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# Replication and study of an agent-based model of industry competition with R&D

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## Abstract

This master thesis studies an agent-based model proposed in Chang (2015a). It describes an economy with firms competing by improving efficiency. Each firm has its own technology and makes investments in research to reduce costs. After describing the original model and presenting results, this paper experiments with two modifications. One regards the occurrence of abnormally large shocks; another relates to the existence of multiple optimal technologies.



## Acknowledgments

This study received a huge contribution from my supervisor, Professor Angelo Secchi. It is not an exaggeration to say that I would have not been able to implement an agent-based model if not for Secchi's commitment to his students. His expertise on economic models gave me guidance to overcome the obstacles; his patience and interest gave me motivation to put the effort required to write this paper in such a short time; and his unmatched logical precision and clarity of reasoning serves as a reminder to study Economics as a rigorous science.


# 1 Introduction

The model studied by this paper is an application of agent-based computational economics (ACE) and was described in Chang (2015a). That work modeled heterogeneous firms with individual production methods that can be improved by investing in R&D. Total attention is given to the producer, its objective function, its learning process and the evolution of individual firms through time. Differently, consumers are not detailed as individual agents. They are, instead, represented by a demand function.

Although Chang made his work available in his personal website [1], I decided to not base this master thesis on his code. Instead, I replicated his study with my own imple-

[1] See http://academic.csuohio.edu/changm

mentation using the language Python and making use of object-oriented programming (OOP). There are three reasons for that decision. First, it allowed me to have a deep understanding of the mechanisms in the model, since I had to code every component. Second, Chang used the language C++, which, although is the optimal choice in processing performance, presents a steeper learning curve to understand the syntax. As this simulation is not very demanding in terms of computational power, I believe that the loss in code readability is not compensated. Third, the original code was written in a procedural way and did not use OOP, which would have allowed a separation between the economic model and purely technical components. If implemented in Python with OOP, I believe that the code is easier to be understood by economists without a computer-science background, and is amenable to future extensions in the model. Every part of my implementation is an exact replica of the model described by Chang, except for section 4 where I experiment with two simple modifications to the original model.

# 2 Agent-based computational economics

Economies in the real world are composed by a large number of economic agents interacting repeatedly among themselves in a local context. Each of these local interactions contributes to global consequences such as employment, growth rates, income distribution, market institutions and social conventions. These resulting structures, in turn, affect how agents will act subsequently (Tesfatsion, 2006). Agent interactions are simulated by agent-based models, and each outcome emerges endogenously as a result of individual decisions. What bases each agent's decisions is its particular set of characteristics, which is contingent on its past experiences and learning processes. As quoted in Chang (2015a), a clear explanation of agent-based computational economics (ACE) is given by Tesfatsion and Judd (2006):

> ACE is the computational study of economic processes modeled as dynamic systems of interacting agents who do not necessarily possess perfect rationality and information. Whereas standard economic models tend to stress equilibria, ACE models stress economic processes, local interactions among

> traders and other economic agents, and out-of-equilibrium dynamics that may or may not lead to equilibria in the long run. Whereas standard economic models require a careful consideration of equilibrium properties, ACE models require detailed specifications of structural conditions, institutional arrangements, and behavioral dispositions.
>
> (Tesfatsion & Judd, 2006)

ACE receives criticism regarding its validity in interpreting and generalising results. Richiardi (2004) presents a survey on the advantages and disadvantages of ACE. He argues that agent-based models have potential to help in the economic analysis, and that there are methodological solutions for the aforementioned problems. Tesfatsion (2006) divides the application of ACE between descriptive and normative. To the former, ACE helps to explain the emergence of global structures "despite the absence of top-down planning and control". In other words, how local interactions between autonomous agents generate some specific regularities instead of others. Regarding normative aspects, ACE can give insights about how each mechanism affects social outcomes when agents behave autonomously according to different incentives.

## 2.1 Other studies using agent-based models

Several studies have applied ACE in their modelling. Related to industrial organization, Chang (2010) makes a case for agent-based modelling to study out-of-equilibrium dynamics. He suggests some potential lines of research that could start with an empty industry and generate entering and exititing firms endogenously. Furthermore, Chang (2010) suggests a study on the response of those endogenous variables to external shocks and on the relation between firm size, growth and survival rates.

