Coupled Phenomena☆ (3rd year, LEAer)

Academic Year 23/24

Topic 5: Atmospheric Models

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Abstract

Exponential Atmospheric model, discussion and validity, multi-layer exponential models, scale factors, dependencies of the most complete models, review of existing atmospheric models and their properties, applications: time of falling of a satellite

Contents

1. Introduction

The exploration of atmospheric models has played a pivotal role in advancing the aerospace industry, contributing significantly to technological breakthroughs in aviation and space exploration. These advancements have had far-reaching effects beyond aerospace, impacting fields such as meteorology, telecommunications, Earth's topography, and space exploration. Additionally, it is fundamental to represent the atmosphere adequately since, without precise data, the task of modeling the trajectory of re-entry vehicles becomes much more complicated. This topic, even though it will not be thoroughly discussed in this report, will be the central point behind the software.

This report will delve into the diverse approaches scientists have taken in studying Earth's atmosphere by, firstly, focusing on the analysis of one specific Model−the Exponential Atmosphere. This section will explain how this Model is defined, as well as the advantages and disadvantages related to this approach.

Subsequently, the scale factors in atmospheric models will be discussed. The way they are defined and their use will be the main point of this part. Lastly, due to the abundance and variety of existing atmospheric models, a range of models applicable to specific altitude levels will be presented. Consequently, following the examination of various atmospheric models, the report will detail how factors such as weather, climate, solar cycle, and atmospheric phenomena influence the precision of these models, as well as compare them to the Exponential Atmosphere Model.

Taking everything into consideration, the primary goals of this report are to understand the different atmospheric models used nowadays, identify their advantages and disadvantages, and comprehend how factors, such as solar cycles, affect their precision.

2. Exponential Atmosphere Model

After presenting the motivation of this report, it clearly denoted the need to deduce a mathematical model that, with some simplifications, can represent the atmosphere and its behavior. The name of the section is explained by the fact that in all the models (from the simplest to the most complex), there will be an exponential relation.

2.1. Physical Foundations of an Atmospheric Model

The goal of this project is to present to the reader some atmospheric models that differ between them depending on the approximations chosen. Before starting to enumerate and deduce them, let's take a quick look into some physical foundations we consider crucial to the reader in order to understand the following sections easily.

Starting with Kinetic and Molecular Temperature definitions:

1. **Kinetic Temperature:** Measure of kinetic energy (identical to thermometer temperature at low altitudes).
2. **Molecular Temperature:** Approximation where the molecular weight is assumed constant to its sea-level value throughout the atmosphere.

Some Altitude definitions:

1. **Absolute altitude (r):** Distance from the center of the Earth to a specific point.
2. **Geometric altitude (Z):** Distance from a point to the surface below.
3. **Geopotential altitude (h):** Approximation where the value measured is compatible with assuming acceleration of gravity doesn't change with altitude.

After pointing out some differences between altitude definitions, let's deduce an expression for gravitational acceleration. From Newton's law of universal gravitation[1] it's easily determined that the gravitational acceleration in a specific geometric altitude is:

$$g = g_0\left(\frac{r_{earth}}{r_{earth} + Z}\right)^2 \tag{1}$$

where $g_0 = 9.80665$ m/s^2 (at sea level). It's interesting to note that at Low-Earth orbit ($\simeq 6000\ km$ [ASL]), $g = 2.60\ m/s \simeq 26.52\ \%\ \cdot g_0$, and at the "end of the atmosphere" ($\simeq 100\ km$ [ASL]), $g = 9.5059\ m/s \simeq 96.93\ \%\ \cdot g_0$. These values are interesting in order to evaluate possible approximations in the models presented next.

Analyzing an infinitesimal block of air, we can deduce a relation between pressure and altitude, and it is evident that:

$$dp = -\rho g dZ \tag{2}$$

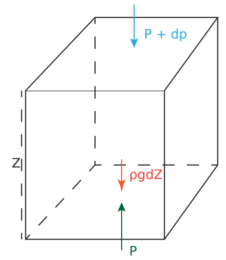

Figure 1: Infinitesimal block of air

[1] $F_g = m \cdot a_g = G\frac{Mm}{r^2}$

In the specific case of the Geopotential Altitude, where we consider g = constant, it's possible to affirm that $dp = -\rho g_0 dh$.

From those equations, we can deduce a relation between Geometric and Geopotential height:

$$dh = (\frac{g}{g_0})\ dZ = (\frac{r_{earth}}{r_{earth} + Z})^2\ dZ \tag{3}$$

2.2. Atmospheric Model with Constant Temperature

This is the simplest Model that will be presented in this report. Let's set it as an approximation that the air is a perfect gas, the temperature is constant, and the molecular weight is constant for different altitudes. According to Dalton's law, the total pressure is equal to the sum of the pressure of all the components. At the temperature and pressure found in the atmosphere, nitrogen and oxygen, the majority of the constitution of the atmosphere, show almost ideal gas behavior [1]. The atmospheric pressure will end up being equal to the dry atmosphere pressure plus water vapor pressure. Obviously, water vapor will not have perfect behavior, ignoring its effect, in earth-specific cases, will lead to a result with 1% error. This way, it's a good approximation to say that the atmosphere behaves like an ideal gas. With that, it's possible to use the Perfect Gas equation:

$$PV = nRT \Longrightarrow \rho = \frac{PM}{RT} \tag{4}$$

and from the last section, we can easily say that: $\frac{dp}{dZ} = -\rho g$.
With these two results, it's easy to compute a result for pressure and density:

$$\begin{cases} p = p_0\ e^{\frac{-MZg}{RT}} \\ \rho = \rho_0\ e^{\frac{-MZg}{RT}} \end{cases} \tag{5}$$

It's possible to see the pressure and the density decrease exponentially with the geometric altitude. Clearly, this is not the most precise Model of the atmosphere since approximations like constant temperature are not acceptable for the use of an atmospheric model.

2.3. Atmospheric Model with Temperature layers

As previously explained, it's not a good approximation to consider temperature constant with height. At the same time, it is known that the temperature sometimes doesn't change linearly throughout the atmosphere. This way, it is a good idea to divide the atmosphere into different layers, where every single one of them has its own temperature change model. While in some layers, a linear approach between temperature and altitude may be precise enough (compared to real-world data), above 86km of altitude, it is verified that a non-linear approach may be needed. The layers will be selected in a way that "altitudes at which the thermal gradients change can be designated as breakpoints" [2]. The atmosphere will be divided into 7 different layers for altitudes till 86 km of geometric altitude, and 13 layers above 86 km [Table 1]. Looking closely to the table, it's possible to see a new numerical quantity "Lapse Rate". This is going to be used to perform calculations of temperature in specific heights.

Layer	Geopotential Altitude h (km)	Geometric Altitude Z (km)	Molecular Temperature T_M (K)	Kinetic Temperature (K)	Molecular Weight	Lapse Rate L_h (K/km)
0	0.0	0.0	288.150	-	-	-6.5
1	11.0	11.0102	216.650	-	-	0.0
2	20.0	20.0631	216.650	-	-	+1.0
3	32.0	32.1619	228.650	-	-	+2.8
4	47.0	47.3501	270.650	-	-	0.0
5	51.0	51.4125	270.650	-	-	-2.8
6	71.0	71.8020	214.650	-	-	-2.0
7	84.8520	86.0	186.946	186.946	28.9644	+1.6481
8	100.0	210.65	210.02	210.02	28.88	+5.0
9	110.0	260.65	257.00	257.00	28.56	+10.0
10	120.0	360.65	349.49	349.49	28.08	+20.0
11	150.0	960.65	892.79	892.79	26.92	+15.0
12	160.0	1110.65	1022.2	1022.2	26.66	+10.0
13	170.0	1210.65	1103.4	1103.4	26.49	+7.0
14	190.0	1350.65	1205.4	1205.4	25.85	+5.0
15	230.0	1550.65	1322.3	1322.3	24.70	+4.0
16	300.0	1830.65	1432.1	1432.1	22.65	+3.3
17	400.0	2160.65	1487.4	1487.4	19.94	+2.6
18	500.0	2420.65	1506.1	1506.1	16.84	+1.7
19	600.0	2590.65	1506.1	1506.1	16.84	+1.1
20	700.0	2700.65	1507.6	1507.6	16.17	-

