Learning Structured Predictors

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Supervised (Structured) Prediction

- Learning to predict: given training data

  $$\{(\mathbf{x}^{(1)}, \mathbf{y}^{(1)}), (\mathbf{x}^{(2)}, \mathbf{y}^{(2)}), \ldots, (\mathbf{x}^{(m)}, \mathbf{y}^{(m)})\}$$

  learn a predictor $\mathbf{x} \to \mathbf{y}$ that *works well* on unseen inputs $\mathbf{x}$

- Non-Structured Prediction: outputs $\mathbf{y}$ are atomic
  - Binary prediction: $\mathbf{y} \in \{-1, +1\}$
  - Multiclass prediction: $\mathbf{y} \in \{1, 2, \ldots, L\}$

- Structured Prediction: outputs $\mathbf{y}$ are structured
  - Sequence prediction: $\mathbf{y}$ are sequences
  - Parsing: $\mathbf{y}$ are trees
  - ...

Named Entity Recognition

$\mathbf{y}$	PER	-	QNT	-	-	ORG	ORG	-	TIME
$\mathbf{x}$	Jim	bought	300	shares	of	Acme	Corp.	in	2006

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$\mathbf{y}$	PER	PER	-	-	LOC
$\mathbf{x}$	Jack	London	went	to	Paris

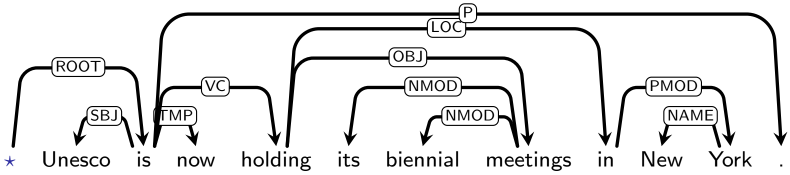
$\mathbf{y}$	PER	PER	-	-	LOC
$\mathbf{x}$	Paris	Hilton	went	to	London

$\mathbf{y}$	PER	-	-	LOC
$\mathbf{x}$	Jackie	went	to	Lisdon

Part-of-speech Tagging

$\mathbf{y}$	NNP	NNP	VBZ	NNP	.
$\mathbf{x}$	Ms.	Haag	plays	Elianti	.

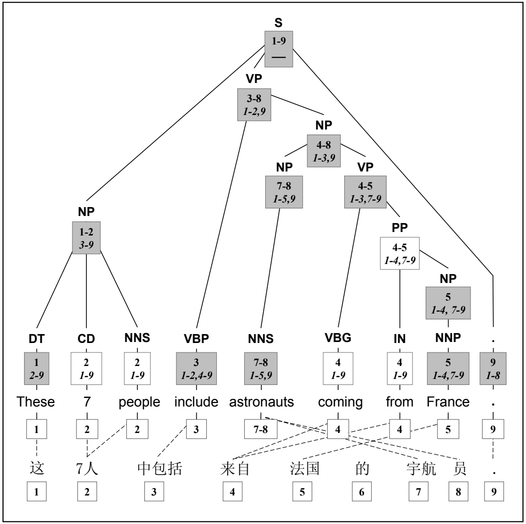
Syntactic Parsing



$\mathbf{x}$ are sentences

$\mathbf{y}$ are syntactic dependency trees

Machine Translation



(?)

Object Detection

(?)

$\mathbf{x}$ are images

$\mathbf{y}$ are grids labeled with object types

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Today's Goals

- Introduce basic concepts for structured prediction
  - We will restrict to sequence prediction
- What can we can borrow from standard classification?
  - Learning paradigms and algorithms, in essence, work here too
  - However, computations behind algorithms are prohibitive
- What can we borrow from HMM and other structured formalisms?
  - Representations of structured data into feature spaces
  - Inference/search algorithms for tractable computations
  - E.g., algorithms for HMMs (Viterbi, forward-backward) will play a major role in today's methods

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Sequence Prediction

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Sequence Prediction

- $\mathbf{x} = x_1 x_2 \dots x_n$ are input sequences, $x_i \in \mathcal{X}$
- $\mathbf{y} = y_1 y_2 \dots y_n$ are output sequences, $y_i \in \{1, \dots, L\}$

- **Goal:** given training data

$$\left\{(\mathbf{x}^{(1)}, \mathbf{y}^{(1)}), (\mathbf{x}^{(2)}, \mathbf{y}^{(2)}), \dots, (\mathbf{x}^{(m)}, \mathbf{y}^{(m)})\right\}$$

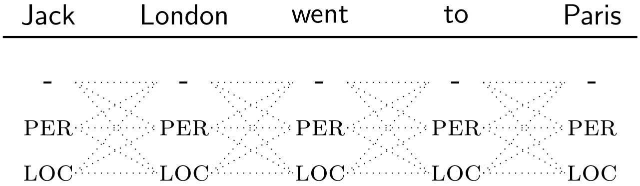
  learn a predictor $\mathbf{x} \rightarrow \mathbf{y}$ that works well on unseen inputs $\mathbf{x}$

- What is the form of our prediction model?

Exponentially-many Solutions

- Let $\mathcal{Y} = \{\text{-}, \textsc{per}, \textsc{loc}\}$

- The solution space (all output sequences):



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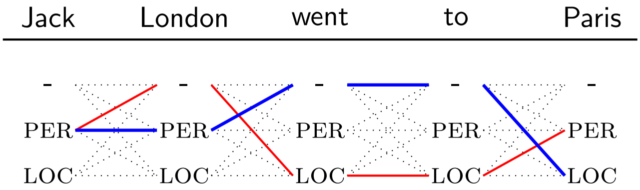

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Approach 1: Local Classifiers

?

Jack London went to Paris

Decompose the sequence into n classification problems:

- A classifier predicts individual labels at each position

$$\hat{y}_i = \underset{l \in \{\text{LOC, PER, -}\}}{\operatorname{argmax}} \ \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, l)$$

- $\mathbf{f}(\mathbf{x}, i, l)$ represents an assignment of label l for x_i
- $\mathbf{w}$ is a vector of parameters, has a weight for each feature of $\mathbf{f}$
  - Use standard classification methods to learn $\mathbf{w}$
- At test time, predict the best sequence by
  a simple concatenation of the best label for each position

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Indicator Features

- $\mathbf{f}(\mathbf{x}, i, l)$ is a vector of d features representing label l for x_i

$$[\ \mathbf{f}_1(\mathbf{x}, i, l), \ldots, \mathbf{f}_j(\mathbf{x}, i, l), \ldots, \mathbf{f}_d(\mathbf{x}, i, l)\ ]$$

- What's in a feature $\mathbf{f}_j(\mathbf{x}, i, l)$?
  - Anything we can compute using $\mathbf{x}$ and i and l
  - Anything that indicates whether l is (not) a good label for x_i
  - Indicator features: binary-valued features looking at:
    - a simple pattern of $\mathbf{x}$ and target position i
    - and the candidate label l for position i

$$\mathbf{f}_j(\mathbf{x}, i, l) = \begin{cases} 1 & \text{if } x_i = \text{London and } l = \textsc{loc} \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{f}_k(\mathbf{x}, i, l) = \begin{cases} 1 & \text{if } x_{i+1} = \text{went and } l = \textsc{loc} \\ 0 & \text{otherwise} \end{cases}$$

Feature Templates

- Feature templates generate many indicator features mechanically
- A feature template is identified by a type, and a number of values
  - Example: template $\textsc{word}$ extracts the current word

$$\mathbf{f}_{\langle \textsc{word}, a, w \rangle}(\mathbf{x}, i, l) = \begin{cases} 1 & \text{if } x_i = w \text{ and } l = a \\ 0 & \text{otherwise} \end{cases}$$

  - A feature of this type is identified by the tuple $\langle \textsc{word}, a, w \rangle$
  - Generates a feature for every label $a \in \mathcal{Y}$ and every word w

e.g.:	$a = \textsc{loc}$	$w =$ London,	$a =$ -	$w =$ London
	$a = \textsc{loc}$	$w =$ Paris	$a = \textsc{per}$	$w =$ Paris
	$a = \textsc{per}$	$w =$ John	$a =$ -	$w =$ the

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- In feature-based models:
  - Define feature templates manually
  - Instantiate the templates on every set of values in the training data
    $\rightarrow$ generates a very high-dimensional feature space
  - Define parameter vector $\mathbf{w}$ indexed by such feature tuples
  - Let the learning algorithm choose the relevant features

More Features for NE Recognition

$$\begin{array}{ccccc} & \text{PER} & & & \\ \text{Jack} & \text{London} & \text{went} & \text{to} & \text{Paris} \end{array}$$

In practice, construct $\mathbf{f}(\mathbf{x}, i, l)$ by ...

