Leakage Inductance

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THE principle of leakage inductance or reactance will be quite familiar to transformer designers, but due to a possible ambiguity as to definition we here define the quantity as it will be determined from the data sheet on page 133. Due to current in each of two windings there will be a quantity of magnetic flux which will link with the winding causing it, but not with the other winding, and due to each of these groups of flux there will be an E.M.F. induced when the current and flux change. The total leakage flux could be regarded as due to combined effect of the current in both windings, and the E.M.F. induced in each winding treated separately, or the flux due to the current in each winding could be treated separately and the E.M.F.'s in each winding derived and integrated. For the purpose of the calculations made from this data sheet, the leakage inductance from one winding to another can be defined as the inductive component of the impedance of one winding, the other winding being short-circuited.

Derivation of Formula

If we consider an element of one of the windings of a simple double wound transformer in cross section, the element being taken parallel to the division between the two windings so that the leakage flux throughout the length of the element is constant, then the leakage flux density, B (Fig. 1) will be proportional to the distance from the outside of the winding, the maximum occuring in the space between the windings, where the leakage ampere-turns are maximum. The E.M.F. induced in the whole winding due to the element of flux will be proportional to the number of turns which the flux couples, and the value of the flux. Thus, if x measures the distance from the outside of the winding, the E.M.F. induced in the winding due to the flux passing through the element will be proportional to x^2dx. The total E.M.F. throughout the winding will be proportional to $\int_0^{a_1} x^2dx$, or $a_1^3/3$.

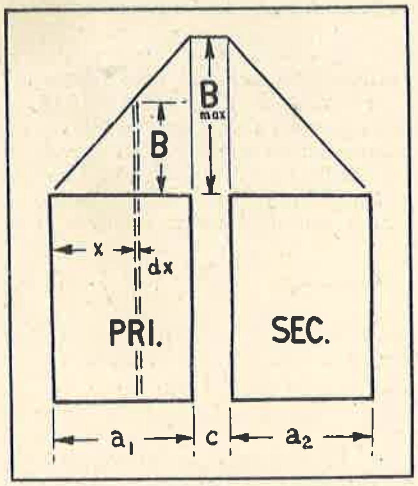


Fig. 1. Derivation of leakage inductance formula

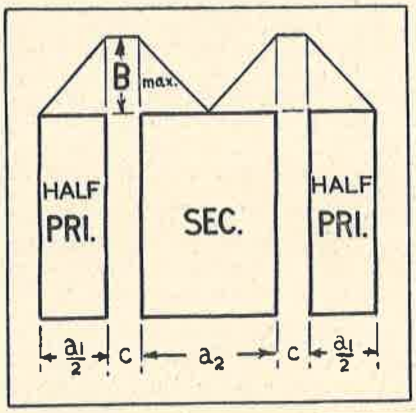


Fig. 2. Effect of elementary mixing

Thus, the leakage inductance due to the winding space will be proportional to one-third of the thickness as compared with the leakage inductance due to the maximum flux in the space between the windings. If a_1 and a_2 are the thicknesses of the windings, and c is the space between the windings, the leakage inductance will be proportional to

$$\frac{a_1 + a_2}{3} + c.$$

If one of the windings is divided in two, each half being disposed on opposite sides of the other winding, then the maximum leakage flux ampere-turns at the spaces between the windings, responsible for producing B_{max}, will be half that of the simple winding case, *see* Fig. 2. The leakage inductance due to each half winding will, according to the integration above, be one-eighth of the previous value for that winding. The leakage inductance for the whole winding will be reduced to one-quarter, or by a factor of four, ignoring the space between the windings. To simplify the conception as windings are further subdivided, it will be noted that there are two points of maximum flux in this second case, so we coin the term "leakage flux area" to express the integration for each pair of winding

