```
from typing import Callable, List, Tuple
from dataclasses import dataclass, field
import matplotlib.pyplot as mpl
import numpy as np
```

If running python on Windows operating system, copy the file

https://raw.githubusercontent.com/joaochenriques/MCTE_2022/main/libs/mpl_utils.py

to the working folder.

```
import pathlib
if not pathlib.Path("mpl_utils.py").exists():
  !curl -O https://raw.githubusercontent.com/joaochenriques/MCTE_2022/main/libs/mpl_utils.py &> /dev/null

import mpl_utils as mut
mut.config_plots()
mpl.rcParams["figure.figsize"] = (12, 3)
%config InlineBackend.figure_formats = ['svg']
```

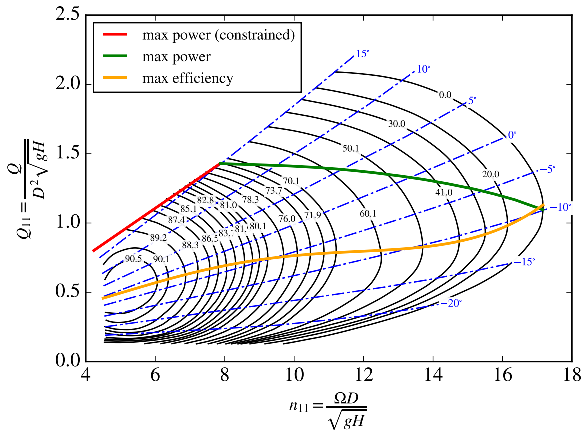
Algebraic models

Barrage simulator in ebb mode

The following models were implemented to simulate the tidal power plant:

- The basin
- The tide
- Hydraulic turbines
- Electrical generators
- Sluice gates
- Power plant controller

Turbine hill map and turbine operating curve



Turbine mode

Turbine dimensionless numbers

- Rotational speed

$$n_{11} = \frac{\Omega D}{\sqrt{gh}}.$$

- Flow rate

$$Q_{11} = \frac{Q}{D^2\sqrt{gh}}.$$

- Efficiency

$$\eta_\text{turb} = \frac{P_\text{turb}}{P_\text{avail}}.$$

The power available to the turbine is given by

$$P_\text{avail} = \rho g h Q,$$

where

$$Q = D^2\sqrt{gh}\,Q_{11}(n_{11}).$$

The turbine is to be operated at constant rotational speed due to the use of a synchronous generator (see generator class).

The available energy is

$$\frac{\mathrm{d}E_{\mathrm{avail}}}{\mathrm{d}t} = P_{\mathrm{avail}}.$$

The energy harvest by the turbine is

$$\frac{\mathrm{d}E_{\mathrm{turb}}}{\mathrm{d}t} = \eta_{\mathrm{turb}}\left(n_{11}\right)\,P_{\mathrm{avail}},$$

The mean turbine efficiency is

$$\overline{\eta_{\mathrm{turb}}} = \frac{E_{\mathrm{turb}}}{E_{\mathrm{avail}}}.$$

Sluicing mode

The in sluicing mode the "turbine" is modelled as

$$Q_{\mathrm{turb}}^{\mathrm{sluice}} = C_{\mathrm{d}}A_{\mathrm{turb}}\sqrt{2gh}$$