- Define a number of simple patterns of $\mathbf{x}$ and i
  - current word x_i
  - is x_i capitalized?
  - x_i has digits?
  - prefixes/suffixes of size 1, 2, 3, ...
  - is x_i a known location?
  - is x_i a known person?
  - next word
  - previous word
  - current and next words together
  - other combinations
- Define feature templates by combining patterns with labels l
- Generate actual features by instantiating templates on training data

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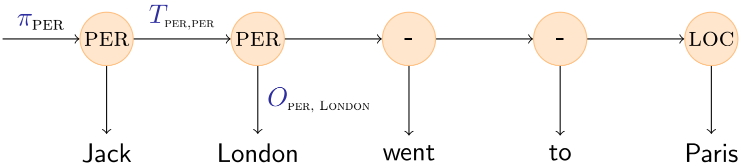
Main limitation: features can't capture interactions between labels!

Approach 2: HMM for Sequence Prediction

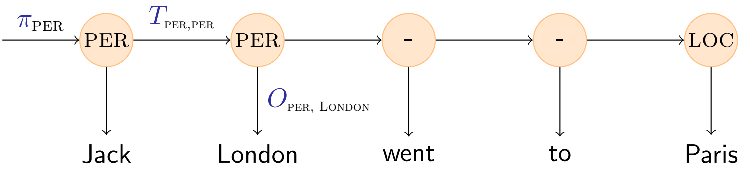


- Define an HMM were each label is a state
- Model parameters:
  - π_l : probability of starting with label l
  - $T_{l,l'}$: probability of transitioning from l to l'
  - $O_{l,x}$: probability of generating symbol x given label l
- Predictions:

$$p(\mathbf{x}, \mathbf{y}) = \pi_{y_1} O_{y_1,x_1} \prod_{i>1} T_{y_{i-1},y_i} O_{y_i,x_i}$$

- Learning: relative counts + smoothing
- Prediction: Viterbi algorithm

Approach 2: Representation in HMM



- Label interactions are captured in the transition parameters
- But interactions between labels and input symbols are quite limited!
  - Only $O_{y_i,x_i} = p(x_i \mid y_i)$
  - Not clear how to exploit patterns such as:
    - Capitalization, digits
    - Prefixes and suffixes
    - Next word, previous word
    - Combinations of these with label transitions
- Why? HMM independence assumptions:

  given label y_i, token x_i is independent of anything else

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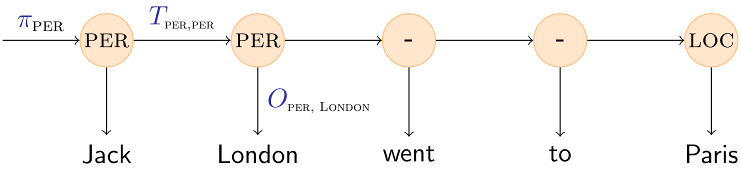

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Local Classifiers vs. HMM

Local Classifiers

- Form:

$$\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, l)$$

- Learning: standard classifiers
- Prediction: independent for each x_i
- Advantage: feature-rich
- Drawback: no label interactions

HMM

- Form:

$$\pi_{y_1} O_{y_1, x_1} \prod_{i>1} T_{y_{i-1}, y_i} O_{y_i, x_i}$$

- Learning: relative counts
- Prediction: Viterbi
- Advantage: label interactions
- Drawback: no fine-grained features

Approach 3: Global Sequence Predictors

$\mathbf{y}$:	PER	PER	-	-	LOC
$\mathbf{x}$:	Jack	London	went	to	Paris

Learn a single classifier from $\mathbf{x} \rightarrow \mathbf{y}$

$$\text{predict}(\mathbf{x}_{1:n}) = \underset{\mathbf{y} \in \mathcal{Y}^n}{\text{argmax}}\, \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})$$

Next questions: ...

- How do we represent entire sequences in $\mathbf{f}(\mathbf{x}, \mathbf{y})$?
- There are exponentially-many sequences $\mathbf{y}$ for a given $\mathbf{x}$, how do we solve the argmax problem?

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Factored Representations

$\mathbf{y}$:	PER	PER	-	-	LOC
$\mathbf{x}$:	Jack	London	went	to	Paris

- How do we represent entire sequences in $\mathbf{f}(\mathbf{x}, \mathbf{y})$?
  - Look at individual assignments y_i (standard classification)
  - Look at bigrams of outputs labels $\langle y_{i-1}, y_i \rangle$
  - Look at trigrams of outputs labels $\langle y_{i-2}, y_{i-1}, y_i \rangle$
  - Look at n-grams of outputs labels $\langle y_{i-n+1}, \ldots, y_{i-1}, y_i \rangle$
  - Look at the full label sequence $\mathbf{y}$ (intractable)
- A factored representation will lead to a tractable model

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Bigram Feature Templates

	1	2	3	4	5
$\mathbf{y}$	PER	PER	-	-	LOC
$\mathbf{x}$	Jack	London	went	to	Paris

- A template for word + bigram:

$$\mathbf{f}_{\langle \mathrm{WB},a,b,w\rangle}(\mathbf{x}, i, y_{i-1}, y_i) = \begin{cases} 1 & \text{if } x_i = w \text{ and} \\ & \quad y_{i-1} = a \text{ and } y_i = b \\ 0 & \text{otherwise} \end{cases}$$

e.g., $\mathbf{f}_{\langle \mathrm{WB},\mathrm{PER},\mathrm{PER},\text{London}\rangle}(\mathbf{x}, 2, \mathrm{PER}, \mathrm{PER}) = 1$
$\mathbf{f}_{\langle \mathrm{WB},\mathrm{PER},\mathrm{PER},\text{London}\rangle}(\mathbf{x}, 3, \mathrm{PER}, \text{-}) = 0$
$\mathbf{f}_{\langle \mathrm{WB},\mathrm{PER},\text{-},\text{went}\rangle}(\mathbf{x}, 3, \mathrm{PER}, \text{-}) = 1$

More Templates for NER

	1	2	3	4	5
$\mathbf{x}$	Jack	London	went	to	Paris
$\mathbf{y}$	PER	PER	-	-	LOC
$\mathbf{y}'$	PER	LOC	-	-	LOC
$\mathbf{y}''$	-	-	-	LOC	-
$\mathbf{x}'$	My	trip	to	London	...

$\mathbf{f}_{\langle \text{W},\text{PER},\text{PER},\text{London}\rangle}(\ldots) = 1$ iff $x_i =$"London" and $y_{i-1} = \text{PER}$ and $y_i = \text{PER}$

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Representations Factored at Bigrams

$\mathbf{y}$:	PER	PER	-	-	LOC
$\mathbf{x}$:	Jack	London	went	to	Paris

- $\mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$
  - A d-dimensional feature vector of a label bigram at i
  - Each dimension is typically a boolean indicator (0 or 1)
- $\mathbf{f}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$
  - A d-dimensional feature vector of the entire $\mathbf{y}$
  - Aggregated representation by summing bigram feature vectors
  - Each dimension is now a count of a feature pattern

Linear Sequence Prediction

$$\text{predict}(\mathbf{x}_{1:n}) = \underset{\mathbf{y} \in \mathcal{Y}^n}{\text{argmax}}\, \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})$$

where

$$\mathbf{f}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$$

- Note the linearity of the expression:

$$\begin{aligned} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y}) &= \mathbf{w} \cdot \sum_{i=1}^{n} \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i) \\ &= \sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i) \end{aligned}$$

- Next questions:
  - How do we solve the argmax problem?
  - How do we learn $\mathbf{w}$?