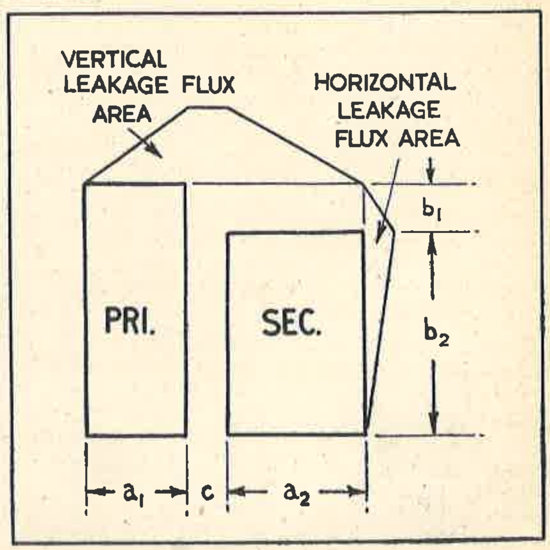


Fig. 3. Components of leakage flux in two directions

sections. This leakage flux area is represented graphically by each rhomboid at the top of Figs. 1 and 2. The leakage flux in each space between windings has half its original value, but there are now two such spaces, so the total component of leakage inductance due to the spaces will be the same, if each space is equal to the single space in the original case.

Thus the leakage inductance can be written as proportional to $a/3N^2 + c$, where a is the total thickness of all winding sections, N represents the number of leakage flux areas, and c is the thickness of each insulation section.

Considering the leakage flux now on the basis of the reluctance of its magnetic circuit: the reluctance of the whole of the circuit outside of the windings may be ignored; the reluctance within the windings will be proportional to the length of the flux paths, or dimension "b" of the winding (*see* Fig. 8) and inversely proportional to the area of flux. The area of flux will be proportional to dimension "a" of the windings, as already considered, and also to the length of the mean turn of the windings, which we will denote by d. Thus, the whole expression for leakage inductance, if T is the number of turns in the winding at which it is measured, can be written:

$$L \propto \frac{d}{b}\left(\frac{a}{3N^2} + c\right)T^2 \quad \text{............} \quad (1)$$

Windings without Common Dimension in one direction

In the foregoing consideration the assumption has been made that both windings are of equal dimension in direction "b". Where this is not true, the simplest method of predicting leakage inductance will be to resolve the leakage flux into two components at right angles. In the simple case where the windings have equal dimensions in the one direction there is zero component of leakage flux at right angles to the main direction.

As the E.M.F. produced by a rate of change of current in a winding is proportional to the square of the component of flux being considered (as shown by the term N^2 in Equation 1) the total E.M.F. due to two fluxes at right angles will be equal to the sum of the E.M.F's due to the component fluxes. Therefore the total leakage inductance will be equal to the sum of the values given due to each component of flux. Where the windings are not of equal length, or having non-coincident ends, the leakage flux in the second direction will be proportional to the unbalance of ampere turns. Fig. 3 illustrates this for a simple case The leakage inductance due to the flux in a vertical direction will be calculated directly from the formula given above, while the leakage inductance due to the flux in a horizontal direction can be calculated from the same formula using a modified value of turns, T_a, given by $T_a = T\left(\frac{b_1}{b_1 + b_2}\right)$. For this component of flux, the positions of b and a will be reversed and there will be no c (Fig. 8).

Various degrees of mixing

By increasing the number of equivalent leakage flux areas, N, the leakage inductance can be reduced for a given number of turns and winding cross section, until the limit is reached where $a/3N^2$ is less than c, when c becomes the limiting factor. Thus, where a high degree of mixing is used, the thickness of insulation between windings becomes important, and it may be necessary to use the absolute minimum permissible. The table of winding arrangements shows values of N^2 for various ways of dividing up primary and secondary. It will be noticed that the best arrangement, *i.e.*, that giving the highest value of N^2 for a given number of sections, is always that having a half area section at each end of the arrangement. This is sometimes two half area primary sections and sometimes one half area primary section and one half area secondary section.

