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Monetary policy and credibility: an agent-based model and a VAR for Brazil's economy

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Chapter 1

An agent-based model of inflation targeting

Abstract

This chapter replicates an agent-based model of a new Keynesian economy. A firm seeks profit maximization indirectly through a slow adjustment of quantities. Households choose how much inflation to index into their reservation wage and how much to substitute consumption in reaction to monetary policy. Households also try to increase utility by learning with the others. A central bank pursues an inflation target by setting the nominal interest rate according to a Taylor rule. Expectations about future inflation are formed by a combination between the central bank's communication of the target and households past experiences, and directly impact actual inflation rates. Scenarios with different levels of credibility are simulated. We find that credibility plays an important role in stabilizing inflation and reducing uncertainty about monetary policy effects.

Contents

Introduction

Agent-based models (hereafter ABM) have been used to simulate economic scenarios with a bottom-up approach by modelling agents' behaviour (analogous to the microfoundations of DSGE models) and studying the emergent macroeconomic variables. In the specific field of economics, ABM has also been called agent-based computational economics, or ACE[1]. A concise description is given by Tesfatsion and Judd (2006):

> ACE is the computational study of economic processes modeled as dynamic systems of interacting agents who do not necessarily possess perfect rationality and information. Whereas standard economic models tend to stress equilibria, ACE models stress economic processes, local interactions among traders and other economic agents, and out-of-equilibrium dynamics that may or may not lead to equilibria in the long run. Whereas standard economic models require a careful consideration of equilibrium properties, ACE models require detailed specifications of structural conditions, institutional arrangements, and behavioral dispositions.
>
> (Tesfatsion & Judd, 2006)

ACE receives criticism regarding its validity in interpreting and generalising results. Richiardi (2004) presents a survey on the advantages and disadvantages of ACE. He argues that agent-based models have potential to help in the economic analysis, and that there are methodological solutions for the aforementioned problems. Tesfatsion (2006) divides the application of ACE between descriptive and normative. To the former, ACE helps to explain the emergence of global

[1] In this paper we use ABM and ACE interchangeably

structures “despite the absence of top-down planning and control”. In other words, how local interactions between autonomous agents generate some specific regularities instead of others. Regarding normative aspects, ACE can give insights about how each mechanism affects social outcomes when agents behave autonomously according to different incentives.

Other studies using agent-based models

Several studies have applied ACE in their modelling. Related to industrial organization, Chang (2010) makes a case for agent-based modelling to study out-of-equilibrium dynamics. He suggests some potential lines of research that could start with an empty industry and generate entering and exiting firms endogenously. Furthermore, Chang (2010) suggests a study on the response of those endogenous variables to external shocks and on the relation between firm size, growth and survival rates.

Dosi, Pereira, and Virgillito (2015) apply agent-based modelling to simulate stylized facts of industrial dynamics. With an evolutionary model of learning processes, the study was able to replicate some phenomena found in the empirical literature such as productivity asymmetries, skewed distribution of size, a negative relation between size and growth variance, and fat-tailed distribution of growth rates.

From a macroeconomic perspective, Napoletano, Dosi, Fagiolo, and Roventini (2012) study dynamics of investment, both when they are determined by past profits and when they are based on expected future demand. They demonstrate that low inequality is essential to a steady growth with low unemployment. Furthermore, they study the impact of increasing wage-flexibility on growth and unemployment.

Dosi, Fagiolo, and Roventini (2010) were able to reproduce some macro-stylized facts using agent-based modelling. Their study relates “Schumpeterian theories of technology-driven economic growth with Keynesian theories of demand generation”. It demonstrates that innovation alone cannot generate a growth with full employment path. It is necessary, instead, to have a Keynesian demand generation by means of fiscal policy.

Still in macroeconomics, Dosi, Fagiolo, Napoletano, and Roventini (2012) created an agent-based model that includes a banking sector and a monetary authority. It was able to reproduce some features of current and past recessions and concluded that more unequal societies are more vulnerable to severe cycles, exhibits higher unemployment and has higher probability of crises.

In this chapter we describe our replication of an ABM originally devised by Salle, Yıldızoğlu, and Sénégas (2013) and present the results from simulations using the Python programming language. The model aims at analysing the impact of the central bank’s credibility when conducting an inflation targeting regime.

Salle et al. (2013) demonstrated with an agent-based model that economies with more credible central banks will have, on average, lower inflation and volatility. They accomplished that by simulating different scenarios, varying a parameter that determines how much households should rely on the central bank’s target to form their expectations about future inflation.

We replicated the model of Salle et al. (2013) almost in its exact description. Nevertheless, some details were understandably omitted from the reference paper, which required us to make additional assumptions in order to build a functional computer simulation. Moreover, we had a different question in mind, which was not explored in Salle et al. (2013). Although they showed several results from an aggregate perspective, we explored a new angle by analysing the dynamics of inflation after temporary shocks to other variables. To that end, more modifications were made to the original model.

As a result of our short departure from Salle et al. (2013), we obtained average results that were different in magnitude, although reaching the same conclusion that credibility facilitates the role of the central bank by anchoring expectations and demanding a less fierce intervention of monetary policy.

This chapter is presented as follows. Section 1 presents the structure of the model and the behavior of the agents. Section 2 explains the method used by Salle et al. (2013) to define the parameters. Section 3 presents aggregate results and compares with the reference paper. In turn, section 4 estimates impulse responses of inflation and compares scenarios with high and low central bank credibility. Finally, section 5 presents the conclusion remarks.

1 The model

Salle et al. (2013) describe three types of agents following the new Keynesian framework. The firm produces final goods and uses labour as input, the households supply labour and consume final goods and, lastly, the central bank follows an inflation targeting regime by setting the interest rate. As usual in mainstream new Keynesian models—and explained throughout this chapter—, the agents' behaviour create downwardly-rigid wages and sticky prices.

1.1 Firm

The firm is owned equally by all households, each receiving a share of the profits. It attempts to maximize profits by an indirect method: instead of choosing the optimal production, the firm acts with bounded rationality. To understand this process, we shall describe step-by-step how this agent behaves in Salle's model.

The first constraint facing the firm is the production function at time t. Given a quantity of hired labour H_t and a technology factor A (constant), the firm will produce an aggregate supply of Y_t^s goods:

$$Y_t^s = AH_t^{1-\alpha} \tag{1.1.1}$$

where parameter $\alpha \in [0, 1[$ confers decreasing marginal returns.

Considering that we have n households (described later), the quantity of hired labour will be the sum of the labour employed by all individuals at time t: $H_t = \sum_{i=1}^{n} h_{i,t}$.

Employing a labour quantity of H_t will generate a total cost equal to:

$$\Psi_t = \sum_{i=1}^{n} h_{i,t} w_{i,t} \tag{1.1.2}$$

where $w_{i,t}$ is the wage required by household i at time t (more in section 1.2).

Deriving algebraically, we obtain the marginal cost (see Salle (2012) for details) as a function of total cost and production:

$$\Psi'_t = \frac{\Psi_t}{(1-\alpha)Y_t^s} \tag{1.1.3}$$

Finally, the firm sets a fixed mark-up μ on the marginal cost. The selling price will be:

$$P_t = \frac{(1+\mu)}{(1-\alpha)} \frac{\Psi_t}{Y_t^s} \tag{1.1.4}$$

The firm will supply its production at the goods market (section 1.3). The quantity effectively sold, Y_t, will generate a profit:

$$\Pi_t = P_t Y_t - \Psi_t \tag{1.1.5}$$

Salle et al. (2013) further assumes that the goods are perishable, i.e. the excess supply at time t cannot be sold at $t+1$.

Learning process of the firm

The firm will choose its demand for labour, H_t^d, with bounded rationality. To model this behavior we depart from Salle et al. (2013)—where the firm adjusts according to profitability—to reproduce Salle, Sénégas, and Yıldızoğlu (2018), where the firm adjusts according to excess supply:

$$H_t^d = \begin{cases} H_t \times (1-\epsilon) & \text{if} \quad Y_t^s > Y_t; \\ H_t \times (1+\epsilon) & \text{if} \quad Y_t^s = Y_t. \end{cases} \tag{1.1.6}$$

Therefore, the firm will increase labour demand if all of its previous production was sold at the goods market, and decrease it otherwise. This slow process of adjustment towards a market-clearing situation gives rise to *price-stickiness.*

1.2 Households

The model represents many households in the form of heterogeneous agents, with idiosyncratic preferences, that learn to increase their individual utility. There are n households, indexed by $i \in [1, n]$.

Households have two roles in the model. First, they provide labour to the firm and receive wages in exchange. Second, they buy the final good produced by the firm.

Regarding the labour supply, each household supplies—inelastically—1 unit of labour. However, they choose their reservation wage w, the minimum amount they are willing to receive, independently from each other. Households will index part of the expected inflation by increasing their reservation wage when $\pi^e_{i,t+1} > 0$. However, they will not adjust negatively when inflation is absent (or deflation is expected), therefore creating *downwardly-rigid wages*:

$$w_{i,t} = \begin{cases} w_{i,t-1} \times \left(1 + \gamma^w_{i,t} . \pi^e_{i,t+1}\right) & \text{if} \quad \pi^e_{i,t+1} > 0; \\ w_{i,t-1} & \text{otherwise.} \end{cases} \tag{1.2.1}$$

where $\gamma^w_{i,t}$ is named *indexation strategy* and is the share of inflation that the household i will add to its reservation wage. $\gamma^w_{i,t}$ is defined heterogeneously by a learning process explained later on.

In addition to its wage, the household will receive a share $\frac{1}{n}$ of the firm's profits in the previous period—Π_{t-1}. Furthermore, any income that is not spent will become savings and generate interest in the next period. As a result, household i will have an income equal to:

$$y_{i,t} = w_{i,t}h_{i,t} + \frac{\Pi_{t-1}}{n} + b_{i,t-1}\left(1 + i_{t-1}\right) \tag{1.2.2}$$

where $b_{i,t-1}$ is the income received by household i that was not spent at period $t-1$, and i_{t-1} is the interest rate. Considering $c_{i,t}$ the goods consumed, the savings balance will be updated as:

$$b_{i,t} = y_{i,t} - c_{i,t}.P_t \tag{1.2.3}$$

To make decisions regarding consumption, the household will look at its permanent income $\tilde{y}_{i,t}$ defined—in real terms—as a linear combination between current real income and the time-discounted sum of past real incomes:

$$\tilde{y}_{i,t} = (1-\rho).\frac{y_{i,t}}{P_t} + \rho.\tilde{y}_{i,t-1} = (1-\rho)\sum_{l=0}^{t}\rho^{t-l}\frac{y_{i,l}}{P_l} \tag{1.2.4}$$

Having defined income, we proceed to analyse consumption. Considering $k_{i,t}$ the consumption share, household i will, at time t, demand c^d_i:

$$c^d_i = k_{i,t}.\tilde{y}_{i,t} \tag{1.2.5}$$

Consumption share, in turn, adjusts to the real interest rate $(i_t - \pi^e_{i,t+1})$:

$$k_{i,t} = k_{i,t-1} - \gamma^k_{i,t}\left(i_t - \pi^e_{i,t+1}\right) \tag{1.2.6}$$

where $\gamma^k_{i,t}$ is named *substitution strategy* and represents the strength of the transmission mechanism of monetary policy[2]. A higher *substitution strategy* implies a stronger reaction to a change in interest rate. The learning process through which households choose their *substitution strategy* is explained later.

Finally, consumption of goods will give some utility to the household—the logarithm of the quantity consumed:

$$u(c_{i,t}) = ln(c_{i,t}) \tag{1.2.7}$$

Learning process of the households

The households choose, individually, a pair of strategies at every period t. The *indexation strategy* $\gamma^w_{i,t}$ will define how strongly the household i will demand an increase in wage in response

[2]Salle et al. (2013) consider the gap between real interest rate and the natural rate. Therefore, they define $k_{i,t} = k_{i,t-1} - \gamma^k_{i,t}\left(i_t - \pi^e_{i,t+1} - r^n_t\right)$. However, without loss of generality, Salle et al. (2013) set r^n_t equal to 0. For simplicity, we omitted this term altogether.

to expected inflation; the *substitution strategy* $\gamma^k_{i,t}$, on the other hand, will define how strongly the household will decrease consumption after an increase in interest rate.

The determinant variable for the household learning is the smoothed utility—a linear combination between current utility and the time-discounted sum of past utilities:

$$\tilde{u}_{i,t} = (1-\rho)u_{i,t} + \rho\tilde{u}_{i,t-1} = (1-\rho)\sum_{l=0}^{t}\rho^{t-l}u_{i,l} \tag{1.2.8}$$

According to two parameters P_{imit} (probability of imitating) and P_{mut} (probability of mutation), each household will imitate the pair of strategies of another agent or will mutate its strategies to random new ones, respectively. On the other hand, with a probability $1 - P_{imit} - P_{mut}$ the household will keep its current strategies.

Imitation

If a given household decides to imitate another agent, it will do so by means of a genetic algorithm, which is simply a method to build a probability distribution according to some fitness criterion. In Salle et al. (2013), the fitness of each household is measured by the ratio of the exponential of its utility over the sum of all other households' fitness. The idea becomes clearer with formalization. When any household, at period t, decides to imitate, another household j will have a probability of being selected given by:

$$\frac{exp(\tilde{u}_{j,t-1})}{\sum_{l=1}^{n} exp(\tilde{u}_{l,t-1})} \tag{1.2.9}$$

After a household j is selected to be imitated by household i, $\gamma^w_{i,t}$ assumes the value of $\gamma^w_{j,t-1}$ and $\gamma^k_{i,t}$ assumes the value of $\gamma^k_{j,t-1}$. Therefore, strategies that yielded higher utilities in the past will tend to be more diffused.

Mutation

On the alternative case where a household i decides to mutate, its pair of strategies are randomly drawn from a normal distribution with mean equal to the average in the previous period and constant standard variance:

$$\gamma^w_{i,t} \sim \mathcal{N}\left(\frac{\sum_{l=1}^{n}\gamma^w_{l,t-1}}{n}, \sigma^2_{mutW}\right) \tag{1.2.10}$$

$$\gamma^k_{i,t} \sim \mathcal{N}\left(\frac{\sum_{l=1}^{n}\gamma^k_{l,t-1}}{n}, \sigma^2_{mutK}\right) \tag{1.2.11}$$

1.3 Labour market

Considering all households as perfectly substitutes for labour supply, Salle et al. (2013) assumes that the firm will satisfy its labour demand by hiring workers in ascending order of reservation wages. Since the model imposes a limit of n households, the effectively quantity of labour hired by the firm will be:

$$H_t = min\left(H_t^d, H_t^s\right) \tag{1.3.1}$$

where $H_t^s = \sum_{i=1}^{n} h_{i,t}^s = n$, since each household supplies 1 unit of labour (see 1.2).

If $H_t^d < H_t^s$, some households will be unemployment at period t. We compute unemployment as $u_t = \frac{n-H_t}{n}$. Aggregate wage level, in turn, is computed as the weighted average of individual wages:

$$W_t = \frac{\sum\limits_{i=1}^{n} h_{i,t} w_{i,t}}{H_t} \tag{1.3.2}$$

Finally, the real wage rate is simply $\omega_t = \frac{W_t}{P_t}$.

1.4 Goods market

We have already defined aggregate supply Y_t^s in section 1.1. On the consumption side, aggregate demand is trivially given by the sum of the consumption chosen by all households:

$$Y_t^d = \sum_{i=1}^{n} c_{i,t} \tag{1.4.1}$$

A possible situation is the occurrence of excess demand, $Y_t^s < Y_t^d$. In that case, households are prioritized in descending order of quantity demanded. The good is sold at the price set by the firm. Finally, inflation is computed as $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$

1.5 Central bank and inflation targeting

To set monetary policy, we model a central bank (CB) under an inflation targeting regime. Salle et al. (2013) assumes a committed CB that reacts to inflation and unemployment with fixed parameters after a simulation starts—i.e. the CB does not change its preferences during a given simulation.

Accordingly, the CB sets the nominal interest rate i_t following a Taylor rule (first proposed in Taylor (1993)). Additionally, and differentiating from Salle et al. (2013), we add a random noise to monetary policy. This random noise is purely exogenous and allows the posterior identification of orthogonal impulse responses to a shock in interest rate as defined by Sims (1992). The interest rate becomes:

$$1 + i_t = \left(1 + \pi^T\right)\left(1 + r_t^n\right)\left(\frac{1 + \pi_{t-1}}{1 + \pi^T}\right)^{\phi_\pi}\left(\frac{1 + u^*}{1 + u_{t-1}}\right)^{\phi_u} + \epsilon_t \tag{1.5.1}$$

where the inflation target π^T, the natural real interest rate r_t^n, and the natural level of unemployment u^* are parameters set equal to 0. On the other hand, the preferences of the CB, aversion to inflation (ϕ_π) and aversion to unemployment (ϕ_u), are parameters that, although being constant for a given simulation, receive different values according to the experiment under analysis (see Section 2).

Lagged inflation π_{t-1} and unemployment rate u_{t-1} have been defined in sections 1.4 and 1.3, respectively. Differently in Salle et al. (2013), the CB considers current values of inflation π_t and unemployment u_t instead of lagged ones. We argue in section 1.7 that, from a practical point of view due to the simulation algorithm, the CB cannot observe current values.

Finally, ϵ_t follows a normal distribution with mean zero and standard deviation equal to 8% of the total standard deviation of past values of i_t. This is approximately the same proportion that we found in the residuals when estimating the VAR for Brazil's economy in Chapter 2.

1.6 Inflation expectations

The expectations about inflation is the main feature that varies across different simulation scenarios. There are 5 alternative scenarios with varying levels of credibility, noise and coordination of agents about the inflation target perception. In this section, we mention the corresponding scenario number of each feature. The details of all scenarios are presented later in section 2.

Agents will form their expectation about future inflation $\pi^e_{i,t+1}$ as a linear combination between past values ($\tilde{\pi}_t$) and their perception of the target communicated by the central bank (π^p_i):

$$\pi^e_{i,t+1} = \chi.\pi^p_i + (1-\chi).\tilde{\pi}_t \tag{1.6.1}$$

where the credibility parameter χ is common to all households. Therefore, if $\chi = 1$, households believe completely in their target perception (scenarios 1, 2 and 3). Conversely, if $\chi = 0$, households ignore the target and form their expectations based solely on past values (scenario 5). In the intermediate case, households balance the two options (scenario 4)

Additionally, households are not always correct in their assessment of the inflation target π^T. They perception embeds a noise:

$$\pi^p_i = \pi^T + \xi_i, \text{ where } \xi_i \sim \mathcal{N}\left(0, \sigma^2_\xi\right) \tag{1.6.2}$$

which represents imprecisions in the communications of the central bank if $\sigma^2_\xi > 0$ (scenarios 2 and 3). Conversely, in scenarios where $\sigma^2_\xi = 0$ (scenarios 1, 4 and 5), there is no perception noise (perfect precision) and $\pi^p_i = \pi^T$.

Another possibility is that the households coordinate among themselves the perception of the inflation target. In that case, a single draw of ξ_i is obtained for all agents (scenarios 1, 2, 4 and 5).

1.7 Additional assumptions

Departing from Salle et al. (2013), we made additional assumptions that were not explicit in the reference paper but were necessary from a practical point of view to enable the simulation to run.