Linear Sequence Prediction

$$\text{predict}(\mathbf{x}_{1:n}) = \operatorname*{argmax}_{\mathbf{y} \in \mathcal{Y}^n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})$$

where

$$\mathbf{f}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$$

- Note the linearity of the expression:

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- Next questions:
  - How do we solve the argmax problem?
  - How do we learn $\mathbf{w}$?

Predicting with Factored Sequence Models

- Consider a fixed $\mathbf{w}$. Given $\mathbf{x}_{1:n}$ find:

$$\underset{\mathbf{y}\in\mathcal{Y}^n}{\operatorname{argmax}} \sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$$

- Use the Viterbi algorithm, takes $O(n|\mathcal{Y}|^2)$

- Notational change: since $\mathbf{w}$ and $\mathbf{x}_{1:n}$ are fixed we will use

$$s(i, a, b) = \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, a, b)$$

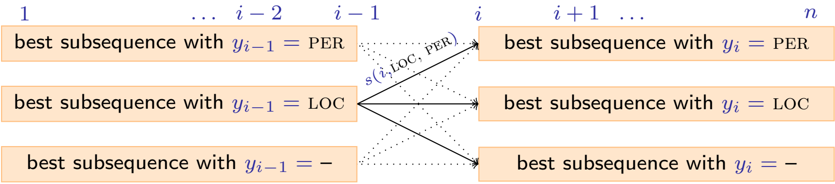
Viterbi for Factored Sequence Models

- Given scores $s(i, a, b)$ for each position i and output bigram a, b, find:

$$\operatorname*{argmax}_{\mathbf{y} \in \mathcal{Y}^n} \sum_{i=1}^{n} s(i, y_{i-1}, y_i)$$

- Use the Viterbi algorithm, takes $O(n|\mathcal{Y}|^2)$

- Intuition: output sequences that share bigrams will share scores



Intuition for Viterbi

- Assume we have the best sub-sequence up to position $i-1$ ending with each label:

1	$\cdots$	$i-1$	i
best subsequence with $y_{i-1} = \text{PER}$			
best subsequence with $y_{i-1} = \text{LOC}$			
best subsequence with $y_{i-1} = -$			

- What is the best sequence up to position i with $y_i = $LOC?

Intuition for Viterbi

- Assume we have the best sub-sequence up to position $i-1$ ending with each label:

$1 \quad \cdots \quad i-1 \quad i$

best subsequence with $y_{i-1} = \text{PER}$

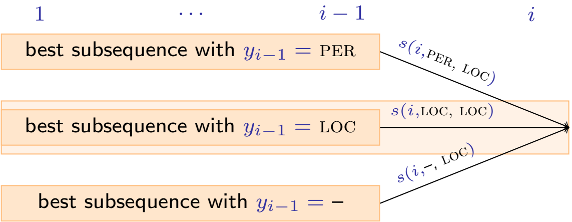
best subsequence with $y_{i-1} = \text{LOC}$

best subsequence with $y_{i-1} = $ –

- What is the best sequence up to position i with $y_i = $LOC?

Intuition for Viterbi

- Assume we have the best sub-sequence up to position $i-1$ ending with each label:



- What is the best sequence up to position i with $y_i = $LOC?

Viterbi for Linear Factored Predictors

$$\hat{\mathbf{y}} = \operatorname*{argmax}_{\mathbf{y} \in \mathcal{Y}^n} \sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$$

- **Definition:** score of optimal sequence for $\mathbf{x}_{1:i}$ ending with $a \in \mathcal{Y}$

$$\delta(i, a) = \max_{\mathbf{y} \in \mathcal{Y}^i : y_i = a} \sum_{j=1}^{i} s(j, y_{j-1}, y_j)$$

- Use the following recursions, for all $a \in \mathcal{Y}$:

$$\begin{aligned} \delta(1, a) &= s(1, y_0 = \text{NULL}, a) \\ \delta(i, a) &= \max_{b \in \mathcal{Y}} \delta(i-1, b) + s(i, b, a) \end{aligned}$$

- The optimal score for $\mathbf{x}$ is $\max_{a \in \mathcal{Y}} \delta(n, a)$
- The optimal sequence $\hat{\mathbf{y}}$ can be recovered through *back-pointers*

Linear Factored Sequence Prediction

$$\text{predict}(\mathbf{x}_{1:n}) = \underset{\mathbf{y} \in \mathcal{Y}^n}{\text{argmax}}\, \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})$$

- Factored representation, e.g. based on bigrams
- Flexible, arbitrary features of full $\mathbf{x}$ and the factors
- Efficient prediction using Viterbi
- Next, learning $\mathbf{w}$:
  - Probabilistic log-linear models:
    - Local learning, *a.k.a.* Maximum-Entropy Markov Models
    - Global learning, *a.k.a.* Conditional Random Fields
  - Margin-based methods:
    - Structured Perceptron
    - Structured SVM

The Learner's Game

Training Data

PER	-	-
Maria	is	beautiful
LOC	-	-
Lisbon	is	beautiful
PER	-	-	LOC
Jack	went	to	Lisbon
LOC	-	-
Argentina	is	nice
PER	PER	-	-	LOC	LOC
Jack	London	went	to	South	Paris
ORG	-	-	ORG
Argentina	played	against	Germany

Weight Vector $\mathbf{w}$

The Learner's Game

Training Data

PER	-	-			
Maria	is	beautiful			
LOC	-	-			
Lisbon	is	beautiful			
PER	-	-	LOC		
Jack	went	to	Lisbon		
LOC	-	-			
Argentina	is	nice			
PER	PER	-	-	LOC	LOC
Jack	London	went	to	South	Paris
ORG	-	-	ORG		
Argentina	played	against	Germany		

Weight Vector $\mathbf{w}$

$$\mathbf{w}_{\langle \textsc{Lower}, \text{-} \rangle} = +1$$

The Learner's Game

Training Data

- PER Maria, - is, - beautiful
- LOC Lisbon, - is, - beautiful
- PER Jack, - went, - to, LOC Lisbon
- LOC Argentina, - is, - nice
- PER Jack, PER London, - went, - to, LOC South, LOC Paris
- ORG Argentina, - played, - against, ORG Germany

Weight Vector $\mathbf{w}$

$$\mathbf{w}_{\langle \textsc{Lower}, \text{-} \rangle} = +1$$
$$\mathbf{w}_{\langle \textsc{Upper}, \textsc{per} \rangle} = +1$$

The Learner's Game

Training Data

PER	-	-			
Maria	is	beautiful			
LOC	-	-			
Lisbon	is	beautiful			
PER	-	-	LOC		
Jack	went	to	Lisbon		
LOC	-	-			
Argentina	is	nice			
PER	PER	-	-	LOC	LOC
Jack	London	went	to	South	Paris
ORG	-	-	ORG		
Argentina	played	against	Germany		

Weight Vector $\mathbf{w}$

$$\mathbf{w}_{\langle \text{LOWER}, - \rangle} = +1$$
$$\cancel{\mathbf{w}_{\langle \text{UPPER}, \text{PER} \rangle} = +1}$$
$$\mathbf{w}_{\langle \text{UPPER}, \text{LOC} \rangle} = +1$$

The Learner's Game

Training Data

- PER - -
  Maria is beautiful
- LOC - -
  Lisbon is beautiful
- PER - - LOC
  Jack went to Lisbon
- LOC - -
  Argentina is nice
- PER PER - - LOC LOC
  Jack London went to South Paris
- ORG - - ORG
  Argentina played against Germany

