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BENDING STIFFENER DESIGN THROUGH STRUCTURAL OPTIMIZATION

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ABSTRACT

Bending stiffeners are very important ancillary equipments of umbilicals or flexible risers, since they protect the lines from overbending. Their design however is a complex task, since many load cases must be taken into account; the structure itself has a section that is variable with curvilinear coordinate. To aid the designer in this task, optimization algorithms can be used to automate the search for the best design.

In this work an optimization algorithm is applied to the design of the bending stiffener. First, a bending stiffener model is created, which is capable of simulating different load case conditions and provide, as output, results of interest such as maximum curvature, deformation along the stiffener, shear forces and so on. Then, a bending stiffener design procedure is written as an optimization problem and, for that, objective function, restrictions and design variables defined.

Study cases were performed, comparing a regular design with its optimized counterpart, under varying conditions.

INTRODUCTION

The design of marine umbilicals and flexible risers is known to be a difficult and time consuming task. The prediction of extreme loads on these structures requires several environmental conditions to be considered, generating several load cases, which must be all numerically simulated. The bending stiffener attached to the riser's top will be submitted to the same load cases.

In the bending stiffener viewpoint, as it will be discussed in the next session, it is possible to represent a dynamic load in a quasi-static manner, in such a way that a tension/angle pair represents a load case for bending stiffener design. A Matlab® code was developed to model the behavior of the bending stiffener, with certain assumptions, providing results such as curvatures, moments and strains in each section.

This model was coupled to an optimization algorithm, which is capable of searching automatically through a set of possible designs, to find the one that has the best value for a given quantity (called objective function) while satisfying certain requirements (called constraints). This is discussed in the "optimization" section.

Previous works ([1] to [5]) addressed the design of a riser, using optimization tools to speed up the process. The same kind of design difficulties is encountered in the design of a bending stiffener, which motivated the use of a similar approach.

The selected optimization algorithm is the Genetic Algorithm (GA), based on the process of natural selection. It is capable of addressing a wide class of problems and its robustness has been extensively tested in several types of problems, including structural and riser synthesis ([1] to [5]).

PHYSICAL AND MATHEMATICAL MODEL

When hanging from a floating unit, submerged cables are installed at a certain nominal angle at the top, usually in the range between 5deg and 15deg from the vertical. This angle is imposed by the top end connection, which may be an I-tube, bell-mouth, fixed support, etc.

In this scenario, submerged cables are usually subjected to dynamic loads from waves and also from the movements imposed by the floating unit. When this happens, the cables experience angle variations at the top end and this could lead to its failure due to large curvatures (e.g. "necking") which could induce large bending stresses, not to mention fatigue. In order to avoid this situation, an ancillary component called bending stiffener is used.

In a riser system, bending stiffeners play a vital role in the top end as they must protect the cable from excessive curvature.

Physically, that is accomplished by making a smooth transition from the small cable bending stiffness to the ideally infinite top connection bending stiffness.

Bending stiffeners are usually composed of two parts, one polymeric and the other metallic. The polymeric part is made of polyurethane in most of the cases and has a conic section; hence, this geometry allows a variable moment of inertia and thus a variable bending stiffness. The metallic part is responsible for the interface between the polymeric part and the connection interface at the floating unit; for this reason, the geometry of this part may vary.

Top angle variation has two main components. The first one is related to offsets and second order motions of the floating unit, which have a time scale of an order of magnitude greater than the wave periods. The second one is related to the first order motions of the floating unit, which have a time scale of the wave periods. Ideally, bending stiffeners should be designed taking into account the whole time series of the tension/angle behavior as predicted by dynamic analysis.

In practice, however, only a "few" combinations tension/angle are chosen to represent the critical situations that the bending stiffener may face involving, for example, maximum tension, angle or shear force. This quasi-static procedure assumes that the polymeric material will not be highly influenced by the frequency of the external excitations, which are of the same magnitude order of the wave periods. This assumption is consistent to the industry experiences.

Moreover, the problem can be seen as two-dimensional as the bending stiffener is axysimmetric and thus any plane passing through its longitudinal axis can be the main flexural plane. A reasonable choice is then the plane containing the bending stiffener axis and the force at its tip.