Window of observation

The first assumption is about the window of observation of agents when looking at past values. Specifically, households observe past utilities to compute $\tilde{u}_{i,t}$, past inflation rates to compute $\tilde{\pi}_t$, and past incomes to compute $\tilde{y}_{i,t}$. Salle et al. (2013) consider the time-discounted sum of all past periods, which we found to be computationally expensive and unnecessary, since the time discount (exponentiating by $\rho < 1$) renders all values in the distant past insignificant. Instead, we programmed agents to observe past values only to the point where, after the time discount, they are larger than 0.00005.

Initial values

We set the initial state of the economy as follows:

- $\pi_0 = \pi^T$: initial inflation is equal to the target.
- $H_0^d = n$: initial labour demand is equal to the number of households.
- To draw the initial pair of strategies, each household assumes a population mean of 1.
- $w_{i,0} = 10 \; \forall i \in [1, n]$: initial reservation wage of all households is equal to 10.
- $b_{i,0} = 0 \; \forall i \in [1, n]$: initial savings balance of all households is equal to 0.

Central bank

The reference paper states that the central bank sets current interest rate according to current inflation and unemployment. The current interest rate is consulted by households to make decisions of consumption in current period. Therefore, we need an interest rate before we can even calculate current inflation and current unemployment. For that reason, we assume that the central bank looks at the previous values of inflation and unemployment.

Expected inflation

According to the reference paper, for workers to form their expected inflation, they consult the inflation trend at time t (using all past and current inflation rates, analogous to the formulas for permanent income and smoothed utility). However, before we can calculate current inflation, we need the result of the whole process from the wage bargain until production. Since expected inflation is an input parameter in the reservation wage function, we can only use past values of inflation to form the trend.

Labour supply

Each household can supply fractional amounts of labour, i.e. they are not restricted to choose between working full-time and not working at all.

The case of zero consumption

When a household decides not to consume any amount of goods $c_{i,t} = 0$, we specify a negative utility, $u(c_{i,t}) = -2$. This is necessary since the logarithmic function is undefined in 0.

Zero lower bound to nominal interest rate
We assume that the central bank has a practical lower bound to monetary policy. We set a minimum of -0.5%.

2 Parameters

Salle et al. (2013) specifies 5 simulation scenarios with different levels of credibility and noise in inflation target perception. For all scenarios, we use a common set of 17 different parametrizations (which we call experiments), spanning the entire domain of all parameters. These parametrizations are chosen according to a design of experiments (see Salle (2012) for a clear explanation). The objective of such method is to explore all relevant cases and see how each scenario, averaging across the 17 experiments, generates different results.

We show the differences in the scenarios in Table 1.1. The list with all 17 experiments can be found in Salle et al. (2013).

Scenario	Credibility	Precision	Coordination
1	Full ($\chi = 1$)	Perfect ($\pi_i^p = \pi^T$)	Yes
2	Full ($\chi = 1$)	Noisy perception of target	Yes
3	Full ($\chi = 1$)	Noisy perception of target	No
4	Partial ($\chi \in (0, 1)$)	Perfect ($\pi_i^p = \pi^T$)	Yes
5	None ($\chi = 0$)	Perfect ($\pi_i^p = \pi^T$)	Yes

Table 1.1: Scenarios

3 Results

Similarly to Salle et al. (2013), we ran all the 17 experiments for each scenario, with 20 independent replications of each experiment. Therefore, for each scenario we run 340 simulations with $T = 800$ time periods and $n = 500$ households. For each simulation, we collect data at every 50 periods, discarding the first 100 to mitigate the effects of the initial state, therefore collecting 15 observations per simulation. In total, we have 5,100 observations for each scenario.

We report the aggregate results in Table 1.2. We observe that scenarios 4 (imperfect credibility) and 5 (zero credibility) have higher inflation gap than the others (0.0358 and 1.7243, respectively), with scenario 5 presenting the highest. Furthermore, the standard deviation of inflation gap is also much higher in these two scenarios (1.7768 and 118.1149, respectively), evidencing the occasional occurrence of explosive hyper-inflation in the absence of credibility. Have in mind that these values are shown in decimal, therefore scenario 5 presented a standard deviation of inflation gap of 11811.49 p.p.. Our results for scenarios 1, 2 and 3 are close to those in Salle et al. (2013). Differently, for scenarios 4 and 5 we obtained much more extreme results.

As for the unemployment rate, Table 1.2 reports higher values for scenarios 2 (noisy perception of inflation target) and 5—the highest—, with scenario 4 in third place (the same happens with the standard deviation of unemployment rate). In Salle et al. (2013), the difference is that scenario 4 has the same unemployment rate as scenario 2. Moreover, our results had higher unemployment rate overall.

These differences in results are understandable. Firstly, our model has a different learning process for the firm (see 1.1). Secondly, we were forced to make several additional assumptions to have a functioning simulation algorithm (see 1.7).

Table 1.2: Aggregate results of all scenarios.

	Scenario				
	1	2	3	4	5
$\pi_t - \pi^T$	-0.0003 (0.0103)	0.0040 (0.0354)	-0.0005 (0.0102)	0.0358 (1.7768)	1.7243 (118.1149)
u_t	0.1498 (0.2107)	0.2851 (0.3507)	0.1497 (0.2106)	0.1761 (0.2261)	0.4757 (0.3436)
$mean(\gamma^w_{i,t})$	0.9915 (0.1549)	0.9883 (0.2170)	0.9894 (0.1475)	0.9573 (0.1438)	0.8644 (0.1758)
$mean(\gamma^k_{i,t})$	1.0128 (0.1309)	0.9887 (0.1711)	1.0130 (0.1276)	1.0064 (0.1425)	0.9933 (0.1585)
$var(\gamma^w_{i,t})$	0.0486 (0.0110)	0.0475 (0.0108)	0.0485 (0.0110)	0.0486 (0.0110)	0.0489 (0.0110)
$var(\gamma^k_{i,t})$	0.0499 (0.0115)	0.0489 (0.0114)	0.0500 (0.0116)	0.0501 (0.0116)	0.0498 (0.0117)

4 Impulse responses

Additionally to the analysis performed by Salle et al. (2013), we study some dynamic effects by means of a structural vector autoregressive model (VAR). Estimating our empirical model with three variables, i_t, π_t and output gap ($\frac{Y^s_t - Y^*}{Y^*}$, where Y^* is the potential output if unemployment reaches its natural level), and choosing the autoregressive lags according to Akaike information criterion (AIC, first presented in Akaike (1974)), we obtain a VAR estimation for each experiment. We test successfully the VAR stability by confirming that all roots of the coefficient matrix lie within the unit circle. Finally, we use the Cholesky decomposition to obtain orthogonal impulse responses.

4.1 Result 1

Credibility reduces the persistence of a shock to inflation

We simulate once all 17 experiments, first with high credibility ($\chi = 0.9$), then with low credibility ($\chi = 0.1$). In sequence, for both levels of credibility, we compute the average impulse response of inflation after an exogenous shock to itself. Additionally, we also compute the middle 95% interval of impulse responses across the 17 experiments in each period after the shock. We report the results in figure 1.1. In simulations with high credibility, inflation responds with a lower magnitude, cycles symmetrically around zero, and dissipates quickly. Differently, under low credibility, inflation reacts with a higher magnitude, cycles predominantly above zero, and persists for a long time.

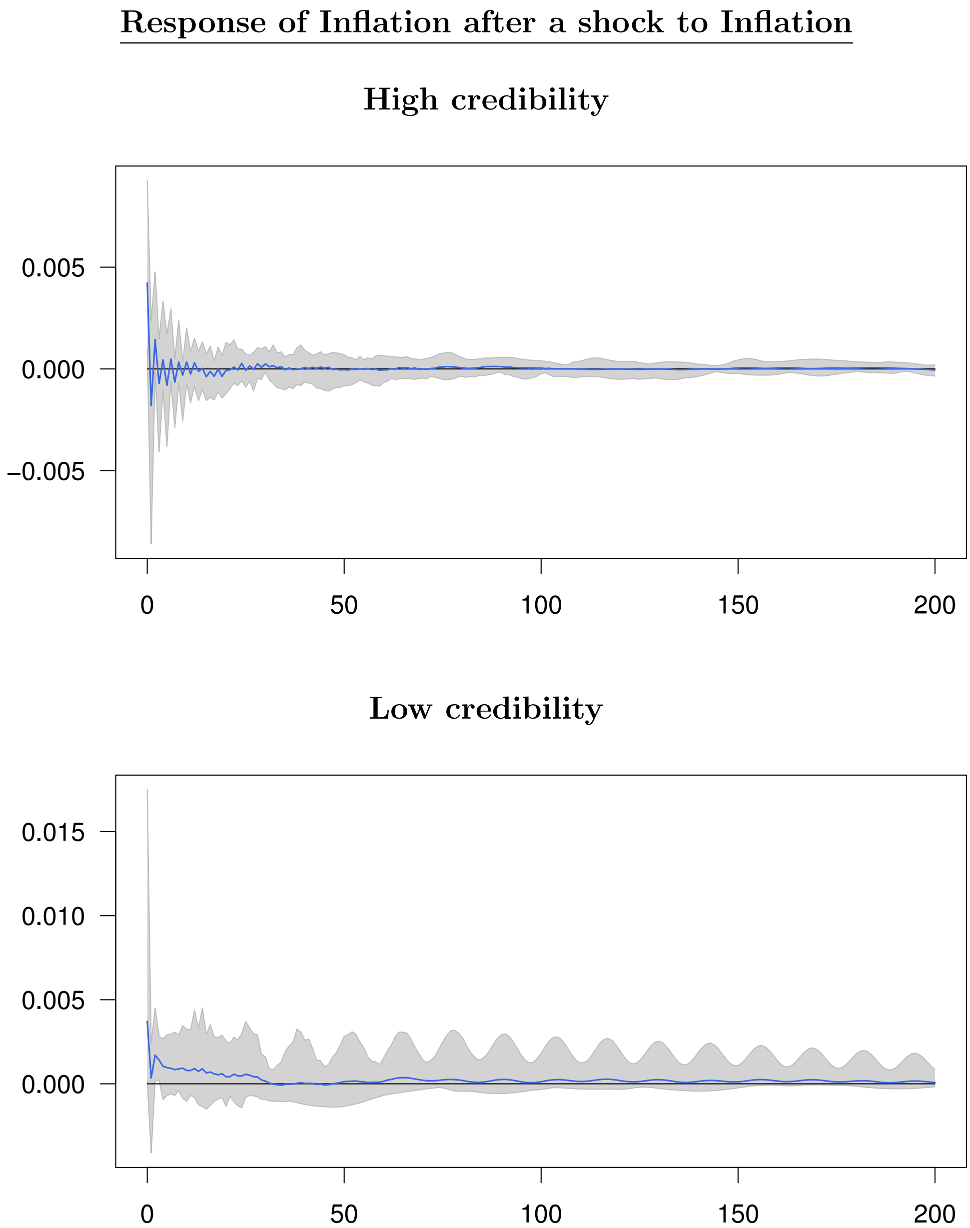


Figure 1.1: Average results of 17 replications with varying parameters. The same parameters are used first with high credibility ($\chi = 0.9$), then with low credibility ($\chi = 0.1$). Blue line: average impulse response. Grey area: middle 95% interval.

4.2 Result 2

Credibility reduces uncertainty

We simulate a specific set of parameters to study the response of inflation to an exogenous shock to monetary policy. We set $\phi_\pi > 1$ to follow the Taylor principle (Woodford, 2003). Table 1.3 reports the relevant parameters. The basic parameters common to all experiments were reported in section 2 and maintain the same values. Credibility is set first with $\chi = 0.9$ (high credibility), then with $\chi = 0.1$ (low credibility). In both cases we assume

Table 1.3: Parameters to simulate Result 2.

P_{mut}	P_{imit}	ρ	σ_{mutK}	σ_{mutW}	ϕ_π	ϕ_u	σ_ξ
0.05	0.1	0.45	0.3	0.3	1.3	0.8	0

We execute five independent simulations with the same parameters setting $\chi = 0.9$, and do the same procedure with $\chi = 0.1$. As with Result 1, we estimate a VAR and verify the standard diagnostic tests. Figure 1.2 reports the response of inflation after a shock to nominal interest rate. We observe that under high credibility all replications generated the same impulse responses (no grey area). Therefore, with fixed parameters, the agent-based model reacts every time in the exact same way. Differently, under low credibility, we notice that the dynamic response of inflation presents uncertainty. The grey area evidences that different simulations with the same parameters generated different impulse responses.

Additionally, under high credibility, inflation has a strong negative reaction shortly after the shock and then cycles symmetrically around zero. Under low credibility, the initial reaction might be either stronger or weaker, and presents a cycle predominantly above zero after some time. It should be noted that, contrary to what the figure may suggest, cycles still occur under low credibility, but they disappear visually when we average the impulse responses of five simulations.

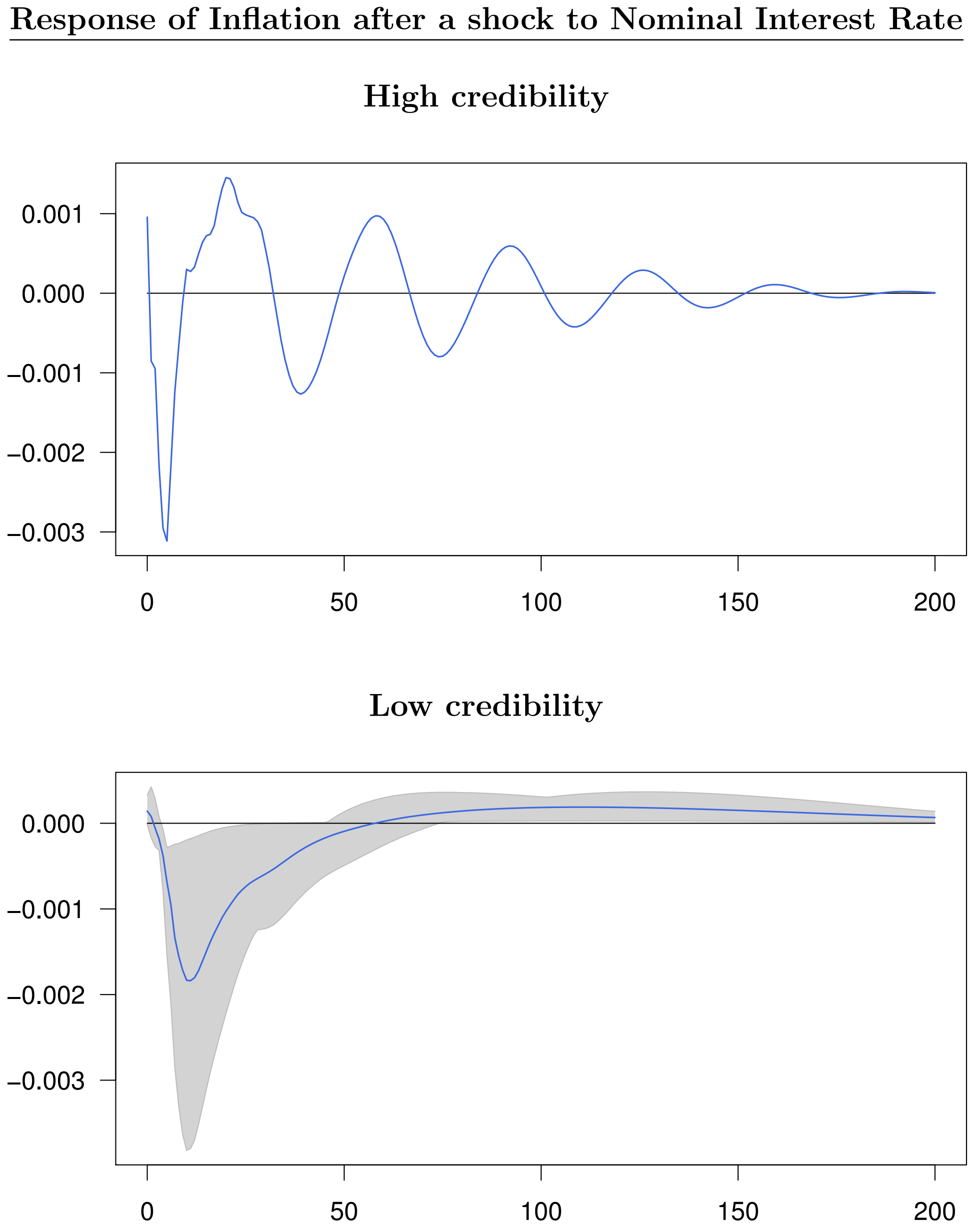


Figure 1.2: Average results of 5 replications with constant parameters. The same parameters are used first with high credibility ($\chi = 0.9$), then with low credibility ($\chi = 0.1$). Blue line: average impulse response. Grey area: middle 95% interval.

5 Conclusion

In this chapter we replicated an agent-based model described in Salle et al. (2013). We programmed a simulation using the Python language with some modifications and additional assumptions, and compared our average results with those presented in the reference paper.

As in Salle et al. (2013), our simulation generated lower inflation gap and volatility in scenarios with high central bank credibility. As for unemployment, both a noisy perception of the inflation target or a low credibility of the central bank cause higher rates. Nevertheless, our results were more extreme than Salle's. Specifically, we had episodes of explosive hyper-inflation when simulating scenarios with low credibility.

Finally, in this chapter we estimated impulse responses to compare the dynamic reaction of inflation after a shock to interest rate and after shock to inflation itself. We compared the results between scenarios with high and low credibility. We found that credibility reduces the persistence of inflation after an exogenous inflationary pressure. Furthermore, credibility removes uncertainty about the reaction of inflation after a shock to interest rate.

Chapter 2

Monetary policy shocks in Brazil: a VAR estimation

Abstract

This chapter estimates several VAR models using Brazil's economic data. We alternate among four measures of economic activity: real GDP growth, output gap, industrial production, and IBC-br. We run diverse diagnostic tests and compare model performance according to objective criteria. Our best impulse response estimates suggest a negative reaction of inflation after a shock to monetary policy between 8 and 23 months ahead. We also estimate how monetary policy reacts to inflation and economic activity, finding an immediate response of the central bank. Finally, we use a measure of credibility to estimate its effects on Brazil's inflation targeting regime.

Contents

Introduction

Brazil is a difficult country to study monetary policy shocks for two main reasons. First, it possess a history of monetary instabilities, with new currencies frequently replacing each other, in a total of 9 since its independence. Therefore, any study that goes past 1994, the year when the Brazilian Real was created, will face a challenge to isolate the effects of the actions of the central bank from those arising from several political crises, episodes of hyperinflation, and divergent macroeconomic policies. Second, even after 1994, some statistical series had their methodology altered (e.g. industrial production has three different versions), and others are very new (e.g. the economic activity index, IBC-br, starts in 2003[1]).

The *Banco Central do Brasil* (Brazil's central bank) was created in 1964, assuming the nation's monetary policy, which was hitherto distributed among three entities, the *Superintendência da Moeda e do Crédito* (SUMOC), the *Banco do Brasil*, and *Tesouro Nacional*. The central bank went through important institutional changes up to 1988, with the establishment of Brazil's last constitution. Brazil formally adopted an inflation targeting regime in 1999, with the establishment of a target value of inflation and tolerance margins motivated by a currency crisis that hit Brazilian Real earlier that year. Together with the inflation targeting regime, Brazil formally adopted a free-floating exchange rate and a mandatory budget surplus to be defined yearly by the Congress.