Weight Vector $\mathbf{w}$

$\mathbf{w}_{\langle \text{LOWER},\text{-}\rangle} = +1$

$\cancel{\mathbf{w}_{\langle \text{UPPER},\text{PER}\rangle} = +1}$

$\mathbf{w}_{\langle \text{UPPER},\text{LOC}\rangle} = +1$

$\mathbf{w}_{\langle \text{WORD},\text{PER},\text{Maria}\rangle} = +2$

The Learner's Game

Training Data

PER	-	-
Maria	is	beautiful

LOC	-	-
Lisbon	is	beautiful

PER	-	-	LOC
Jack	went	to	Lisbon

LOC	-	-
Argentina	is	nice

PER	PER	-	-	LOC	LOC
Jack	London	went	to	South	Paris

ORG	-	-	ORG
Argentina	played	against	Germany

Weight Vector $\mathbf{w}$

$\mathbf{w}_{\langle \text{LOWER},\text{-} \rangle} = +1$

$\cancel{\mathbf{w}_{\langle \text{UPPER},\text{PER} \rangle} = +1}$

$\mathbf{w}_{\langle \text{UPPER},\text{LOC} \rangle} = +1$

$\mathbf{w}_{\langle \text{WORD},\text{PER},\text{Maria} \rangle} = +2$

$\mathbf{w}_{\langle \text{WORD},\text{PER},\text{Jack} \rangle} = +2$

The Learner's Game

Training Data

- PER Maria, - is, - beautiful
- LOC Lisbon, - is, - beautiful
- PER Jack, - went, - to, LOC Lisbon
- LOC Argentina, - is, - nice
- PER Jack, PER London, - went, - to, LOC South, LOC Paris
- ORG Argentina, - played, - against, ORG Germany

PER	-	-
Maria	is	beautiful

LOC	-	-
Lisbon	is	beautiful

PER	-	-	LOC
Jack	went	to	Lisbon

LOC	-	-
Argentina	is	nice

PER	PER	-	-	LOC	LOC
Jack	London	went	to	South	Paris

ORG	-	-	ORG
Argentina	played	against	Germany

Weight Vector $\mathbf{w}$

$\mathbf{w}_{\langle \textsc{Lower}, \text{-} \rangle} = +1$

$\cancel{\mathbf{w}_{\langle \textsc{Upper}, \textsc{per} \rangle} = +1}$

$\mathbf{w}_{\langle \textsc{Upper}, \textsc{loc} \rangle} = +1$

$\mathbf{w}_{\langle \textsc{Word}, \textsc{per}, \text{Maria} \rangle} = +2$

$\mathbf{w}_{\langle \textsc{Word}, \textsc{per}, \text{Jack} \rangle} = +2$

$\mathbf{w}_{\langle \textsc{NextW}, \textsc{per}, \text{went} \rangle} = +2$

The Learner's Game

Training Data

- PER - -
  Maria is beautiful
- LOC - -
  Lisbon is beautiful
- PER - - LOC
  Jack went to Lisbon
- LOC - -
  Argentina is nice
- PER PER - - LOC LOC
  Jack London went to South Paris
- ORG - - ORG
  Argentina played against Germany

Weight Vector $\mathbf{w}$

$\mathbf{w}_{\langle \text{Lower},\text{-} \rangle} = +1$

$\cancel{\mathbf{w}_{\langle \text{Upper},\text{PER} \rangle} = +1}$

$\mathbf{w}_{\langle \text{Upper},\text{LOC} \rangle} = +1$

$\mathbf{w}_{\langle \text{Word},\text{PER},\text{Maria} \rangle} = +2$

$\mathbf{w}_{\langle \text{Word},\text{PER},\text{Jack} \rangle} = +2$

$\mathbf{w}_{\langle \text{NextW},\text{PER},\text{went} \rangle} = +2$

$\mathbf{w}_{\langle \text{NextW},\text{ORG},\text{played} \rangle} = +2$

The Learner's Game

Training Data

PER	-	-
Maria	is	beautiful

LOC	-	-
Lisbon	is	beautiful

PER	-	-	LOC
Jack	went	to	Lisbon

LOC	-	-
Argentina	is	nice

PER	PER	-	-	LOC	LOC
Jack	London	went	to	South	Paris

ORG	-	-	ORG
Argentina	played	against	Germany

Weight Vector $\mathbf{w}$

$\mathbf{w}_{\langle \textsc{Lower}, \text{-} \rangle} = +1$

$\cancel{\mathbf{w}_{\langle \textsc{Upper}, \textsc{per} \rangle} = +1}$

$\mathbf{w}_{\langle \textsc{Upper}, \textsc{loc} \rangle} = +1$

$\mathbf{w}_{\langle \textsc{Word}, \textsc{per}, \text{Maria} \rangle} = +2$

$\mathbf{w}_{\langle \textsc{Word}, \textsc{per}, \text{Jack} \rangle} = +2$

$\mathbf{w}_{\langle \textsc{NextW}, \textsc{per}, \text{went} \rangle} = +2$

$\mathbf{w}_{\langle \textsc{NextW}, \textsc{org}, \text{played} \rangle} = +2$

$\mathbf{w}_{\langle \textsc{PrevW}, \textsc{org}, \text{against} \rangle} = +2$

The Learner's Game

Training Data

PER	-	-
Maria	is	beautiful

LOC	-	-
Lisbon	is	beautiful

PER	-	-	LOC
Jack	went	to	Lisbon

LOC	-	-
Argentina	is	nice

PER	PER	-	-	LOC	LOC
Jack	London	went	to	South	Paris

ORG	-	-	ORG
Argentina	played	against	Germany

Weight Vector $\mathbf{w}$

$\mathbf{w}_{\langle \text{LOWER},\text{-}\rangle} = +1$

$\cancel{\mathbf{w}_{\langle \text{UPPER},\text{PER}\rangle} = +1}$

$\mathbf{w}_{\langle \text{UPPER},\text{LOC}\rangle} = +1$

$\mathbf{w}_{\langle \text{WORD},\text{PER},\text{Maria}\rangle} = +2$

$\mathbf{w}_{\langle \text{WORD},\text{PER},\text{Jack}\rangle} = +2$

$\mathbf{w}_{\langle \text{NEXTW},\text{PER},\text{went}\rangle} = +2$

$\mathbf{w}_{\langle \text{NEXTW},\text{ORG},\text{played}\rangle} = +2$

$\mathbf{w}_{\langle \text{PREVW},\text{ORG},\text{against}\rangle} = +2$

$\ldots$

$\mathbf{w}_{\langle \text{UPPERBIGRAM},\text{PER},\text{PER}\rangle} = +2$

$\mathbf{w}_{\langle \text{UPPERBIGRAM},\text{LOC},\text{LOC}\rangle} = +2$

$\mathbf{w}_{\langle \text{NEXTW},\text{LOC},\text{played}\rangle} = -1000$

Log-linear Models

for Sequence Prediction

$\mathbf{y}$	PER	PER	-	-	LOC
$\mathbf{x}$	Jack	London	went	to	Paris

Log-linear Models for Sequence Prediction

- Model the conditional distribution:

$$\Pr(\mathbf{y} \mid \mathbf{x}; \mathbf{w}) = \frac{\exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\}}{Z(\mathbf{x}; \mathbf{w})}$$

  where
  - $\mathbf{x} = x_1 x_2 \ldots x_n \in \mathcal{X}^*$
  - $\mathbf{y} = y_1 y_2 \ldots y_n \in \mathcal{Y}^*$ and $\mathcal{Y} = \{1, \ldots, L\}$
  - $\mathbf{f}(\mathbf{x}, \mathbf{y})$ represents $\mathbf{x}$ and $\mathbf{y}$ with d features
  - $\mathbf{w} \in \mathbb{R}^d$ are the parameters of the model
  - $Z(\mathbf{x}; \mathbf{w})$ is a normalizer called the *partition function*

$$Z(\mathbf{x}; \mathbf{w}) = \sum_{\mathbf{z} \in \mathcal{Y}^*} \exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{z})\}$$

- To predict the best sequence

$$\text{predict}(\mathbf{x}_{1:n}) = \underset{\mathbf{y} \in \mathcal{Y}^n}{\operatorname{argmax}} \Pr(\mathbf{y} | \mathbf{x})$$

Log-linear Models: Name

- Let's take the $\log$ of the conditional probability:

$$
\begin{aligned}
\log \Pr(\mathbf{y} \mid \mathbf{x}; \mathbf{w}) &= \log \frac{\exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\}}{Z(\mathbf{x}; \mathbf{w})} \\
&= \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y}) - \log \sum_{y} \exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\} \\
&= \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y}) - \log Z(\mathbf{x}; \mathbf{w})
\end{aligned}
$$

- Partition function: $Z(\mathbf{x}; \mathbf{w}) = \sum_{\mathbf{y}} \exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\}$
- $\log Z(\mathbf{x}; \mathbf{w})$ is a constant for a fixed $\mathbf{x}$
- In the $\log$ space, computations are linear,
  i.e., we model log-probabilities using a linear predictor