Table 1: Atmosphere's Layers and respective values. [2]

Let's introduce the following expression for temperature variations:

$$T_M = T_{M_i} + L(l - l_i), l \in \{Z, h\} \tag{6}$$

. This relationship allows us to compute temperatures in any layer i of the atmosphere. From the equations 2 and 4 deduced before, we can compute a new model by substituting T with the the expression given above. This is done by integrating each term of the equation. From that, we can obtain the following atmospheric model:

$$P = P_0 \cdot \left[1 + \frac{L \cdot (Z - Z_i)}{T_{M_i}}\right]^{-\frac{GM}{LR}} \tag{7}$$

2.3.1. Considering the variation of the gravitational acceleration, g.

For a more complex analysis, we might consider the variation of the gravitational acceleration, g. As seen previously, the correlation between pressure and Geometric Altitude is given by $\frac{dp}{dZ} = -\rho g$. From the Perfect Gas equation, we have $P = \frac{\rho R^* T}{M_0}$. It is important to remember that the temperature is now a function of the Geometric Altitude, Z, so $T_M(Z) = T_{M_i} + L_Z(Z - Z_i)$. Starting from the first relation, passing the term dZ to the other side, and dividing both sides by P, however only substituting P in the right side, we obtain:

$$\frac{dP}{P} = \frac{-\rho g dZ}{P} \implies \frac{dP}{P} = \frac{-\rho g M_0 dZ}{\rho R^* T_M(Z)}$$

Another simplification involves replacing the universal gas constant with the gas constant for air $\frac{R^*}{M_0} = R$. We have, finally:

$$\implies \frac{dP}{P} = \frac{-g\, dZ}{R\, T_M(Z)} \tag{8}$$

To calculate the barometric formulas, we followed the concept of the book [2].
If we recall equation 1 we can do an approximation of the gravitational acceleration:

$$\begin{cases} g = g_0(\frac{r_{earth}}{r_{earth}+Z})^2 \\ b = \frac{2}{r_{earth}} = 3.139 \times 10^-7 \text{ /m} \end{cases} \implies g \approx g_0 \left[1 - \left(\frac{2}{R_E}\right) Z\right] = g_0[1 - bZ] \tag{9}$$

This way we can express it conveniently in the integral:

$$\int_{P_i}^{P} \frac{\mathrm{d}P}{P} = -\frac{g_0}{R} \int_{Z_i}^{Z} \frac{(1 - bZ)\mathrm{d}Z}{T_{M_i} + L_Z(Z - Z_i)} \tag{10}$$

For isothermal layers, such as tropopause, stratopause, and mesopause, considering L = 0, we have $T_M = T_{M_i}$. Therefore, Using software to solve the integral, we have::

$$P = P_i\, e^{-\left[\frac{g_0(Z-Z_i)}{RT_{M_i}}\right]\left[1-\frac{b}{2}(Z-Z_i)\right]} \tag{11}$$

$$\rho = \rho_i\, e^{-\left[\frac{g_0(Z-Z_i)}{RT_{M_i}}\right]\left[1-\frac{b}{2}(Z-Z_i)\right]} \tag{12}$$

However, most of the layers are not isothermal, and we must consider the Lapse rate. So, using the geometric altitude, we have $T_M(Z) = T_{M_i} + L_Z(Z - Z_i)$. The integral to solve will then be:

$$\int_{P_i}^{P} \frac{\mathrm{d}P}{P} = -\frac{g_0}{RL_{Z_i}} \int_{Z_i}^{Z} \frac{(1 - bZ)\mathrm{d}Z}{\frac{T_{M_i}}{L_{Z_i}} + (Z - Z_i)} \tag{13}$$

Again, resorting to software to solve this integral, we have:

$$P = P_i \left[\left(\frac{L_{Z_i}}{RT_{M_i}}\right)(Z - Z_i) + 1\right]^{-\left\{\left(\frac{g_0}{RZ_{Z_i}}\right)\left[1+b\left(\frac{T_{M_i}}{L_{Z_i}}-Z_i\right)\right]\right\}} e^{\left\{\left(\frac{g_0 b}{RL_{Z_i}}\right)(Z-Z_i)\right\}} \tag{14}$$

$$\rho = \rho_i \left[\left(\frac{L_{Z_i}}{RT_{M_i}}\right)(Z - Z_i) + 1\right]^{-\left\{\left(\frac{g_0}{RZ_{Z_i}}\right)\left[\frac{RL_{Z_1}}{g_0}+1+b\left(\frac{T_{M_i}}{L_{Z_i}}-Z_i\right)\right]\right\}} e^{\left\{\left(\frac{g_0 b}{RL_{Z_i}}\right)(Z-Z_i)\right\}} \tag{15}$$

2.4. Multi-layer exponential models and Scale factors

We are going to start by defining Scale Factor (or Scale Height). Scale height serves as a universal metric for characterizing the gradual diminishment of a parameter, often employed to delineate a planet's atmospheric properties. Defined as the vertical span over which density and pressure decrease by a factor of 1/e, subsequent reductions occur by an additional factor of 1/e for each added scale height, illustrating the extent to which the atmosphere envelops the planet [3].

Assuming an entirely isothermal atmosphere, we can approximate the temperature by the sea-level value, $T_{M_0} = T_{sea\,level} = 15^\circ = 288.15$ k. The gas constant for air is $R = 287$ J/kg-K and the Sea-level gravity acceleration is $g_0 = 9.81 m/s^2$. Therefore, we have:

$$\begin{cases} \rho = \rho_o \, e^{-\frac{Z g_o}{R T_{M_0}}} \\ H = \frac{R T_{M_0}}{g_o} \end{cases} \implies \begin{cases} \rho = \rho_0 \, e^{-\frac{Z}{H}} \\ H = \frac{R T_{M_0}}{g_0} = \frac{287\,(\mathrm{J/kg\text{-}K}) \times 288.15\,(\mathrm{K})}{9.81\,(\mathrm{m/s^2})} \approx 8430\,\mathrm{m} \end{cases} \tag{16}$$

The Earth's atmosphere exhibits a complex structure, comprising various layers each characterized by distinct thermal behaviors. While layers such as the tropopause, stratopause, and mesopause exhibit isothermal conditions, most of the atmosphere is non-isothermal.

This non-isothermal nature introduces complexities in modeling, necessitating simplifications for analytical purposes. To delve further into the atmospheric profile, we turn to the barometric formula, which depends on the molecular temperature $T_M(Z)$. To facilitate calculations, we define $T_M(Z)$ for each of the 20 layers. In the accompanying image illustrating the first 7 layers, a noticeable pattern emerges. The non-isothermal layers show a linear relationship with geometric altitude. Although the simplicity of this linear approximation may diminish beyond the initial layers, for the scope of our analysis, we shall employ this approximation to gain valuable insights into the atmospheric structure.

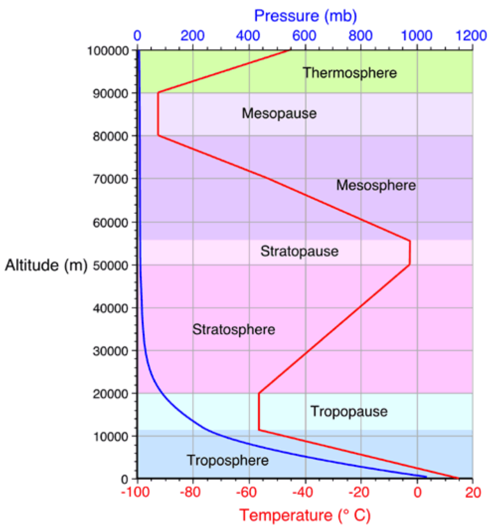

Figure 2: Earth's thermal profile [4]

To determine the scale factor for each of the 20 layers, we must consider their initial molecular temperature, geometric altitude, and lapse rate. For isothermal layers, we will be able to determine a value for the scale height, but for the non-isothermal, we will present an expression with the variable Z: $T_M(Z) = T_{M_i} + L(Z - Z_i)$. The scale factor can then be defined as: $H = \frac{R T_M}{g_o}$. We can now determine each layer's scale factor:

$$\begin{cases} \rho = \rho_o \, e^{-\frac{Z g_o}{R T_M(Z)}} \\ H = \frac{R T_M(Z)}{g_o} \end{cases} \implies \begin{cases} \rho = \rho_0 \, e^{-\frac{Z}{H}} \\ H = \frac{R[T_{M_i} + L(Z - Z_i)]}{g_o} = \frac{R T_i}{g_o} \end{cases} \tag{17}$$

Where T_i is the temperature formula for each i-layer. To simplify the understanding of this, we present the following table. So far, this will be the most complete model to evaluate atmospheric properties.