Dosi, Pereira, and Virgillito (2015) applies agent-based modeling to simulate stylized facts of industrial dynamics. With an evolutionary model of learning processes, the study was able to replicate some phenomena found in the empirical literature such as productivity assimetries, skewed distribution of size, a negative relation between size and growth variance, and fat-tailed distribution of growth rates.

From a macroeconomic perspective, Napoletano, Dosi, Fagiolo, and Roventini (2012) studies dynamics of investment, both when they are determined by past profits and when they are based on expected future demand. They demonstrate that low inequality is essential to a steady growth with low unemployment. Furthermore, it studies the impact of increasing wage-flexibility on growth and unemployment.

Dosi, Fagiolo, and Roventini (2010) was able to reproduce some macro-stylized facts using agent-based modelling. It relates "Schumpeterian theories of technology-driven economic growth with Keynesian theories of demand generation". It demonstrates that innovation alone cannot generate a growth with full employment path. It is necessary, instead, to have a Keynesian demand generation by means of fiscal policy.

Still in macroeconomics, Dosi, Fagiolo, Napoletano, and Roventini (2012) created an agent-based model that includes a banking sector and a monetary authority. It was able to reproduce some features of current and past recessions and concluded that more unequal societies are more vulnerable to severe cycles, exhibits higher unemployment and has higher probability of crises.

# 3 An ACE model of industry dynamics

This section presents an ACE model created by Myong-Hun Chang and published in his book *A Computational Model of Industry Dynamics* (2015). I based on descriptions contained in the book to replicate his experiments. In this section I present the model and the results I obtained—which are consistent with Chang's results—and, in section 4, my two original modifications. Most of the mathematical formalization was borrowed from his book or his papers.

## 3.1 Description

Chang (2015a) builds a model of an industry under Cournot competition. Every firm produces the same homogeneous good and chooses its output to maximize profits. The only equilibrium price is, therefore, set endogenously according to a demand curve. Firms are heterogeneous with respect to production efficiency, which translates into lower or higher marginal costs. Consumers, on the other hand, are not individually represented; they are replaced by a single inverse demand function. This dynamic is repeated over 5000 periods, with each period's state variables being carried over to the next. Therefore, firms that survive period $t$ continue to exist in $t+1$.

Regarding each firm's efficiency, Chang (2015a) uses "six building blocks of the model":

> 1. The production process is viewed as a system of activities, where each activity can be accomplished using one of the finite number of methods (practices) specifically available for that activity.
> 2. A firm's technology is defined by its vector of chosen methods, one method for each component activity of the production process. Assuming two possible methods per activity, a technology is defined by a vector of zeroes and ones.
> 3. The efficiency of a firm is determined by how closely its technology matches an ex ante unknown "optimal technology" uniquely defined for a given technological environment.
> 4. The degree of a firm's efficiency, as defined above, determines its marginal cost of production.
> 5. Firms are technologically diverse at any given point in time. The variation in technology choices leads to asymmetry in firms' marginal costs and, consequently, to differential profits attained through market competition.
> 6. Firms may pursue R&D by experimenting with their technologies – i.e., by altering the method(s) for carrying out one or more of the activities in the production process.
>
> (Chang, 2015a)

Chang (2015a) formalizes the concept of production process as a set of $N$ tasks[2], where each of them can be done using one of two methods represented by 0 or 1. A task

[2]Chang uses the terms "task" and "activity" interchangeably.

is then characterized by a bit. The technology of firm $i$ in period $t$ is represented by a binary vector of $N$ dimensions, $\underline{z}_i^t \in \{0,1\}^N$, such that $\underline{z}_i^t \equiv (z_i^t(1), z_i^t(2), ..., z_i^t(N))$ and $z_i^t(h) \in \{0,1\}$ is the method used in task $h$. Any two technologies can be compared to obtain their degree of heterogeneity. This measure is defined by the "Hamming distance", which in this context means the number of tasks for which the chosen methods are different:

$$D\left(\underline{z}_i^t, \underline{z}_j^t\right) \equiv \sum_{h=1}^{N} \left| z_i^t(h) - z_j^t(h) \right|. \tag{3.1.1}$$