The forces that could induce changes in the flexural plane are gravitational and hydrodynamic. However, as they are generally much smaller than the force at the tip of the bending stiffener, the assumption of a two-dimensional problem holds.

The bending stiffener basic operation can be seen in the sketch presented in Fig. 1.

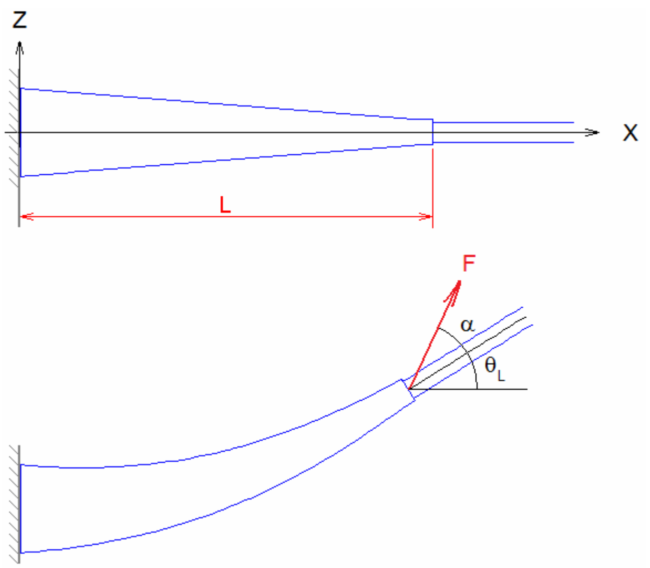

Figure 1 – Bending stiffener operation sketch

In the upper portion, the basic bending stiffener geometry is presented along with a two-dimensional coordinate system with origin at the base root diameter; a curvilinear coordinate s has the same origin. In the lower portion, the basic operating condition is presented; notice that three parameters completely define one condition: the force F, the bending stiffener angle θ_L and the angle between the bending stiffener axis and the force direction α. Let L be the bending stiffener total length.

The section prior to the conic portion was not modeled since it has a steelwork which makes it considerably more rigid than the rest of the bending stiffener.

The bending stiffener mathematical model is based on the equations derived for a two-dimensional inextensible cable submitted to large displacements as presented by Silveira & Martins [6]. According to that model,

$$\begin{cases} \dfrac{dx}{ds} = \cos\theta \\ \dfrac{dz}{ds} = \sin\theta \\ \dfrac{dF_x}{ds} = -c_x \\ \dfrac{dF_z}{ds} = -c_z + \gamma_{ef} \\ \dfrac{d\theta}{ds} = \dfrac{M}{EI} \\ \dfrac{dM}{ds} = F_x \sin\theta - F_z \cos\theta \end{cases} \tag{1}$$

In Eq. (1), (x,z) are the horizontal and vertical coordinates, (F_x, F_z) are the horizontal and vertical forces, M is the bending moment, θ is the angle between the cable's tangent and the horizontal plane, (c_x, c_z) are the horizontal and vertical hydrodynamic forces, γ_{ef} is the effective weight and EI is the cable's bending stiffness (a linear relationship between bending moment and curvature was considered).

At that time, the authors showed that when trying to directly (numerically) integrate this set of ODEs, Eq. (1) for a "large" length, numerical issues appeared and lead to erroneous solutions. Typically, this "large" value is a few times the flexural length λ_f as defined by Pesce [7]:

$$\lambda_f = \sqrt{EI/T} \tag{2}$$

In Eq. (2), T is the tension force in the cable. Silveira & Martins [6] recalled that the flexural length is associated to the boundary-layer correction from zero to finite cable bending stiffness and probably related to the numerical integration errors. Once these authors were interested in integrating the whole suspended length of the submerged cable, they have used $3\lambda_f$ in order to keep the accumulated errors as a minimum. However, in the same paper one can see that actually lengths over $50\lambda_f = 50 \times 1.2m = 60m$ could be numerically integrated. Additionally, a bending stiffener has a typical bending stiffness much greater than a pipe's bending stiffness. Therefore, it seems reasonable to try using the same set of equations to model and perform direct numerical integration scheme to the bend stiffener.