In the same table the winding sections are shown connected in series for convenience. This is not essential, and parallel or series parallel connexions can equally well be used. Where series connexion is used a half area section should have half the number of turns in each unit area section, and the whole number of turns in the winding will, of course, be the sum of the turns in the individual sections. Where parallel connexion is used, the turns in all parallel groups of winding must be equal, and if a half area section is parallel with full sections, then the wire gauge of the half section should be such that the winding occupies half the area and has twice the resistance.

Use of the information given here will show that for a push-pull transformer design the best coupling will be obtained by increasing the number of layer sections in the winding rather than by using a complicated cross connected winding using a divided bobbin. It is quite possible to produce an arrangement which will give identical resistance and leakage inductance for each half of a centre tapped push-pull winding, for example, the seventh arrangement in the left-hand column in the table, using the centre primary section for one half and the two outside half area sections for the other half. These will each have identical D.C. resistance and identical leakage inductance to the whole of the secondary, whether the secondary be series or parallel connected. The only possibility of lack of balance with this arrangement occurs in the winding capacity and inter-winding capacity, which will be considered in a later data sheet.

In general, the best shaped winding cross section from the viewpoint of achieving a low leakage inductance will be one having a long layer length corresponding to dimension, b, and a small winding depth (dimension a). The standard "*wastefree*" type laminations give a winding area which is practically ideal from the viewpoint of combining a high primary, or shunt, inductance with a low leakage inductance. The thicker stacks of laminations give increased ratio of primary inductance to leakage inductance, because the primary inductance is proportional to the cross sectional area of the iron circuit, while the leakage inductance is proportional to the length mean turn, and increasing the stack of laminations increases the cross sectional area by a greater ratio than it does the length mean turn. The limit in this direction will usually be a practical one imposed by the difficulty of producing a winding with long, flat sides.

Putting leakage inductance to use

The author has produced several is put to practical use by making it an inductive element in a filter.* Use of leakage inductance for this purpose has the great advantage that the associated capacitive elements can be transformed by the turns ratio of the winding, so that conveniently small values of capacitance may be used where the theoretical circuit values may be many times larger. A typical bobbin used to produce two of the inductance elements in a low pass m-derived filter is shown at Fig. 4. The winding located in the slot lettered a_3 will have a slightly higher leakage inductance to that in a_2 than will the one located in slot a_1, due to the fact that b_2 is greater than b_1. It should be realised that the leakage inductance can only be varied by adjusting the number of turns in the input winding or by rearranging the spacing and dimensions, although the inductance referred to any winding can be adjusted by variation of the turns of that individual winding. The input and output of the filter may be designed for 600 ohms or any other convenient impedance and the turns of the various windings adjusted so that all the capacitors used may be of a standard value, say, 0.01 μF. Fig. 5 shows another arrangement used for producing two absorption dips in a response curve by simply shunting windings 3 and 4 by a standard value capacitor. The actual size of a complete transformer with its associated condensers can be made very much smaller than such an absorption filter designed to operate at the line impedance of the network.

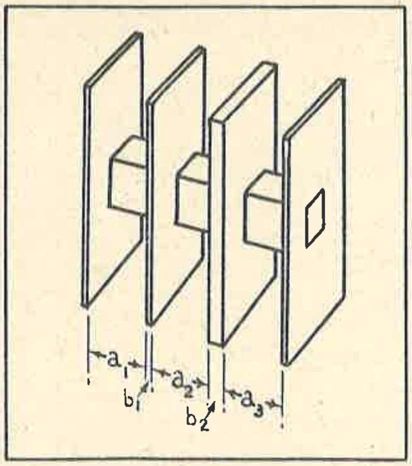


Fig. 4. Bobbin for m-derived low pass filter using leakage inductance.