All firms operate in the same technological environment. This means that, for each period, there exists an optimal technology, unknown to the firms, for which the marginal cost is minimum—zero. We can therefore calculate the degree of heterogeneity between any technology and the optimal. To formalize this concept, Chang (2015a) specifies a vector $\underline{\hat{z}}^t \in \{0,1\}^N$ as the optimal technology for the entire industry in period $t$. The marginal cost of firm $i$ in period $t$ is then defined by:

$$c_i^t\left(\underline{z}_i^t, \underline{\hat{z}}^t\right) = 100 \cdot \frac{D\left(\underline{z}_i^t, \underline{\hat{z}}^t\right)}{N}. \tag{3.1.2}$$

Since we are dividing the number of differing tasks, $D \in [0, N]$, by the total number of tasks $N$, and multiplying by 100, it is clear that $c_i^t \in [0, 100]\ \forall i$.

Two concepts are very important for this model. First, the optimal technology might change from one period to another. This is a result of technological shocks that occur exogenously. I explain the mechanism in section 3.3.1. Second, each firm can modify its own technology by experimenting with a different method for any chosen task. This process happens either by innovation or imitation. I devote section 3.3.2 to explain this process of R&D.

## 3.2 Market competition

Firms operate in a Cournot competition. All firms choose their quantity simultaneously to maximize profits. Although the optimal technology is unknown, all firms have perfect information about their own marginal cost.

Chang (2015a) defines the following:

**Inverse demand function**

$$P^t(Q^t) = a - \frac{Q^t}{s^t} \tag{3.2.1}$$

where $Q^t = \sum_{j=1}^{m^t} q_j^t$ is the total industry output, $m^t$ is the number of firms producing a positive amount in period $t$, and $s_t$ denotes the current size of the market. Parameter $s_t$ is exogenous and assumed to be fixed through all periods for the baseline model. In a later chapter, Chang (2015a) experiments with an oscillating $s_t$ to represent business cycles. I show the results of both situations in sections 3.4.1 and 3.4.2 respectively. Finally, parameter $a$ is the demand intercept and is kept constant in all scenarios.

**Total cost function**

$$C_i^t\left(q_i^t, c_i^t\right) = f + c_i^t \cdot q_i^t \tag{3.2.2}$$

where $c_i^t$ is the marginal cost defined in equation 3.1.2, and $f$ is the fixed cost assumed to be constant for all firms through all periods. Substituting marginal cost into the total cost function gives:

$$C_i^t\left(q_i^t, \underline{z}_i^t, \hat{\underline{z}}^t\right) = f + 100 \cdot \frac{D\left(\underline{z}_i^t, \hat{\underline{z}}^t\right)}{N} \cdot q_i^t. \tag{3.2.3}$$

**Profit function**

The profit earned by firm $i$ in period $t$, given the inverse demand function, is:

$$\pi_i^t\left(Q^t, q_i^t, c_i^t\right) = \left(a - \frac{Q^t}{s^t}\right) \cdot q_i^t - f - c_i^t \cdot q_i^t - I_i^t \tag{3.2.4}$$

where $I_i^t$ is the investment in R&D (see section 3.3.2). Maximizing each firm's profit function with respect to $q_i^t$ gives the following:

$$\frac{\partial \pi_i^t}{\partial q_i^t} = 0 \implies \left(a - \frac{Q^t}{s^t}\right) - q_i^t\left(\frac{1}{s^t}\right) - c_i^t = 0 \implies q_i^t = s^t\left(a - c_i^t\right) - Q^t. \tag{3.2.5}$$

#### Short-term equilibrium

Solving the algebra for $m^t$ firms gives a short-term equilibrium (period $t$):

$$\bar{Q}^t = \left(\frac{s^t}{m^t+1}\right)\left(m^t a - \sum_{j=1}^{m^t} c_j^t\right) \tag{3.2.6}$$

$$\bar{P}^t = \left(\frac{1}{m^t+1}\right)\left(a + \sum_{j=1}^{m^t} c_j^t\right) \tag{3.2.7}$$

$$\bar{q}_i^t = s^t\left[\left(\frac{1}{m^t+1}\right)\left(a + \sum_{j=1}^{m^t} c_j^t\right) - c_i^t\right] \Leftrightarrow \bar{q}_i^t = s^t\left[\bar{P}^t - c_i^t\right] \tag{3.2.8}$$

The profit earned by the firm will be:

$$\bar{\pi}_i^t = \frac{1}{s^t}\left(\bar{q}_i^t\right)^2 - f - I_i^t. \tag{3.2.9}$$

The wealth of firm $i$ is updated to $w_i^t = w_i^{t-1} + \bar{\pi}_i^t$. As will be detailed in section 3.3.6, the firm stays in the market for period $t+1$ if its wealth is greater than a parameter representing the minimum necessary to survive: $w_i^t \geq \underline{W}$.