In this chapter we seek to study the causal relations between monetary policy and other macroeconomic variables in Brazil. To that objective, we use several specifications of vector autoregressive models (VAR), using different measures of economic activity, and estimate the dynamic responses of inflation after a shock to monetary policy. We use several measures to compare the quality of the models.

Other authors have already studied the effects of monetary policy shocks in Brazil, obtaining, sometimes, contradictory results. Arquete and Jayme-Jr (2003) estimated a VAR between 1994 and 2002 and found that monetary policy is not effective against inflation, and sometimes causes opposite effects. With similar results, Minella (2003) does not find a strong effect on inflation but suggests an increase in efficacy after the Brazilian Real was created. He also observes that monetary policy does not react quickly to inflation and output gap.

On the other hand, some authors have found a significant reaction of inflation to shocks in interest rate using Brazil's data. Céspedes, Lima, and Maka (2008) used a SVAR model with two sub-samples. The first one, from 1996 to 1998 generated ambiguous responses due to its small size. The second one, from 1993 to 2004—this time under the inflation targeting regime—, generated negative responses on price level after 6 months. Another study, Cardeal, Roberto, and Resumo (2009), confirms a significant negative effect.

When comparing our results with those of previous studies, it is important to differentiate between impulse responses of inflation (presented in this paper) and those of the price level (as in Céspedes et al. (2008) and Cardeal et al. (2009)). For the two to be equivalent, inflation must be analysed in cumulative values, which is not our present focus.

This chapter is organized as follows. Section 1 enumerates the data sources. Section 2 explains the relevant variables, with diagnostics and necessary transformations. Section 3 details the empirical strategy and the VAR specifications. Section 4 contains the results of diagnostic tests. Section 5 presents several criteria to compare the performance of different models. Section 6 estimates impulse responses of inflation and economic activity after a shock to monetary policy. Section 7 presents a VAR estimation with credibility shocks. Finally, section 8 presents concluding remarks.

[1]The economic activity index is calculated by the Central Bank of Brazil under the name IBC-br.

1 Data sources

We obtain the relevant time series from three Brazilian institutions and the IMF:

- *Banco Central do Brasil* (BCB): the Brazil's central bank.
- *Instituto Brasileiro de Geografia e Estatística* (IBGE): the official agency for producing data about Brazil's economy and society.
- *Instituto de Pesquisa Econômica Aplicada* (IPEA): the main Brazilian federal institution for economic research and technical support for public policies.
- International Monetary Fund (IMF)

From the BCB, we used the application *Sistema Gerenciador de Séries Temporais* (SGS), which consolidates and publishes several economic and financial data, and is the main data source for reports written by the central bank. From the IBGE, we used the application *Sistema de Recuperação Automática* (SIDRA), which concentrates the aggregate data collected by the agency. From the IPEA, we downloaded the time series directly in their website. Lastly, from the IMF, we used the well-known International Financial Statistics (IFS).

2 Variables

Adjusting for seasonality

Before delving into the description of variables, we shall discuss a common problem when analysing causality in time series data. Some variables present natural cycles, which we call seasonality, varying predictably with a given periodicity, usually within a year. Those variables, such as inflation, output and unemployment, have to be adjusted in order to avoid spurious regressions and omitted variable biases. We used the *X-13ARIMA-SEATS Seasonal Adjustment Program*[2] (hereafter X13) offered by the United States Census Bureau to detect and remove seasonal effects. Specifically, we used the *SEATS* function, which makes the adjustment based on a seasonal ARIMA model [3] and shows the significance of dummies for each quarter. Accordingly, we used the adjusted series when seasonality was present.

2.1 Inflation rate

The inflation rate is calculated by IBGE and made available in monthly rates (% change of price index[4]).

When needed in monthly frequency (π_t^m), we annualize the original series π_t^{ori} as follows:

$$\pi_t^m = (1 + \pi_t^{ori})^{12} - 1 \tag{2.1.1}$$

[2]Available in R with the package *seasonal*

[3]The order of the seasonal ARIMA is chosen according to the Bayesian Information Criterion (BIC)

[4]The most widely used price index in Brazil is the Índice de Preços ao Consumidor Amplo (IPCA).

Analogously, when needed in quarterly frequency (π_t^q), we first aggregate by quarter (composing the monthly index) and annualize. As a result, our quarterly annualized inflation rate becomes:

$$\pi_t^q = \left(\prod_{m=1}^{3} (\pi_m^{ori} + 1) \right)^4 - 1 \tag{2.1.2}$$

where $m \in [1, 3]$ are the months in each quarter.

Visual inspection

From visual inspection (Figure 2.1) there is no clear trend in inflation, and except for two peaks in 1995 and 2002, it appears be covariance-stationary. However, there seems to be deterministic cycles, which suggests the presence of seasonality.

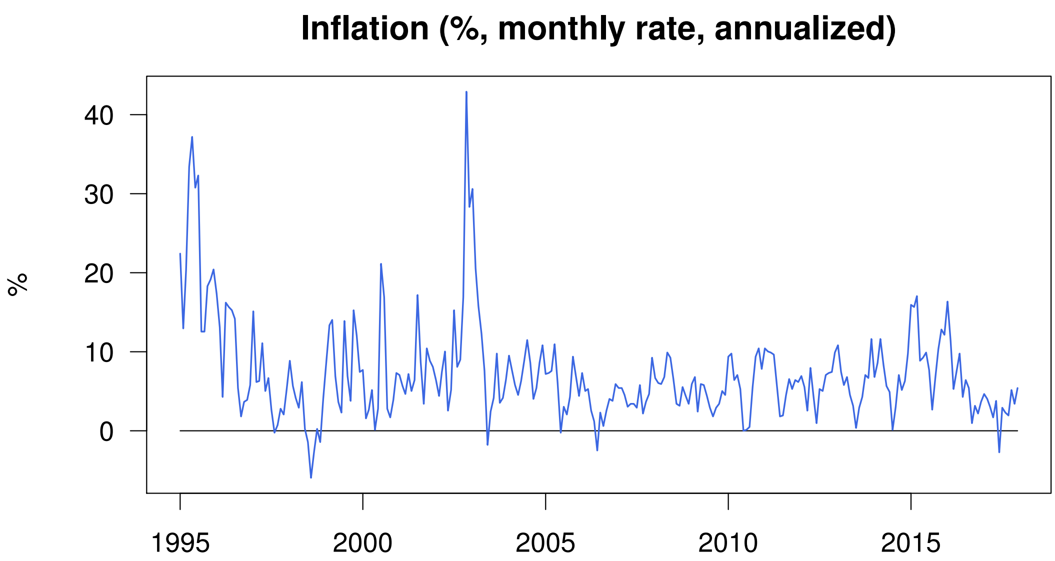


Figure 2.1: Inflation (% monthly, annualized). Data source: IBGE.

Stationarity

To confirm stationarity, we use the Augmented Dickey Fuller Test (ADF, first presented in Dickey and Fuller (1979)), and report the results in Table 2.1, which rejects the hypothesis of a unit root.

Seasonality

Inflation exhibits a significant seasonal component with most of the peaks occurring in the first or fourth quarters. The result of the test is displayed in Table 2.2. X13 detected a significant seasonal component with monthly dummies (MA-Seasonal-12). The original and the adjusted series are shown together in Figure 2.2.

Null-hypothesis:	presence of unit root
Test statistic:	-6.2334
Lag order:	6
P-value:	< 0.01
Conclusion:	Unit root rejected

Table 2.1: Test: Augmented Dickey-Fuller Test for Inflation

Table 2.2: Test: X-13ARIMA-SEATS Seasonal Adjustment

Dummy	Coef.	Std. Error
MA-Seasonal-12	0.83137***	0.03342
Obs.: 276		
SEATS ARIMA: (1 0 0)(0 1 1)		
AICc: -979.5, BIC: -961.8		

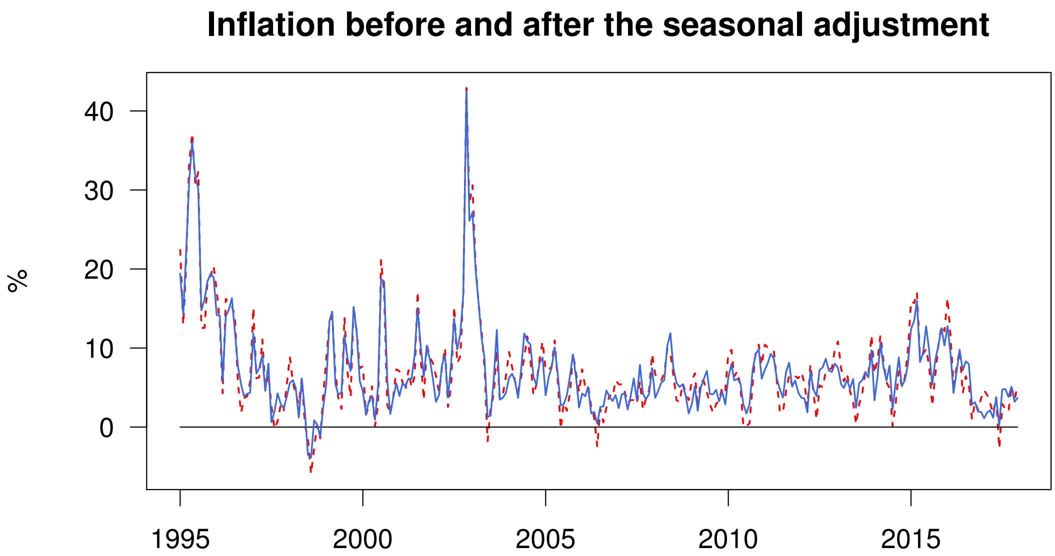


Figure 2.2: Blue: inflation after the seasonal adjustment. Red: original inflation series

Conclusion: In accordance with our tests, we use henceforth inflation rate to denote its seasonally-adjusted version.

2.2 Nominal interest rate

We downloaded the monetary policy rate—i.e. the target interest rate set by the central bank—from the BCB. The series contains the annual rate with daily frequency; we took the values at each last day of the month or the quarter according to the needed frequency.

Visual inspection

Nominal interest rate appears to have a downward trend as reported in Figure 2.3.

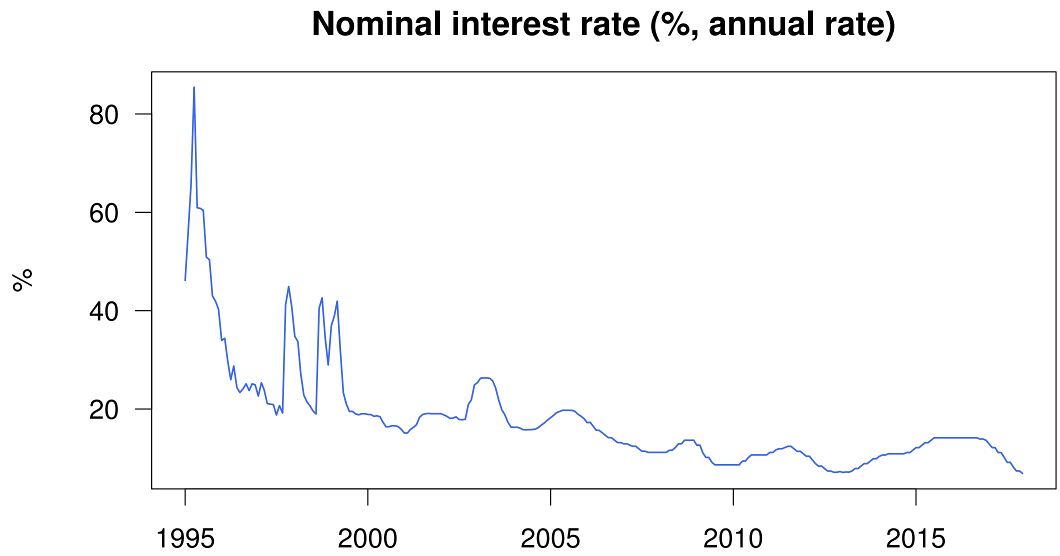


Figure 2.3: Nominal interest rate (%, daily rate). Data source: BCB.

Stationarity

Although a visual inspection suggests the presence of a unit root, the ADF test rejects such hypothesis (Table 2.3).

Null-hypothesis:	presence of unit root
Test statistic:	-6.0185
Lag order:	6
P-value:	< 0.01
Conclusion:	Unit root rejected

Table 2.3: Test: Augmented Dickey-Fuller Test for Nominal interest rate

Seasonality: The X13 did not detect a seasonal component in nominal interest rate.

Conclusion: We use the original series of nominal interest rate.

2.3 Money supply (M1)

Money supply is obtained from the BCB in levels, monthly frequency.

Visual inspection
A brief visual inspection evidences the presence of a unit root when M1 is considered in levels (Figure 2.4). Therefore, we transform the variable to log differences, which appears to become stationary (Figure 2.5).

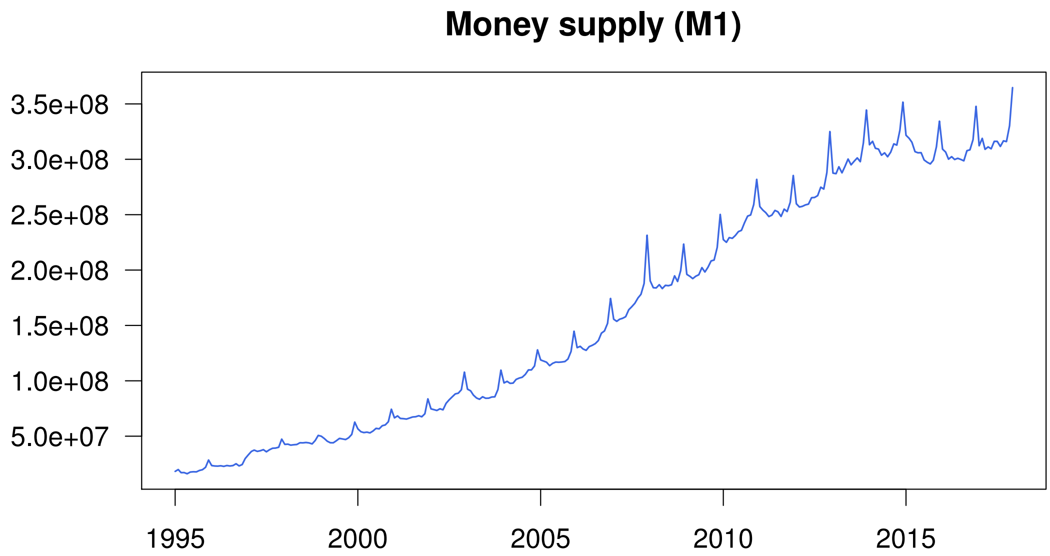


Figure 2.4: Money supply (M1) (levels). Data source: BCB.

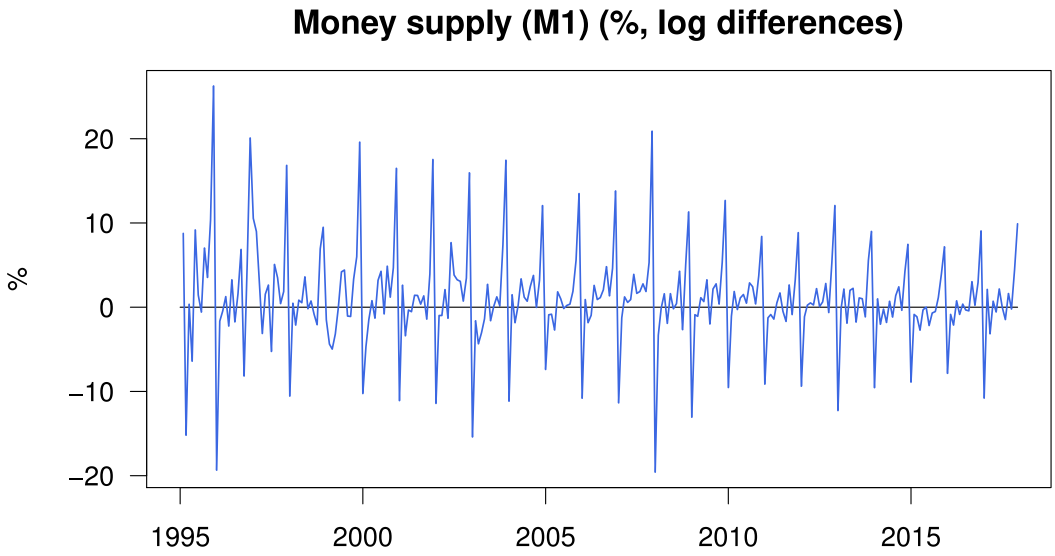


Figure 2.5: Money supply (M1) (log differences). Data source: BCB.

Stationarity

This series in its original form exhibits a clear positive trend, which is confirmed by the ADF test (Table 2.4). On the contrary, as suggested by the visual inspection, the transformation to log differences becomes, indeed, stationary (Table 2.5).

Null-hypothesis:	presence of unit root
Test statistic:	-2.4586
Lag order:	6
P-value:	0.3828
Conclusion:	Unit root cannot be rejected

Table 2.4: Test: Augmented Dickey-Fuller Test for M1 in levels

Null-hypothesis:	presence of unit root
Test statistic:	-8.4801
Lag order:	6
P-value:	< 0.01
Conclusion:	Unit root rejected

Table 2.5: Test: Augmented Dickey-Fuller Test for M1 in log differences

Seasonality

Money supply presented a significant seasonal component. Table 2.6 reports the results obtained with the X13. The resulting series after the seasonal adjustment is shown in Figure 2.6.

Table 2.6: Test: Augmented Dickey-Fuller Test

Dummy	Coef.	Std. Error
MA-Seasonal-12	0.686474***	0.043663
Obs.: 275 SEATS ARIMA: (0 0 0)(0 1 1) AICc: -1284, BIC: -1236		

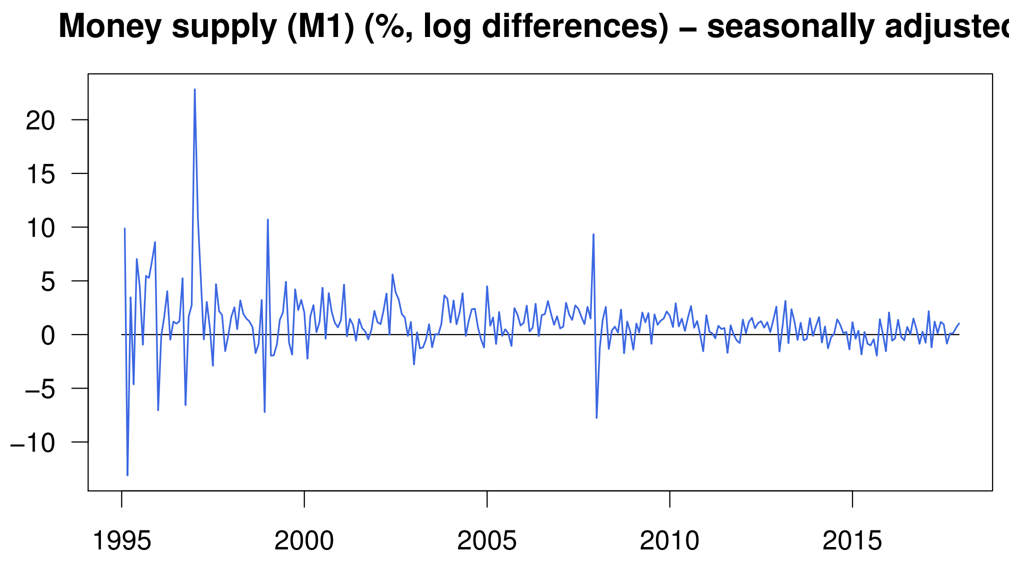


Figure 2.6: Money supply (M1), log differences, after the seasonal adjustment.