In Eq. (1), by making the hypothesis that the hydrodynamic and gravitational forces acting over the bending stiffener are much smaller than the tension forces involved, as the bending stiffener length is much smaller than the riser's length, one may disregard (c_x, c_z) and γ_{ef}. Also, by analyzing the moment equation in Eq. (1), one may see that $F_x \sin\theta - F_z \cos\theta$ corresponds to the shear force. Thus,

$$\begin{cases} \dfrac{dx}{ds} = \cos\theta \\ \dfrac{dz}{ds} = \sin\theta \\ \dfrac{d\theta}{ds} = \kappa \\ \dfrac{dM}{ds} = -V \end{cases} \tag{3}$$

In Eq. (3), κ is the curvature and V is the shear force.

If now a free-body diagram of the bending stiffener is constructed, it is noticeable that the reaction loads at the base (which will be transferred to the platform) depend on the force at the tip of the bending stiffener and also depends on the angles θ_L and α as shown in Fig. 1. The free-body diagram is illustrated in Fig. 2.

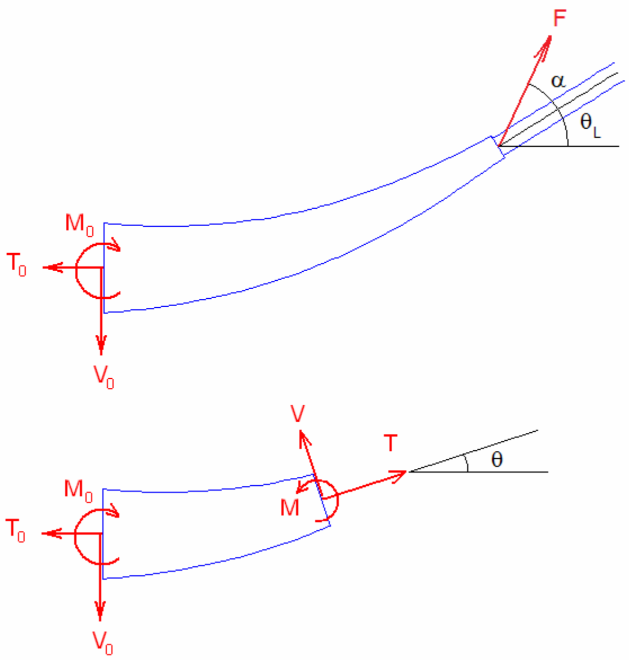

Figure 2 – Bending stiffener free-body diagram

From the upper and lower portions of Fig. 2, one has, respectively:

$$\begin{cases} T_0 = F\cos(\theta_L + \alpha) \\ V_0 = F\sin(\theta_L + \alpha) \end{cases} \tag{4}$$

$$\begin{cases} T_0 = T\cos\theta - V\sin\theta \\ V_0 = T\sin\theta + V\cos\theta \end{cases} \tag{5}$$

From Eq. (4) and Eq. (5), it is straightforward to see that:

$$V = F\sin(\theta_L + \alpha - \theta) \tag{6}$$

Eq. (6) is the relation between the shear V and the force F at the tip of the bending stiffener, which is an important relation to be used in Eq. (3). Before, by making the assumption that, for $s > \lambda_f$ the shear forces in the cable are very small when compared to the tension forces at the same location, one has $\alpha \to 0$, i.e., $F \to T$.

Combining Eq. (4), Eq. (6) and the constitutive relation $M = EI\kappa$ in Eq. (1) one has:

$$\begin{cases} \dfrac{dx}{ds} = \cos\theta \\ \dfrac{dz}{ds} = \sin\theta \\ \dfrac{d\theta}{ds} = \kappa \\ \dfrac{d\kappa}{ds} = -\dfrac{1}{EI}\left[\dfrac{dEI}{ds}\kappa + T\sin(\theta_L - \theta)\right] \end{cases} \tag{7}$$

An additional hypothesis of no slip among cable and bending stiffener is made and, as consequence, the bending stiffness of the assembly (bending stiffener + cable) may be taken as the sum of both bending stiffnesses.