## 3.3 Dynamics of the model

This section describes 6 steps through which the simulation passes for every period $t \in [1, T]$, where $T$ is a constant parameter.

### 3.3.1 Step 1: Turbulence in technological environment

In the beginning of each period there is a probability of change in the optimal technology. Chang (2015a) uses the term "technological shock" to denote this unpredictable turbulence that affect firms' marginal costs. This shock has a limit in magnitude defined by the parameter $g$. In other words, the new optimal technology can have a maximum of $g$ tasks (bits) different from the previous one.

Consider a binary vector $\underline{\hat{z}}^{t-1} \in \{0,1\}^N$ representing the optimal technology in period $t-1$. Define $\delta\left(\underline{\hat{z}}^{t-1}, g\right) \subset \{0,1\}^N$ as the set of points that are at an exact Hamming distance $D = g$ from $\underline{\hat{z}}^{t-1}$. Therefore, the set of points that are within a Hamming distance $D \in [0, g]$ is:

$$\Delta\left(\underline{\hat{z}}^{t-1}, g\right) \equiv U_{i=0}^{g} \delta\left(\underline{\hat{z}}^{t-1}, i\right) \tag{3.3.1}$$

Furthermore, define the parameter $\gamma$ as the probability of a technological shock occurring. From Chang (2015a) the optimal technology at period $t$ will be:

$$\underline{\hat{z}}^{t} = \begin{cases} \underline{\hat{z}}' & \text{with probability } \gamma \\ \underline{\hat{z}}^{t-1} & \text{with probability } 1-\gamma \end{cases} \tag{3.3.2}$$

where $\underline{\hat{z}}' \in \Delta\left(\underline{\hat{z}}^{t-1}, g\right)$ is randomly selected according to the uniform distribution. Parameters $g$ and $\gamma$ are constant through all periods.

### 3.3.2 Step 2: R&D decisions

After the occurrence of a technological shock has been processed, we move to the decisions about R&D. Firms that survived from period $t-1$ (see section 3.3.6) continue in the market in period $t$. Their first decision in each period regards to if and how they will invest in research.

Each firm when entering the market for the first time is endowed with a random technology that can be more or less similar to the optimal. To reduce their costs (i.e., to have a technology more similar to the optimal), firms can invest in R&D. Chang (2015a) implemented this mechanism in two ways:

#### (A) Innovation

A firm can innovate by choosing a task and changing its method. Since there are two possible methods (0 or 1) for each task, two scenarios can happen. First, if the chosen task was different from the optimal, when the firm modifies it, it becomes equal to the optimal. Second, if the task was equal to the optimal, when the firm modifies it, it becomes different from the optimal.

To illustrate, consider that the firm's technology in period $t$ is 1110010110, where each digit is a bit that defines if a task is done by method 0 or 1. Assume that the optimal technology in period $t$ is 1000101001. If the firm decides to change task number 2 (from left to right), it will flip from 1 to 0. As a result, the firm's technology is now 1**0**10010110 and it became more similar to the optimal. Before the change, the firm's technology Hamming distance to the optimal was 8 (i.e., 8 tasks was being done with the non-optimal method). After the change, the Hamming distance decreased to 7. Since in this example the total number of tasks $N$ is 10, the marginal cost $c_i^t$ went from 80 to 70. The firm succeeded in experimenting with innovation.

Differently, if the firm decided to change task number 4, it would flip from 0 to 1. However, in the optimal technology, task 4 had method 0. Therefore, task 4 was optimal before the change. As a result, the Hamming distance, which was 8 before the change, became 9. The marginal cost rose from 80 to 90. This means that the firm failed in experimenting with innovation.

Regardless of whether the innovation is successful or not, the firm has a fixed cost in the project. It is defined by a parameter $K_{IN}$ and kept constant for all firms through all periods. In case of success, the firm keeps the new method. If, however, it fails, the firm reverts the change and keeps the technology it had before the experiment, although the R&D cost is irrecoverable.