Layer	Temperature Formula
0	$T_0 = 288.15 - 6.50Z$
1	$T_1 = 216.650$
2	$T_2 = 216.650 + 1.0(Z - 20.0631)$
3	$T_3 = 228.650 + 2.80(Z - 32.1619)$
4	$T_4 = 270.650$
5	$T_5 = 270.650 - 2.80(Z - 51.4125)$
6	$T_6 = 214.650 - 2.0(Z - 71.8020)$
7	$T_7 = 186.946 + 1.6481(Z - 86.0)$
8	$T_8 = 210.020 + 5.0(Z - 210.6500)$
9	$T_9 = 257.00 + 10.0(Z - 260.6500)$
10	$T_{10} = 349.490 + 20.0(Z - 360.650)$

(a) Table 1

Layer	Temperature Formula
11	$T_{12} = 892.790 + 15.0(Z - 960.650)$
12	$T_{12} = 1022.20 + 10.0(Z - 1110.650)$
13	$T_{13} = 1103.40 + 7.0(Z - 1210.650)$
14	$T_{14} = 1205.40 + 5.0(Z - 1350.650)$
15	$T_{15} = 1322.30 + 4.0(Z - 1550.650)$
16	$T_{16} = 1432.10 + 3.30(Z - 1830.650)$
17	$T_{17} = 1487.40 + 2.60(Z - 2160.650)$
18	$T_{18} = 1506.10 + 1.70(Z - 2420.650)$
19	$T_{19} = 1506.10 + 1.10(Z - 2590.650)$
20	$T_{20} = 1507.60$

(b) Table 2

Figure 3: Temperature Formulas

To enhance the comprehensiveness of our study, we have extended our investigation beyond Earth and explored the scale factor (H) for various planets within our solar system. To facilitate comparison, we employed the simplification of an isothermal and homogeneous atmosphere. In this case, we used another way to define the scale factor. This approach allows us to calculate H using the given constants for each planet: the atmospheric temperature (T), the Boltzmann constant (($k = 1.380649 \times 10^{-23}\,\mathrm{m^2\,kg\,s^{-2}\,K^{-1}}$), the gravitational acceleration (g), and the mean particle mass (μ).

$$\begin{cases} \rho = \rho_o \, e^{-\frac{Z}{H}} \\ H = \frac{KT}{g\mu} \end{cases} \tag{18}$$

The following table presents the calculated scale factors for the Solar System planets, providing valuable insights into the atmospheric characteristics of these celestial bodies. To determine the scale factor, we used g, the indicated T_{eff} temperature, and the ratio $\frac{\mu}{\mu_H}$, where μ_H is the mean particle mass per hydrogen molecule ($\mu_H = 1.67 \times 10^{-27}\,\mathrm{kg}$).

Object	T_{ground} [K]	T_{eff} [K]	μ/μ_H	g [m/s^2]	H [km]
Mercury	-	440	-	3.7	-
Venus	730	328	44	8.9	6.9
Earth	288	263	28	9.8	8.4
Mars	215	227	44	3.7	11
Jupiter	-	124	2.3	23.1	19
Saturn	-	95	2.3	9.0	38
Uranus	-	59	2.3	8.7	24
Neptune	-	59	19	11.0	19

Table 2: Properties of Planets in our Solar System [5] [6]

3. Other Models

3.1. International Reference Ionosphere (IRI)

3.1.1. Ionosphere

The Ionosphere ranges from 48 km (30 mi) to 965 km (600 mi) above sea level, and it consists of the ionized part of the upper Earth's atmosphere. As such, it includes all of the thermosphere, as well as some parts of the exosphere and mesosphere. It exists primarily because of the ultraviolet radiation from the Sun. Forming the inner part of the Magnetosphere develops an important role in atmospheric electricity. Among other functions, it influences radio propagation due to the number of its free electrons.[7] [8]

It is composed of different layers, these being layers D, E, and F. The D layer is the innermost layer, 48 km (30 mi) to 90 km (56 mi) above the surface of the Earth. E layer is the middle layer, 90 km (56 mi) to 150 km (93 mi) above the surface of the Earth. Finally, the F layer, also known as the Appleton–Barnett layer, extends from about 150 km (93 mi) to more than 500 km (310 mi) above the surface of Earth. During nighttime, the F layer stands out as the sole layer with substantial ionization, whereas the ionization levels in the E and D layers are notably minimal. Throughout daylight hours, heightened ionization occurs in the D and E layers, along with the F layer, which undergoes the formation of an extra, less intense region of ionization referred to as the F1 layer. The F2 layer remains present continuously, both day and night, serving as the primary region accountable for the refraction and reflection of radio waves.[9]

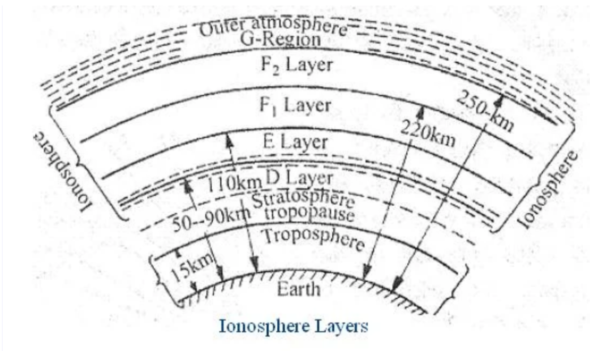

Figure 4: Ionosphere's Layers[10]

3.1.2. IRI

The International Reference Ionosphere (IRI) is a scientific project of international scope, sponsored by the Committee on Space Research (COSPAR) and the International Union of Radio Science (URSI). In the late sixties, these organizations established a Working Group to create an empirical standard model for the Earth's ionosphere. Over the years, the model has undergone various improvements, leading to the release of new editions. [11] [12] [13]

IRI focuses on providing average monthly values for electron density, electron temperature, ion temperature, and ion composition in the altitude range from 50 km to 2000 km, specifically tailored to a given geographic location, time, and date. [14] [13]

This model is based on a comprehensive analysis of a wide variety of available data sources, such as the global network of ionosondes, incoherent scatter radars such as Jicamarca, Arecibo, Millstone Hill, Malvern, and St. Santin, as well as the ISIS and Alouette topside sounders. Additionally, in situ instruments on numerous satellites and rockets contribute valuable data. [11] [15] [14]

Updates to the IRI model are conducted annually during special IRI Workshops, often coinciding with events like the COSPAR general assembly, and are guided by the decisions of the IRI Working Group. The IRI community stays informed about model updates, workshops, publications, and other relevant matters through the IRI listserver.

Required INPUTS: [11]

- solar indices (F10.7 daily, 81-day, and 12-month running mean; sunspot number 12-month running mean);
- ionospheric index (ionosonde-based IG index 12-month running mean);
- magnetic index (3-h ap, daily ap).

