1
6275076242034024952375744479495186907817445463389916996241797768405719343204351 4
5257742154022687397839042663919527013208432012608204990431354202780771335427538 3
7146809854507436078210538715424994495758386976837494663367033869729779348749511 9
4250277031336265090460531899933281164515415465789648298122217939244029193610681 4
7873824697431532755948632166251887562274480242458569523073380960285818923187270 0
5759040203449594459075551431709671760868163573328647113493055325569761381075968 5
9730351304846686217274369043113609226328181014695596465091256187560560838721759 0
3765863391039848855188584374600451779026999580824084399853282182131950584480906 9
7045593471074255980995143938274160486459895359664670991964001882930256174559216 0
9921907949196367633621927882712319320617229331297604043940258994153007483590857 6
9698293762095198076701526632309189471824590240885826870912403836674328903686133 7
9696998469956875181532918640300637033270179357326196137756127588823561072737519 2
1033669769310165589588120015690292005452034953418776425561854234363502732818155 4
0159573790327698999479327714896456654817708672677205524583129655063443420248293 3
1034231593486994662116314423575219684385926375605433748842826608395380284674509 7
7109988002905162517733748118182940791509318995383511545870026145498757263919497 9
4079191073599885257380530194909887130283346654797002032631389302378003977049720 4
1150303088788467614912993296871867720715335381911351697220828761586304898002548 4
6374000053069962836250832015271906604064560388436653216508222932587559423254048 1
1216144316245845775118028799644129516902821598945562567226917292958189778688279 0
7039038602453223584073233149866157897788362787940697971367789068463613875171581 6
0207294432087218604413818068410383238650815496931085252694956924982495485855053 0
8593385624641189239624788463993125354695877511053575212934813519479937763446281 2
5393158448306881629784334626327768010016882432625109307563053405946191630088787 1
3233550356007850896842033709299351422946163329586637741418390775671541253115567 5
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