The boundary conditions associated to Eq. (7) and with the present problem is

$$\begin{cases} x(0) = 0 \\ z(0) = 0 \\ \theta(0) = 0 \\ \theta(L) - \theta_L = 0 \end{cases} \tag{8}$$

Notice that this mathematical problem is a two-point boundary value problem as described by Keller [8]. This type of problem may be solved by a shooting algorithm or by a collocation method. This paper adopts the second alternative.

Actually, the mathematical problem as defined by Eq. (7) and Eq. (8) is the same as presented previously by Boef & Out [9] and Vaz & Lemos [10]. Souza & Ramos [11] have also revisited this problem recently and based their parametric study of bending stiffener dimensions in the work of DeRuntz [12].

OPTIMIZATION

Optimization is a research field which aims to create ways to minimize or maximize a certain quantity, which is a function (called objective function) of some variables (called design variables) and is subject to some requirements (called constraints).

These three parameters (objective function, design variables and constraints) define an optimization problem. A great number of algorithms for solving optimization problems exist, one of which must be chosen depending on the characteristics of a given problem. In this work the Genetic Algorithm (GA), described in the next session, was used.

In a structural design problem, typical objective functions are the cost, size or weight of a structure (to be minimized) or its resistance to given set of loads (to be maximized). Typical design variables are the dimensions of structural components and their materials. Typical constraints include minimum and maximum dimensions of components, maximum allowable stresses, tensions, curvatures or strains.

The above are only typically found examples, but any quantifiable criteria may be used either for objective function or as a constraint.

GENETIC ALGORITHM

The Genetic Algorithm mimics the process of natural selection, guided by random decisions, which makes it robust and capable of, most times, escape local minima.

In GA the design variables are lined up resulting in a string of values, which is treated as a chromosome. To each chromosome a set of design variables is associated, the solution corresponding to those values is evaluated and then its objective function value calculated.

The chromosomes are then selected to mate and breed, with greater probabilities of being selected for those which represent a better solution (as measured by the objective function value).

The selected chromosomes are then combined via crossover when, just as in nature, part of the chromosome of one parent is connected to part of the chromosome of the other, generating two pairs of chromosomes, which represent two solutions that combine features of both parents.

A mutation operator is also applied, randomly changing some values of the chromosome, to keep the genetic diversity of the population.

In this process a new population with different solutions is created. These new solutions have their objective function values calculated and the process is repeated until either convergence or a given number of generations is achieved.

No specific knowledge about the problem (like gradients or convexity) is required, only the capability to calculate the objective function in each point of the design space.

For further information on GAs, the reader is referred to [13] and [14].

OPTIMIZATION PROBLEM DEFINITION

As previously discussed three things define an optimization problem: design variables, objective function and constraints.

Design Variables

The design variables are the variables used by the optimizer to interact with the structure. It is by changing them that a new design which fulfills all requirements and minimizes or maximizes the objective function will be found. Therefore, these variables shall be able to represent all main dimensions of a bending stiffener.

Due to manufacturing process reasons, the inner diameter is considered given depending only on the cable external diameter and is therefore not left free to vary. The same applies to the material of the bending stiffener, which is also considered given.

Figure 3 presents the basic sketch of the studied bending stiffener and the four design variables: length of the conical section L_c, length of the tip L_t, diameter at the root of the stiffener conical section D_r and diameter at the tip D_t.

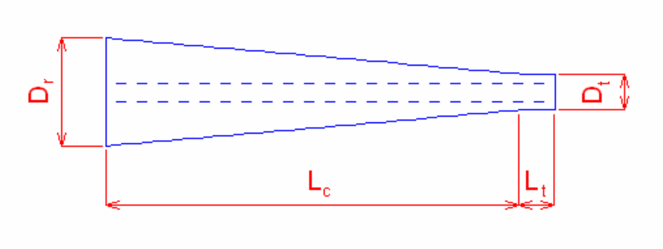

Figure 3 – Bending stiffener dimensions

Objective Function

The polymer volume will be used as objective function, since smaller bending stiffeners tend to present advantages such as:

- Easier to install and interface with other equipment;
- Smaller moments transmitted to platform structure;
- Cheaper;

Since the volume does not depend on the load cases, they are only important to check if a particular design violates or not the constraints.