The firm chooses a task to innovate randomly according to the uniform distribution.

**(B) Imitation**

Another way to invest in R&D is through imitation. A firm $i$ observes some competitor $k$ and copy one of its tasks. To choose a competitor, firm $i$ considers only the profitable incumbents in period $t-1$ (i.e., $\pi_k^{t-1} > 0$). Given a set $S^{t-1}$ containing all firms in the market in period $t-1$, consider the subset $S_+^{t-1} \subset S^{t-1}$ with only those firms earning profits in $t-1$. Firm $i$ will choose a competitor from $S_+^{t-1}$ using a random selection with a Roulette Wheel algorithm. The algorithm assigns a probability $p_{ij}^t$ for each competitor $j \in S_+^{t-1}, j \neq i$ to be observed by firm $i$ in period $t$. Probability $p_{ij}^t$ will be the share of firm $j$ in the sum of profits of all competitors $k \in S_+^{t-1}, k \neq i$. Therefore, the likelihood of being imitated increases with profits—relative to other competitors—in the preceding period.

$$p_{ij}^t = \frac{\pi_j^{t-1}}{\displaystyle\sum_{\forall k \in S_+^{t-1}, k \neq i} \pi_k^{t-1}} \qquad \forall j \in S_+^{t-1}, j \neq i \tag{3.3.3}$$

This implies that $\displaystyle\sum_{\forall j \in S_+^{t-1}, j \neq i} p_{ij}^t = 1 \forall i \in S^{t-1}$. In my implementation, I used a Roulette Wheel algorithm to create a probability density function with all competitors and select one random point.

After a competitor $k$ is selected for imitation, firm $i$ chooses a random task according to the uniform distribution and copy its method. As in the innovation, firm $i$ will adopt the new technology if it is more efficient (i.e., results in a lower marginal cost). Otherwise, firm $i$ reverts to the technology it had before the imitation. There is an irrecoverable fixed cost $K_{IM}$ incurred in imitating a competitor.

It is assumed that imitation is cheaper than innovation, i.e. $K_{IM} < K_{IN}$

**Adopting a new technology**

Chang (2015a) formalizes the adoption or rejection of the technology obtained by R&D as follows:

Define $\underline{\tilde{z}}_i^t$ as the new technology being evaluated for adoption—obtained either by innovation or imitation—, $\underline{z}_i^{t-1}$ as technology possessed by firm $i$ in the previous period, $\underline{\hat{z}}^t$ as the optimal technology in current period, $c_i^t$ as the marginal cost function of firm $i$ defined in equation 3.1.2. Then, the technology adopted by firm $i$ in period $t$ will be

$$\underline{z}_i^t = \begin{cases} \underline{\hat{z}}_i^t & \text{if and only if } c_i^t\left(\underline{\hat{z}}_i^t, \underline{\hat{z}}^t\right) < c_i^t\left(\underline{z}_i^{t-1}, \underline{\hat{z}}^t\right); \\ \underline{z}_i^{t-1} & \text{otherwise.} \end{cases} \tag{3.3.4}$$

In Chang (2015a) the investment in R&D of firm $i$ in period $t$ will be:

$$I_i^t = \begin{cases} 0 & \text{if no R\&D was pursued;} \\ K_{IN} & \text{if R\&D was pursued and innovation was chosen;} \\ K_{IM} & \text{if R\&D was pursued and imitation was chosen.} \end{cases} \tag{3.3.5}$$

**Learning and decision-making**

Each firm has to make, in every period $t$, decisions regarding R&D. Since investment in research is optional, a firm decides firstly to either pursue R&D or not. Subsequently, if deciding to proceed, the firm chooses between *innovation* and *imitation*. Both decision processes use probabilities assigned to each firm individually. I explain in sequence those probability functions.

**1. Firm $i$ decides if invests in research**

Firm $i$ will consider investing in R&D only if it has enough wealth to cover the most costly scenario. The cost would be $K_{IN}$ to innovate or $K_{IM}$ to imitate. Therefore, a necessary condition is:

$$w_i^{t-1} \geq max\{K_{IN}, K_{IM}\} \tag{3.3.6}$$

where $w_i^{t-1}$ is the wealth possessed by firm $i$ after competing (i.e., profits earned or losses incurred) in previous period.