Conclusion: In accordance with our tests, we use hereafter money supply to denote its log differences, seasonally adjusted.

2.4 Real exchange rate

Series calculated by the Central Bank of Brazil under the name *Real effective exchange rate index*. Spanning from 1994 to 2017, this series represents the variation of Brazilian Real (rise = depreciation) against an aggregate measure of the foreign currencies of 15 trade partners weighted by their participation in Brazil's commerce—adjusted by inflation.

Visual inspection

Plotting the real exchange rate (Figure 2.7) we observe several peaks: 1999, 2001, 2002, 2008, 2015. This behavior raises suspicion about non-stationarity. Accordingly, we transform the series to first differences and plot the result in Figure 2.8.

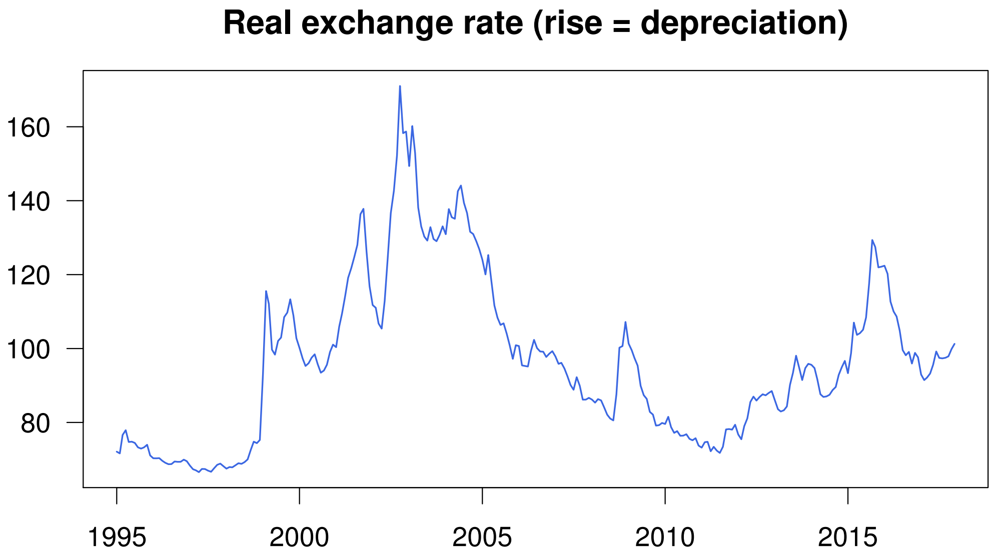


Figure 2.7: Real exchange rates: index measured as the value of Brazilian real against the currencies of Brazil's 15 largest trade partners, adjusted by inflation. Variable in levels. Data source: BCB.

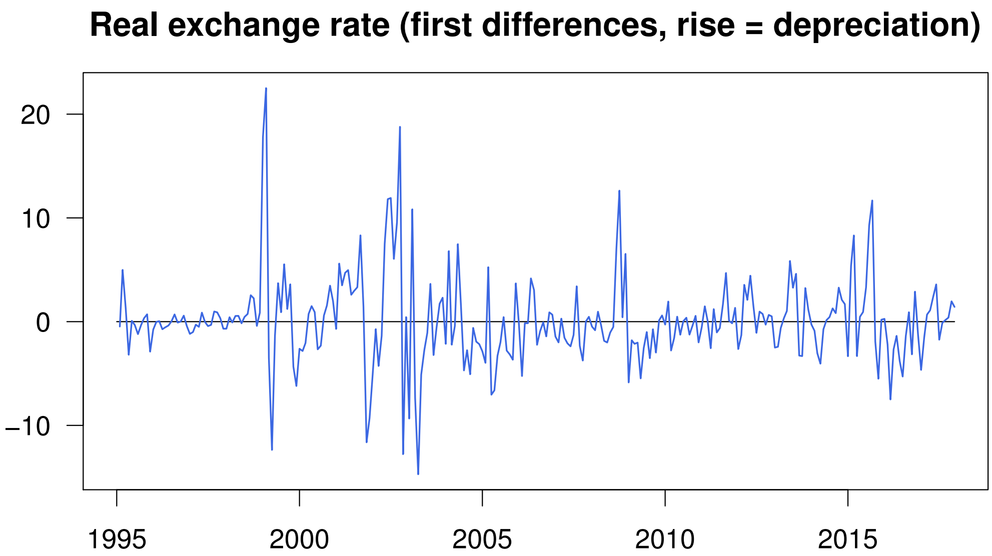


Figure 2.8: Real exchange rate after transformation to first differences. Data source: BCB.

Stationarity

This series in its original form suggests the existence of a unit root, which is confirmed by the

ADF test (Table 2.7). On the contrary, as suggested by the visual inspection, the transformation to first differences becomes, indeed, stationary (Table 2.8).

Null-hypothesis:	presence of unit root
Test statistic:	-2.1247
Lag order:	6
P-value:	0.5235
Conclusion:	Unit root cannot be rejected

Table 2.7: Test: Augmented Dickey-Fuller Test for real exchange rate in levels

Null-hypothesis:	presence of unit root
Test statistic:	-6.6966
Lag order:	6
P-value:	< 0.01
Conclusion:	Unit root rejected

Table 2.8: Test: Augmented Dickey-Fuller Test for real exchange rate in first differences

Seasonality: The X13 did not detect a seasonal component in real exchange rate.

Conclusion: In accordance with our tests, we will use real exchange rate in first differences.

2.5 Commodity prices index

This series represents the evolution of the price of commodities. As pointed out by Sims (1992), the central bank may react to this index in order to anticipate inflation.

Visual inspection

A brief visual inspection evidences the presence of a unit root when commodity prices index is considered in levels (Figure 2.4). Therefore, we transform the variable to log differences, which appears to become stationary (Figure 2.5).

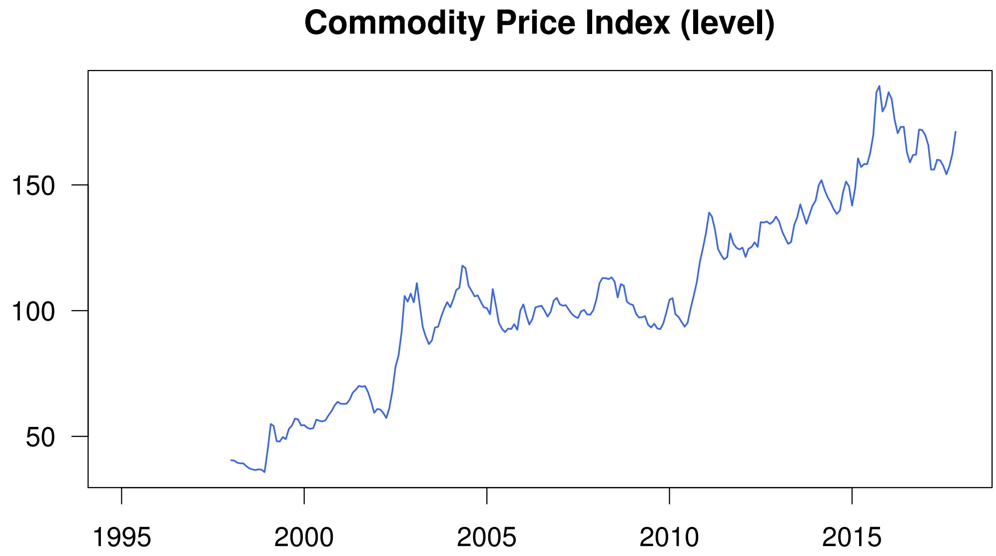


Figure 2.9: Commodity prices index in levels. Data source: BCB.

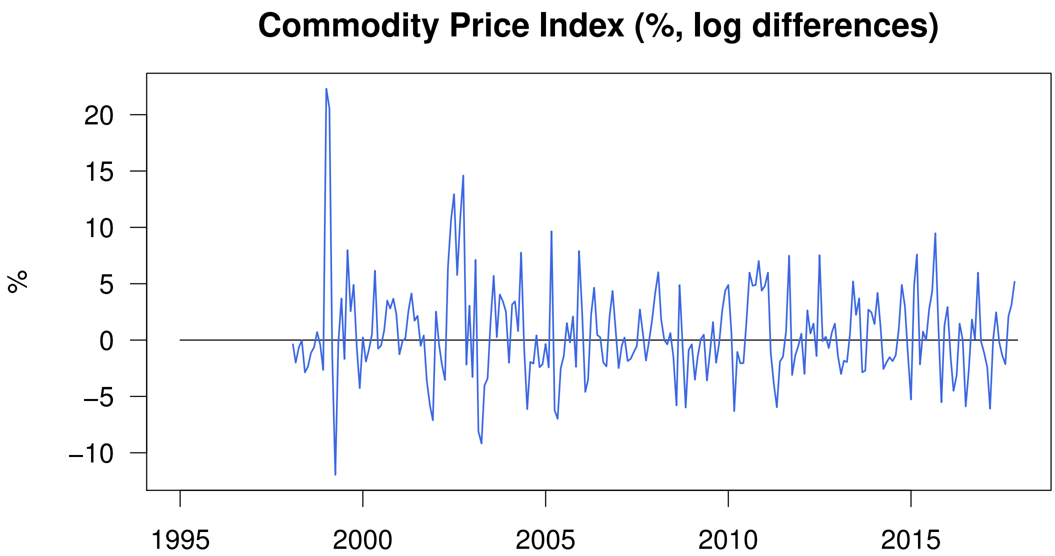


Figure 2.10: Commodity prices index in log differences. Data source: BCB.

Stationarity

This series in its original form exhibits a clear positive trend caused by inflation over time. This is confirmed by the ADF test (Table 2.9) who fails to reject a unit root at $p < 5\%$. On the contrary, as suggested by the visual inspection, the transformation to log differences becomes, indeed, stationary (Table 2.10).

Null-hypothesis:	presence of unit root
Test statistic:	-3.1708
Lag order:	6
P-value:	0.09347
Conclusion:	Unit root cannot be rejected

Table 2.9: Test: Augmented Dickey-Fuller Test for Commodity Prices Index in levels.

Null-hypothesis:	presence of unit root
Test statistic:	-6.4527
Lag order:	6
P-value:	< 0.01
Conclusion:	Unit root rejected

Table 2.10: Test: Augmented Dickey-Fuller Test for Commodity Prices Index in log differences.

Seasonality: The X13 did not detect a seasonal component in commodity prices index.

Conclusion: We use the stationary transformation of commodity prices index into log differences.

2.6 Economic activity

For robustness, we analysed four different measures of economic activity. In quarterly data we used real GDP (variation, %) and output gap; in monthly data we used industrial production (variation, %) and IBC-br (variation, %), an index calculated by the Central Bank of Brazil explained in section 2.6.4.

2.6.1 Real GDP growth

Brazil's real GDP growth was obtained from the IMF. We downloaded the series between 1995 and 2017 in quarterly frequency, which granted 90 observations.

Visual inspection

From visual inspection (Figure 2.1) there is no clear trend in real GDP growth, and except for extreme volatility in 1995-1996, it appears to be covariance-stationary. Nevertheless, there seems to exist deterministic cycles, which suggests the presence of seasonality.

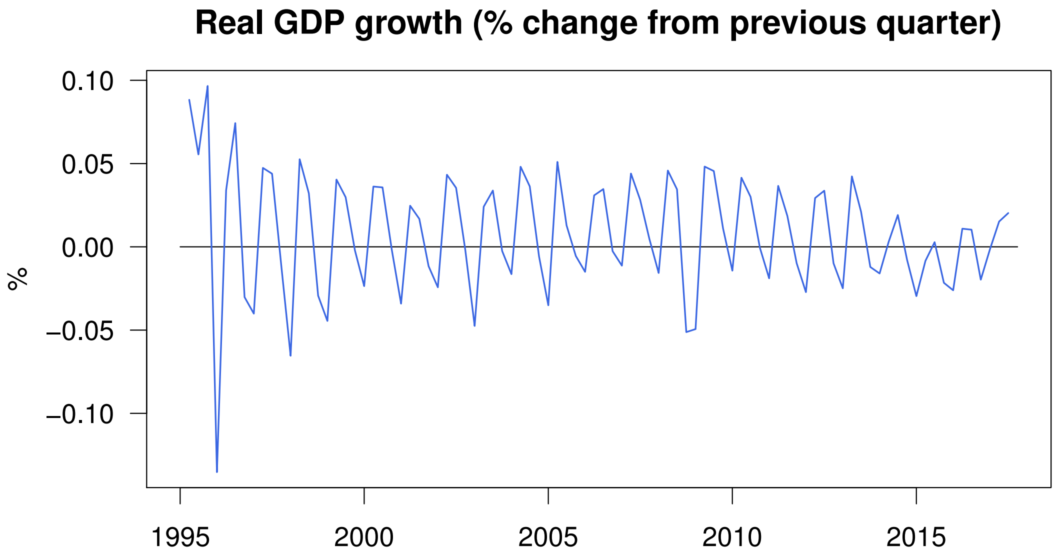


Figure 2.11: Real GDP growth. % change in comparison to previous quarter. Data source: IMF.

Stationarity

As we could infer from a visual inspection, the ADF test rejects the presence of a unit root (Table 2.11).

Null-hypothesis:	presence of unit root
Test statistic:	-3.5503
Lag order:	4
P-value:	0.042
Conclusion:	Unit root rejected at 5%

Table 2.11: Test: Augmented Dickey-Fuller Test for Real GDP Growth

Seasonality

Real GDP growth exhibits a significant seasonal component with most of the peaks occurring in the first quarters every year. The result of the test is displayed in Table 2.12. X13 detected a significant seasonal component with quarterly dummies (MA-Seasonal-04). The adjusted series is plotted in Figure 2.12.

Table 2.12: Test: X-13ARIMA-SEATS Seasonal Adjustment Program

Dummy	Coef.	Std. Error
MA-Seasonal-04	0.55716***	0.08491
Obs.: 90 SEATS ARIMA: (0 0 0)(0 1 1) AICc: -479.2, BIC: -467.6		

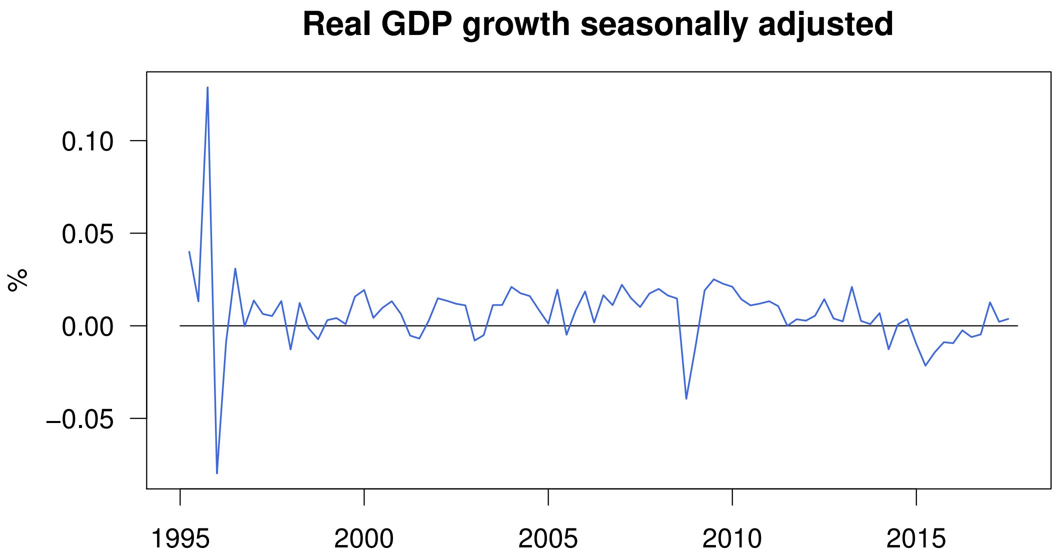


Figure 2.12: Real GDP growth after the seasonal adjustment.

Conclusion: In accordance with our tests, we use real GDP growth seasonally adjusted.

2.6.2 Output gap

The output gap, calculated by IPEA, represents the % gap between actual output and its potential value. The potential output is obtained from the estimation of the non-accelerating inflation rate of unemployment (NAIRU) and the aggregate production function. The data is already seasonally adjusted from IPEA, available between 1995 and 2017 in monthly frequency. The methodology is described in Souza-Júnior (2005).

Visual inspection

The plot in Figure 2.13 shows two strong recessions, 2009 and 2016, besides some other periods when the actual output was below the potential.

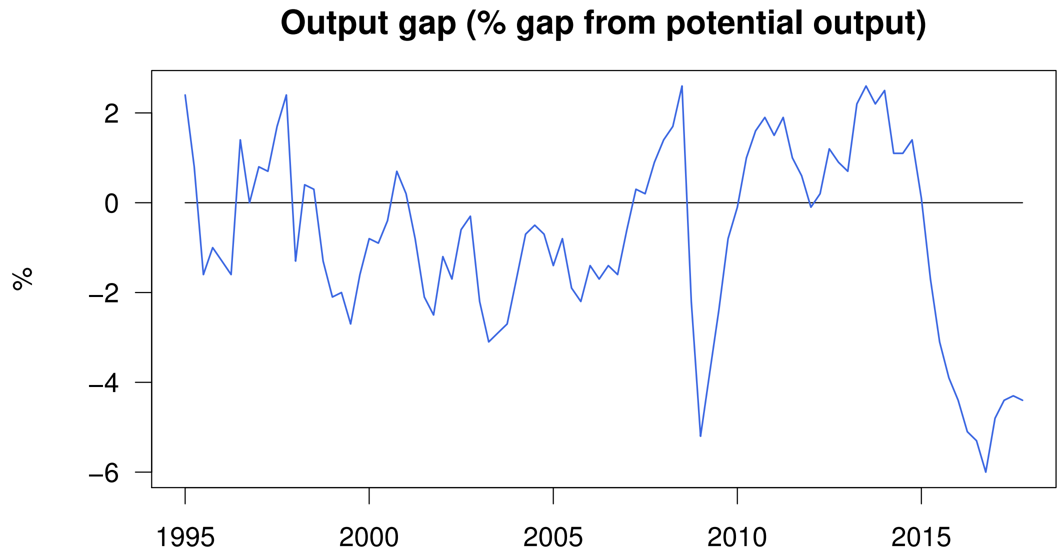


Figure 2.13: Output gap. % gap between actual and potential output. Data source: IPEA.

Stationarity
An ADF test reveals that the series is not stationary (Table 2.13). However, lest we incur a loss in interpretability, we decided not to transform the series. Instead, we ensure that the VAR is stable when using output gap.