Making Predictions with Log-Linear Models

- For tractability, assume $\mathbf{f}(\mathbf{x}, \mathbf{y})$ decomposes into bigrams:

$$\mathbf{f}(\mathbf{x}_{1:n}, \mathbf{y}_{1:n}) = \sum_{i=1}^{n} \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$$

- Given $\mathbf{w}$, given $\mathbf{x}_{1:n}$, find:

$$\begin{aligned}
\operatorname*{argmax}_{\mathbf{y}_{1:n}} \Pr(\mathbf{y}_{1:n} | \mathbf{x}_{1:n}; \mathbf{w}) &= \operatorname*{amax}_{\mathbf{y}} \frac{\exp\left\{\sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)\right\}}{Z(\mathbf{x}; \mathbf{w})} \\
&= \operatorname*{amax}_{\mathbf{y}} \exp\left\{\sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)\right\} \\
&= \operatorname*{amax}_{\mathbf{y}} \sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)
\end{aligned}$$

- We can use the Viterbi algorithm

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\end{aligned}$$

- We can use the Viterbi algorithm

Parameter Estimation in Log-Linear Models

$$\Pr(\mathbf{y} \mid \mathbf{x}; \mathbf{w}) = \frac{\exp\left\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\right\}}{Z(\mathbf{x}; \mathbf{w})}$$

- Given training data

$$\left\{(\mathbf{x}^{(1)}, \mathbf{y}^{(1)}), (\mathbf{x}^{(2)}, \mathbf{y}^{(2)}), \ldots, (\mathbf{x}^{(m)}, \mathbf{y}^{(m)})\right\} \quad ,$$

- How to estimate $\mathbf{w}$?
- Define the conditional log-likelihood of the data:

$$L(\mathbf{w}) = \sum_a k = 1^m \log \Pr(\mathbf{y}^{(k)} | \mathbf{x}^{(k)}; \mathbf{w})$$

- $L(\mathbf{w})$ measures how well $\mathbf{w}$ explains the data. A good value for $\mathbf{w}$ will give a high value for $\Pr(\mathbf{y}^{(k)} | \mathbf{x}^{(k)}; \mathbf{w})$ for all $k = 1 \ldots m$.
- We want $\mathbf{w}$ that maximizes $L(\mathbf{w})$

Parameter Estimation in Log-Linear Models

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- Given training data

$$\left\{(\mathbf{x}^{(1)}, \mathbf{y}^{(1)}), (\mathbf{x}^{(2)}, \mathbf{y}^{(2)}), \ldots, (\mathbf{x}^{(m)}, \mathbf{y}^{(m)})\right\} \quad ,$$

- How to estimate $\mathbf{w}$?
- Define the conditional log-likelihood of the data:

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- We want $\mathbf{w}$ that maximizes $L(\mathbf{w})$

Learning Log-Linear Models: Loss + Regularization

- Solve:

$$\mathbf{w}^* = \operatorname*{argmin}_{\mathbf{w}\in\mathbb{R}^d} \overbrace{-L(\mathbf{w})}^{\text{Loss}} + \overbrace{\frac{\lambda}{2}||\mathbf{w}||^2}^{\text{Regularization}}$$

  where
  - The first term is the negative conditional log-likelihood
  - The second term is a regularization term, it penalizes solutions with large norm
  - $\lambda \in \mathbb{R}$ controls the trade-off between loss and regularization
- Convex optimization problem $\rightarrow$ gradient descent
- Two common losses based on log-likelihood that make learning tractable:
  - Local Loss (MEMM): assume that $\Pr(\mathbf{y} \mid \mathbf{x}; \mathbf{w})$ decomposes
  - Global Loss (CRF): assume that $\mathbf{f}(\mathbf{x}, \mathbf{y})$ decomposes

Learning Log-Linear Models: Loss + Regularization

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$$\mathbf{w}^* = \underset{\mathbf{w}\in\mathbb{R}^d}{\operatorname{argmin}} \overbrace{-L(\mathbf{w})}^{\text{Loss}} + \overbrace{\frac{\lambda}{2}||\mathbf{w}||^2}^{\text{Regularization}}$$

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Maximum Entropy Markov Models (MEMM)

?

- Similarly to HMMs:

$$
\begin{aligned}
\Pr(\mathbf{y}_{1:n} \mid \mathbf{x}_{1:n}) &= \Pr(y_1 \mid \mathbf{x}_{1:n}) \times \Pr(\mathbf{y}_{2:n} \mid \mathbf{x}_{1:n}, y_1) \\
&= \Pr(y_1 \mid \mathbf{x}_{1:n}) \times \prod_{i=2}^{n} \Pr(y_i | \mathbf{x}_{1:n}, \mathbf{y}_{1:i-1}) \\
&= \Pr(y_1 | \mathbf{x}_{1:n}) \times \prod_{i=2}^{n} \Pr(y_i | \mathbf{x}_{1:n}, \mathbf{y}_{i-1})
\end{aligned}
$$

- Assumption under MEMMs:

$$
\Pr(y_i | \mathbf{x}_{1:n}, \mathbf{y}_{1:i-1}) = \Pr(y_i | \mathbf{x}_{1:n}, y_{i-1})
$$

Parameter Estimation in MEMM

$$\Pr(y_{1:n} \mid \mathbf{x}_{1:n}) \quad = \quad \Pr(y_1 \mid \mathbf{x}_{1:n}) \times \prod_{i=2}^{n} \Pr(y_i | \mathbf{x}_{1:n}, i, y_{i-1})$$

- The log-linear model is normalized locally (i.e. at each position):

$$\Pr(y \mid \mathbf{x}, i, y') = \frac{\exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y', y)\}}{Z(\mathbf{x}, i, y')}$$

- The log-likelihood is also local :

$$L(\mathbf{w}) = \sum_{k=1}^{m} \sum_{i=1}^{n^{(k)}} \log \Pr(\mathbf{y}_i^{(k)} | \mathbf{x}^{(k)}, i, \mathbf{y}_{i-1}^{(k)})$$

$$\frac{\partial L(\mathbf{w})}{\partial \mathbf{w}_j} = \frac{1}{m} \sum_{k=1}^{m} \sum_{i=1}^{n^{(k)}} \left[\overbrace{\mathbf{f}_j(\mathbf{x}^{(k)}, i, \mathbf{y}_{i-1}^{(k)}, \mathbf{y}_i^{(k)})}^{\text{observed}} - \overbrace{\sum_{y \in \mathcal{Y}} \Pr(\mathbf{y} | \mathbf{x}^{(k)}, i, \mathbf{y}_{i-1}^{(k)}, y) \; \mathbf{f}_j(\mathbf{x}^{(k)}, i, \mathbf{y}_{i-1}^{(k)}, y)}^{\text{expected}} \right]$$

Conditional Random Fields

(?)

- Log-linear model of the conditional distribution:

$$\Pr(\mathbf{y}|\mathbf{x};\mathbf{w}) = \frac{\exp\{\mathbf{w}\cdot\mathbf{f}(\mathbf{x},\mathbf{y})\}}{Z(\mathbf{x})}$$

  where
  - $\mathbf{x}$ and $\mathbf{y}$ are input and output sequences
  - $\mathbf{f}(\mathbf{x},\mathbf{y})$ is a feature vector of $\mathbf{x}$ and $\mathbf{y}$ that decomposes into factors
  - $\mathbf{w}$ are model parameters
- To predict the best sequence

$$\hat{\mathbf{y}} = \underset{\mathbf{y}\in\mathcal{Y}^*}{\operatorname{argmax}} \Pr(\mathbf{y}|\mathbf{x})$$

- Log-Likelihood at the global (sequence) level:

$$L(\mathbf{w}) = \sum_{k=1}^{m} \log \Pr(\mathbf{y}^{(k)}|\mathbf{x}^{(k)};\mathbf{w})$$