Passing the wealth condition, firm $i$ decides randomly according to its own individual probability function $\alpha_i^t$:

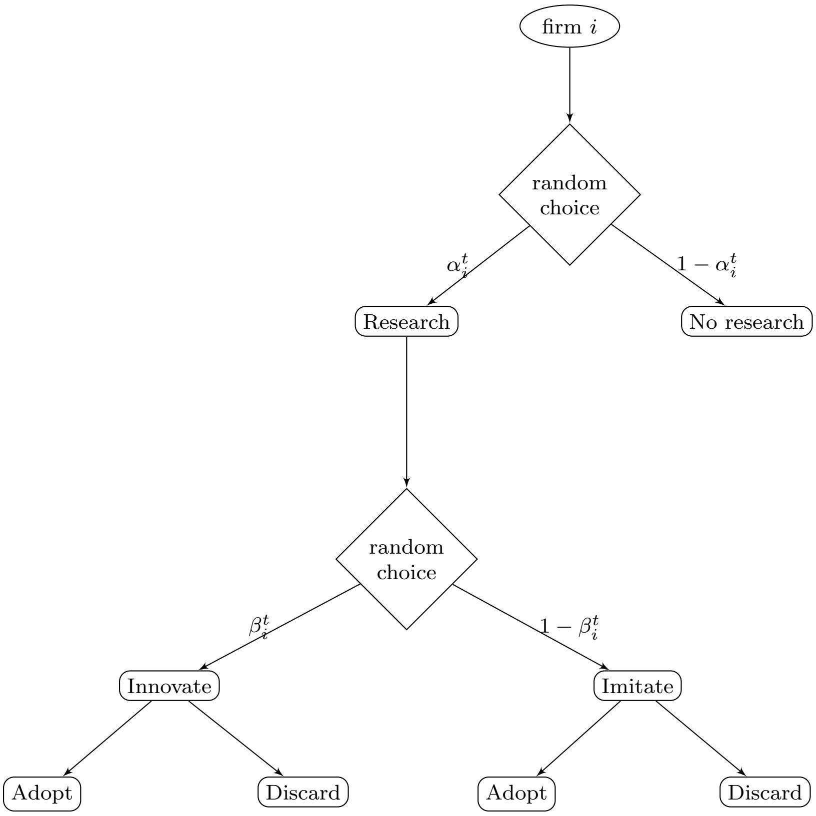



**Figure 1:** Flow chart representing the decision process of a firm regarding R&D. Based on a similar chart from Chang (2015a).

$$\text{firm } i \text{ will invest in research} = \begin{cases} \mathit{YES} & \text{with probability } \alpha_i^t; \\ \mathit{NO} & \text{with probability } 1-\alpha_i^t. \end{cases} \tag{3.3.7}$$

If firm $i$ decides to pursue R&D, it passes to the next decision process.

**2. Firm $i$ decides between *innovation* and *imitation***

Firm $i$ decides between the two options according to its individual probability function $\beta_i^t$:

$$\text{firm } i\text{'s choice} = \begin{cases} \mathit{Innovation} & \text{with probability } \beta_i^t; \\ \mathit{Imitation} & \text{with probability } 1-\beta_i^t. \end{cases} \tag{3.3.8}$$

The complete decision process is illustrated in figure 1 .

**Learning**

Both probabilities $\alpha_i^t$ and $\beta_i^t$ are defined according to firm $i$'s past experiences. Chang (2015a) adopts a version of a learning rule called *Experience-Weighted Attraction (EWA)*, which was described in Camerer and Hua Ho (1999). This rule assigns to each firm attractions to determined paths.

To understand the EWA rule, consider a situation in which an agent $j$ has two alternative paths $X$ and $\bar{X}$ to choose from in period $t$, and its current attraction to each path are $X_j^t$ and $\bar{X}_j^t$ respectively. If $j$ chooses $X$ and has a positive payoff, its attraction to $X$ in the next period will be $X_j^{t+1} = X_j^t + 1$. If, otherwise, $j$ has a negative payoff, its attraction to the alternative path $\bar{X}$ in the next period will be $\bar{X}_j^{t+1} = \bar{X}_j^t + 1$. In a decision process taking place in period $t$, the probability that agent $j$ will choose path $X$ is $p_j^t(X) = X_j^t/(X_j^t + \bar{X}_j^t)$, and the probability that it will choose the alternative path $\bar{X}$ is $p_j^t(\bar{X}) = \bar{X}_j^t/(X_j^t + \bar{X}_j^t) = 1 - p_j^t(X)$.