Null-hypothesis:	presence of unit root
Test statistic:	-2.2438
Lag order:	4
P-value:	0.4757
Conclusion:	Unit root cannot be rejected

Table 2.13: Test: Augmented Dickey-Fuller Test for output gap.

Seasonality: The series is already seasonally adjusted in the data source.

Conclusion: We use the original series of output gap.

2.6.3 Industrial production

Industrial production is calculated by IBGE with the series named PIM-PF. We have downloaded it from the IBGE's website in monthly % change, seasonally adjusted. Importantly, though, this series has been modified two times since its conception in 1985. The first modification, in 2004, incorporated data retrospectively up to 1991, with the older series, from 1991 to 2004, being deprecated. The second modification—current version—came to light in 2014, and incorporates data retrospectively up to 2002. As a result, we have three distinct series for different time periods (see Instituto Brasileiro de Geografia e Estatística - IBGE (2015) for detailed methodology): version 1 (from 1985 to 2004), version 2 (from 1991 to 2014), and version 3 (from 2002 onwards). Our estimations used the last version.

Visual inspection

An inspection of Figure 2.14 suggests that the series is covariance stationary, except for a sharp economic downturn in 2008.

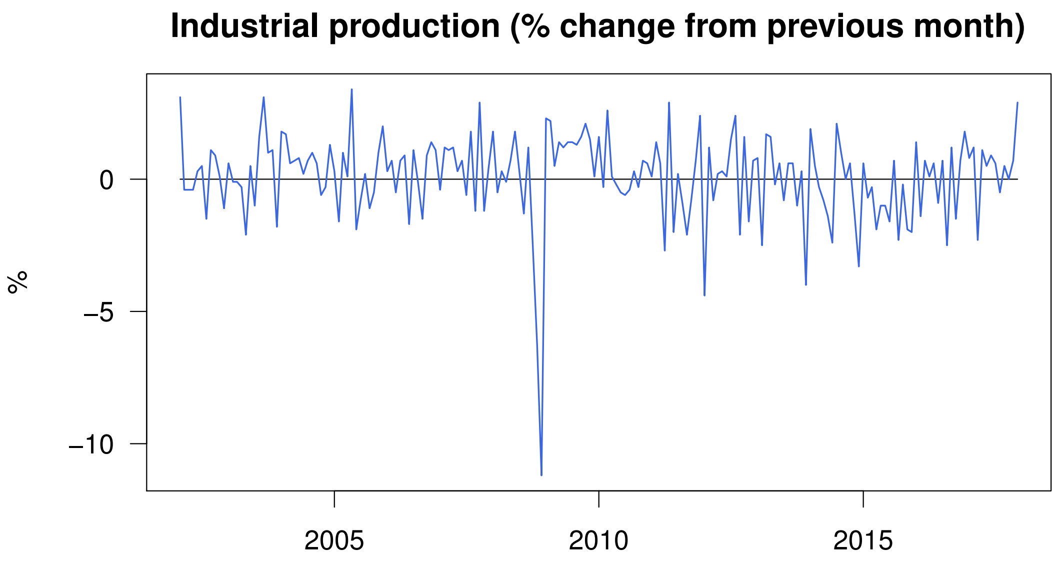


Figure 2.14: Industrial production, % change in comparison to previous month. Already seasonally adjusted. Data source: IBGE.

Stationarity

We run the ADF test to formally verify stationarity. Table 2.14 shows the rejection of a unit root hypothesis.

Null-hypothesis:	presence of unit root
Test statistic:	-5.4341
Lag order:	5
P-value:	< 0.01
Conclusion:	Unit root rejected

Table 2.14: Test: Augmented Dickey-Fuller Test for Industrial production

Seasonality: The series is already seasonally adjusted in the data source.

Conclusion: We use the original series of industrial production.

2.6.4 IBC-br

The IBC-br is a monthly index calculated by the Central Bank of Brazil. It is an aggregate of 21 variables of economic activity intended to be used as an anticipation of the real GDP, which is computed in quarterly frequency. We downloaded the series from the Central Bank's website in monthly level, seasonally adjusted, spanning from 2003 to 2017.

Visual inspection

The plot in Figure 2.15 suggests the presence of a unit root when IBC-br is considered in levels. Therefore, we transform the variable to log differences, which appears to become stationary (Figure 2.16).

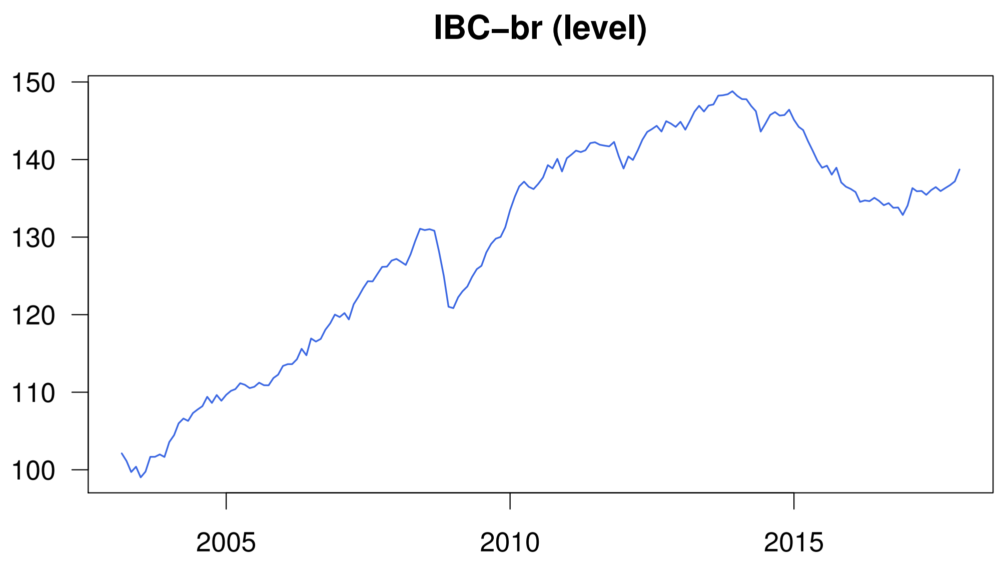


Figure 2.15: IBC-br, levels. Data source: BCB.

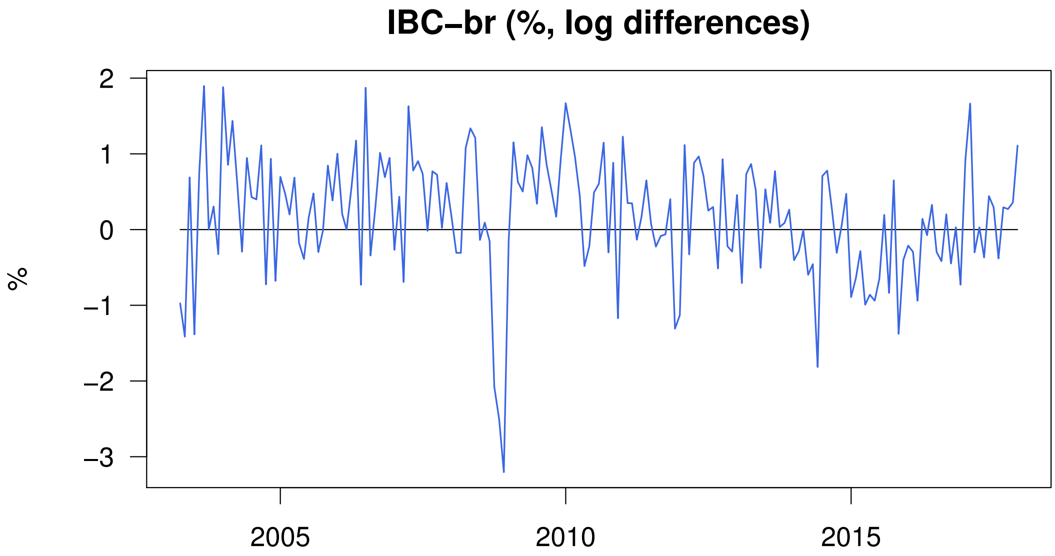


Figure 2.16: IBC-br, log differences. Data source: BCB.

Stationarity

This series in its original form exhibits a clear positive trend caused by the long-term GDP growth, which is confirmed by the ADF test (Table 2.15). Differently, as suggested by Figure 2.16, the transformation to log differences becomes stationary (Table 2.16), which is evidenced by a rejection of ADF test.

Null-hypothesis:	presence of unit root
Test statistic:	-1.3765
Lag order:	5
P-value:	0.8367
Conclusion:	Unit root cannot be rejected

Table 2.15: Test: Augmented Dickey-Fuller Test for IBC-br in levels

Null-hypothesis:	presence of unit root
Test statistic:	-5.0604
Lag order:	5
P-value:	< 0.01
Conclusion:	Unit root rejected

Table 2.16: Test: Augmented Dickey-Fuller Test for IBC-br in log differences

Seasonality: The series is already seasonally adjusted in the data source.

Conclusion: We use the transformation in log differences of IBC-br.

3 Empirical model

Starting from the seminal work of Sims (1992), we estimate a VAR (using least squares in each individual equation) and obtain a structural form using Cholesky decomposition to restrict contemporaneous interactions among the variables. The recursive ordering is necessary to obtain dynamic responses from an exogenous shock—specifically, to assess the effects of an unexpected monetary policy shock on the other variables of the system. We estimate two alternative orderings. In the first one, interest rate precedes other variables, which assumes a lagged reaction of the Central Bank to economic conditions, but a contemporaneous reaction of the other variables to monetary policy. In the second one, interest rate comes last, which assumes a contemporaneous reaction of the Central Bank, but a lagged response of the other variables.

The two orderings depend on strong assumptions. The central bank is expected to react in advance to the actual increase in inflation by matters of forecasting tools. Therefore, for the first ordering to be valid, the VAR must contain variables that represent the set of information available to the central bank. As a matter of fact, we cannot simplify the decision making of the monetary authority to a small VAR. Nevertheless, our sample size would present a challenge if we were to increase complexity. As for the second ordering, assuming that economic conditions do not reflect agents anticipation of the central bank's decision is likely to be invalid as well (Woodford, 2005). To mitigate these limitations, we estimated the VARs with both alternative orderings and found similar impulse responses. Therefore, we report in this paper only one ordering.

As a consequence of limitations in data availability, we were forced to use a different time period in each VAR specification. When including real GDP growth or output gap, the estimation spans from 1995 to 2017 in quarterly frequency, with 90 and 92 observations, respectively. When including industrial production, the estimation goes from 2002 to 2017 in monthly frequency, with 191 observations. Finally, when including the IBC-br (explained in section 2.6.4), the estimation encompasses data from 2003 to 2017 in monthly frequency, with 177 observations. Importantly, the samples with industrial production and IBC-br comprises only the period after the inflation targeting regime was established in Brazil.

Finally, we used the Akaike Information Criterion (AIC) to choose the optimal number of lags in each VAR specification. We chose AIC based on advice by Ivanov and Kilian (2005). That study simulated several time series with a known lag order, and estimated some VARs according to different information criteria. They found that choosing lags according to AIC produced the most accurate impulse responses for monthly data (lowest RMSE in comparison to the VAR with the true lag order). Although Ivanov and Kilian (2005) favored the use of Schwarz Information Criterion (SIC) for small samples in quarterly data, that method assigned only 1 lag for our corresponding specifications, which significantly increased the statistic of the serial correlation tests. Therefore, we used AIC for all specifications: 4 lags in monthly and 2 lags in quarterly data.

3.1 Specifications

3.1.1 Estimating a structural VAR

Our goal is to find a structural VAR, which will allows us to estimate impulse responses. To illustrate, let us assume that our VAR has order 1, with the form:

$$AY_t = B_0 + B_1 Y_{t-1} + u_t \tag{3.1.1}$$

$$Y_t = A^{-1}B_0 + A^{-1}B_1 Y_{t-1} + A^{-1}u_t \tag{3.1.2}$$

We first estimate the reduced-form *VAR 1* with the following specification:

$$Y_t = G_0 + G_1 Y_{t-1} + \epsilon_t \tag{3.1.3}$$

where, making the correspondence with the structural VAR, $G_0 = A^{-1}B_0$, $G_1 = A^{-1}B_1$, and $\epsilon_t = A^{-1}u_t$. Vectors G_1 contains the coefficients of the lagged values, and ϵ_t contains the regression residuals.

Since we obtain G_0, G_1, and ϵ_t from the regression, it is sufficient to specify A^{-1} to compute the structural VAR of equation 3.1.1. From the reduced-form estimation, we use the Cholesky decomposition on the covariance matrix of the residuals (Σ_ϵ) to find a lower triangular matrix, which we will designate A^{-1}.

3.1.2 Quarterly data

Using the two measures of economic activity in quarterly data, GDP growth and output gap, we estimate a basic VAR with the three main variables, which we call *VAR 1*. Denoting Y_t^1 the vector of endogenous variables, *VAR 1* will have:

$$Y_t^1 = \begin{bmatrix} i_t \\ y_t \\ \pi_t \end{bmatrix} \tag{3.1.4}$$

where i_t is the nominal interest rate, y_t is either GDP growth or output gap, and π_t is the inflation rate. Using the Akaike criterion, we found an optimal order of 2 lags.

Additionally, we estimate the VAR 1 with output gap after June/1999 (we name it VAR 1 after IT), when the inflation targeting regime was formally established in Brazil. To mitigate the limitations from a small sample, we restricted this VAR by significance. This is an iterated method in which the VAR is re-estimated with the least significant regressor restricted to 0. The method is repeated until all regressors are significant at 10%.

3.1.3 Monthly data

Similarly, using the alternative two measures of economic activity in monthly data, industrial production and IBC-br, we estimated the same basic *VAR 1* (3.1.4) with the three main variables. However, in this case the Akaike criterion suggested an optimal order of 4 lags.

We then proceed to three additional specifications, increasing progressively the complexity of the model:

VAR 2: VAR 1 + money supply M_t

$$Y_t^2 = \begin{bmatrix} i_t \\ M_t \\ y_t \\ \pi_t \end{bmatrix} \tag{3.1.5}$$

Subsequently, following Sims (1992), we added, in turn, two extra variables that might affect monetary policy decisions: real effective exchange rate and commodities price index.

VAR 3: VAR 2 + real exchange rate r_t

$$Y_t^3 = \begin{bmatrix} i_t \\ r_t \\ M_t \\ y_t \\ \pi_t \end{bmatrix} \tag{3.1.6}$$

VAR 4: VAR 3 + commodity prices index C_t

$$Y_t^4 = \begin{bmatrix} i_t \\ r_t \\ C_t \\ M_t \\ y_t \\ \pi_t \end{bmatrix} \tag{3.1.7}$$

In all four specifications, y_t was either industrial production or IBC-br. Additionally, we estimated the VAR 3 with IBC-br restricted by significance.

In total, we estimated 12 VARs: 2 with quarterly data, 8 with monthly data, an additional VAR 1 after 1999 with output gap, and an additional VAR 3, restricted by significance, with industrial production.

4 Diagnostics

4.1 Method for testing stability of a VAR

The most important diagnostic on the validity of a VAR is the test of stability. To estimate impulse responses, we need to transform our VAR into a moving average (MA) of infinite order. We follow the procedure first introduced by Wold (1954). Consider a system of order p in the form:

$$AY_t = B_1Y_{t-1} + B_2Y_{t-2} + ... + B_pY_{t-p} + u_t \tag{4.1.1}$$

Using the backward shift operator L, equation 4.1.1 can be represented as:

$$(A - B_1L - B_2L^2 - ... - B_pL^p)Y_t = u_t \tag{4.1.2}$$

Assuming that the matrix in parenthesis is invertible, our VAR is stable if all roots of $(A - B_1L - B_2L^2 - ... - B_pL^p)$ lie outside of the unit circle:

$$Y_t = (A - B_1L - B_2L^2 - ... - B_pL^p)^{-1}u_t \tag{4.1.3}$$

The package *vars* in *R* provides the same test from a different perspective. As explained in Pfaff (2008), the stability of a VAR(p) can be analysed by transforming it into a VAR(1):

$$\xi_t = \mathbf{A}\xi_{t-1} + \mathbf{v}_t \tag{4.1.4}$$

where:

$$\xi_t = \begin{bmatrix} \mathbf{y}_t \\ \vdots \\ \mathbf{y}_{t-p+1} \end{bmatrix}, \mathbf{A} = \begin{bmatrix} A_1 & A_2 & \cdots & A_{p-1} & A_p \\ I & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & I \end{bmatrix}, \mathbf{v}_t = \begin{bmatrix} \mathbf{u}_t \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} \tag{4.1.5}$$

If all eigenvalues of **A** are *within* the unit circle, the VAR is stable. This is the method we used to validate our VAR estimations.

4.2 Test results

Stability

All specifications are stable according to the procedure explained in section 4.1.

Serial correlation in the residuals

The presence of serial correlation is a problem not easily mitigable. In VAR models, where the lags of the dependent variable are used as regressors, serial correlation in the residuals cause a bias in the estimation (known as omitted variable bias). We used two tests that are mainstream in studies with VAR models, and are available in *R* with the package *vars*. The *Portmanteau test* (also known as *Ljung–Box Q test*) is a function of the sum of autocorrelations (adjusted by degrees of freedom) from lags 1 to p, where p is the VAR order. This test follows a chi-squared distribution. Another test is the *Breusch-Godfrey LM test*, which runs auxiliary regressions trying to fit each residual using its lags as regressors. This test computes a statistic that also follows a chi-squared distribution. Results with both tests are reported in Table 2.18, in the Appendix. In the specifications that presented serial correlation, we tried adding more lags, to no avail.

Heteroskedasticity and normality
All specifications presented heteroskedasticity and non normally-distributed errors. Therefore, we must be skeptical when interpreting standard errors. The main problem for our study is that the confidence intervals of the impulse responses are not reliable, since they assume that the errors follow a Normal distribution with constant variance. To mitigate this problem, we use bootstrapped errors in all impulse responses, which assumes only that the sample errors are representative of the population, but does not rely on a specific distribution.

5 Comparing models

We used several criteria to compare the quality of the models. First, we see how well each VAR fits the movements in inflation and economic activity. We also compare statistics of serial correlation. Second, we test the models' performances when forecasting. Specifically, we desire to know the future values of inflation and economic activity. Finally, we compare the variability of the impulse responses when a different sample is used to estimate the VAR.

5.1 Fit and serial correlation

The first criteria (Table 2.22 in the Appendix) compares the fit (adjusted R^2) of the two main equations in each VAR (inflation and economic activity) and the statistics of the serial correlation tests. A higher adjusted R^2 represents a better fit (adjusted to punish the loss of degrees of freedom). A lower statistic of serial correlation suggests a smaller bias.

From the adjusted R^2, we observe that the VARs with industrial production were the best in explaining the variation in inflation (in-sample), while the VARs with IBC-br and the VAR with output gap fared the best in explaining economic activity (the later possessing a much higher statistic).

With respect to serial correlation, both *Portmanteau* and *Breusch-Godfrey LM* statistics increased with model complexity (the VAR 1 was the best with any economic activity variable). The lowest *Portmanteau* and *Breusch-Godfrey LM* was generated by the VAR 1 with industrial production.