where A_{turb} is the area corresponding to the turbine rotor diameter.

```
@dataclass
class TurbineModel:

  # flow rate: efficiency: red line of the map
  poly_CQ1: np.poly1d = np.poly1d( np.array([0.16928201, 0.08989368]) )

  # flow rate: green line of the map
  poly_CQ2: np.poly1d = np.poly1d( np.array([-3.63920467e-04,  9.37677378e-03,
                                             -9.25873626e-02,  1.75687197e+00]) )

  # efficiency: red line of the map
  poly_CE1: np.poly1d = np.poly1d( np.array([-0.02076456, 0.20238444,
                                                  0.48984553]) )
  # efficiency: green line of the map
  poly_CE2: np.poly1d = np.poly1d( np.array([-2.75685709e-04,  2.04822984e-03,
                                                 6.86081825e-04,  7.93083108e-01]) )

  # n11 interpolation domain
  n11_min: float =  4.38
  n11_max: float = 17.17

  # other data
  ga: float = 9.8         # gravity aceleration
  ρw: float = 1025.0      # water density
  CD_sluice: float = 1.0  # discharge coefficient in sluice mode

  #============================================================================
  def __init__( self, D_turb, Omega ) -> None:

    self.Omega = Omega    # we are assuming constant rotational speed model
    self.D_turb = D_turb  # turbine rotor diameter
    self.A_turb = np.pi*(D_turb/2.0)**2

    # constants used in for computing n11 and QT
    self.CT0 = Omega * D_turb / np.sqrt( self.ga )
    self.CT1 = D_turb**2 * np.sqrt( self.ga )

  def n11_range( self ) -> tuple:
    return ( self.n11_min, self.n11_max )

  # dimensionless velocity
  def n11( self, h: float ) -> float:
    # avoid division by zero on h = 0.0
    return self.CT0 / np.sqrt( max( h, 1E-3 ) )

  # dimensionless flow rate
  def Q11( self, n11: float ) -> float:
    assert( n11 >= self.n11_min ), "n11 small than admissable minimum"
    assert( n11 <= self.n11_max ), "n11 greater than admissable maximum"
    if n11 < 7.92193:
      return self.poly_CQ1( n11 )
    else:
      return self.poly_CQ2( n11 )

  # efficiency
  def eta( self, n11: float ) -> float:
    assert( n11 >= self.n11_min ), "n11 small than admissable minimum"
    assert( n11 <= self.n11_max ), "n11 greater than admissable maximum"
    if n11 < 7.92193:
      return self.poly_CE1( n11 ) * 0.912
    else:
      return self.poly_CE2( n11 ) * 0.912

  # computing operational data
  def operating_point( self, h: float ) -> float:
    n11 = self.n11( h )
    QT = self.CT1 * self.Q11( n11 ) * np.sqrt( h )
    PH = self.ρw * self.ga * h * QT
    ηT = self.eta( n11 )
    return QT, PH, ηT
```

```

  # turbine flow rate in sluice mode
  def sluicing( self, h: float ) -> float:
    QS = -self.CD_sluice * self.A_turb * np.sqrt( 2.0 * self.ga * max( -h, 0.0 ) )
    return QS
```

Generator efficiency curve

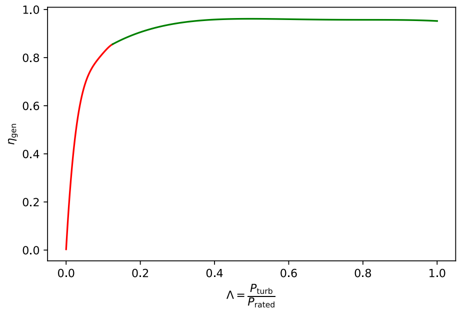
The electrical generator is assumed to be a synchronous machine. The rotational speed is given by

$$\Omega = \frac{2\pi f}{p}$$

where f is the electrical grid frequency and p is the number of pairs of poles.

The generator efficiency is computed as a function of the load

$$\Lambda = \frac{P_{\mathrm{turb}}}{P_{\mathrm{gen}}^{\mathrm{rated}}}$$



Electrical power output is

$$P_{\mathrm{gen}} = \eta_{\mathrm{gen}}(\Lambda)\, P_{\mathrm{turb}},$$

and the converted energy is

$$\frac{\mathrm{d}E_{\mathrm{gen}}}{\mathrm{d}t} = P_{\mathrm{gen}}.$$

The mean generator efficiency is

$$\overline{\eta_{\mathrm{gen}}} = \frac{E_{\mathrm{gen}}}{E_{\mathrm{turb}}}.$$

```
@dataclass
class GeneratorModel:

  # red part of the curve
  poly_C1: np.poly1d = np.poly1d( np.array([-6.71448631e+03,  2.59159775e+03,
                                           -3.80834059e+02,  2.70423225e+01,
                                            3.29394948e-03]) )
  # green part of the curve
  poly_C2: np.poly1d = np.poly1d( np.array([-1.16856952,  3.31172525,
                                           -3.44296217,  1.5416029 ,
                                            0.71040716]) )

  #===========================================================================
  def __init__( self, Pgen_rated: float ) -> None:
    self.Pgen_rated = Pgen_rated

  # efficiency as a function of the load
  def eta( self, Pturb: float ) -> float:
    load = Pturb / self.Pgen_rated

    assert( load >= 0.0 ), "turbine power lower than zero"
    assert( load <= 1.0 ), "generator rated power to low (%f)" % Pturb
    if load < 0.12542:
      return self.poly_C1( load )
    else:
      return self.poly_C2( load )
```