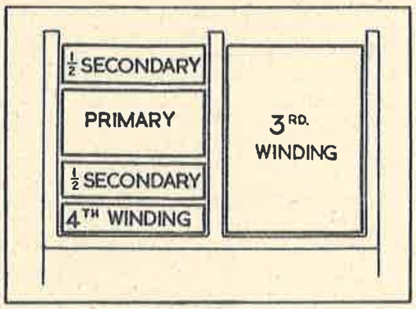


Fig. 5. Bobbin for microphone matching transformer incorporating two absorption circuits

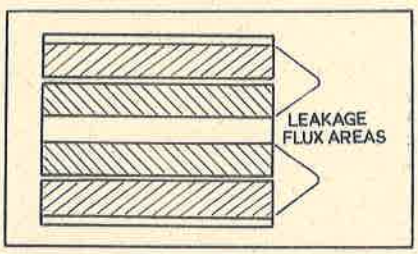


Fig. 6. Mixing arrangements for multi-ratio transformers

Multi-Ratio Transformers

In designing a transformer where parts of the winding will not always be used, the winding layout should be so arranged that portions of winding not in use at any time occupy a space where leakage flux is not induced by the active windings, or is at a minimum, in order to obtain a minimum leakage inductance. With the simple type of winding having only one section each for the primary and secondary, the inactive portion of winding can be arranged so that it is on the side of its own winding remote from the other winding. If sectioning is used, more careful consideration is necessary, but it will always be possible to arrange the winding so that the inactive portion is in a space between leakage flux areas. Fig. 6 shows how this can be achieved in the simplest form of mixing, having a primary in two sections and the secondary in one section. If a portion of the secondary is to be inactive, it should be at the centre of the winding, both electrically and physically, which means that tappings will have to be made at the ends of the active portions, and the inactive portion must be left open circuit. If part of the primary is inactive, then it should be the part remote from the secondary winding. In Fig. 6 the shaded areas of winding represent active portions, while the unshaded areas are inactive.

Auto Transformers

In calculating the leakage inductance for an auto transformer, the series and shunt sections of the winding can be treated in the same way as separate windings. The leakage inductance referred to the input or output terminals can then be calculated in the same way as winding resistance loss is calculated for an auto transformer.

Use of Chart

An appropriate value of N^2 for the winding arrangement being used can be found on the left of the Table overleaf. Referring to the chart on page 132, this value of N^2 is intercepted with the dimension "a" of the winding cross-section at A (*see* Fig. 7). This is then referred vertically to the small space in the chart. Here add dimension "c", the space between sections of winding. Where multisectioning is used this will be the spacing between each primary and secondary section. Refer this sum value vertically to intercept with the length mean turn at B and follow the diagonal scale to intercept with the dimension "b" of the coil at C. From here refer again vertically to intercept with the turns scale at D. The value of leakage inductance is then read off at E. If the number of turns in the winding, the value of dimension "b", or length mean turn, is not accommodated within the range shown, the appropriate value on the chart should be multiplied by a power of ten to bring it to the actual value, and the value of leakage inductance obtained from the chart multiplied by the power of ten obtained from the table below.

Example I: Winding space on bobbin, ⅝ in. long by 3/16 in. deep; mean turn length, 2½ in. Primary turns, 600; secondary turns, 8,000; simple single-section windings in layer arrangement; insulation between windings, 0.020 in.

From this information dimension "a" can be taken as about 0.15 in., allowing for insulation and clearance. The intercept of the 0.15 in. (interpolating between 0.14 in. and 0.16 in.) on the "a" scale with $N^2 = 1$ gives 0.05 in. on the vertical scale. Add to this the 0.020 in. for