5.2 Forecasting

The second criteria (Table 2.23 in the Appendix) compares the forecasting performance. We used the expanding window method, predicting (out-of-sample) inflation and economic activity for 3 and 12 months ahead. A lower RMSE signifies a better performance.

The best forecasts of inflation, both 3-month and 12-month ahead, were attained by the quarterly data (real GDP growth and output gap). This may be, nevertheless, an unfair comparison since, in quarterly frequency, the number of steps ahead are one third of that in monthly data (3 months = 1 quarter). When considering only the two monthly specifications (IBC-br and industrial production), VAR 3 with industrial production fared the best in a 3-month ahead forecast, while its restricted version were the best in 12-month ahead.

When forecasting economic activity, both 3-month and 12-month ahead, the VARs with IBC-br had the lowest RMSE overall. However, we shall be cautious in drawing conclusions. The scale of the variables of economic activity are not necessarily equivalent. Variables with higher magnitudes will generate, naturally, higher RMSE.

5.3 Rolling window impulse responses

Another criterion for comparing models is related to their impulse responses. To our knowledge, this method is original and, therefore, demands a careful explanation. Since we are interested in analysing the effects of monetary policy, the forecasting performance is not an ideal criterion to compare our models. To complement our study, we analysed how reliable the impulse responses are. One primary concern is that our sample is small relatively to studies on developed countries data. As we explained in the introduction, there is little to be done to mitigate this problem. Instead, we measured how sensitive our estimations are with respect to a change in the sample. To accomplish that, we execute a procedure analogous to a rolling window forecast, but with impulse responses. We take each VAR specification at a time and execute the following procedure:

1. Select a sub-sample (75% of the total) starting in the earliest available time period.
2. Estimate the VAR and impulse responses with the sub-sample. This will give us the dynamic response of the variable of interest for each of the 50 periods after a shock to monetary policy.
3. Discard the initial time period of the sub-sample and include one time period ahead.
4. Repeat the process until all observations has been included in at least one sub-sample.

With the procedure above we obtain several impulse response functions for the same model. We then compute the Root Mean Squared Deviation (RMSD) between the upper and lower bounds of the middle 95% interval of each period ahead of the shock. The resulting RMSD is reported in Table 2.24, in the Appendix. We observe that the VARs with IBC-br were the least sensitive to a change in the sample, while the VARs with quarterly data (GDP growth and output gap) were the most. The RMSD is expected to increase with smaller samples. Another interesting result is that, when using industrial production, the RMSD of inflation decreases as we add complexity (from VAR 1 to 4).

6 Impulse responses

In this section we proceed to analyse the dynamic movements of inflation and economic activity as a response to a shock to interest rate. Since our VAR failed to generate normally distributed errors, we used bootstrapping to generate confidence intervals for the impulse responses. Contrary to asymptotic or Monte Carlo methods, bootstrapping does not assume a Gaussian distribution. Instead, it requires only that the fitted residuals be a representative sample of the true data generating process (DGP) residuals. In practice, bootstrapping draws several samples from the VAR's fitted residuals, and take the critical values corresponding to the confidence interval (CI) of choice (usually 95%). Those critical values of residuals will be used to generate synthetic series of the endogenous variables. Each synthetic series will have corresponding impulse response functions, which are assumed to form the upper and lower bound of the original estimation (Runkle, 1987). The magnitude of the impulse is 1 standard deviation of the nominal interest rate errors.

6.1 Real GDP growth

Using real GDP growth as a measure of economic activity, our impulse responses do not reveal much. Due to our small sample size, we were required to keep a simple specification with only three variables. As a result, in Figure 2.17 we observe that the mean response of inflation is almost entirely positive, and the confidence intervals are both positive and negative. As for the response of real GDP growth, it has a negative reaction after the shock, compensating with a rise and cycling around zero. However, one again the confidence intervals are both positive and negative.

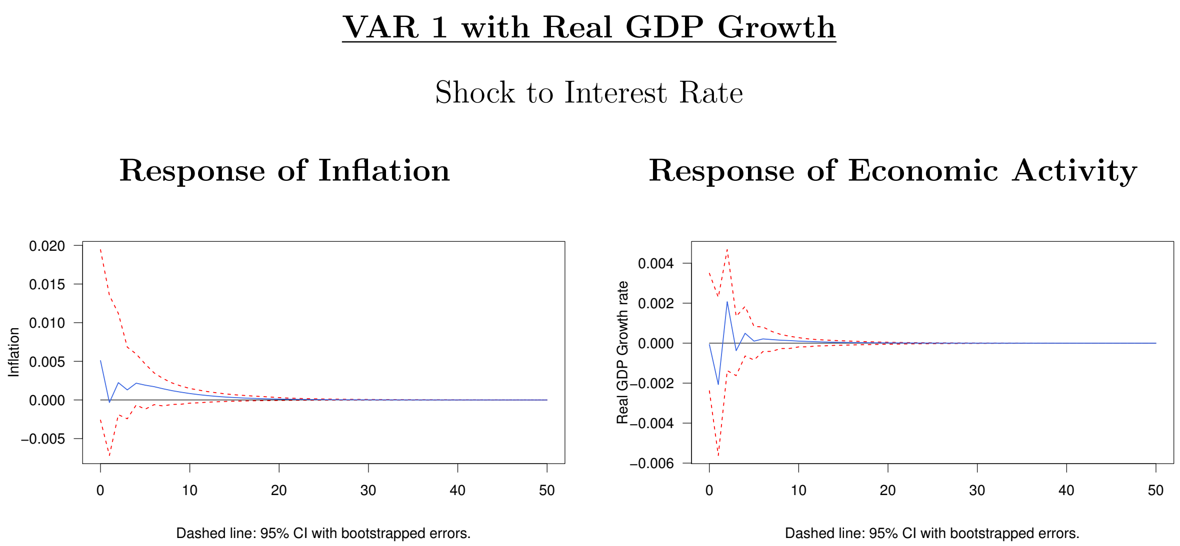


Figure 2.17: VAR 1 with real GDP growth. Impulse responses after an orthogonal shock to nominal interest rate.

6.2 Output gap

Using output gap, we observe a similar response of inflation (Figure 2.18). However, the response of economic activity (the output gap itself) is clearly negative, with a strong negative mean, until it dissipates 20 quarters after. The confidence intervals are predominantly negative.

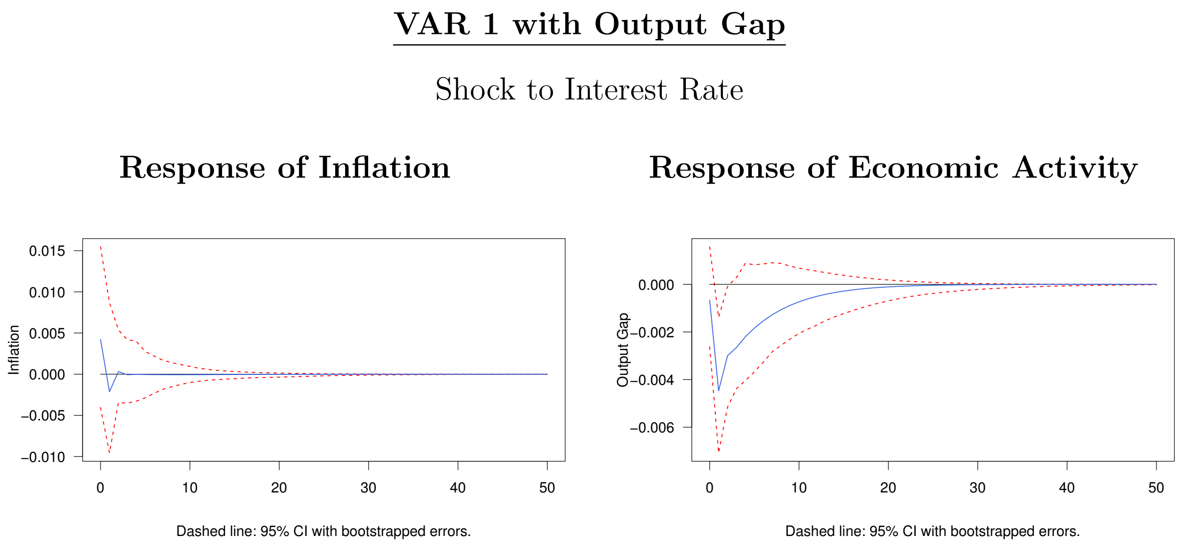


Figure 2.18: VAR 1 with output gap. Impulse responses after an orthogonal shock to nominal interest rate.

Using an alternative VAR 1, in which we consider only the observations after June/1999, when Brazil adopted an inflation targeting regime, and restrict the equations by significance (explained in Section 3.1.2). In Figure 2.19, we observe only a slightly negative mean response around the fourth quarter. However, in that point, the confidence intervals are narrower and predominantly negative. As for the output gap response, we obtain similar results to those in the original VAR 1.

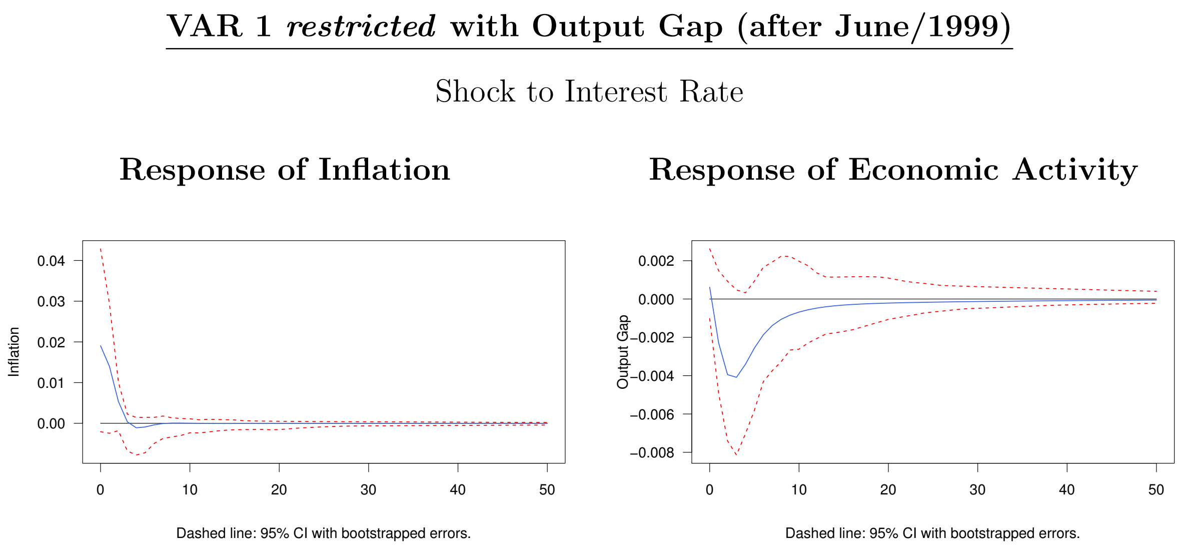


Figure 2.19: VAR 1 with output gap, restricted by significance. Only values after June/1999 (the establishment of inflation targeting regime) were considered. Impulse responses after an orthogonal shock to nominal interest rate.

6.3 Industrial production

Measuring economic activity with industrial production provides a larger time series. Differently from the VARs with output gap and real GDP growth, industrial production starts after the establishment of the inflation targeting regime.

From the previous analysis using output gap, we obtained a suggestion that the strongest negative effects on inflation happen mostly after 4 quarters. To keep this section clear, we concentrate on the VAR 3 *restricted*, which generated the best 12-month ahead forecast. (see Table 2.23). The impulse responses of alternative specifications can be found in the Appendix, section 9.3.1.

Figure 2.20 exhibits narrow confidence intervals for both inflation and economic activity. Inflation presents an initial rise, followed by a negative response between 8 and 23 months ahead, with a minimum at 12 months. The upper bound of CI is also negative between 10 and 15 months. Economic activity is negative from impact until 10 months ahead, and recovers between 10 and 25 months.

VAR 3 (*restricted*) with Industrial Production

Shock to Interest Rate

Response of Inflation **Response of Economic Activity**

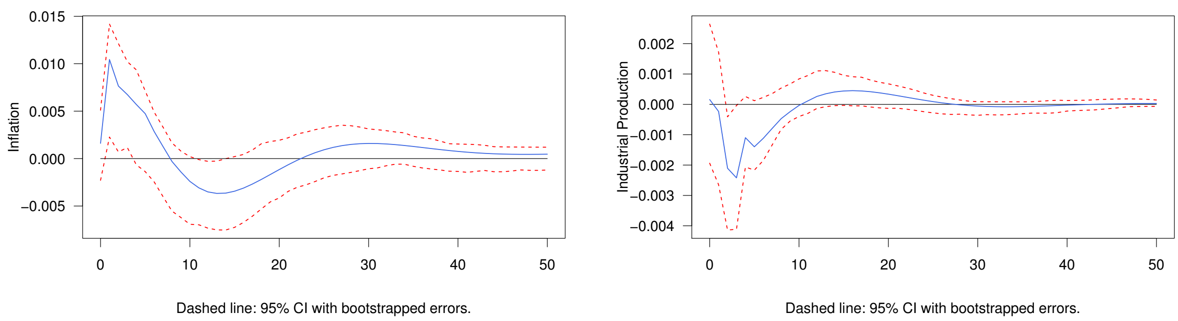


Figure 2.20: VAR 3 (restricted by significance) with industrial production. Impulse responses after an orthogonal shock to nominal interest rate.

Additionally, we estimated the responses of interest rate to shocks in inflation and economic activity (industrial production). Figure 2.21 reports the two impulse response functions. We observe that interest rate reacts positively—and immediately—to both shocks. In case of an inflationary shock, the rise in interest rates lasts 15 months according to the mean impulse response (confidence intervals are strictly positive during 7 months). On the other hand, after a positive shock to industrial production, the central bank increases monetary policy rate for 20 months, although confidence intervals are strictly positive only during the first 7 months.

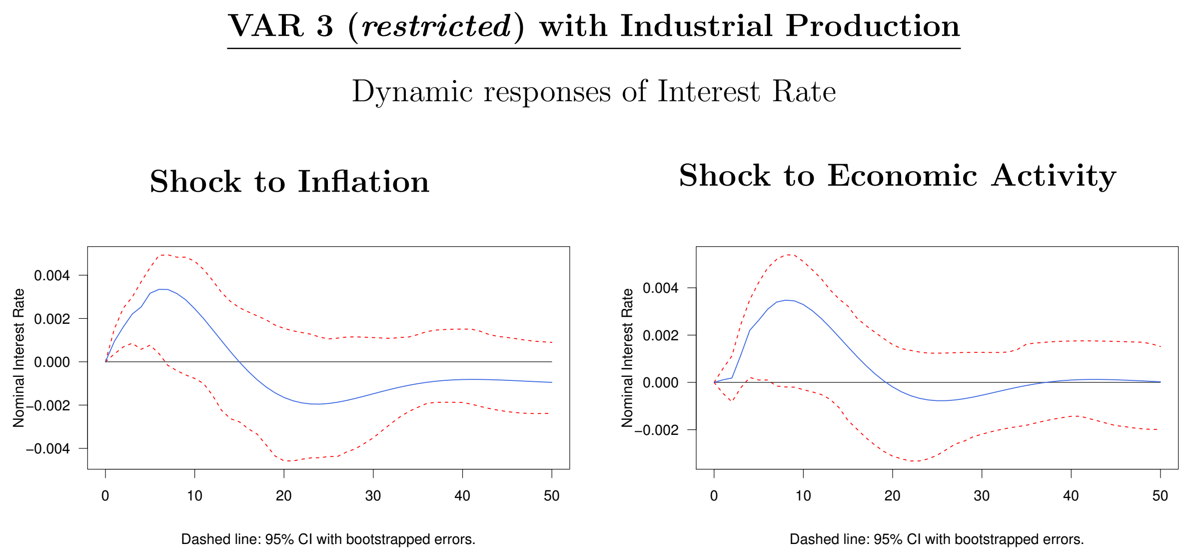


Figure 2.21: VAR 3 (restricted by significance) with industrial production. Impulse responses of interest rate after orthogonal shocks to inflation and economic activity.

6.4 IBC-br

As for the specifications with IBC-br, we concentrate on the VAR 3 to allow for a direct comparison with the previous section. VAR 3 yields the lowest RMSE 3-month ahead forecast of inflation, and the lowest RMSE 12-month ahead forecast of economic activity when using IBC-br (Table 2.23). The alternative specifications (VAR 1, 2, and 4) are presented in the Appendix, section 9.3.2.

When interpreting results, have in mind that this sample (as in the specifications with industrial production) starts when the inflation targeting regime was already established in Brazil. Figure 2.22 exhibits a similar shape to that in the previous section. However, inflation now reacts subtly, with a smaller magnitude than what we observed in the specification with industrial production. The negative mean impulse response lasts from 7 to 28 months after the shock.

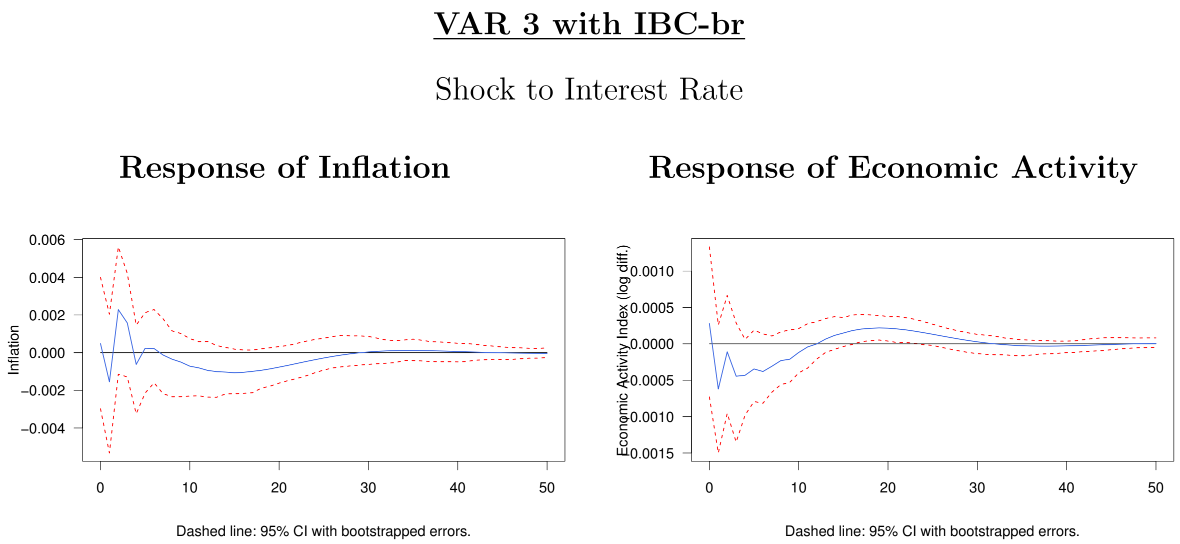


Figure 2.22: VAR 3 with IBC-br. Impulse responses after an orthogonal shock to nominal interest rate.