Computing the Gradient in CRFs

Consider a parameter $\mathbf{w}_j$ and its associated feature $\mathbf{f}_j$:

$$\frac{\partial L(\mathbf{w})}{\partial \mathbf{w}_j} = \frac{1}{m} \sum_{k=1}^{m} \left[\overbrace{\mathbf{f}_j(\mathbf{x}^{(k)}, \mathbf{y}^{(k)})}^{\text{observed}} - \overbrace{\sum_{\mathbf{y} \in \mathcal{Y}^*} \Pr(\mathbf{y} | \mathbf{x}^{(k)}; \mathbf{w}) \; \mathbf{f}_j(\mathbf{x}^{(k)}, \mathbf{y})}^{\text{expected}} \right]$$

where

$$\mathbf{f}_j(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} \mathbf{f}_j(\mathbf{x}, i, y_{i-1}, y_i)$$

- First term: observed value of $\mathbf{f}_j$ in training examples
- Second term: expected value of $\mathbf{f}_j$ under current $\mathbf{w}$
- In the optimal, observed $=$ expected

Computing the Gradient in CRFs

- The first term is easy to compute, by counting explicitly

$$\sum_i \mathbf{f}_j(\mathbf{x}, i, y_{i-1}^{(k)}, y_i^{(k)})$$

- The second term is more involved,

$$\sum_{\mathbf{y} \in \mathcal{Y}^*} \Pr(\mathbf{y}|\mathbf{x}^{(k)}; \mathbf{w}) \sum_i \mathbf{f}_j(\mathbf{x}^{(k)}, i, y_{i-1}, y_i)$$

  because it sums over all sequences $\mathbf{y} \in \mathcal{Y}^n$

- But there is an efficient solution ...

Computing the Gradient in CRFs

- For an example $(\mathbf{x}^{(k)}, \mathbf{y}^{(k)})$:

$$\sum_{\mathbf{y} \in \mathcal{Y}^n} \Pr(\mathbf{y}|\mathbf{x}^{(k)}; \mathbf{w}) \sum_{i=1}^{n} \mathbf{f}_j(\mathbf{x}^{(k)}, i, y_{i-1}, y_i) = \sum_{i=1}^{n} \sum_{a,b \in \mathcal{Y}} \mu_i^k(a,b) \mathbf{f}_j(\mathbf{x}^{(k)}, i, a, b)$$

- $\mu_i^k(a,b)$ is the marginal probability of having labels (a,b) at position i:

$$\mu_i^k(a,b) = \Pr(\langle i, a, b\rangle \mid \mathbf{x}^{(k)}; \mathbf{w}) = \sum_{\mathbf{y} \in \mathcal{Y}^n \;:\; y_{i-1}=a,\; y_i=b} \Pr(\mathbf{y}|\mathbf{x}^{(k)}; \mathbf{w})$$

- The quantities μ_i^k can be computed efficiently in $O(nL^2)$ using the forward-backward algorithm

Forward-Backward for CRFs

- Assume fixed $\mathbf{x}$ and $\mathbf{w}$.
- For notational convenience, define the score of a label bigram as:

$$s(i, a, b) = \exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, a, b)\}$$

  such that we can write

$$\Pr(\mathbf{y} \mid \mathbf{x}) = \frac{\exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\}}{Z(\mathbf{x})} = \frac{\exp\{\sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)\}}{Z(\mathbf{x})} = \frac{\prod_{i=1}^{n} s(i, y_{i-1}, y_i)}{Z}$$

- Normalizer: $Z = \sum_{\mathbf{y}} \prod_{i=1}^{n} s(i, y_{i-1}, y_i)$
- Marginals: $\mu(i, a, b) = \frac{1}{Z} \sum_{\mathbf{y}, \text{s.t.} y_{i-1}=a, y_i=b} \prod_{i=1}^{n} s(i, y_{i-1}, y_i)$

Forward-Backward for CRFs

- **Definition:** forward and backward quantities

$$\alpha_i(a) = \sum_{\mathbf{y}_{1:i} \in \mathcal{Y}^i : y_i = a} \prod_{j=1}^{i} s(j, y_{j-1}, y_j)$$

$$\beta_i(b) = \sum_{\mathbf{y}_{i:n} \in \mathcal{Y}^{(n-i+1)} : y_i = b} \prod_{j=i+1}^{n} s(j, y_{j-1}, y_j)$$

- $Z = \sum_a \alpha_n(a)$
- $\mu_i(a, b) = \{\alpha_{i-1}(a) * s(i, a, b)\} * \beta_i(b) * Z^{-1}\}$

- Similarly to Viterbi, $\alpha_i(a)$ and $\beta_i(b)$ can be computed recursively in $O(n|\mathcal{Y}|^2)$

Forward-Backward for CRFs

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\end{aligned}
$$

- $Z = \sum_a \alpha_n(a)$
- $\mu_i(a, b) = \{\alpha_{i-1}(a) * s(i, a, b)\} * \beta_i(b) * Z^{-1}\}$

- Similarly to Viterbi, $\alpha_i(a)$ and $\beta_i(b)$ can be computed recursively in $O(n|\mathcal{Y}|^2)$

CRFs: summary so far

- Log-linear models for sequence prediction, $\Pr(\mathbf{y}|\mathbf{x};\mathbf{w})$
- Computations factorize on label bigrams
- Model form:

$$\operatorname*{argmax}_{\mathbf{y}\in\mathcal{Y}^*} \sum_i \mathbf{w}\cdot\mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$$

- Prediction: uses Viterbi (from HMMs)
- Parameter estimation:
  - Gradient-based methods, in practice L-BFGS
  - Computation of gradient uses forward-backward (from HMMs)

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- Next Question: MEMMs or CRFs? HMMs or CRFs?

MEMMs and CRFs

MEMMs: $$\Pr(\mathbf{y} \mid \mathbf{x}) = \prod_{i=1}^{n} \frac{\exp\left\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)\right\}}{Z(\mathbf{x}, i, y_{i-1}; \mathbf{w})}$$

CRFs: $$\Pr(\mathbf{y} \mid \mathbf{x}) = \frac{\exp\left\{\sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)\right\}}{Z(\mathbf{x})}$$

- Both exploit the same factorization, i.e. same features
- Same computations to compute $\operatorname{argmax}_{\mathbf{y}} \Pr(\mathbf{y} \mid \mathbf{x})$
- MEMMs locally normalized; CRFs globally normalized
  - MEMM assume that $\Pr(y_i \mid x_{1:n}, y_{1:i-1}) = \Pr(y_i \mid x_{1:n}, y_{i-1})$
  - Leads to “Label Bias Problem”
- MEMMs are cheaper to train (reduces to multiclass learning)
- CRFs are easier to extend to other structures (next lecture)

HMMs for sequence prediction

- $\mathbf{x}$ are the observations, $\mathbf{y}$ are the hidden states
- HMMs model the joint distributon $\Pr(\mathbf{x}, \mathbf{y})$
- Parameters: (assume $\mathcal{X} = \{1, \ldots, k\}$ and $\mathcal{Y} = \{1, \ldots, l\}$)
  - $\pi \in \mathbb{R}^l$, $\pi_a = \Pr(y_1 = a)$
  - $T \in \mathbb{R}^{l \times l}$, $T_{a,b} = \Pr(y_i = b | y_{i-1} = a)$
  - $O \in \mathbb{R}^{l \times k}$, $O_{a,c} = \Pr(x_i = c | y_i = a)$
- Model form

$$\Pr(\mathbf{x}, \mathbf{y}) = \pi_{y_1} O_{y_1, x_1} \prod_{i=2}^{n} T_{y_{i-1}, y_i} O_{y_i, x_i}$$

- Parameter Estimation: maximum likelihood by counting events and normalizing

HMMs and CRFs

- In CRFs: $\hat{\mathbf{y}} = \mathrm{amax}_{\mathbf{y}} \sum_i \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$
- In HMMs:
$$\begin{aligned}\hat{\mathbf{y}} &= \mathrm{amax}_{\mathbf{y}}\, \pi_{y_1} O_{y_1,x_1} \textstyle\prod_{i=2}^{n} T_{y_{i-1},y_i} O_{y_i,x_i} \\ &= \mathrm{amax}_{\mathbf{y}} \log(\pi_{y_1} O_{y_1,x_1}) + \textstyle\sum_{i=2}^{n} \log(T_{y_{i-1},y_i} O_{y_i,x_i})\end{aligned}$$
- An HMM can be expressed as factored linear models:

$\mathbf{f}_j(\mathbf{x}, i, y, y')$	$\mathbf{w}_j$
$i = 1$ & $y' = a$	$\log(\pi_a)$
$i > 1$ & $y = a$ & $y' = b$	$\log(T_{a,b})$
$y' = a$ & $x_i = c$	$\log(O_{a,b})$

- Hence, HMM are factored linear models

HMMs and CRFs: main differences

- Representation:
  - HMM "features" are tied to the generative process.
  - CRF features are **very** flexible. They can look at the whole input $\mathbf{x}$ paired with a label bigram (y_i, y_{i+1}).
  - In practice, for prediction tasks, "good" discriminative features can improve accuracy **a lot**.
- Parameter estimation:
  - HMMs focus on explaining the data, both $\mathbf{x}$ and $\mathbf{y}$.
  - CRFs focus on the mapping from $\mathbf{x}$ to $\mathbf{y}$.
  - A priori, it is hard to say which paradigm is better.
  - Same dilemma as Naive Bayes vs. Maximum Entropy.