Sluice gates

The sluice gates are modelled as a turbulent pressure drop

$$Q_{\mathrm{sluice}} = C_{\mathrm{d}} A \sqrt{2gh}$$

typical discharge coefficients for barrage sluice gates are within the range $0.8 \leq C_{\mathrm{d}} \leq 1.2$. Here we use $C_{\mathrm{d}} = 1.0$.

```
@dataclass
class GateModel:
  ga: float = 9.8
```

```
  CD_sluice: float = 1.0

  #===========================================================================
  def __init__( self, Area: float ) -> None:
    self.Area = Area

  # flow rate as a function of h
  def sluicing( self, h: float ) -> float:
    QS = -self.CD_sluice * self.Area * np.sqrt( 2.0 * self.ga * max( -h, 0.0 ) )
    return QS
```

Tide modelling

The tide level is assumed to be a trignometric series

$$\zeta(t) = \sum_{i}^{n} A_i \cos(\omega_i t + \phi_i).$$

```
@dataclass
class TideModel:

  # A_tide, period and ϕ_tide are vectors to allow simulate multi-component tides
  def __init__( self, A_tide: np.array, ω_tide: np.array, ϕ_tide:np.array ) -> None:
      self.A_tide = A_tide
      self.ω_tide = ω_tide
      self.ϕ_tide = ϕ_tide

  # tide level as a function t
  def level( self, t: float ) -> float:
      return np.sum( self.A_tide * np.cos( self.ω_tide * t + self.ϕ_tide ) )
```

Differential models

Basin modelling

The instantaneous basin volume is computed from

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -Q.$$

The out flow is denoted as positive.

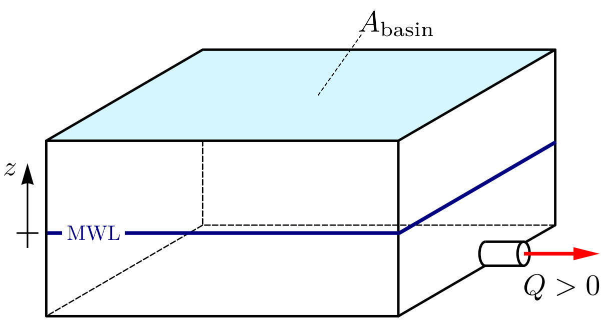
Knowing that $V = A_{\text{basin}} z$, the basin height is computed from

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -q_{\text{basin}}.$$

where z is the water level with respect to the mean water level (MWL), A_{basin} is the basin area and $q_{\text{basin}} = Q/A_{\text{basin}}$.

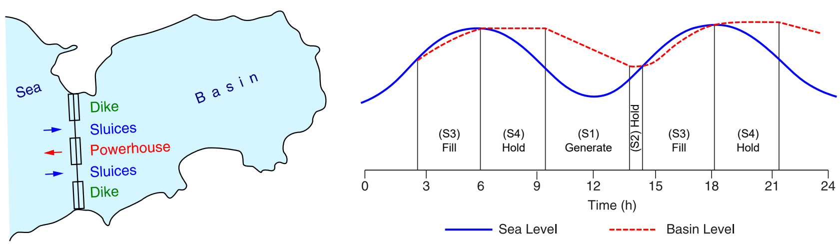
Integrating with the Euler method in Δt, we get

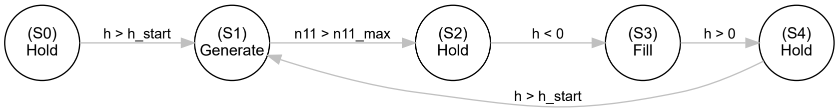
$$z(t + \Delta t) = z(t) + \Delta t\,(-q_{\text{basin}}).$$



Power plant operation model

The power plant modelling and control is based in the following finite state machine