Optional INPUTS: [11] The user can provide several input parameters and the IRI profiles will then be adjusted to these input parameters:

- F2-peak height (hmF2) or propagation factor M3000F2
- F2-peak plasmafrequency (foF2) or electron density (NmF2)
- Bottomside profile parameters B0 (thickness) and B1 (shape)
- F1-ledge height (hmF1)
- F1-ledge plasmafrequency (foF1) or electron density (NmF1)
- E-peak height (hmE)
- E-peak plasmafrequency (foE) or electron density (NmE)
- D-ledge height (hmD)
- D-ledge plasmafrequency (foD) or electron density (NmD)

PROVIDED PARAMETERS: [11]

Electron density, electron temperature, ion temperature, ion composition (O+, H+, He+, N+, NO+, O+2, Cluster ions), equatorial vertical ion drift, vertical ionospheric electron content (vTEC; a user can select the ending height for the integration along the electron density profile), F1 probability, spread-F probability, auroral boundaries, effects of ionospheric storms on F and E peak densities.

IRI is the ionosphere standard endorsed and recommended by nearly all international entities involved with ionospheric matters. Both COSPAR and URSI, the two international unions that instigated the IRI project, have acknowledged IRI as the global standard for the ionosphere, urging its adoption by their member states. The European Cooperation for Space Standardization (ECSS) has chosen IRI as its preferred model for the ionosphere. However, the most significant recognition occurred in April 2014 when IRI was elected as the ISO standard for the ionosphere (ISO 16457, 2014). [14]

3.2. CIRA-86

Following the launch of Sputnik 1 into orbit in October 1957 and later launches of USSR and USA satellites, these data were generated from observations of atmospheric drag effects on satellites. In 1965, a redesigned CIRA was produced in response to the significant growth in data from rockets and satellites. This volume included a mean atmospheric profile from 30 to 300 kilometers, tables of atmospheric structure and variations in the region from 30 11 to 100 kilometers, and tables of mid-latitude atmospheric properties, including diurnal variations, for the region from 120 to 800 kilometers, all based on a theoretical model to supplement the limited observational database available at the time.

The COSPAR International Reference Atmosphere (CIRA) [16] is an empiric atmospheric model that provides temperature and density values from altitudes between 0 and 2000 km. The CIRA 1986 is divided into three sections, each examining a different range of atmospheric altitudes. Ground-based and satellite measurements and radiosonde data from the surface to the upper thermosphere are included in this 1986 version.

CIRA was designed to be a climatology of the lower stratosphere. From the surface to 120 km, the CIRA climatology includes zonal, monthly temperature, 12 wind, and geopotential height profiles. Temperature data from 11 levels (1013, 614, 373, 226, 137, 83, 50, 31, 19, 11, and 7 mbar) was used to determine upper tropospheric and lower stratospheric monthly mean microwave brightness temperature (Tb) from 80ºS to 80ºN at a 10º latitudinal resolution. The remaining fields for calculating microwave Tb were zonal averages of the GFDL mean radiosonde fields.

The inputs of the CIRA model are latitude values in degrees, geopotential heights, and pressures in selected pressure units. The outputs are mean temperature values, pressures, and mean zonal winds.

3.3. The NASA/MSFC Global Reference Atmospheric Model 1999 Version (GRAM-99)

For a long time, Reference or Standard models have been used for designing and planning aerospace missions. However, these models don't provide us a complete global geographical variability, and complete altitude coverage (surface to orbital altitudes) as well as complete seasonal and monthly variability of the thermodynamic variables and wind components. This lack led to the development of the GRAM model which, in addition to providing the geographical, height, and monthly variation of the mean atmospheric state, includes the ability to simulate spatial and temporal perturbations in these atmospheric parameters (e.g. fluctuations due to turbulence and other atmospheric perturbation phenomena).

This model consists of an amalgamation of three empirically-based models that represent different altitude ranges (and the geographical and temporal variations within these altitude ranges). The GUACA[2] (Global Upper Air Climate Atlas) or GGUAS (Global Gridded Upper Air Statistics) data cover the altitude region from 0 to 27 km (in the form of data at the surface and constant pressure levels from 1000 mb to 10 mb).

[2]The Global Upper Air Climatic Atlas (GUACA) Version 1.0 is a two-volume CD-ROM set containing a 12-year (1980-1991) 2.5-degree upper air database obtained from the European Center for Medium-Range Weather Forecasting

The middle atmospheric region (20 to 120 km) data set is compiled from Middle Atmosphere Program (MAP) data and other sources referenced in the GRAM-90 and GRAM-95 reports (the information from the MAP is collected by meteorological balloons). The highest altitude region (above 90 km) is simulated by the Jacchia (1970) model and implemented in the Marshall Engineering Thermosphere (MET) model, specifically the newly-released 1999 version (MET-99). A smooth transition between the altitude regions is provided by fairing techniques. The following picture shows a GRAM-99 summary scheme.

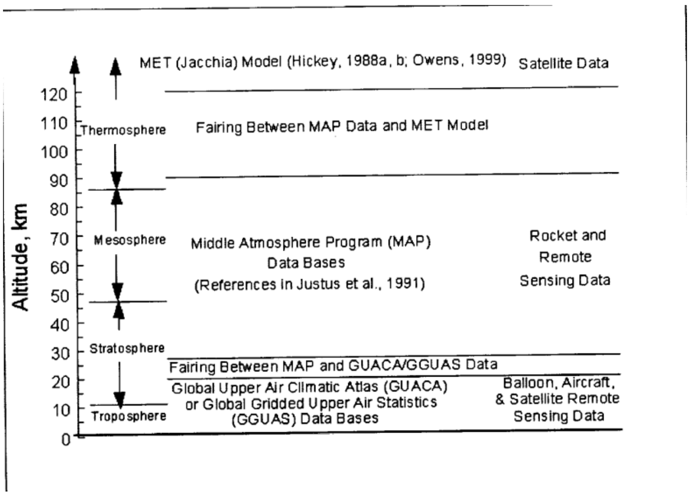

Figure 5: Schematic summary of the atmospheric regions in the GRAM-99 program [17]

It is crucial to clarify what are the Jacchia and the MET models. The first one appeared in the mid-60s with the need to get a model that could help predict the satellite decay time. Using data obtained from the observations of satellite orbital decay, mass spectrometry data, and extreme ultraviolet (EUV) absorption data, Jacchia has developed a series of increasingly accurate models that are a careful blend of empirical and theoretical formulae. In the most recent Jacchia model, the exospheric temperature is assumed to be a function of:

- the average and daily variations in the solar flux;
- the average and three hourly variations in the geomagnetic index;
- the angle between the position vector and the axis of the unsymmetric atmospheric bulge ;
- the angle between the position vector and the geomagnetic pole.

The exospheric temperature is related to the density by the solution of the diffusion equilibrium equations for the different constituents of the atmosphere as a function of altitude. Thus, this model consists of two parts: the first is a basic static model that provides tables with temperature and density profiles for the relevant atmospheric constituents (N2, O2, O, Ar, He, and H), as a function of height from 90 to 2500 km, for a certain exospheric temperature. The second part is a set of formulas to compute this temperature and the expected deviation from the static models, resulting from all of the recognized types of thermospheric variation. A common assumption when using the Jacchia Model is that the atmosphere rotates with the Earth as a rigid body. It's important to point out that over the following years, the Jacchia model suffered modifications and improvements that led to new versions of it, such as the Jacchia 70 (J70) and Jacchia 71 (J71). Major improvements between these models were related to a better understanding of the atmosphere composition. [17]

The second one is a more advanced semi-empirical model built from the Jacchia models. It provides information on atmospheric properties from 90 to 2500 km of altitude, as a function of latitude, longitude, time, solar flux, and geomagnetic indices. The input and output parameters are expressed in the following tables.