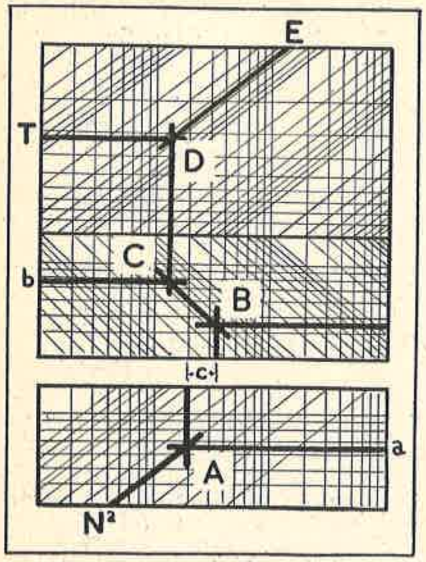


Fig. 7. Showing use of chart

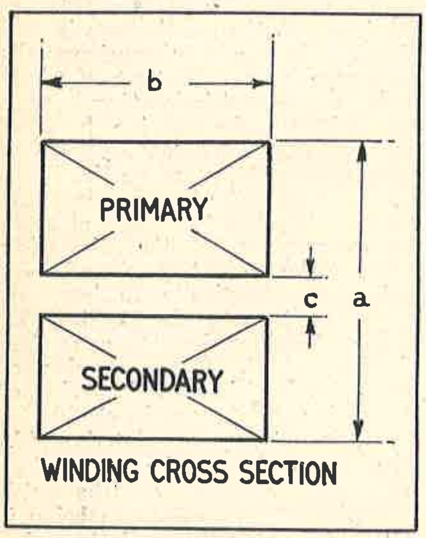


Fig. 8 Dimensions used in chart

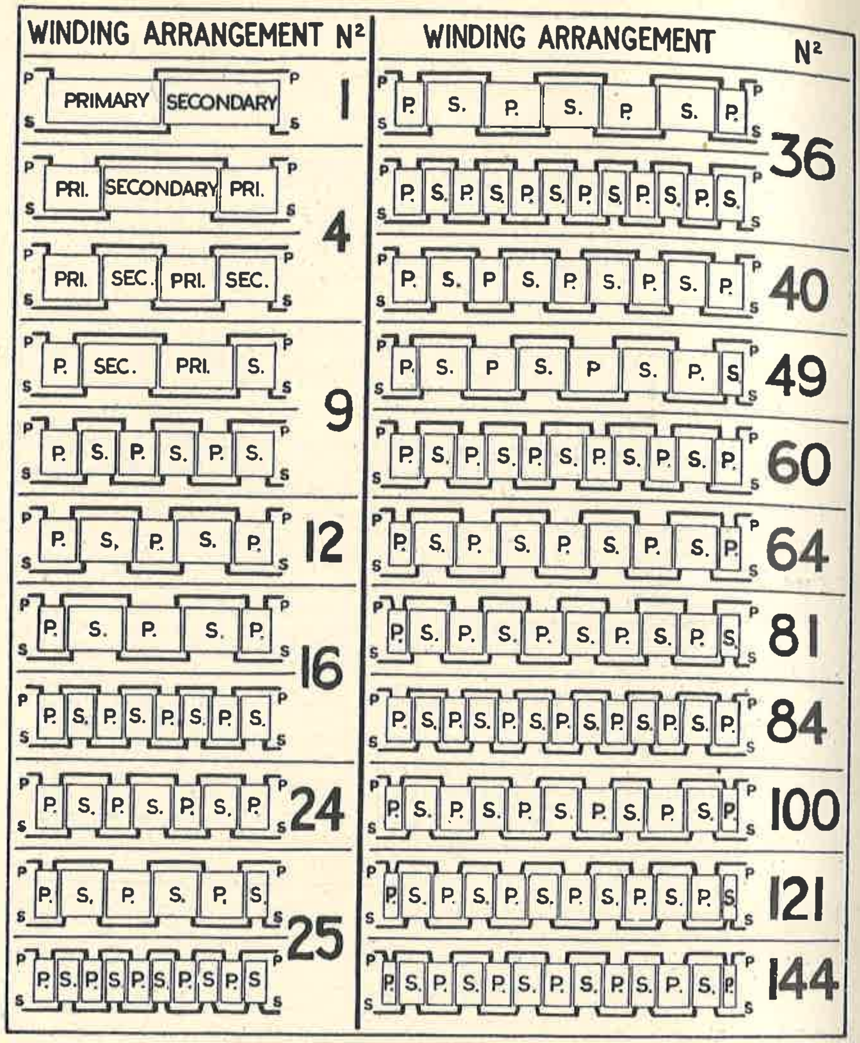


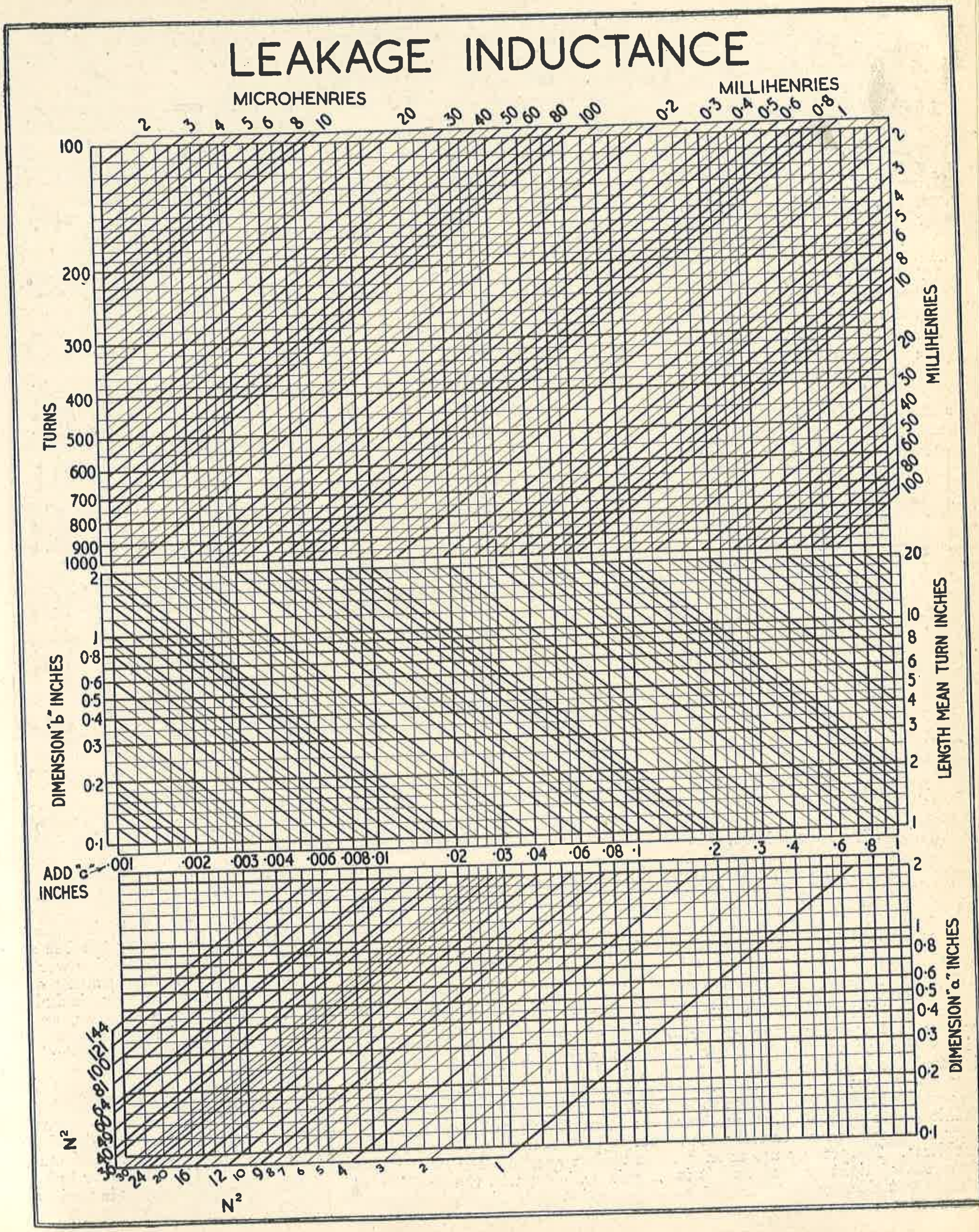
Table of Winding arrangements

insulation, we have 0.07 in. Following the diagonal from the intercept with 2.5 in. length mean turn to the intercept with the 0.625 in. dimension "b", gives about 0.028 on the vertical scale. Following this vertically to intercept with 600 turns, gives leakage inductance referred to the primary as about 1.75 mH. Taking the intercept with 800 turns gives about 2.9 mH. Therefore leakage inductance referred to the secondary of 8,000 turns will be about 290 mH., or 0.29 H.