Finite State Machine simulator

```
func_state = Callable[ [ float, float, np.array ], List[ np.array ] ]
func_transition = Callable[ [ float, np.array, np.array ], int ]


class FSM_simulator:


  def __init__( self, simul_time: float, Delta_t: float, n_vars: int, n_outs: int ) -> None:
    self.dic_states = {}
    self.dic_transitions = {}

    self.n_time = int( simul_time / Delta_t) + 1
    self.n_time = self.n_time
    self.n_vars = n_vars
    self.n_outs = n_outs

    self.time = np.linspace( 0, simul_time, self.n_time )
    self.delta_t = self.time[1]

    self.vars = np.zeros( (n_vars, self.n_time) )
    self.outs = np.zeros( (n_outs, self.n_time) )
    self.states = np.zeros( self.n_time )

    self.t = 0

  def run_simulator( self, state: int, vars: np.array ) -> Tuple[np.array]:
    self.vars[:,0] = vars
    self.states[0] = state
    # outs not available at t=0

    for i in range( 1, self.n_time ):
      self.t = self.time[i-1]
      vars, outs = self.__run_state( state, self.delta_t, self.time[i-1], vars )
      state = self.__test_transitions( state, self.time[i-1], vars, outs )

      self.vars[:,i] = vars
      self.outs[:,i] = outs
      self.states[i] = state

    return ( self.time, self.states, self.vars, self.outs )


  def add_state( self, state: int, func: func_state ) -> None:
    assert state not in self.dic_states.keys(), "state already define"
    self.dic_states[state] = func


  def add_transition( self, state: int, func: func_transition ) -> None:
    if state not in self.dic_transitions.keys():
      self.dic_transitions[state] = [ func ]
    else:
      self.dic_transitions[state].append( func )


  # private member
  def __run_state( self, state: int, delta_t: float, t: float, vars: np.array ) -> List[np.array]:
    return self.dic_states[ state ]( delta_t, t, vars )


  # private member
  def __test_transitions( self, state: int, t: float, vars: np.array, outs: np.array ) -> int:
    for transition in self.dic_transitions[ state ]:
      new_state = transition( t, vars, outs )
      if new_state != state:
        return new_state # first transition that changes the state
    return state
```

Vars used in the simulation

$$\mathbf{x} = \begin{pmatrix} z & E_{\text{avail}} & E_{\text{turb}} & E_{\text{gen}} \end{pmatrix}^{\mathrm{T}}$$