Input Variable	Parameter	Range
INDATA (1)	Altitude	90 to 2,500 km
INDATA (2)	Latitude	–90° to 90°
INDATA (3)	Longitude	–180° to 180°
INDATA (4)	Year	1950 to 2050
INDATA (5)	Month	01 to 12
INDATA (6)	Day	01 to 31
INDATA (7)	Hour	00 to 23
INDATA (8)	Minute	00 to 59
INDATA (9)	Geomagnetic index type	a_p: 2; K_p: 1
INDATA (10)	$F_{10.7}$	0 to 400 $\times 10^4$ Jansky
INDATA (11)	Smoothed $F_{10.7}$	0 to 250 $\times 10^4$ Jansky
INDATA (12)	Geomagnetic index	a_p: 0 to 400; K_p: 0 to 9

Figure 6: MET's input parameter
[18]

Output Variable	Parameter
OUTDATA (1)	Exospheric temperature (K)
OUTDATA (2)	Temperature (K)
OUTDATA (3)	N_2 number density (m^{-3})
OUTDATA (4)	O_2 number density (m^{-3})
OUTDATA (5)	O number density (m^{-3})
OUTDATA (6)	Ar number density (m^{-3})
OUTDATA (7)	He number density (m^{-3})
OUTDATA (8)	H number density (m^{-3})
OUTDATA (9)	Average molecular weight
OUTDATA (10)	Total mass density (kg/m^3)
OUTDATA (11)	Log_{10} mass density (kg/m^3)
OUTDATA (12)	Total pressure (Pa)

Figure 7: MET's output parameters
[18]

The newer versions of this model contemplate slight corrections and changes in the epoch which was changed to J2000 (which corresponds to January 1, 12h, 2000, or Julian date (JD) 2451545.0) at the turning of the century. As we learned in Orbital Mechanics, this change has an impact on the time reference. It is crucial to know the epoch for the celestial coordinates or orbital elements of a celestial body, as they are constantly subjected to disturbances. The main use is to calculate relevant parameters of motion, to predict future positions and velocities.

3.4. US Standard Atmosphere

The US standard atmosphere was first developed in 1958 by the U.S [19]. Committee on Extension to the Standard Atmosphere. Between 1958 and 1976 there were successive updates to the model, as new observational data became available. Taking into account the scope of this report, the focus will primarily be on the most recent, 1976 version.

The US Standard atmosphere is an idealized, steady-state representation of the Earth's atmosphere from the surface to 1000km, as it is assumed to exist in a period of moderate solar activity. The model divides the atmosphere into two regions, one from 0km to 86km, and the second from 86km to 1000km. For altitudes below 86km, hydrostatic equilibrium is assumed and air is treated as a homogeneous mixture of several constituent gases. At these altitudes, the temperature is defined using a linearly segmented temperature-height profile. At greater heights (>86km), dissociation and diffusion processes produce significant departures from homogeneity, the definitions governing the standard are more sophisticated than those used at lower levels. In this high-altitude regime, the hydrostatic equation, as applied to a mixed atmosphere, gives way to the more general equation for the vertical component of flux for individual gas species, which accounts for the relative change in composition with height.

3.4.1. Data Sources and Data Processing

At the time of publishing, 1976, it was difficult to derive accurate estimates of the annual temperature cycle, particularly at higher altitudes (>80km). This was because of the uneven distribution of observations. At any given location, the data for a given month varies from 1 to 20 observations for altitudes between 55 and 80km and from 0 to 12 for altitudes above 80km. Given this limitation, the mean annual temperatures for the various altitudes were obtained by averaging 12 observed and/or interpolated mean monthly values. The mean monthly values for altitudes where there were relatively complete sets of observed mean monthly temperatures were subjected to harmonic analysis for semiannual and annual cycles.

3.4.2. The Models for altitudes above 86km

For altitudes above 86km, there is a decrease in mean molecular weight with increasing height. There are two sources primarily responsible for this: the dissociation of molecular oxygen and diffusive separation. More specifically, the Model considers that atomic oxygen becomes appreciable above 85km and diffusive separation begins to be effective at an average height of 100km. Above 120km, it is relatively safe to assume there is no further large-scale oxygen dissociation and that the simultaneous equations governing molecular diffusion are no longer interdependent and they can be applied separately.

3.4.3. Assumed Sea-level Composition

As the individual flux of each atmospheric constituent is taken into account at higher altitudes, it is important to know the composition of the atmosphere. Thus, principal sea-level constituents are assumed to be (in mole percent):

- N_2 – 78.084%, O_2 – 20.9476%, Ar – 0.934%, $CO2$ – 0.0314%, Ne – 0.001818%, He – 0.000524%, CH_4 – 0.0002%.

4. Validity of Models

The models presented before are developed based on many simplifications, not taking into account many external factors such as the solar cycle, solar eruptions (with massive magnetic waves spread), solar winds, and the Earth's position in its orbit.

Newer models contemplate slight corrections and changes in the epoch. the atmosphere is in constant movement (like a fluid flow through a cylinder in rotation) and therefore companies and agencies that perform rocket launches or that need to compute falling satellite trajectories, require better and more complete models of the atmosphere.

In this section we will be presenting the advantages and disadvantages of the models depicted above as well as comparing them to other complex models and contrasting them with the simpler ones.

4.1. CIRA-86

4.1.1. Advantages and disadvantages

The CIRA-86 model accurately shows how the middle atmosphere is structured based on latitude and season. However, it isn't very reliable for extreme latitudes because there aren't enough measurements to support it. The model gives decent wind measurements in the middle of the atmosphere, but they don't always match with the radar wind measurements, especially at higher latitudes.

4.2. IRI

4.2.1. Advantages

• The Earth's ionosphere is internationally standardized through the International Reference Ionosphere (IRI), acknowledged as the global standard by COSPAR and URSI. Notably, the International Organization for Standardization (ISO), responsible for international standards, has officially recognized and endorsed IRI in this capacity.

• IRI can be driven by characteristic profile parameters either by direct input if data are available locally or by model adjustment and data assimilation for regional or global modeling.

• Future improvements in the IRI model are expected in several areas.

4.2.2. Disadvantages

A critical aspect is the need for an improved depiction of IRI densities and temperatures, especially during periods of extremely low solar activity. This necessity arises due to observed discrepancies identified during the recent and prolonged solar minimum from 2007 to 2009. Furthermore, considering the potential recurrence of very low solar activities in the current solar cycle, a more accurate description becomes paramount. [14]

Notably, during comparisons with satellite measurements from C/NOFS, CHAMP, and GRACE, it has been demonstrated that IRI tends to overestimate the topside electron density [12]. This emphasizes the importance of refining IRI's representation, particularly under conditions of exceptionally low solar activities. [12]

4.3. GRAM-99

4.3.1. Advantages

- **Altitude Range:** GRAM-99 covers a wide range of altitudes ranging from the surface to the thermosphere.
- **Variability:** GRAM-99 accounts for the variability in atmospheric conditions based on geographical location, time of day, and time of year.
- **Applications:** GRAM-99 is used in satellite orbit determination, trajectory analysis, and spacecraft design.

4.3.2. Disadvantages

- **Limited Temporal Resolution:** GRAM-99 doesn't simulate transient events, such as sudden weather changes or atmospheric disturbances.

- **Outdated Data:** GRAM-99 is from 1999. Over time, advances in observational data and improvements in understanding atmospheric processes may cause some aspects of the model to be outdated.

4.4. US Standard Atmosphere

The US Standard atmosphere is a solid Model of the Earth's atmosphere, that although published in 1976 still holds up to the present day. Some of the advantages of the model may include its detailed composition of the Earth's atmosphere and the accuracy of the values for lower temperatures. Disadvantages may include outdated and perhaps scarce temperature and pressure values for higher altitudes and a relatively smaller domain of 1000km.

4.5. Complex vs Simple

4.5.1. Example of a Simple Model

This is an example of a simple atmospheric model, which is very common in introductory books on atmospheric physics.

For starters, three things are assumed:

- The atmosphere is isothermal – a slab of atmosphere all at the same temperature,

- The atmosphere is completely transparent to solar radiation,

- The atmosphere is completely opaque to terrestrial radiation.