7 Credibility

In this section we study the role of credibility for Brazil's inflation targeting regime. Starting with the findings in section 3, we increment the VAR 3, industrial production, with a measure of credibility. Since this is a highly subjective measure, we adopted the method described in Sicsú (2002), where credibility (c_t) is a function of the gap between the markets' expectation of inflation (π_t^e) and the central bank's target (π_t^T), adjusted by the margin of tolerance (m_π) defined by the inflation targeting regime:

$$c_t = 100 - \left(\frac{|\pi_t^e - \pi_t^T|}{m_\pi} * 100 \right) \tag{7.0.1}$$

where $c_t \in \left]-\infty, 100\right]$.

We compute the credibility time series using a measure of market expectations called *relatório Focus*, which is published by the Central Bank of Brazil. Inflation target and the margin of tolerance are the official values downloaded from that same institution.

This time we replace inflation with the gap between actual inflation and the central bank's target π_t^{gap}.

We then specify a model similar to the VAR 3 of section 3, with the addition of credibility and inflation gap, which we now call VAR 5:

VAR 5:

$$Y_t^3 = \begin{bmatrix} c_t \\ M_t \\ r_t \\ i_t \\ y_t \\ \pi_t^{gap} \end{bmatrix} \tag{7.0.2}$$

When estimating the VAR, although AIC suggested the use of 4 lags, we found that using 3 lags, restricting regressors by significance, yielded the lowest RMSE when forecasting inflation and economic activity. We ran the same diagnostic tests, reported in section 9.1.1, in the Appendix.

We also measured the forecasting performance of VAR 5 (Table 2.17) and obtained a RMSE of inflation (3 and 12 months) lower than all other monthly VARs (see Table 2.23 for comparison).

Root Mean Squared Error (RMSE) of forecast

	Inflation		**Economic activity**	
	3 months	**12 months**	**3 months**	**12 months**
VAR 5:	0.0246	0.0292	0.0144	0.0142

Table 2.17: Forecasting performance for model comparison. Based on an out-of-sample expanding window, 3 months or 12 months ahead.

7.1 Impulse responses

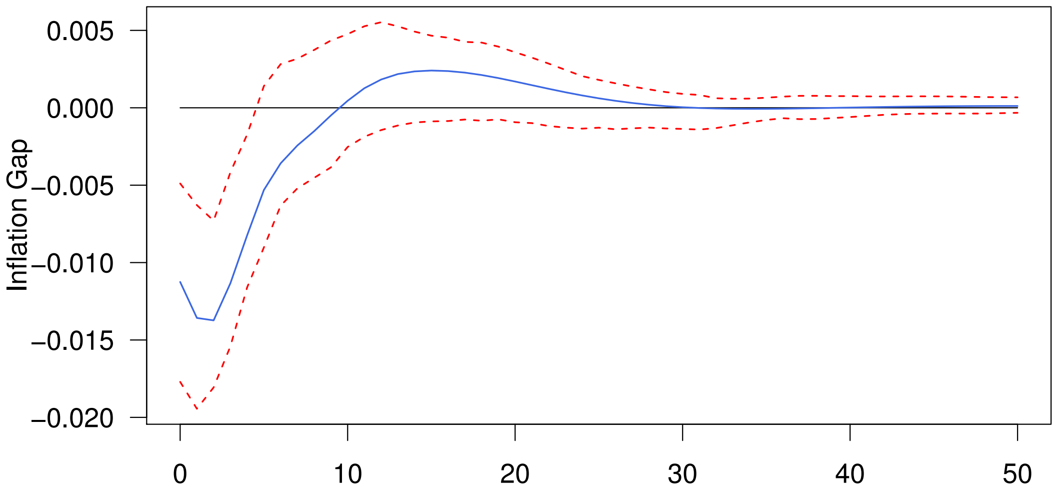
7.1.1 Effects of credibility

Assuming an exogenous shock to credibility, we estimate the dynamic responses of inflation gap, nominal interest rate and industrial production.

After a one-time shock to credibility, inflation gap falls and maintains a negative response for 10 months, and becomes positive (in a much smaller magnitude) between months 10 and 25 (Figure 2.23). This result is expected since a higher credibility represents an anchored expected inflation, which alleviates inflationary pressures.

VAR 5 - shock to Credibility

Response of Inflation Gap



Dashed line: 95% CI with bootstrapped errors.

Figure 2.23: VAR 5. Impulse responses of inflation gap after an orthogonal shock to credibility.

We now proceed to analyse the response of interest rates (Figure 2.24). After a positive shock to credibility, we observe an instantaneous loosening of monetary policy, which prevails for 17 months. This can be explained by the increase in credibility making easier the mission of the central bank. After the initial 17-month period, there is a slightly positive reaction in interest rates, although the confidence interval in that point is both above and below zero.

VAR 5 - shock to Credibility

Response of Nominal Interest Rate

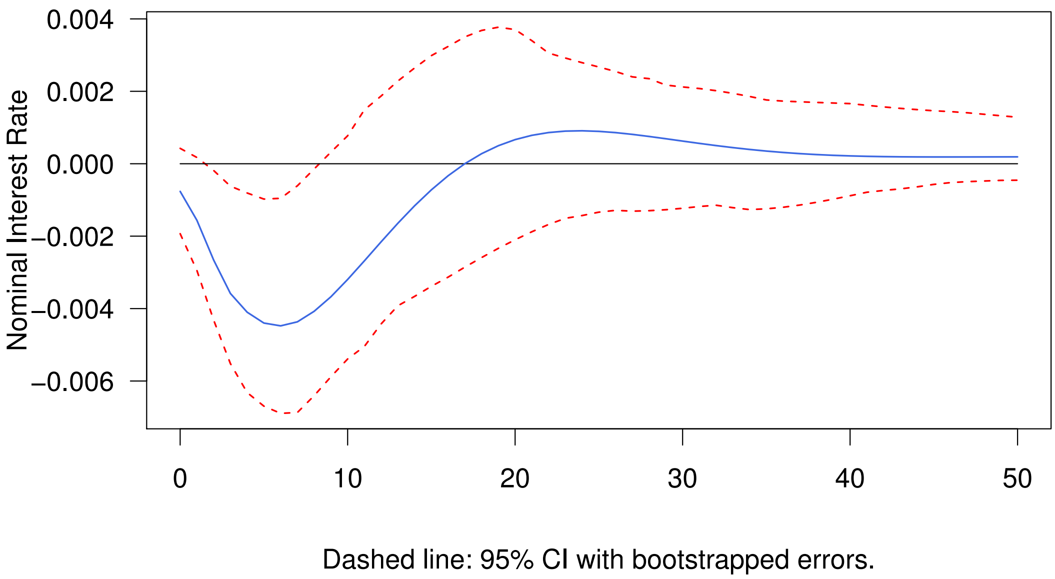


Figure 2.24: VAR 5. Impulse responses of nominal interest rate after an orthogonal shock to credibility.

As for the response of industrial production (Figure 2.25), the confidence intervals include the intercept for 50 months ahead of the shock. Nevertheless, the response is predominantly positive (including the mean) between 1 and 10 months. If our hypothesis of a loosening in monetary policy happening in response to credibility is correct, it is natural to expect an increase in economic activity.

VAR 5 - shock to Credibility

Response of Industrial Production

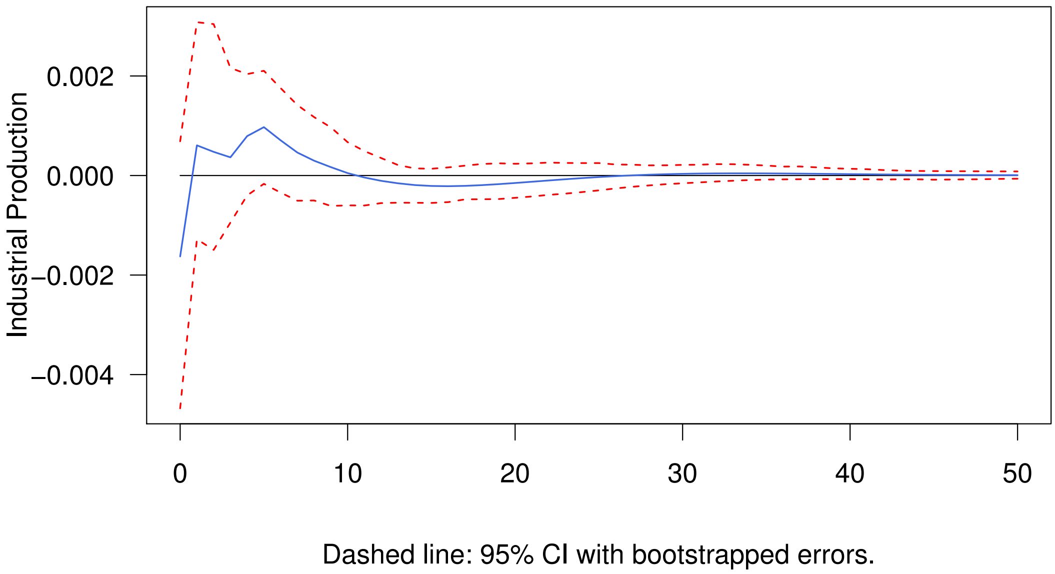


Figure 2.25: VAR 5. Impulse responses of industrial production after an orthogonal shock to credibility.

7.1.2 Reaction of credibility

Without going deep in the determinants of credibility, let us discuss how this variable reacts to inflation gap and interest rate.

Intuition tells us that the agents' expectations about future inflation should depart from the central bank's target as they observe a positive inflation gap. This process of *adaptive expectations* is a middle ground between naive agents and full rationality. One way to test this hypothesis is by estimating the change in credibility (which is a function of the gap between market expectations and the central bank's target) following a shock to inflation gap.

We report the impulse responses in Figure 2.26. After a shock to inflation gap, we observe an initial hump in credibility lasting 3 months, followed by 12 months of negative responses. However, as the confidence intervals include the intercept, our conclusion on this matter requires a more thorough study.

VAR 5 - shock to Inflation Gap

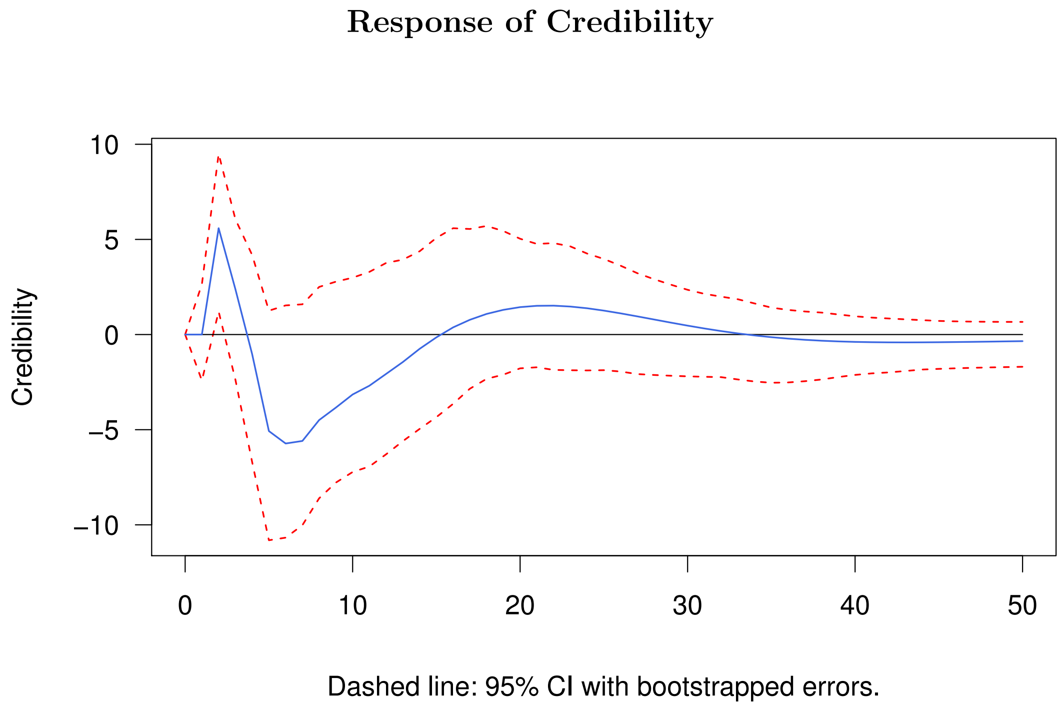


Figure 2.26: VAR 5. Impulse responses of credibility after an orthogonal shock to inflation gap.

Another interesting question is how the central bank's credibility changes after a shock to interest rate. If agents look at past inflation to decide how much to trust the central bank, a shock to interest rate (orthogonal, i.e. not caused by inflation) should not affect credibility in a direct way. Its effects would be indirect if the output gap was positive at the moment of the shock and decreased afterwards. Figure 2.27 reports the impulse responses. After the shock, credibility falls unambiguously for two months. After that, the confidence interval becomes very wide, with a positive mean between 8 and 35 months. Therefore, any conclusion based on this estimation would be purely speculative.

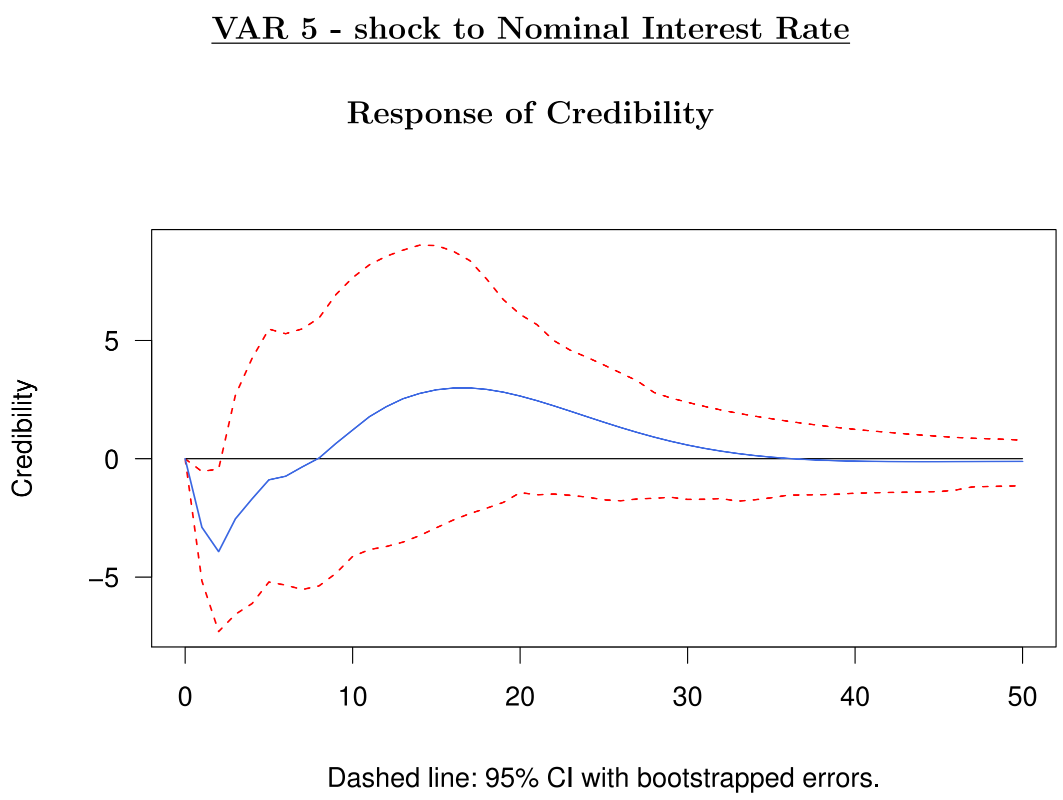


Figure 2.27: VAR 5. Impulse responses of credibility after an orthogonal shock to nominal interest rates.

8 Conclusion

This chapter presented an empirical study of Brazil's economy. Specifically, we sought to estimate the reaction of inflation and economic activity after an orthogonal shock to monetary policy.

We used four different measures of economic activity in our VAR estimations. In quarterly data we employed, separately, real GDP growth and output gap. In monthly data we used industrial production and IBC-br (an index of economic activity of Brazil's central bank).

We verified stationarity and seasonality of all variables and tested the stability of the VARs and the presence serial correlation. All specifications were stable, but several failed in one of the two serial correlation tests (Portmanteau and Breusch-Godfrey). All VARs had heteroskedastic and non-normally distributed errors, although these were mitigated with the use of bootstrapped confidence intervals in the impulse responses.

All models were compared in terms of adjusted R^2, serial correlation statistics, and forecasting performance. We compared the RMSE when predicting inflation and economic activity, 3 and 12 months ahead. Overall, the quarterly specifications fared best in predicting inflation, while the VAR 3 with industrial production had the best results among monthly specifications.

An alternative method for comparing VARs was devised. We used a rolling window to estimate an in-sample VAR for each subset of observations, and compared the RMSD between the upper and lower bounds of the middle 95% mean impulse responses of all sub-samples. A lower RMSD suggests that a given VAR estimation is less sensitive to changes in the sample. For this test, the VARs with IBC-br had the best results.

After validating the empirical models, we analysed the impulse responses. The quarterly specifications showed ambiguous results, possibly because of the small sample. On the other hand, the best model in predicting inflation using monthly data (VAR 3 restricted with industrial production) showed that inflation reacts with an initial hump and then negatively between 8 and 23 months subsequently to a shock to monetary policy. Economic activity reacts negatively and recovers after 10 months.

Furthermore, we estimated a monetary policy reaction to inflationary and activity shocks. We found that exogenous increases in inflation and industrial production prompts an immediate increase in interest rates that lasts 15 and 20 months, respectively.

Finally, this chapter presents an analysis on the effects of credibility to Brazil's inflation targeting regime. We estimate a VAR including a measure of credibility—a function of the gap between market's expectations and the central bank's target. We find that inflation and output gap reacts negatively after an exogenous increase in credibility. Economic activity has a more subtle response, with a positive mean, but confidence intervals both above and below zero.

We also analyse what happens to credibility in reaction to inflation gap. Assuming an exogenous shock to the later, credibility initially increases, but becomes negative between the 4th and 15th months. As for how interest rates affect credibility (after an exogenous shock not caused by inflation), we found insignificant results, as expected.

A study that aims at explaining Brazil's inflation targeting regime must devise ways to overcome the small sample, which limits the complexity of empirical models. For example, using a factor-augmented VAR (FAVAR), which selects the most relevant linear combinations of a large set of variables to use in the regression, might generate more precise results than the ones we obtained in this chapter.

9 Appendix

9.1 Diagnostics

	Portmanteau	**Breusch-Godfrey LM**
Output gap		
VAR 1:	Pass	Pass
VAR 1 *after IT*:	Pass	Reject
Real GDP growth		
VAR 1:	Pass	Rejected
Industrial production		
VAR 1:	Pass	Pass
VAR 2:	Pass	Rejected
VAR 3:	Rejected	Rejected
VAR 3 *restricted*:	Rejected	Pass
VAR 4:	Rejected	Rejected
IBC-br		
VAR 1:	Pass	Rejected
VAR 2:	Rejected	Rejected
VAR 3:	Rejected	Rejected
VAR 4:	Rejected	Rejected

Table 2.18: Results of serial correlation tests with all specifications. *Rejected* means that the test rejected the null-hypothesis of no serial correlation. *Pass* means that serial correlation was not detected.