Structured Prediction

Perceptron, SVMs, CRFs

Learning Structured Predictors

- Goal: given training data
  $$\{(\mathbf{x}^{(1)}, \mathbf{y}^{(1)}), (\mathbf{x}^{(2)}, \mathbf{y}^{(2)}), \ldots, (\mathbf{x}^{(m)}, \mathbf{y}^{(m)})\}$$
  learn a predictor $\mathbf{x} \to \mathbf{y}$ with small error on unseen inputs

- In a CRF:
  $$\begin{aligned} \underset{\mathbf{y} \in \mathcal{Y}^*}{\operatorname{argmax}} P(\mathbf{y}|\mathbf{x}; \mathbf{w}) &= \frac{\exp\left\{\sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)\right\}}{Z(\mathbf{x}; \mathbf{w})} \\ &= \sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i) \end{aligned}$$
  - To predict new values, $Z(\mathbf{x}; \mathbf{w})$ is not relevant
  - Parameter estimation: $\mathbf{w}$ is set to maximize likelihood

- Can we learn $\mathbf{w}$ more directly, focusing on errors?

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The Structured Perceptron

?

- Set $\mathbf{w} = \mathbf{0}$
- For $t = 1 \ldots T$
  - For each training example $(\mathbf{x}, \mathbf{y})$
    1. Compute $\mathbf{z} = \text{argmax}_{\mathbf{z}}\, \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{z})$
    2. If $\mathbf{z} \neq \mathbf{y}$

$$\mathbf{w} \leftarrow \mathbf{w} + \mathbf{f}(\mathbf{x}, \mathbf{y}) - \mathbf{f}(\mathbf{x}, \mathbf{z})$$

- Return $\mathbf{w}$

The Structured Perceptron + Averaging

?; ?

- Set $\mathbf{w} = \mathbf{0}$, $\mathbf{w}^a = \mathbf{0}$
- For $t = 1 \ldots T$
  - For each training example $(\mathbf{x}, \mathbf{y})$
    1. Compute $\mathbf{z} = \mathrm{argmax}_{\mathbf{z}}\, \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{z})$
    2. If $\mathbf{z} \neq \mathbf{y}$
    $$\mathbf{w} \leftarrow \mathbf{w} + \mathbf{f}(\mathbf{x}, \mathbf{y}) - \mathbf{f}(\mathbf{x}, \mathbf{z})$$
    3. $\mathbf{w}^{\mathbf{a}} = \mathbf{w}^{\mathbf{a}} + \mathbf{w}$
- Return $\mathbf{w}^{\mathbf{a}}/mT$, where m is the number of training examples

Perceptron Updates: Example

$\mathbf{y}$	PER	PER	-	-	LOC
$\mathbf{z}$	PER	LOC	-	-	LOC
$\mathbf{x}$	Jack	London	went	to	Paris

- Let $\mathbf{y}$ be the correct output for $\mathbf{x}$.
- Say we predict $\mathbf{z}$ instead, under our current $\mathbf{w}$
- The update is:

$$
\begin{aligned}
\mathbf{g} &= \mathbf{f}(\mathbf{x}, \mathbf{y}) - \mathbf{f}(\mathbf{x}, \mathbf{z}) \\
&= \sum_i \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i) - \sum_i \mathbf{f}(\mathbf{x}, i, z_{i-1}, z_i) \\
&= \mathbf{f}(\mathbf{x}, 2, \text{PER}, \text{PER}) - \mathbf{f}(\mathbf{x}, 2, \text{PER}, \text{LOC}) \\
&\quad + \mathbf{f}(\mathbf{x}, 3, \text{PER}, \text{-}) - \mathbf{f}(\mathbf{x}, 3, \text{LOC}, \text{-})
\end{aligned}
$$

- Perceptron updates are typically very sparse

Properties of the Perceptron

- Online algorithm. Often much more efficient than “batch” algorithms
- If the data is separable, it will converge to parameter values with 0 errors
- Number of errors before convergence is related to a definition of *margin*. Can also relate margin to generalization properties
- In practice:
    1. Averaging improves performance a lot
    2. Typically reaches a good solution after only a few (say 5) iterations over the training set
    3. Often performs nearly as well as CRFs, or SVMs

Averaged Perceptron Convergence

Iteration	Accuracy
1	90.79
2	91.20
3	91.32
4	91.47
5	91.58
6	91.78
7	91.76
8	91.82
9	91.88
10	91.91
11	91.92
12	91.96
...	

(results on validation set for a parsing task)

Margin-based Structured Prediction

- Let $\mathbf{f}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$
- Model: $\mathrm{argmax}_{\mathbf{y} \in \mathcal{Y}^*} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})$
- Consider an example $(\mathbf{x}^{(k)}, \mathbf{y}^{(k)})$:
  $\exists \mathbf{y} \neq \mathbf{y}^{(k)} \ : \ \mathbf{w} \cdot \mathbf{f}(\mathbf{x}^{(k)}, \mathbf{y}^{(k)}) < \mathbf{w} \cdot \mathbf{f}(\mathbf{x}^{(k)}, \mathbf{y}) \Longrightarrow$ error
- Let $\mathbf{y}' = \mathrm{argmax}_{\mathbf{y} \in \mathcal{Y}^* : \mathbf{y} \neq \mathbf{y}^{(k)}} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}^{(k)}, \mathbf{y})$
  Define $\gamma_k = \mathbf{w} \cdot (\mathbf{f}(\mathbf{x}^{(k)}, \mathbf{y}^{(k)}) - \mathbf{f}(\mathbf{x}^{(k)}, \mathbf{y}'))$
- The quantity γ_k is a notion of margin on example k:
  $\gamma_k > 0 \Longleftrightarrow$ no mistakes in the example
  high $\gamma_k \Longleftrightarrow$ high confidence

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Mistake-augmented Margins

?

						$e(\mathbf{y}^{(k)}, \cdot)$
$\mathbf{x}^{(k)}$	Jack	London	went	to	Paris	
$\mathbf{y}^{(k)}$	PER	PER	-	-	LOC	0
$\mathbf{y}'$	PER	LOC	-	-	LOC	1
$\mathbf{y}''$	PER	-	-	-	-	2
$\mathbf{y}'''$	-	-	PER	PER	-	5

- Def: $e(\mathbf{y}, \mathbf{y}') = \sum_{i=1}^{n} [y_i \neq y'_i]$
  e.g., $e(\mathbf{y}^{(k)}, \mathbf{y}^{(k)}) = 0$, $e(\mathbf{y}^{(k)}, \mathbf{y}') = 1$, $e(\mathbf{y}^{(k)}, \mathbf{y}''') = 5$

- We want a $\mathbf{w}$ such that

$$\forall \mathbf{y} \neq \mathbf{y}^{(k)} \;:\; \mathbf{w} \cdot \mathbf{f}(\mathbf{x}^{(k)}, \mathbf{y}^{(k)}) > \mathbf{w} \cdot \mathbf{f}(\mathbf{x}^{(k)}, \mathbf{y}) + e(\mathbf{y}^{(k)}, \mathbf{y})$$

(the higher the error of $\mathbf{y}$, the larger the separation should be)

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$\mathbf{y}'''$	-	-	PER	PER	-	5

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(the higher the error of $\mathbf{y}$, the larger the separation should be)