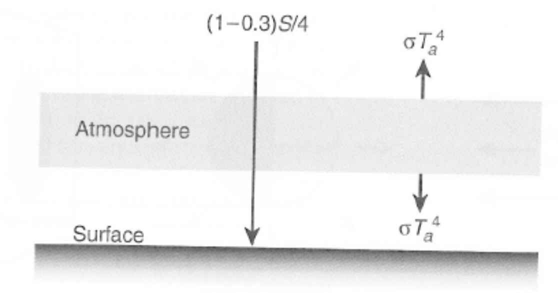

Figure 8: single-slab atmosphere" model [20]

This model helps us see how the temperature of the atmosphere and the surface are related under the simplest of assumptions which are far-fetched as they would not be used in complex models. Despite that it can, and it is used for education purposes as it helps to get a grasp of the basic atmospheric notions important to understanding other models.

4.5.2. Uses of Simple Models

Simple atmospheric models serve various purposes in atmospheric science and related fields [21], like for:

1. **Education and Training:** Simple atmospheric models are valuable tools for teaching basic concepts in atmospheric science as they provide a hands-on approach for students to understand fundamental principles without getting overwhelmed by complex mathematical formulas.

2. **Research Tool for Hypothesis Testing:** Scientists often use simplified models to test hypotheses and explore specific aspects of atmospheric processes. These models allow researchers to isolate and study particular variables or phenomena, gaining insights that may inform more complex models or real-world observations.
3. **Satellite and Remote Sensing Calibration:** Simple models are used to help interpret and calibrate data obtained from satellites and remote sensing instruments. These models can provide a simplified representation of the atmosphere to aid in the analysis of observational data.
4. **Engineering Applications:** Engineers involved in projects related to aviation, construction, and other fields may use simple atmospheric models to estimate parameters such as air density, temperature gradients, and wind patterns. This information is crucial for designing structures that interact with the atmosphere.

4.5.3. Comparison between simple and complex models

While simple atmospheric models have limitations compared to more complex numerical models, they play a valuable role in providing insights, facilitating learning, and addressing specific research questions.

Like the models previously presented, more complex atmospheric models offer a higher level of detail and sophistication compared to simpler models [21]. However, they also come with specific advantages and disadvantages.

- **Spatial and Temporal Resolution:** Complex models can provide detailed information at various spatial and temporal scales, allowing for a more thorough analysis of atmospheric processes. On the other hand, simpler models might miss some information because of the fewer details and lower resolution. This limitation can be a drawback when studying localized or intricate processes.

- **Representation of Physical Processes:** These models often have a wide range of physical processes, such as radiation, convection, and turbulence, providing a more realistic simulation of the atmosphere. In simpler models, many assumptions are made that don't correspond to real-life situations, which represents a risk of oversimplification.

- **Prediction of Extreme Events:** Because of its complexity, the more complex models can predict some atmospheric events like hurricanes or heavy rainfall. On the other hand, due to their simplicity, simpler models may lack the predictive power required for predicting complex atmospheric behavior accurately.

- **Versatility:** Complex models can be used for a variety of applications, including space missions, predicting satellite fall time, weather forecasting, climate research, etc. The simple models may not be suitable for all applications as they are often designed to answer specific questions or address certain aspects of the atmosphere, and their scope may be limited.

The main disadvantage when using complex models is the computational power and resources required to run the simulations based on those models. When we are working with space missions it is extremely important to be able to predict every interaction within the atmosphere and when it comes to satellites, their fall time is heavily dependent on this. Due to that fact, it is important to understand the errors and uncertainties that come with the use of each model, to account for that in the calculations.

Concluding, researchers and engineers must carefully consider the trade-offs when choosing between simple and more complex models based on the goals of their study.

5. Atmosphere phenomena

The Earth's atmosphere is described by a chaotic model which contains a huge range of variables and uncertainties. This complex system incorporates weather patterns, climate, storms, and various occurrences in the upper atmosphere. However, despite the ideal atmospheric model predicting these events, with the amount of information we have about the topic and current technology is impossible to do so. In the following topics, we will discuss the different factors that influence our atmosphere and their impact on satellites' good functioning and lifetime.

5.1. Climate

Climate refers to the long-term patterns and averages of weather conditions in a particular region over an extended period, typically spanning decades to centuries, the WMO (World Meteorological Organization) uses a thirty-year span to define climate. It encircles various elements such as temperature, humidity, precipitation, wind patterns, and atmospheric pressure. Not only that, but climate also contains broader information about sea level, glaciers, thermohaline circulation, etc. Providing a broader perspective compared to weather, which refers to the conditions of one specific moment instead of a deep historical analysis that helps predict future variables.

Nowadays there is a huge variety of classifications for climate which can be associated with biomes since it affects the ecosystem. Nowadays, the most used one around the different countries of the world is the Köppen climate classification scheme, formulated in 1899. [22]

But how does climate influence the satellites' lifetime and good functioning?

The answer to this question is one of the biggest problems we are facing right now, Climate change. This happens because of the high flow of greenhouse gases to the atmosphere such as carbon dioxide that alters the upper atmosphere composition. Having an increased amount of this molecule in the atmosphere, more collisions with the already existent molecules of oxygen will happen, causing a reduction of temperature which leads to a reduction of density.

The consequence of reduced atmospheric density is a depletion of the drag experienced by satellites orbiting the Earth. Satellites, encountering less resistance, can prolong their operational lifetimes. However, despite this seeming to have a good effect, the components of the satellites also have a lifetime and only work for the time they were projected. This extension contributes to the accumulation of space debris once old non-operational satellites fall at slower rates. [23]

5.2. *Solar cycles*

Solar cycles are recurrent events caused by the Sun's magnetic field. These cycles have a duration of 11 years, however, this time can vary from 9 to 14 years each cycle. In every cycle, there are variations in the number of sunspots seen on the surface of the Sun.

Sunspots are temporary dark spots on the Sun's surface caused by magnetic activity. If the number of sunspots is higher, so is the solar activity, therefore, elevated solar radiation. The cycle is a maximum and a minimum and it ends when the reversal of the magnetic north and south poles occurs, normally during peak solar activity. [24]

Solar cycles also influence the occurrence of solar flares and coronal mass ejections (CMEs), releasing energy and charged particles into space. These events can impact space weather and potentially affect Earth's magnetic field, technology, and communication systems. How can solar cycles affect the lifetime of satellites and their normal functioning:

1. Increased solar radiation: During solar maximum, the Sun emits higher levels of solar radiation. This increased radiation can have several effects on satellites, including the potential degradation of solar panels over time. The elevated radiation levels may contribute to a reduction in the efficiency of solar power generation on the satellite.
2. Geomagnetic storms: These storms caused by solar flares and coronal mass ejections can lead to an increased flux of charged particles in Earth's magnetosphere. Satellites in low Earth orbit (LEO) may experience increased atmospheric drag during geomagnetic storms, affecting their orbits and potentially shortening their operational lifetimes.
3. Communication disruptions: Radio signals transmitted between the Earth and satellites may experience increased absorption or scattering due to ionospheric changes caused by solar flares. This can result in signal degradation or interruptions in communication links.
4. Space weather effects: Satellites are exposed to various space weather phenomena such as energetic particle events. High-energy particles from the Sun during solar flares and CMEs can damage satellite electronics over time, affecting their reliability and functionality. [25]

5.3. *Weather*

Weather tells what happens to the psychologist but the climatologist is what he needs to know. Weather is the condition of our atmosphere at any given location and time period. The main factors that affect weather are temperature, precipitation, air pressure, wind direction and strength, humidity level, or relative humidity percentage which tells you how many times lighter clouds will be compared with their original weight per cubic inch.

Also air density - how much space each molecule of gas occupies in relation to another. Weather phenomena mostly occur in the troposphere, which stretches above the Earth's surface and up to 6-8 km altitude. At sensitive times of the year (e.g., spring and autumn) when there is a change in air pressure, thunder and lightning may arise from this layer. High-pressure zones produce fine and sunny weather: the air at the surface is forced down

Numerous atmospheric models do not contain solar cycles as a main influence. Examples of such models include the Exponential Model, the 1976 U.S. Standard Atmosphere Model, and the International Standard Atmosphere.
It is possible to predict future solar cycles when evaluating the past cycles, and predicting their amplitude and time. Therefore, future atmosphere models may incorporate those graphics and be more accurate.