9.1.1 Diagnostics of VAR 5 (credibility)

VAR stability: All roots of the coefficients matrix are within the unit circle. VAR 5 is stable.

Serial correlation in the residuals

The Portmanteau statistic rejected the test of no serial correlation, while Breush-Godfrey LM exhibited a P-value of 23.7%, failing to reject the null hypothesis.

Null hypothesis: No serial correlation in the residuals.		
	Statistic	P-value
Breusch-Godfrey LM test:	118.16	0.237
Portmanteau test:	608.62	$p < 0.01$

Table 2.19: Test of serial correlation in the residuals for VAR 5.

Heteroskedasticity
The ARHC-LM test detected heteroskedasticity in the residuals.

Null hypothesis: No heteroskedastic errors.		
	Statistic	P-value
ARCH-LM test:	2637.4	$p < 0.01$

Table 2.20: Test of heteroskedasticity in the residuals for VAR 5

Errors normality
The Jarque-Bera test rejected the hypothesis of normally distributed errors.

Null hypothesis: Errors are normally distributed.		
	Statistic	P-value
Jarque-Bera test:	1371.3	$p < 0.01$

Table 2.21: Test of normality in the residuals for VAR 5

9.2 Comparing models

Fit and serial correlation

	Adj. R^2 Inflation	Adj. R^2 Economic activity	Portmanteau statistic	Breusch-Godfrey LM statistic
Output gap				
VAR 1 :	0.4475	0.7377	12.1555	71.5105
VAR 1 *after IT*:	0.8405	0.8455	21.6398	61.0838
Real GDP growth				
VAR 1:	0.4228	0.0371	12.4627	102.8905
Industrial production				
VAR 1:	0.5613	0.0022	9.8849	69.2296
VAR 2:	0.5643	0.0055	16.5220	126.9544
VAR 3:	0.6348	0.0667	51.6022	244.5436
VAR 3 *restricted*:	0.8776	0.0762	92.7905	136.6483
VAR 4:	0.6725	0.1163	55.5293	287.8825
IBC-br				
VAR 1:	0.3784	0.1203	21.3342	102.6172
VAR 2:	0.3801	0.1282	28.7509	150.6898
VAR 3:	0.4205	0.1609	46.7938	232.0838
VAR 4:	0.4577	0.1981	58.0994	285.5891

Table 2.22: Results of several statistics for model comparison.

Root Mean Squared Error (RMSE) of forecast

	Inflation		**Economic activity**	
	3 months	**12 months**	**3 months**	**12 months**
Output gap				
VAR 1 :	0.0157	0.0241	0.0101	0.0245
VAR 1 *after IT*:	0.0198	0.0281	0.0108	0.0274
Real GDP growth				
VAR 1:	0.0169	0.0250	0.0150	0.0122
Industrial production				
VAR 1:	0.0323	0.0373	0.0141	0.0139
VAR 2:	0.0340	0.0390	0.0138	0.0140
VAR 3:	0.0271	0.0399	0.0139	0.0140
VAR 3 *restricted*:	0.0295	0.0299	0.0144	0.0142
VAR 4:	0.0293	0.0406	0.0153	0.0146
IBC-br				
VAR 1:	0.0332	0.0378	0.0070	0.0072
VAR 2:	0.0333	0.0376	0.0069	0.0071
VAR 3:	0.0314	0.0392	0.0073	0.0070
VAR 4:	0.0317	0.0398	0.0074	0.0070

Table 2.23: Forecasting performance for model comparison. Based on an out-of-sample expanding window, 3 months or 12 months ahead.

Root Mean Squared Deviation (RMSD)
of impulse responses

	Inflation	**Economic activity**
Output gap		
VAR 1	0.0048	0.0022
VAR 1 *after IT*:	0.0064	0.0028
Real GDP growth		
VAR 1	0.0048	0.0013
Industrial production		
VAR 1	0.0037	0.0006
VAR 2	0.0033	0.0007
VAR 3	0.0032	0.0007
VAR 3 *restricted*	0.0032	0.0007
VAR 4	0.0031	0.0008
IBC-br		
VAR 1	0.0012	0.0003
VAR 2	0.0012	0.0003
VAR 3	0.0013	0.0003
VAR 4	0.0015	0.0003

Table 2.24: Comparing the RMSD of impulse responses of a rolling window sub-sampling within each model.

9.3 Impulse responses

9.3.1 Industrial production

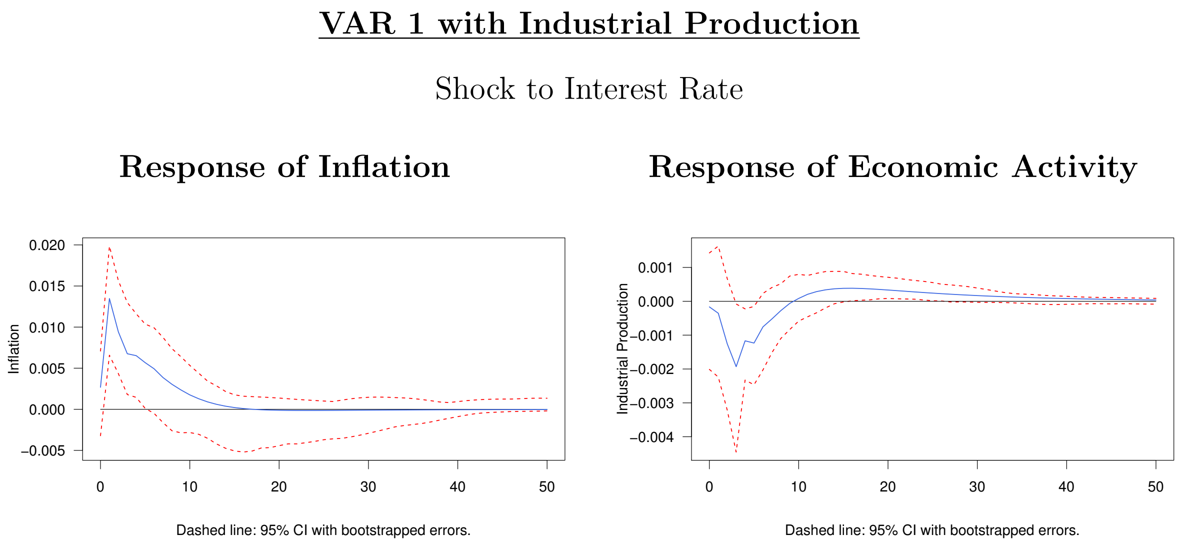


Figure 2.28: VAR 1 with industrial production. Impulse responses after an orthogonal shock to nominal interest rate.

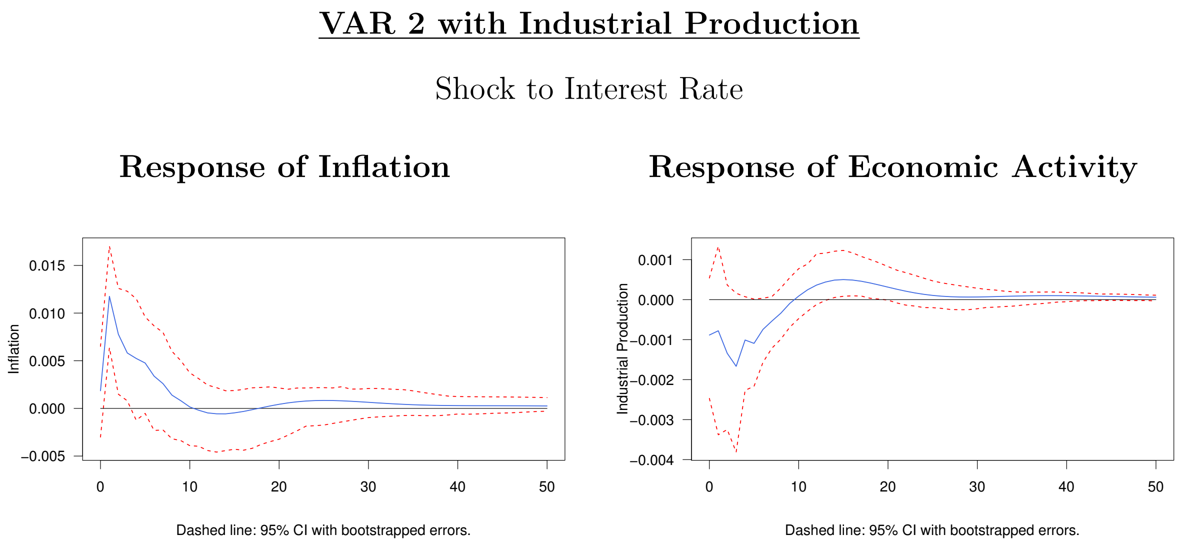


Figure 2.29: VAR 2 with industrial production. Impulse responses after an orthogonal shock to nominal interest rate.

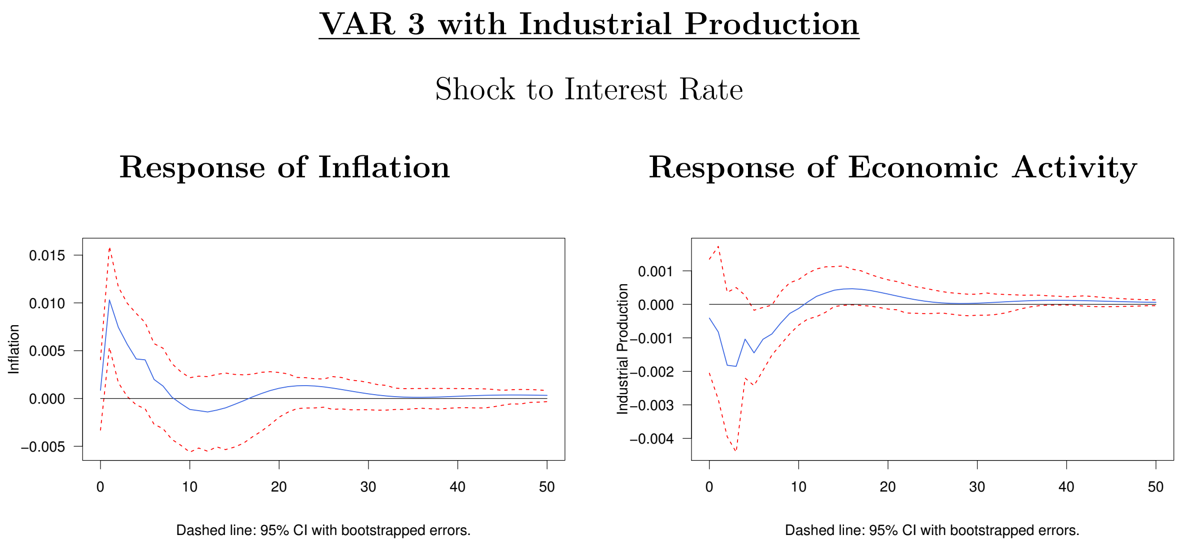


Figure 2.30: VAR 3 with industrial production. Impulse responses after an orthogonal shock to nominal interest rate.

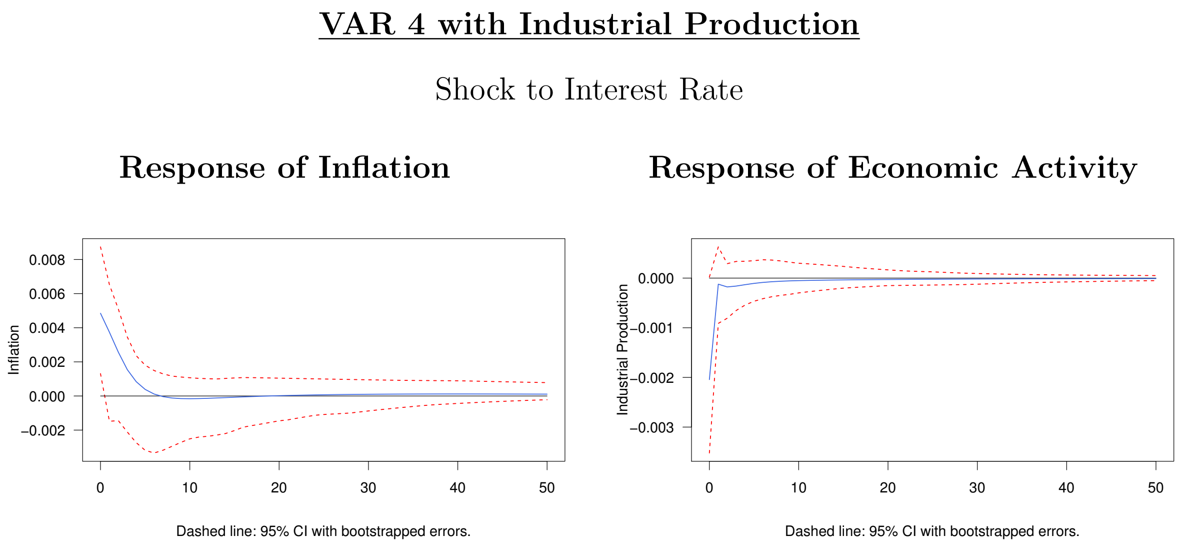


Figure 2.31: VAR 4 with industrial production. Impulse responses after an orthogonal shock to nominal interest rate.

9.3.2 IBC-br

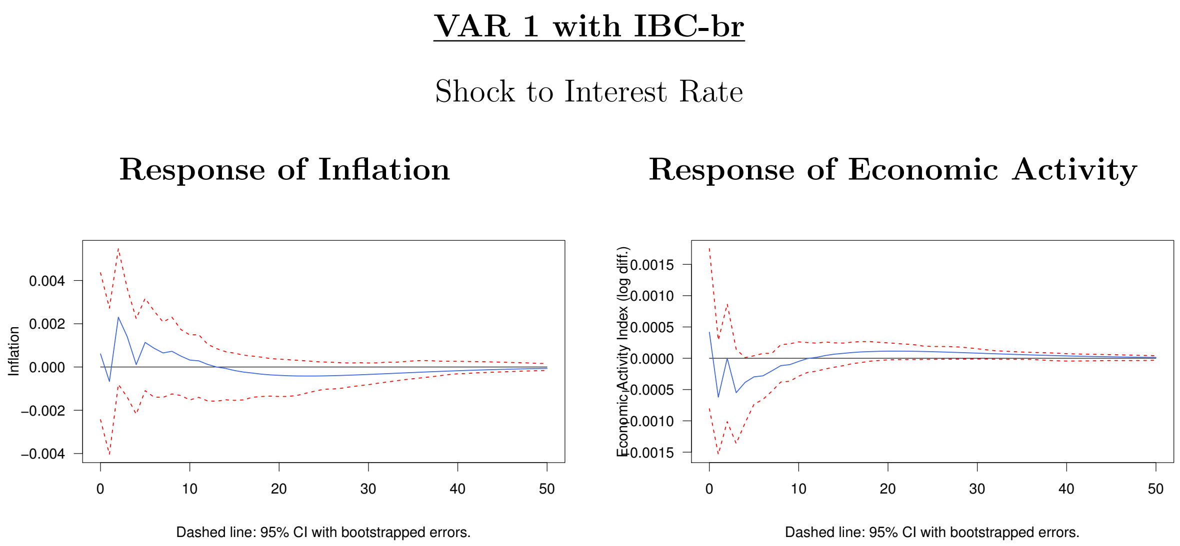


Figure 2.32: VAR 1 with IBC-br. Impulse responses after an orthogonal shock to nominal interest rate.

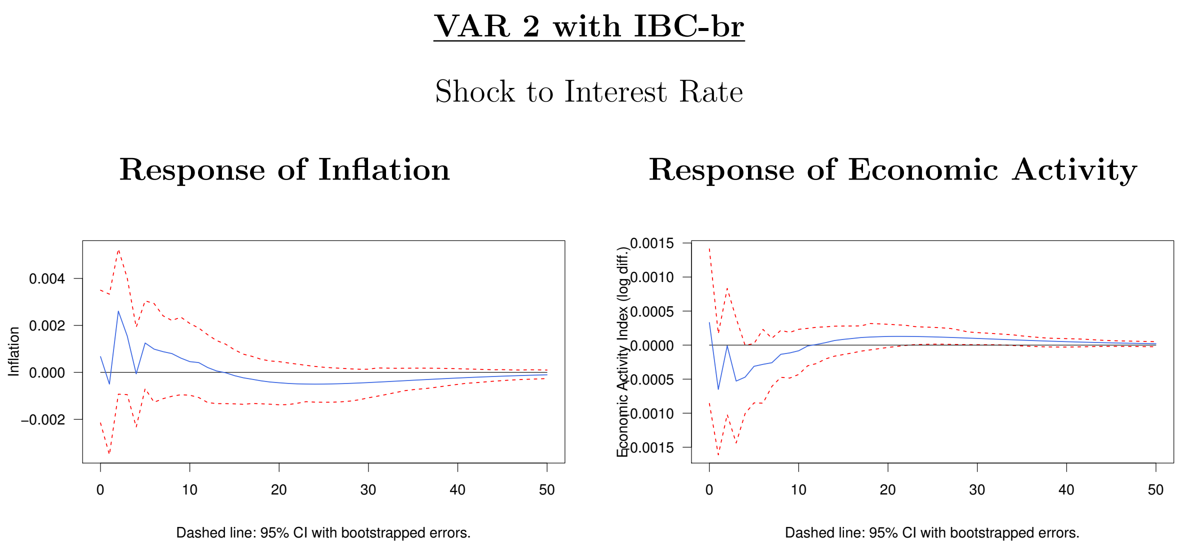


Figure 2.33: VAR 2 with IBC-br. Impulse responses after an orthogonal shock to nominal interest rate.

VAR 4 with IBC-br

Shock to Interest Rate

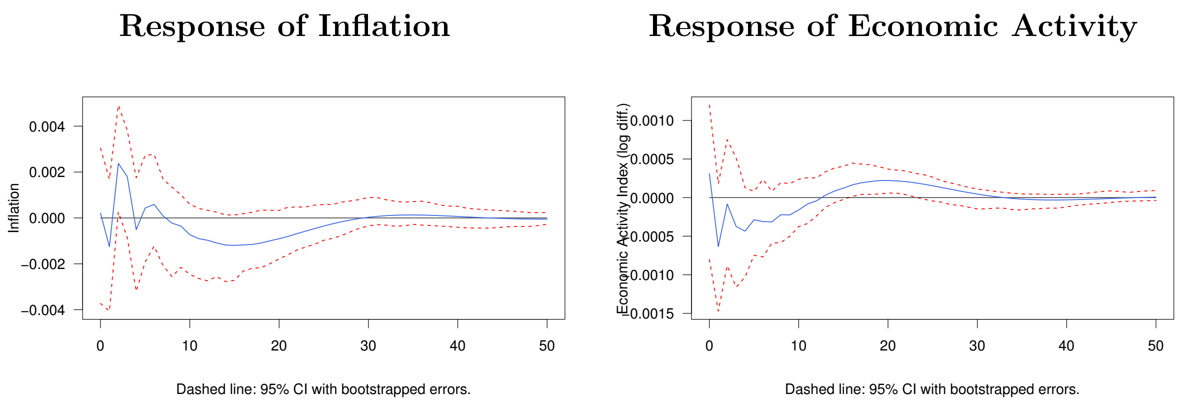


Figure 2.34: VAR 4 with IBC-br. Impulse responses after an orthogonal shock to nominal interest rate.

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