Structured Hinge Loss

- Define a mistake-augmented margin

$$\gamma_{k,\mathbf{y}} = \mathbf{w} \cdot \mathbf{f}(\mathbf{x}^{(k)}, \mathbf{y}^{(k)}) - \mathbf{w} \cdot \mathbf{f}(\mathbf{x}^{(k)}, \mathbf{y}) - e(\mathbf{y}^{(k)}, \mathbf{y})$$
$$\gamma_k = \min_{\mathbf{y} \neq \mathbf{y}^{(k)}} \gamma_{k,\mathbf{y}}$$

- Define loss function on example k as:

$$L(\mathbf{w}, \mathbf{x}^{(k)}, \mathbf{y}^{(k)}) = \max_{\mathbf{y} \in \mathcal{Y}^*} \left\{ \mathbf{w} \cdot \mathbf{f}(\mathbf{x}^{(k)}, \mathbf{y}) + e(\mathbf{y}^{(k)}, \mathbf{y}) - \mathbf{w} \cdot \mathbf{f}(\mathbf{x}^{(k)}, \mathbf{y}^{(k)}) \right\}$$

- Leads to an SVM for structured prediction
- Given a training set, find:

$$\operatorname*{argmin}_{\mathbf{w} \in \mathbb{R}^D} \sum_{k=1}^{m} L(\mathbf{w}, \mathbf{x}^{(k)}, \mathbf{y}^{(k)}) + \frac{\lambda}{2} \|\mathbf{w}\|^2$$

Regularized Loss Minimization

- Given a training set $\left\{(\mathbf{x}^{(1)}, \mathbf{y}^{(1)}), \ldots, (\mathbf{x}^{(m)}, \mathbf{y}^{(m)})\right\}$.
  Find:

$$\underset{\mathbf{w} \in \mathbb{R}^D}{\operatorname{argmin}} \quad \sum_{k=1}^{m} L(\mathbf{w}, \mathbf{x}^{(k)}, \mathbf{y}^{(k)}) + \frac{\lambda}{2} \|\mathbf{w}\|^2$$

- Two common loss functions $L(\mathbf{w}, \mathbf{x}^{(k)}, \mathbf{y}^{(k)})$:
  - Log-likelihood loss (CRFs)

$$-\log P(\mathbf{y}^{(k)} \mid \mathbf{x}^{(k)}; \mathbf{w})$$

  - Hinge loss (SVMs)

$$\max_{\mathbf{y} \in \mathcal{Y}^*} \left(\mathbf{w} \cdot \mathbf{f}(\mathbf{x}^{(k)}, \mathbf{y}) + e(\mathbf{y}^{(k)}, \mathbf{y}) - \mathbf{w} \cdot \mathbf{f}(\mathbf{x}^{(k)}, \mathbf{y}^{(k)}) \right)$$

Learning Structure Predictors: summary so far

- Linear models for sequence prediction

$$\underset{\mathbf{y}\in\mathcal{Y}^*}{\operatorname{argmax}} \sum_i \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$$

- Computations factorize on label bigrams
  - Decoding: using Viterbi
  - Marginals: using forward-backward
- Parameter estimation:
  - Perceptron, Log-likelihood, SVMs
  - Extensions from classification to the structured case
  - Optimization methods:
    - Stochastic (sub)gradient methods (**??**)
    - Exponentiated Gradient (**?**)
    - SVM Struct (**?**)
    - Structured MIRA (**?**)

Beyond Linear Sequence Prediction

Factored Sequence Prediction, Beyond Bigrams

- It is easy to extend the scope of features to k-grams

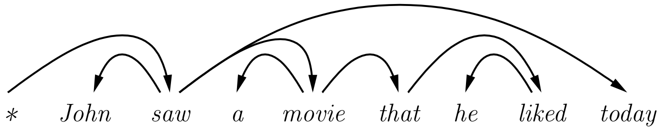
$$\mathbf{f}(\mathbf{x}, i, y_{i-k+1:i-1}, y_i)$$

- In general, think of state σ_i remembering relevant history
  - $\sigma_i = y_{i-1}$ for bigrams
  - $\sigma_i = y_{i-k+1:i-1}$ for k-grams
  - σ_i can be the state at time i of a deterministic automaton generating $\mathbf{y}$
- The structured predictor is

$$\operatorname*{argmax}_{\mathbf{y} \in \mathcal{Y}^*} \sum_i \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, \sigma_i, y_i)$$

- Viterbi and forward-backward extend naturally, in $O(nL^k)$

Dependency Structures



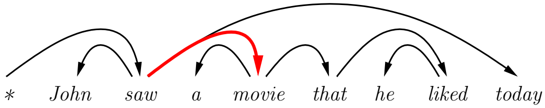
- Directed arcs represent dependencies between a head word and a modifier word.

- E.g.:

  movie *modifies* saw,
  John *modifies* saw,
  today *modifies* saw

Dependency Parsing: arc-factored models

?



- Parse trees decompose into single dependencies $\langle h, m \rangle$

$$\underset{\mathbf{y} \in \mathcal{Y}(\mathbf{x})}{\operatorname{argmax}} \sum_{\langle h, m \rangle \in y} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, h, m)$$

- Some features: $\mathbf{f}_1(\mathbf{x}, h, m) = [\ \text{"saw"} \ \rightarrow \text{"movie"}\ ]$
  $\mathbf{f}_2(\mathbf{x}, h, m) = [\ \text{distance} = +2\ ]$
- Tractable inference algorithms exist (tomorrow's lecture)

Linear Structured Prediction

- Sequence prediction (bigram factorization)

$$\operatorname*{argmax}_{\mathbf{y}\in\mathcal{Y}(\mathbf{x})} \sum_{i} \mathbf{w}\cdot\mathbf{f}(\mathbf{x}, i, \mathbf{y}_{i-1}, \mathbf{y}_i)$$

- Dependency parsing (arc-factored)

$$\operatorname*{argmax}_{\mathbf{y}\in\mathcal{Y}(\mathbf{x})} \sum_{\langle h,m\rangle\in y} \mathbf{w}\cdot\mathbf{f}(\mathbf{x}, h, m)$$

- In general, we can enumerate parts $r \in \mathbf{y}$

$$\operatorname*{argmax}_{\mathbf{y}\in\mathcal{Y}(\mathbf{x})} \sum_{r\in\mathbf{y}} \mathbf{w}\cdot\mathbf{f}(\mathbf{x}, r)$$

Factored Sequence Prediction: from Linear to Non-linear

$$\mathrm{score}(\mathbf{x}, \mathbf{y}) = \sum_i \mathrm{s}(\mathbf{x}, i, y_{i-1}, y_i)$$

- Linear:

$$\mathrm{s}(\mathbf{x}, i, y_{i-1}, y_i) = \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, \mathbf{y}_{i-1}, \mathbf{y}_i)$$

- Non-linear, using a feed-forward neural network:

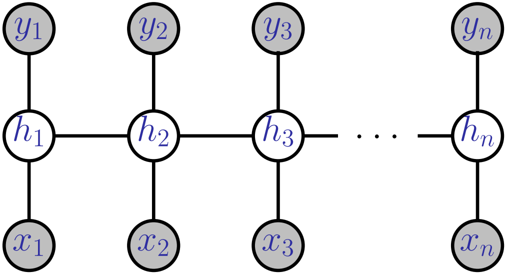
$$\mathrm{s}(\mathbf{x}, i, y_{i-1}, y_i) = \mathbf{w}_{y_{i-1}, y_i} \cdot h(\mathbf{f}(\mathbf{x}, i))$$

  where:

$$h(\mathbf{f}(\mathbf{x}, i)) = \sigma(W^2 \sigma(W^1 \sigma(W^0 \mathbf{f}(\mathbf{x}, i))))$$

- Remarks:
  - The non-linear model computes a hidden representation of the input
  - Still factored: Viterbi and Forward-Backward work
  - Parameter estimation becomes non-convex, use backpropagation

Recurrent Sequence Prediction



- Maintains a state: a hidden variable that keeps track of previous observations and predictions
- Making predictions is not tractable
  - In practice: greedy predictions or beam search
- Learning is non-convex
- Popular methods: RNN, LSTM, Spectral Models, ...

Thanks!

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