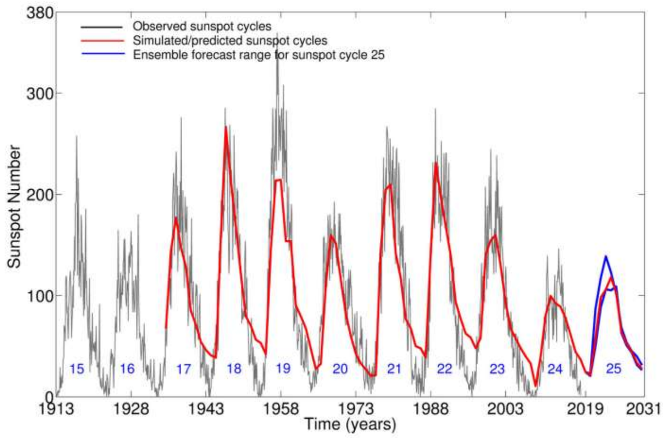

Figure 9: Past solar cycles and future prediction [26]

by high pressure and becomes dry. When there are no clouds to block it, you get a clear blue sky. Low-pressure zones have the opposite effect: here air streams up-equals a fall in temperature and the formation of many dense, grey, and thick clouds seems more likely. To gain an understanding of the complexity and sensitivity of weather prediction, there is a field at the crossroads of mathematics and physics-Chaos theory, that has many applications in weather forecasting. This is complex because small discrepancies in initial conditions, like measurement errors, can have significant differences in the outcome of the forecast.

The differences in temperature between equatorial and polar regions happen because of the varying angles of sunlight, which is dictated by the latitude. This temperature difference creates an instability zone between 23 and 66 degrees, south or north, and can generate extreme weather events in the atmosphere, like typhoons, hurricanes, cyclones, and tornadoes. [27]

The density profile is also relevant, as the weather also affects the density of the atmosphere. As the density varies with the altitude, an atmospheric model needs to consider the differences of atmospheric layers, because they also influence the drag force. The wind also affects the drag force, introducing additional lateral forces.

Finally, the weather can also cause electromagnetic disturbances through lightning and electrical storms, so this also has to be taken into consideration when using an atmospheric model.

To summarize, weather has a major influence on atmospheric models. From different temperatures around the world to different air pressures and wind in different periods in time, the weather dictates how favorable the atmospheric models are to maneuvers at a certain point in time. It is also important to note that sudden weather changes can affect the calculations done prior to this change and the success of the re-entry.

5.4. Atmospheric Phenomena

As said in the topic before, there are many atmospheric phenomena that have an influence on atmospheric models, such as typhoons, hurricanes, cyclones, and tornadoes. In this topic, we will cover these phenomena in more detail. Cyclones are storms whose winds revolve around a center of low atmospheric pressure. Hurricanes and typhoons are terms used for tropical cyclones in different places. It is called a hurricane in the Atlantic Ocean and a typhoon in the Indian Ocean. A tornado on the other hand is a rotating column of air extending from the sky to the ground. These events are very dangerous if spacecraft get caught in them, and that is the reason why we should be cautious of them. [27]

There are more complex phenomena that can not be explained by current atmosphere models, which consider the atmosphere a fluid, based on the Navier-Stokes equations and other laws of physics. Auroras and Transient Luminous Events (TLE) are some examples of these unpredictable events. Auroras are optical phenomena resulting from the interaction of solar wind with Earth's atmosphere. The Auroras show a colorful light, predominantly green but occasionally with shades of pink. This occurs at latitudes typically from 10º to 20º from the geomagnetic poles, happening because of high-energy electrons (1 to 15 keV) colliding with nitrogen and oxygen atoms. Transient Luminous Events are also known as upper-atmospheric lightning or ionospheric lightning and include events such as Sprites, Elves, and blue jets. A lightning flash can abruptly alter the electric field, triggering secondary flashes. A large alteration in the electric field can also initiate sprites, which occur during extensive horizontal lightning strikes. Near the top of storm complexes, there are also changes, when positively charged pools become negatively charged, which creates a strong electric field. This happens approximately at 100 kilometers above the Earth's surface. Even though auroras and TLE are not particularly dangerous, they show us how fast the atmosphere can change its properties and how difficult these changes are to predict. Even so, there are some entities who give us a short-term forecast of auroras (one example is NOAA, in the following web page [28])[29] [30]

To conclude, there are many ways that atmospheric phenomena can be created, and we still can't predict or understand them totally. For this reason, we need to be careful with them when making a re-entry, because these phenomena can alter the calculations made prior and make the re-entry fail, and also because all of these phenomena can corrupt data gathered by satellites. Because of these reasons, atmospheric phenomena have to be considered when creating (and choosing) an atmospheric model for re-entry.

6. Rotation of the atmosphere

The Earth's atmosphere is bound to it by gravity, and most of it moves along with the planet due to friction with the ground and the stickiness of the different layers of air above it. This movement of air relative to the surface is what we call "wind". If the atmosphere were uniform and its conditions did not vary with altitude, we would expect the application of the Navier-Stokes equations together with Bernoulli's equation with the following boundary conditions: the speed of the air at the surface would match the speed of the Earth at that point $v_\theta(R_E) = \omega_E R_E$, in which w_E corresponds to the angular velocity of the Earth and R_E represents the Radius of the Earth; and the speed at the furthest zone would be zero $v_\theta(R_{atm}) = 0$. Here, R_{atm} represents the Radius of the atmosphere.

$$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0 \tag{19}$$

$$\rho(\frac{\partial v}{\partial t} + v.\nabla v) = -\nabla p + \nabla.\tau + \rho g \tag{20}$$

However, above 200km, the incredibly thin atmosphere spins faster than the Earth. This is because its conditions vary, such as temperature and pressure, especially from layer to layer, and so the hypothesis above does not explain what happens in reality:

1. In the Troposphere, since temperature and density decrease, the air slows down with increasing altitude due to friction with the Earth's surface.
2. As you move into the Stratosphere, the speed generally increases with altitude due to the presence of the ozone layer, which absorbs the Sun's ultraviolet radiation and causes temperature inversions (in this layer temperature increases as mentioned before). These inversions create stable conditions that allow for smoother and faster movement of air.
3. In the Mesosphere and Thermosphere, these upper layers have varying compositions and experience changes due to solar activity. The air in these layers can move at high speeds due to the effect of solar radiation and the heating of gases in the Thermosphere. As mentioned before, around 200km-300km of altitude, in the upper atmosphere, air exceeds the surface rotational velocity. This so-called phenomenon of super-rotation refers to the remarkably swift eastward winds found here, particularly in the equatorial region.
   Super-rotation occurs due to a combination of factors: [31]
   - Absorption of Solar Radiation: The equatorial regions receive more solar radiation than the poles, leading to temperature differences. This variation creates a gradient in the atmosphere, causing air to move from warmer regions near the equator toward the poles, carrying heat and angular momentum;
   - Coriolis Effect: As air filled with heat and momentum moves from the equator towards the poles due to the thermal tide, the Coriolis effect, which is caused by the Earth's rotation, deflects this air movement, causing it to curve eastward;
   - Wave Interactions: Atmospheric waves, such as Rossby waves or gravity waves, play a role in redistributing energy and momentum within the atmosphere. These waves can contribute to the maintenance or amplification of the phenomenon of super-rotation. In Earth's atmosphere, Kelvin waves generate eastward along the equator, playing a vital role in phenomena like the El Niño-Southern Oscillation (irregular periodic variation in winds and sea surface temperatures over the tropical eastern Pacific Ocean, affecting the climate of much of the tropics and subtropics);
   - Stratospheric Jet Streams: The presence of strong jet streams in the stratosphere, particularly the subtropical and polar jet streams, also influences the super-rotation. These high-altitude, fast-moving air currents contribute to the overall dynamics of the upper atmosphere.

4. The outermost layer of the atmosphere, the Exosphere, where air particles are extremely sparse, has varying speeds depending on interactions with solar winds and other cosmic forces.
   In general, as we move up in altitude and gas molecules get further from each other, wind speed becomes null as no mass of air could collectively move at any rate. This way, the speed of Earth's atmosphere doesn't become null at a specific altitude, but it gradually thins out as you go higher into the atmosphere.