Constraints

The definition of the constraints is a problem specific decision and thus prior knowledge of the problem is of great importance. Based on standard design practice, four constraints were enforced:

- Maximum allowable curvature (in the cable);
- Maximum allowable strain in the bending stiffener polymeric part;
- Maximum bending moment at the conic section base;
- Limit values in the bending stiffener overall dimensions, in order to avoid unrealistically large or small values.

Solutions that did not respect the imposed constraints are discarded by assigning a very high objective function value, which virtually makes it impossible for them to be selected for the next generation.

CASE STUDY

In order to test the implementation two case studies with different environmental conditions and platforms were performed.

For both cases, sets of global extreme analysis were simulated using wide-used commercial non-linear finite-element structural software. These extreme analyses are based on centenary waves combined to decenary currents and vice-versa and aim to predict the highest loads and movements that an umbilical will face during its operational life.

As stated previously, the global results of interest at the top end are the tension force magnitude and its angle with respect to the bending stiffener longitudinal axis (or its support neutral axis, e.g. I-tube). Combinations of the tension/angle pairs resulting from the global extreme analysis are used as input for the bending stiffener design and optimization.

The first condition refers to an umbilical connected to a semi-submersible platform in 1000m water depth in a free-hanging configuration. A total of 50 load cases were simulated, resulting in tensions forces varying in the range 180kN to 334kN and angles varying between 5deg and 22deg. The most critical pairs are presented in Table 1. These are the pairs in which either maximum curvature or strain was closer to the imposed maximum values.

Each load case was simulated for two different elastic modulus of the polymer, to simulate two different temperature conditions. Umbilical bending stiffness is $7.16 kNm^2$ for both cases. A maximum bending moment at root of 300kNm constraint was imposed.

Table 1 – Critical Pairs (Case 1)

Pair	Tension (kN)	Angle (degrees)
1	192.2	14.3
2	333.9	21.0
3	321.2	21.4
4	247.3	22.2

The design variables were left free to vary in the predefined range given by Table 2. The range was based on experience with previous designs, but maximum values were increased to allow for different designs to be selected if they are optimum. The inner diameter of the bending stiffener is 0.14 m.

Table 2 – Design Variable Range (Case 1)

Variable	Min(m)	Max.(m)
L_c	1.0	5.0
L_t	0.10	3.0
D_r	0.20	1.0
D_t	0.17	Root diameter

The second condition refers to an umbilical connected to a FPSO platform in a shallow water depth of 100m with the cable in a lazy-S configuration. Again, a total of 50 load cases times 2 elastic modulus conditions were taken into account, combining tensions varying in the range of 15kN to 34kN and angles varying between 16deg and 57deg. The most critical pairs are presented in Table 3.

Table 3 – Critical Pairs (Case 2)

Pair	Tension (kN)	Angle (degrees)
1	33.7	48.5
2	30.7	50.8
3	26.4	52.4
4	16.4	54.7
5	19.1	56.8

The design variables were left free to vary in the predefined range given by Table 4. The inner diameter of the bending stiffener is 0.14 m.

Table 4 – Design Variable Range (Case 2)

Variable	Min(m)	Max.(m)
L_c	1.0	4.0
L_t	0.10	1.0
D_r	0.20	1.0
D_t	0.17	Root diameter

Maximum strain was limited in both conditions to 5% and maximum curvature to 0.25.

The parameters chosen for the Genetic Algorithm are shown in Table 5. They were chosen based on [5].

Table 5 – Genetic Algorithm parameters

Population	20
Crossover rate	0.75
Mutation Rate	0.1
Selection type	Roulette Wheel + Elite (2 per generation)

RESULTS

Table 6 presents the design variable values for the optimized stiffener in the first case study (Semi-submersible + 1000 m depth). The original design is presented for comparison purposes only; it does not affect the outcome of the optimization.