Example II: Using the same bobbin as above, but with a partition vertically, 0.025 in. thick, the primary and secondary are to be wound either side of the partition.

Dimension "a" will now be 0.6 in., "b" approximately 0.17 in. (allowing 0.0175 in. clearance at the top), and "c" 0.025 in. Intercept with "a" = 0.6 in. and $N^2 = 1$ gives a vertical of 0.2 in. Adding "c" this becomes 0.225 in. Intercept with length mean turn of 2.5 in. and "b" of 0.17 in. (in opposite direction along the diagonal line this time) brings us to the vertical reference at about 0.35. Following this up to intercept with 600 turns gives leakage inductance referred to the primary of about 20 mH. The 800 intercept gives 36 mH, so the leakage inductance referred to the secondary will be about 3.6 H.

Example III: A push pull output transformer is to be wound on a bobbin of length $1\frac{1}{2}$ in. and depth, allowing for top clearance, of $\frac{3}{8}$ in. The length mean turn is 6 in. Primary turns 4,000; secondary turns 75. First consider completely layer mixing arrangement using the 7th arrangement in the left hand column. Insulation between sections can be as low as 0.015 in. Required leakage inductance between each primary and secondary and between whole primary and secondary. Each primary half-winding to occupy 30 per cent of the available space, and the secondary 40 per cent.



LEAKAGE INDUCTANCE
MICROHENRIES
2 3 4 5 6 8 10 20 30 40 50 60 80 100
MILLIHENRIES
0·2 0·3 0·4 0·5 0·6 0·8 1
2 3 4 5 6 8 10 20 30 40 50 60 80 100
MILLIHENRIES
TURNS
100 200 300 400 500 600 700 800 900 1000
DIMENSION "b" INCHES
2 1 0·8 0·6 0·5 0·4 0·3 0·2 0·1
LENGTH MEAN TURN INCHES
20 10 8 6 5 4 3 2 1
ADD "c" INCHES
·001 ·002 ·003 ·004 ·006 ·008 ·01 ·02 ·03 ·04 ·06 ·08 ·1 ·2 ·3 ·4 ·6 ·8 1
DIMENSION "a" INCHES
2 1 0·8 0·6 0·5 0·4 0·3 0·2 0·1
N²
144 121 100 84 81 64 60 49 40 36
36 30 24 20 16 12 10 9 8 1 6 5 4 3 2 1
N²

Four thicknesses of 0.015 in. take 0.060 in., leaving just over 0.3 in. actual winding depth. Each primary should occupy a total of 0.09 in. and the secondary 0.12 in.

Leakage inductance between each primary and secondary: "a" = 0.21 in., $N^2 = 4$. This intercept gives vertical of about 0.017 in. Adding 0.015 in. for "c" gives 0.032 in., which intercepted with length mean turn 6 in. and "b" of 1.5 in. brings us to about .013. For 2,000 turns, read off for 200 and multiply by 100. This gives leakage inductance from each primary to secondary of about 8.5 mH.

Between whole primary and secondary: "a" = 0.3 in., $N^2 = 16$. This intercept gives vertical, 0.0063 in. Adding "c", 0.0213. (There would be considerable advantage here if winding insulation thickness could be reduced.) Intercepts with length mean turn and "b" of 6 in. and 1.5 in. respectively give vertical of about .0085, and intercept with 400 turns gives inductance of 0.22 mH, or 22 mH for 4,000 turns.

Example IV: For the same transformer as the previous example, the bobbin is divided in the centre by a partition 0.1 in. thick, and each side is wound with two primary sections and one secondary section. Required, leakage inductance between each primary and secondary and between whole primary and secondary, when the primary sections making up each half are (a) both in the same side of the bobbin, and (b) when they are cross-connected.