As in Chang (2015a), consider $A_i^t$ as the attraction of firm $i$ to pursue R&D, and $\bar{A}_i^t$ as the attraction *not* to pursue R&D. The probability of firm $i$ choosing to do research in period $t$ is:

$$\alpha_i^t = \frac{A_i^t}{A_i^t + \bar{A}_i^t} \tag{3.3.9}$$

Similarly, if firm $i$ chooses to do research, the probability that it will choose innovation instead of imitation is:

$$\beta_i^t = \frac{B_i^t}{B_i^t + \bar{B}_i^t} \tag{3.3.10}$$

Conversely, the probability that firm $i$ will choose imitation is $1 - \beta_i^t \equiv \frac{\bar{B}_i^t}{B_i^t + \bar{B}_i^t}$.

After making a decision, we have the following possible scenarios and their respective learning consequences according to whether the research is successful—i.e., generates a technology with a lower marginal cost—or not:

**A. Firm $i$ chooses research with innovation and is successful:**

$$A_i^{t+1} = A_i^t + 1 \text{ and } B_i^{t+1} = B_i^t + 1. \qquad (3.3.11)$$

**B. Firm $i$ chooses research with innovation and is unsuccessful:**

$$\bar{A}_i^{t+1} = \bar{A}_i^t + 1 \text{ and } \bar{B}_i^{t+1} = \bar{B}_i^t + 1. \qquad (3.3.12)$$

**C. Firm $i$ chooses research with imitation and is successful:**

$$A_i^{t+1} = A_i^t + 1 \text{ and } \bar{B}_i^{t+1} = \bar{B}_i^t + 1. \qquad (3.3.13)$$

**D. Firm $i$ chooses research with imitation and is unsuccessful:**

$$\bar{A}_i^{t+1} = \bar{A}_i^t + 1 \text{ and } B_i^{t+1} = B_i^t + 1. \qquad (3.3.14)$$

**E. Firm $i$ chooses not to invest in research:** no changes in attractions.

Chang (2015a) also sets all firms attractions equal to 10 for their first period in the market:

$$A_i^t = \bar{A}_i^t = B_i^t = \bar{B}_i^t = 10 \, \forall i \quad \text{when } t = \text{ firm } i\text{'s first period.} \qquad (3.3.15)$$

Therefore, firms are initially indifferent between the paths and have initial probabilities $\alpha_i^t = 0.5$ and $\beta_i^t = 0.5$. These attractions will change over time according to each firm's learning process.

### 3.3.3 Step 3: Entry decisions

We now direct our attention to the new firms that will potentially enter the market. Consider $R^t$ the set of those firms. For every period $t$, $R^t$ will be composed by a fixed number $r$ of new firms. Each firm $i \in R^t$ enters the market if they expect to make a profit in current period. Firm $i$ makes a prediction about the equilibrium price by obtaining information from the market in the previous period. We derived the equilibrium price in period $t$—see equation 3.2.7—as depending only on the number of incumbent firms $m^t$ and on each incumbent $j$'s marginal cost $c_j^t$.

Consider a set $S^{t-1}$ containing all incumbent firms during period $t-1$. Consider also a subset $S_A^{t-1} \subset S^{t-1}$ such that $S_A^{t-1} = \{j \in S^{t-1} | q_j^{t-1} > 0\}$ containing only active firms (i.e., producing a positive amount of goods) in period $t-1$. In order to estimate price in period $t$, a potential entrant $i$ assumes $m^t = |S_A^{t-1}| + 1$. In other words, firm $i$ believes that only active firms from the previous period plus itself will compete in period $t$. Therefore, firm $i$ assumes to be the only new entrant.

The second necessary information in estimating the price is the marginal cost of each incumbent. Two assumptions are made: (1) potential entrant $i$ knows the exact equilibrium price in $t-1$ and (2) it can observe the exact quantity produced $q_j^{t-1} \forall j \in S_A^{t-1}$. With these information and the knowledge of the economy's parameter $s^{t-1}$ (market size), equation 3.2.8 can be solved to give marginal cost $c_j^{t-1} \forall j \in S_A^{t-1}$. Potential entrant $i$ assumes that every incumbent will maintain its marginal cost from period $t-1$—this will likely be inaccurate since a technological shock might have occurred (section 3.3.1), and incumbents might have successfully pursued R&D (section 3.3.2).