Table 6 – Design variables before and after optimization (Case 1)

Variable	Original(m)	Optimized (m)
L_c	2.8	2.1
L_t	0.10	0.30
D_r	0.60	0.62
D_t	0.17	0.19

As one can see, the optimized stiffener is considerably shorter than the original, 25% in the conic section and 18% for the stiffener as a whole. The root diameter was slightly increased. In order to compensate for the reduction in conic section length, the optimizer enlarged the tip, both its diameter and length.

Both stiffeners were simulated in Orcabend® [15] to compare the results with those of the developed code. The results are presented in Fig. 4 for the original bending stiffener and in Fig. 5 for the optimized one. The two lines (red and blue) represent the tension/angle pairs in which maximum admissible curvature is reached, for two bending stiffener material Young modulus conditions, related to a higher (less stiff, red line) and a lower temperature (more stiff, blue line). The Young modulus values used were 210 and 150MPa. Each one of the points is a tension/angle pair, which result from a global analysis and is used for design purposes. It can be noted that in the optimized stiffener, the maximum curvature line is closer to the load cases, that is, the material usage is better.

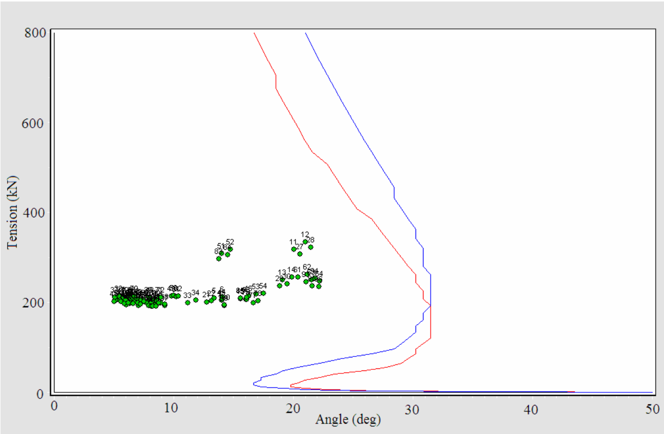

Figure 4 – Orcabend simulation, original stiffener (Case 1)

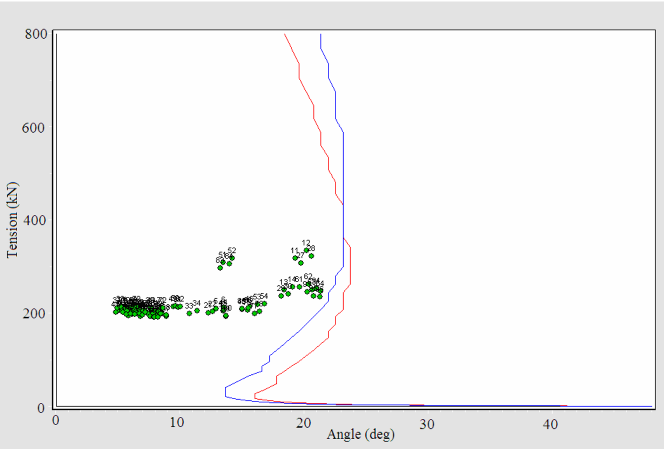

Figure 5 – Orcabend simulation, optimized stiffener (Case 1)

Maximum strain along the bending stiffener for all load cases is 4.2% for the optimized stiffener, against a 4.7% for the original one. Those results are depicted in Fig. 6 and Fig. 7 respectively. This smaller strain happened in spite of the increase in root diameter.

Maximum bending moment at the start of the conical section is slightly smaller for the optimized bending stiffener, with a value of 153.6kNm against 161.3kNm of the original.

As for the objective function, the volume of the original bending stiffener was 0.317 m^3. This was reduced to 0.269m^3 in the optimized design, a 15% decrease. Since strain levels and maximum bending moment are far from the imposed maximum, while curvature is close to the limit, it can be concluded that maximum curvature is the limiting factor, which prevents further reduction of the bending stiffener.