Outputs

$$\mathbf{y} = \begin{pmatrix} h & \zeta & Q_{\text{turb}} & Q_{\text{sluice}} & P_{\text{avail}} & P_{\text{turb}} & P_{\text{gen}} & \eta_{\text{turb}} & \eta_{\text{gen}} \end{pmatrix}^{\mathrm{T}}$$

```
@dataclass
```

```
class Models:

    n_turbs: int = 24
    Dturb: float = 6.0

    grid_freq: float = 50.0
    ppoles: int = 32
    Pgen_rated: float = 25E6

    n_gates: int = 6
    Agates: float = 10.0*15.0

    #==========================================================================
    # post init variables

    # tide components
    ζ: np.array = field(init=False) # amplitudes
    ω: np.array = field(init=False) # frequencies (rad/hour => rad/s)
    ϕ: np.array = field(init=False) # phases

    Omega: float = field(init=False)
    n11_max: float = field(init=False)

    turbine: TurbineModel = field(init=False)
    generator: GeneratorModel = field(init=False)

    gate: GateModel = field(init=False)
    tide: TideModel = field(init=False)


    def __init__( self, ζ: np.array, ω: np.array, ϕ: np.array ) -> None:

      self.ζ = ζ
      self.ω = ω
      self.ϕ = ϕ

      # synchronous rotational speed
      self.Omega = 2 * np.pi * self.grid_freq / self.ppoles

      self.turbine = TurbineModel( D_turb = self.Dturb, Omega = self.Omega )
      self.generator = GeneratorModel( Pgen_rated = self.Pgen_rated )

      _, self.n11_max = self.turbine.n11_range()

      self.gate = GateModel( Area = self.Agates )
      self.tide = TideModel( self.ζ, self.ω, self.ϕ )


    ## HELPER FUNCTIONS ########################################################
    # minimum turbine head that starts turbine generation
    def turbine_starting_head( self, t: float ) -> float:
      return 5.0

    def basin_area( self, z: float ) -> float:
      return (-0.102996*z**2+1.272972*z+23.31)*1E6

    ## STATES ##################################################################
    def S1_Generate( self, delta_t: float, t: float, vars: np.array ) -> List[np.array]:

      z = vars[0]
      ζ = self.tide.level( t )
      h = z - ζ

      Q_sluice = 0.0
      Q_turb, P_avail, η_turb = self.turbine.operating_point( h )
      P_turb = η_turb * P_avail

      η_gen = self.generator.eta( P_turb )
      P_gen  = η_gen * P_turb

      q_basin = Q_turb * self.n_turbs / self.basin_area( z )
      RHS = np.array( ( -q_basin, P_avail, P_turb, P_gen ) )

      # variables at ( t + delta_t )
      vars = vars + delta_t * RHS

      #======================================================================
      # outputs at ( t + delta_t )
      z = vars[0]
      ζ = self.tide.level( t + delta_t )
      h = z - ζ

      Q_turb, P_avail, η_turb = self.turbine.operating_point( h )
      P_turb = η_turb * P_avail

      η_gen = self.generator.eta( P_turb )
      P_gen  = η_gen * P_turb

      outs = np.array( ( h, ζ, Q_turb, Q_sluice, P_avail, P_turb, P_gen, η_turb, η_gen ) )

      return ( vars, outs )
```

```python


  def SX_Hold( self, delta_t: float, t: float, vars: np.array ) -> List[np.array]:

    #======================================================================
    # outputs at ( t + delta_t )
    z = vars[0]
    ζ = self.tide.level( t + delta_t )
    h = z - ζ

    Q_turb = Q_sluice = P_avail = P_turb = P_gen = η_turb = η_gen = 0.0

    outs = np.array( ( h, ζ, Q_turb, Q_sluice, P_avail, P_turb, P_gen, η_turb, η_gen ) )

    # vars do not change
    return ( vars, outs )


  def S3_Fill( self, delta_t: float, t: float , vars: np.array ) -> List[np.array]:

    z = vars[0]
    ζ = self.tide.level( t )
    h = z - ζ

    Q_sluice = self.gate.sluicing( h )
    Q_turb = self.turbine.sluicing( h )

    P_avail = P_turb = P_gen = η_turb = η_gen = 0.0

    q_sluice = Q_sluice * self.n_gates
    q_turb = Q_turb * self.n_turbs
    q_basin = ( q_sluice + q_turb ) / self.basin_area( z )
    RHS = np.array( ( -q_basin, 0.0, 0.0, 0.0 ) )

    # variables at ( t + delta_t )
    vars = vars + delta_t * RHS

    #======================================================================
    # outputs at ( t + delta_t )
    z = vars[0]
    ζ = self.tide.level( t + delta_t )
    h = z - ζ

    Q_sluice = self.gate.sluicing( h )
    Q_turb = self.turbine.sluicing( h )
    outs = np.array( ( h, ζ, Q_turb, Q_sluice, P_avail, P_turb, P_gen, η_turb, η_gen ) )

    return ( vars, outs )


  ## TRANSITIONS ################################################################
  def T_S0_S1( self, t: float, vars: np.array, outs: np.array ) -> int:
    h = outs[0]
    h_start = models.turbine_starting_head( t )
    return 1 if h > h_start else 0

  def T_S1_S2( self, t: float, vars: np.array, outs: np.array ) -> int:
    h = outs[0]
    n11 = self.turbine.n11( h )
    return 2 if n11*1.1 > self.n11_max else 1

  def T_S2_S3( self, t: float, vars: np.array, outs: np.array ) -> int:
    h = outs[0]
    return 3 if h < 0.0 else 2

  def T_S3_S4( self, t: float, vars: np.array, outs: np.array ) -> int:
    h = outs[0]
    return 4 if h > 0.0 else 3

  def T_S4_S1( self, t: float, vars: np.array, outs: np.array ) -> int:
    h = outs[0]
    h_start = models.turbine_starting_head( t )
    return 1 if h > h_start else 4
```

```python
ζ = np.array( [ 4.18, 1.13 ] )                # amplitudes
ω = np.array( [ 0.5058/3600, 0.5236/3600 ] )  # frequencies (rad/hour => rad/s)
ϕ = np.array( [ -3.019, -3.84 ] )             # phases

if len( ω ) == 2:
  tide_period = 1.0 / ( np.max( ω[1] ) - np.min( ω[0] ) ) * 2.0*np.pi
else:
  tide_period = 1.0 / ω[0] * 2.0*np.pi

simul_time = 4.0*tide_period
delta_t = 100.0

z0_basin = 1.0 # initial basin level
```

```python
models = Models( ζ, ω, ϕ )

```

```
n_vars = 4
n_outs = 9

simul = FSM_simulator( simul_time, delta_t, n_vars, n_outs )

simul.add_state( 0, models.SX_Hold )
simul.add_state( 1, models.S1_Generate )
simul.add_state( 2, models.SX_Hold )
simul.add_state( 3, models.S3_Fill )
simul.add_state( 4, models.SX_Hold )

simul.add_transition( 0, models.T_S0_S1 )
simul.add_transition( 1, models.T_S1_S2 )
simul.add_transition( 2, models.T_S2_S3 )
simul.add_transition( 3, models.T_S3_S4 )
simul.add_transition( 4, models.T_S4_S1 )
```