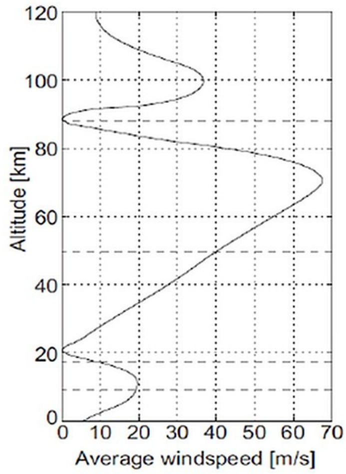

Figure 10: Evolution of atmosphere's speed relative to Earth with altitude [32]

7. Practical example

For a practical application of the models we´ve studied before, we present an approximation performed by THE AUSTRALIAN SPACE WEATHER AGENCY. The model is applied to satellites with orbits below 500 Kilometers. These orbits can be regarded as circular. The density, ρ, is specified by a simple exponential with variable scale height, H. Fixing the exosphere temperature, T, H varies with the altitude h through the use of an effective atmospheric molecular mass m. This m includes both the actual variation in molecular mass with height and a compensation term for the variation in temperature over the considered range from 180 to 500 km. The variation in density due to the space environment is introduced through T which is specified as a function of the solar radio flux $F10.7$ and the geomagnetic index Ap. The set of equations defining the model follows:

$$\begin{cases} T & = 900 + 2.5(F10.7 - 70) + 1.5Ap \quad \text{(Kelvin)} \\ m & = \begin{cases} 27 - 0.012(h - 200), & \text{if } 180 < h < 500 \\ \text{undefined}, & \text{otherwise} \end{cases} \\ H & = \frac{T}{m} \quad \text{(km)} \\ \rho & = 6 \times 10^{-10} \exp\left(-\frac{h-175}{H}\right) \quad (\text{kg m}^{-3}) \end{cases} \tag{21}$$

All constants used were determined empirically.
To simulate the drag force, let's consider the following equation:

$$D = \frac{1}{2}\rho A v^2 C_d \tag{22}$$

where D is the drag force, ρ is the atmospheric density, v is the speed of the satellite, A is its cross-sectional area perpendicular to the direction of motion, and C_d is the drag coefficient. Due to the difficulty of separating out independent variations in the cross-sectional area from the variations in the drag coefficient, we shall henceforth use an effective cross-sectional area $Ae = ACd$. Using the expression that relates the period of an elliptical orbit with its semimajor axis (we learned this in Orbital Mechanics):

Satellite - Test-1

Mass = 100.0 kg
Area = 1.0 m^2
Initial height = 300.0 km
F10.7 = 70 Ap = 0

TIME (days)	HEIGHT (km)	PERIOD (mins)	MEAN MOTION (rev/day)	DECAY (rev/day^2)
0.0	300.0	90.5	15.9139	2.66E-03
10.5	289.9	90.3	15.9463	3.50E-03
19.5	279.9	90.1	15.9823	4.62E-03
26.3	269.9	89.9	16.0182	6.11E-03
31.5	259.9	89.7	16.0546	8.11E-03
35.4	249.9	89.5	16.0908	1.08E-02
38.4	239.8	89.3	16.1279	1.44E-02
40.6	229.8	89.1	16.1642	1.93E-02
42.3	219.6	88.9	16.2018	2.60E-02
43.6	209.1	88.7	16.2406	3.55E-02
44.5	199.3	88.5	16.2768	4.75E-02
45.2	189.2	88.3	16.3146	6.45E-02
45.7	179.5	88.1	16.3507	8.65E-02

Re-entry after 46 days (0.13 years)

Figure 11: Simulation of a satellite decay [33]

$$T^2\mu = 4\pi^2 a^3 \tag{23}$$

where T is the period, and μ is the gravitational parameter.

The reduction in the period due to atmospheric drag is given by:

$$\frac{dT}{dt} = -3\pi a\rho(Ae/m) \tag{24}$$

It's important to note that the previous equation is obtained using orbital dynamics and fluid dynamics laws. The last two equations, together with the equations modeling the atmospheric density, can be iterated from the starting satellite altitude and time with a program. The output of a simulation performed by THE AUSTRALIAN SPACE WEATHER AGENCY is presented in Figure 11.

8. Conclusion

In conclusion, atmospheric models play a crucial role in both scientific and engineering communities by providing a means to represent the atmosphere's behavior through mathematical models, albeit with some simplifications. Despite their limitations, simpler models are valuable for addressing specific challenges, such as interpreting satellite and remote sensing data and aiding in calibration and analysis. On the other hand, more complex models, which delve into physics problems in greater detail, offer a closer representation of the atmosphere to reality but come with the trade-off of requiring computationally expensive simulations. Therefore, the choice between different models should be based on the specific goals of the study and the available resources.

Atmospheric phenomena, including climate, solar cycles, and weather, pose challenges when predicting Earth's atmosphere. For instance, phenomena like solar cycles can influence the lifetimes and functionalities of satellites, emphasizing the importance of considering these factors in modeling and predicting the behavior of space technology and related projects where atmospheric conditions are crucial.

The practical application of atmospheric models is exemplified by the simulation conducted by the Australian Space Weather Agency, where satellite decay due to drag is simulated. This serves as a demonstration of the practical utility of atmospheric models, underscoring the significance of accurate predictions for the success of a space mission.

The complexity of Earth's atmosphere presents challenges for engineers, but advancements in atmospheric models have contributed to a better understanding of these complexities, enabling improved decision-making, design, and predictions for space missions. Researchers and engineers continue to enhance and study these models, tailoring them to the specific goals and challenges encountered in each mission.

Individual Contribution

- **João Carvalho, 102686:**
  Report: 2.1 - 2.3; **Slides:** 9-20 **Software** Python - Main Script, Connection between all the atmospheric models developed with the main script, Read data from TXT file, Integration of the ode system, event function, find local peaks function, Density function for the US 1976 Model, Data plot for each atmospheric Model, Review.

- **Álvaro Caridade, 103526:**
  Report: 2.3.1 and 2.4; **Slides:** 21-36 **Software** Python - Density function using Scale Factors based on the approximations presented in the report, Integration of ode system, Review

- **António Reis, 102473:**
  Report: 3.1.1, 3.1.2 and 4.2; **Slides:** 70-78 **Software** Python - Density function Gram and Jachia 1500K, Review

- **Gonçalo Silva, 102995:**
  Report: 3.3 and 7; **Slides:** 79-86 and 100-103 **Software** Python - Density function Gram and Jachia 2400k, Review

- **Fernando Vicente, 103048:**
  Report: 6; **Slides:** 57-65 **Software** Python - Review, User Manual

- **Tomás Coelho, 102805:**
  Report: 1,8; **Slides:** 2-6 ; 104-105 **Software** Python - Integration of ode system, CIRA model integration, Review.

- **Ricardo Sousa, 102498:**
  Report: 1,8; **Slides:** 100-103 ;104-105 **Software** Python - CIRA model integration, Review

- **Eduardo Helena, 102793:**
  Report: 3.2, 4.1, 4.3 and 4.5 ; **Slides:** 66-69 **Software** Python - Density function MET, Review

- **João Alegrete, 103676:**
  Report: 5.1 and 5.2 ; **Slides:** 37-39; 44-47; 51-56 **Software** Python - Density function Gram and Jachia 600k, Review

- **Tiago Ruge, 102551:**
  Report: 5.3 and 5.4 ; **Slides:** 40-43; 48-50; 97-99 **Software** Python - Density function MET, Review

- **João Vilaça, 103966:**
  Report: 3.2, 3.4 and 4.4; **Slides:** 87-96 **Software** Python - Density function MET, Review

Qualitative Evaluation:

We believe that the distribution of tasks and responsibilities within our group was well-balanced, and each member actively fulfilled their designated role. Consequently, our qualitative evaluation reflects a unanimous score of 100% for every group member

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