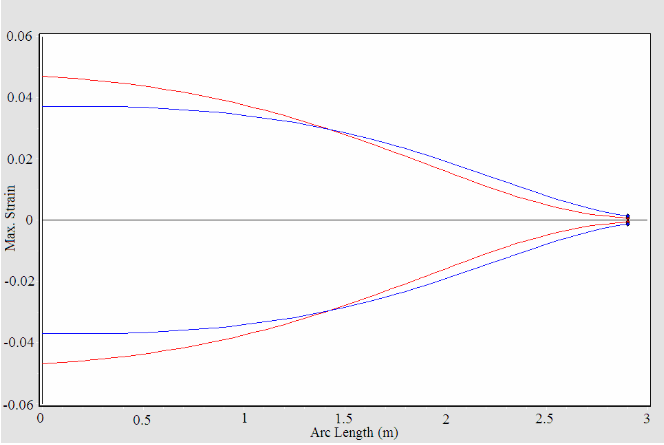

Figure 6 – Orcabend simulation, original stiffener (Case 1), strain at maximum strain case.

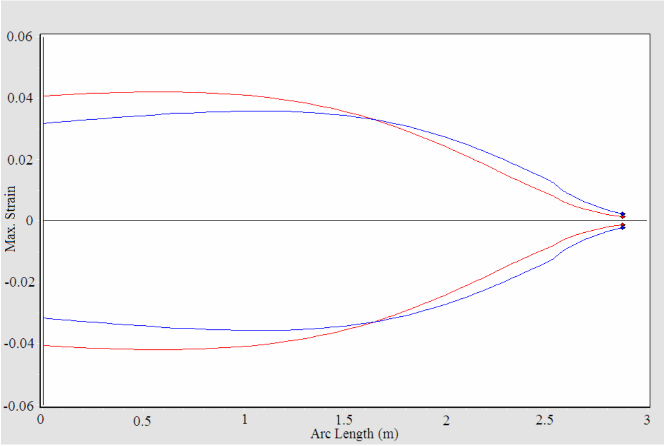

Figure 7 – Orcabend simulation, optimized stiffener (Case 1), strain at maximum strain case.

As for the second case study (FPSO + 100 m depth), again a previous design was used as initial design for the optimization process. Table 7 presents the design variable values for the original and both optimized stiffeners.

Table 7 – Design variables before and after optimization (Case 2)

Variable	Original (m)	Optimized (m)
L_c	3.4	2.7
L_t	0.30	0.25
D_r	0.54	0.38
D_t	0.18	0.177

As one can see, the optimized stiffener is shorter than the original, 15% in the conic section and 16% for the stiffener as a whole. But the major difference is in the root diameter which was decreased by 36%. The tip diameter was increased by 4%. This led to an impressive reduction in volume of 60%, from 0.324 m^3 to 0.131 m^3.

Both stiffeners were simulated in Orcabend [15] to compare the results with those of the developed code. The results are presented in Fig. 8 for the original bending stiffener and in Fig. 9 for the optimized one. The two lines (red and blue) represent the maximum admissible curvature as a function of the tension and angle. It can be noted that in the optimized stiffener, the maximum curvature line is closer to the load cases, that is, the material usage is better. Also, notice that the ratio base diameter to conic section length was reduced; this means that a smoother stiffness gradient was achieved.

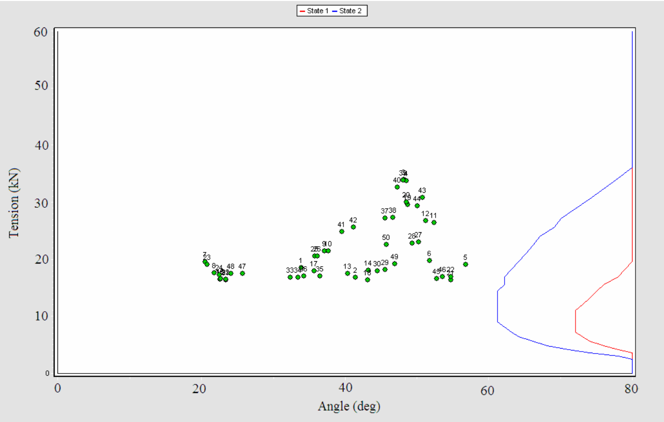

Figure 8 – Orcabend simulation, original stiffener (Case 2)