Simulate the power plant

```
vars = np.array( (z0_basin, 0.0, 0.0, 0.0) )

time_vec, states_vec, vars, outs = simul.run_simulator( 0, vars )

#================================
z_vec = vars[0]

E_avail_vec = vars[1]
E_turb_vec  = vars[2]
E_gen_vec   = vars[3]

#================================
h_vec = outs[0]
ζ_vec = outs[1]

Q_turb_vec   = outs[2]
Q_sluice_vec = outs[3]

P_avail_vec = outs[4]
P_turb_vec  = outs[5]
P_gen_vec   = outs[6]
η_turb_vec  = outs[7]
η_gen_vec   = outs[8]
```

```
hours_vec = time_vec / 3600.0
period_hours = tide_period / 3600.0

# Number of points of each period. Required to make the mean of last period
pp = int( tide_period / delta_t )

P_turb_max = np.max( P_turb_vec )
P_turb_mean = np.mean( P_turb_vec[-pp:] )
P_gen_mean = np.mean( P_gen_vec[-pp:] )
C_fac = P_gen_mean / models.generator.Pgen_rated

print( "Max instantaneous power per turbine = %.2f MW" % (P_turb_max/1E6) )
print()
print( "Mean turbine power    = %.2f MW" % (P_turb_mean*models.n_turbs/1E6) )
print( "Mean electrical power = %.2f MW" % (P_gen_mean*models.n_turbs/1E6) )
print()
print( "Capacity factor = %.2f" % C_fac )
```

```
mpl.plot( hours_vec, h_vec, label='Tide level [m]', dashes=(9,1) )
mpl.plot( hours_vec, z_vec, label='Basin level [m]', dashes=(7,1,1,1) )
mpl.plot( hours_vec, states_vec, label='State $S_i$ [-]' )
mpl.xlim( 3*period_hours, 4*period_hours )
mpl.xlabel( 'time [hours]' )
mpl.legend(loc='lower left')
mpl.grid();
```

```
mpl.plot( hours_vec, P_turb_vec/1E6, label='Power per turbine [MW]'  )
mpl.plot( hours_vec, P_gen_vec/1E6, label='Power per generator [MW]', dashes=(9,1) )
mpl.xlim( 3*period_hours, 4*period_hours )
mpl.xlabel( 'time [hours]' )
mpl.legend(loc='lower left')
mpl.grid();
```

```
mpl.plot( hours_vec, Q_turb_vec, label='Flow rate per turbine [m$^3$/s]' )
mpl.plot( hours_vec, Q_sluice_vec, label='Flow rate per sluice gate [m$^3$/s]', dashes=(9,1) )
mpl.xlim( 3*period_hours, 4*period_hours )
mpl.xlabel( 'time [hours]' )
mpl.legend(loc='lower left')
mpl.grid();
```

```
mpl.plot( hours_vec, η_turb_vec, label='$\eta_\mathrm{turb}$' )
```

```
mpl.plot( hours_vec, η_gen_vec, label='$\eta_\mathrm{gen}$', dashes=(9,1) )
mpl.plot( hours_vec, η_turb_vec*η_gen_vec, label='$\eta_\mathrm{turb}\,\eta_\mathrm{gen}$', dashes=(7,1,1,1) )
mpl.xlim( 3*period_hours, 4*period_hours )
mpl.xlabel( 'time [hours]' )
mpl.legend(loc='lower left')
mpl.gca().set_yticks(np.arange( 0, 1.01, 0.1) )
mpl.grid();
```

```
if len( ω ) == 2:
  X1 = ζ[0]
  X2 = ζ[1]
  ωm = ω[0] - ω[1]
  ϕm = ϕ[0] - ϕ[1]
  ev = np.sqrt(X1**2 + X2**2 + 2*X1*X2*np.cos( ωm*time_vec + ϕm ) )

  mpl.plot( hours_vec, ζ_vec, label="tide level" )
  mpl.plot( hours_vec, ev, 'r-', lw=2, label="envelop" )
  #mpl.xlim( 3*period_hours, 4*period_hours )
  mpl.xlabel( 'time [hours]' )
  mpl.legend(loc='lower left');
else:
  print( "No envelop to plot" );
```

```

```