Between each primary and whole secondary: (a) both primary sections same side: first due to axial flux, "a" = 0.3 in., $N^2 = 4$; "b" for primary 0.7 in., for secondary 1.4 in. As primary occupies 60 per cent of depth and secondary 40 per cent, mean can be taken as 1.12 in. Thus inductance due to axial flux is given at 13.5 mH. Due to radial flux, ampere turns are 50 per cent of total, thus this portion can be referred to 1,000 turns; "a" = 1.4 in., "b" = 0.25 in. (computed as above for axial flux), $N^2 = 1$, "c" = 0.1 in. Thus inductance due to radial flux in given as about 180 mH. Total leakage inductance from each primary to whole secondary is thus nearly 200 mH. (Given above as 193.5 mH.)

Between each primary and whole secondary (b) cross-connected arrangement: this can practically be regarded as two leakage flux areas only, both having only axial flux, each having a physical area of cross-section "a" = 0.21 in. and "b" = 0.7 in. Treating the two together as a composite winding such that "a" = 0.42 in., $N^2 = 4$, "b" = 0.7 in., the inductance is now found to be about 28 mH, about one-seventh of the value given by the other method of connexion.

Between whole and primary and secondary:—This value is not affected by the method of connexion, since the whole area is used in each case; "a" = 0.3 in., $N^2 = 4$, "b" = 1.4 in., c = 0.015 in. (as before). Inductance thus given is 44 mH referred to 4,000 turns.

In examples III and IV any of the values of leakage inductance can be referred to the secondary by taking the appropriate intercept with 750 turns and dividing by 100 to obtain the inductance referred to 75 turns.

The Use of the Decca Navigator System for Under-Water Oil Exploration

THE world's consumption of mineral oil has been rapidly rising over the past 10 years, and the existing land resources are inadequate to meet this growing demand. Oil experts have, for a number of years, been studying the possibility of recovering oil from the ocean bed. It is known that the sea areas of the Caribbean Sea and the Persian Gulf are probably rich in oil resources, and where the water is not unduly deep, present-day techniques permit drilling to be carried out.

An essential feature of oil recovery is the preliminary exploration and survey. When this work is carried out on land, normal survey techniques can be employed, but when the same problem is to be tackled at sea, the surveyor finds himself faced with entirely new problems. The work must often be done out of sight of land and it is quite unsatisfactory to rely on astronomical observations. Fortunately, recent developments in radio position fixing provide an entirely new method. The Decca Navigator System, developed primarily as a radio aid to marine navigation, has been found by many survey authorities, including the Hydrographer of the Royal Navy, to possess an accuracy unequalled by any other radio device. It can provide, over thousands of square miles of sea, a fix of position with an accuracy adequate for purposes of hydrographic survey and oil exploration.

A chain of Decca navigator stations, consisting of a master and two slave stations, lays down a pattern of radio position lines which, interpreted by a radio receiver carried in the survey launch, enables the surveyor to plot his position with very great accuracy. Thus, over a featureless area of sea, is provided a grid which marks the position of any point as simply and as accurately as though it were on land.

The Decca Navigator Company, appreciating the value of the System in this new field, have developed a transportable Decca transmitting station which can be rapidly erected in any part of the world.

The decision of the Bahrein Petroleum Company to employ the Decca System in their oil exploration work was taken after a careful study of all existing electronic aids, including several American systems, and it may therefore be assumed that British progress in this field is in advance of American.

The application of Decca to under-water oil exploration is arousing very considerable interest among oil surveyors throughout the world, and it may be expected that a number of other systems will shortly be employed for similar work in various parts of the world.

An inve

WHILE many field of electro linear elements th the strict current proportionality of (containing resona iron-cored inducta mained the m Although the devel circuits has inev empirically, a rati lifts them out of mysterious should applications, if no formance. Howev previous attempts functioning of su presented almost culties suggests th to approach the p mathematical mea is required is a theory and pract simple mathemati exact graphical measurements. I two concrete ex simplest case of next an actual fr of more compl attacked, and in semi-graphical an logical, satisfying

Ferro-resonance

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* Telephone Manufactur