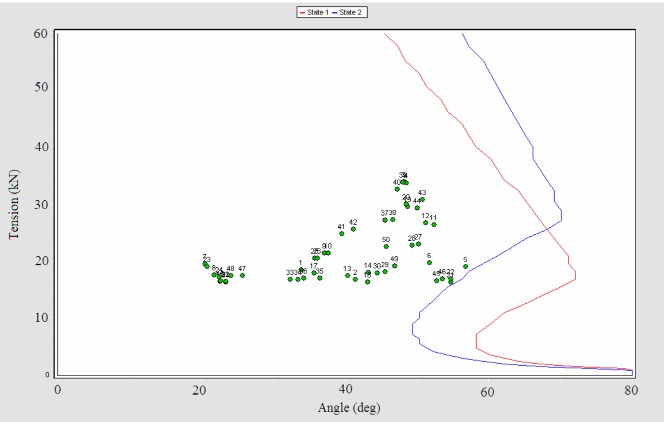

Figure 9 – Orcabend simulation, optimized stiffener (Case 2)

Maximum strain along the bending stiffener for all load cases is 5.0% for the optimized stiffener (equal to maximum allowable value), against a 4.3% for the original one.

Maximum bending moment at the start of the conical section is smaller for the optimized bending stiffener, with a value of 46.38 kNm against 69.35 kNm of the original. This is a significant 33% reduction in bending moment at the top.

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

Souza & Ramos [11] present several charts in which they analyze the effect of the variations of the design variables treated in the present work. The presented results are of great help in the design of the bending stiffener, since they provide a qualitative idea of the effect of each variable in the component’s behaviour. The analysis of those authors, however, occurs for each variable independently and it is not straightforward to optimize a bending stiffener that way. Nevertheless, this work brings an important description that helps understanding why an optimization procedure in the bending stiffener design is useful.

Therefore, the proposed optimization approach to bending stiffener design in this work has a potential use in the offshore industry. It was shown that this approach has led to very good results, with improved designs being generated and significant reduction in volume of the bending stiffener. The use of optimization has the advantages inherent to an automatic process, that is, it is much less error-prone and can be made in less time than would be necessary for a regular design, leaving the designer free to perform other tasks during its execution.

Two situations were studied, one covering shallow water, in which tensions are small but angles are high and one for deep water, which present higher tensions combined with moderate angles. In both cases the final optimized stiffener is adequate according to all criteria.

Based on Fig. 4 and Fig. 5, one can notice that the “original” design was relatively close to the “optimum” design in the first case study. Anyway, the optimization algorithm was able to find a solution with an objective function 15% smaller. On the other hand, based on Fig. 8 and Fig. 9, one can notice that the “original” design was not close to the best material usage in the second case study. Even so, the algorithm was able to find an “optimum” design that lead to a significant reduction in the objective function of 56%. It seems, although a more detailed study is necessary to assure, that no requirements are necessary to the initial design in order to make the algorithm work.

Comparison of the results for both load cases shows that the optimum bending stiffener for shallow water is both longer and thinner than the one for deeper water. This is due to larger top angles combined with smaller tensions and confirms the authors’ experience in previous designs.

As a general rule, reduction in volume is obtained through a reduction in root diameter and conic length; this leads to a stiffness reduction and therefore to smaller bending moments at the top, which is also desirable in order not to overload the structure that interfaces the bending stiffener.

In this work, only normal extreme conditions have been addressed. When abnormal conditions are taken into account, strains and curvatures tends to increase and geometry modifications might be necessary, so that the limits on these criteria are respected. This can be done in a future work.

The same is true of fatigue conditions. If a life assessment algorithm is integrated to the existing code, it will be possible to calculate the expected life of the cable and to impose a restriction on its minimum value. For this to be done it is also necessary to incorporate a local model of the studied cable, which allows stresses on the structural elements to be calculated.

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