After obtaining the number of firms—including itself—and each firm's marginal cost—including its own—, potential entrant $i$ estimates equilibrium price $\bar{P}^{te}$, its optimal output $\bar{q}_i^t$ and its expected profits $\bar{\pi}_i^{te}$. Considering parameters $b$ and $\underline{W}$ as representing, respectively, the initial wealth and the minimum wealth to survival, firm $i$ enters the market if:

$$\bar{\pi}_i^{te} + b \geq \underline{W} \tag{3.3.16}$$

Observe that parameter $b$ can be negative to represent an entry cost. Firms do not perform R&D the same period in which they enter the market.

### 3.3.4 Step 4: Shutdown decisions

At this point our simulation have processed the technological shock, the R&D of previous incumbents, and the entry of new firms. However, not all firms will produce a positive amount. Remember that each new firm assumed it was the only one to enter, although we had a pool of $r$ potential entrants. It is likely that many potential entrants will overestimate the equilibrium price and, by deciding to enter, will exert a downwards pressure. At the same time, previous incumbents might be taken by surprise by a technological shock, by competitors' successful R&D, and by the entry of new competitors. All these uncertain factors can result in an equilibrium price below of some incumbents' marginal cost. These inefficient competitors will shutdown for the current period—i.e., become *inactive*. Even if a firm decides to shutdown, it cannot avoid the fixed cost $f$. Chang (2015a) provides a concise description of the algorithm:

> Starting from the initial set of active firms, compute the equilibrium outputs for each firm. If the outputs for one or more firms are negative, then deactivate the least efficient firm from the set of currently active firms, i.e., set $q_i^t = 0$ where $i$ is the least efficient firm. Redefine the set of active firms (as the previous set of active firms minus the deactivated firms) and recompute the equilibrium outputs. Repeat the procedure until all active firms are producing non-negative outputs.
>
> Chang (2015a)

### 3.3.5 Step 5: Production decisions

Only the firms that decided to be *active* in period $t$ will produce a positive amount $q_i^t > 0$ and receive revenues. They might still incur in a loss if revenues $\bar{P}^t \cdot q_i^t$ minus variable costs $c_i^t \cdot q_i^t$ are not enough to cover fixed costs $f$.

All firms, *active* and *inactive*, have their wealth updated according to the short-term equilibrium in section 3.2, and proceed to the final step in which they will decide if continue in the market during next period.

#### 3.3.6 Step 6: Exit decisions

Each incumbent firm $i$ in period $t$ leaves the market if $w_i^t < \underline{W}$. Otherwise, firm $i$ survives to period $t+1$ and maintains its technology, wealth and attractions.

The simulation then stores the data for analysis and return to Step 1 for the next period.

### 3.4 Main results

This section presents some results obtained from the software I implemented using Chang's model. Although my understanding of the model came from studying Chang (2009, 2010, 2011, 2015a, 2015b), I focus most of my analysis on different variables than those of the references.

#### 3.4.1 Baseline parameters

This baseline simulation was run according to the parameters set in Chang (2015a), chapter 4.2:

| Notation | Description | Value |
|---|---|---|
| $N$ | Number of tasks | 96 |
| $r$ | Number of potential entrants per period | 40 |
| $b$ | Initial wealth of a new entrant | 0 |
| $\underline{W}$ | Minimum wealth to survival | 0 |
| $a$ | Demand intercept | 300 |
| $f$ | Fixed cost | 200 |
| $K_{IN}$ | Fixed cost of innovation | 100 |
| $K_{IM}$ | Fixed cost of imitation | 50 |
| $A_i$ *initial* | Attraction to pursue R&D | 10 |
| $\bar{A}_i$ *initial* | Attraction *not* to pursue R&D | 10 |
| $B_i$ *initial* | Attraction to innovation | 10 |
| $\bar{B}_i$ *initial* | Attraction to imitation | 10 |
| $T$ | Time horizon of the simulation | 5000 |
| $s^t = \hat{s}$ *constant* | Market size | 4 |
| $\gamma$ | Probability of a technological shock to occur | 0.1 |
| $g$ | Maximum magnitude of technological change during a shock | 8 |

Table 1: baseline parameters.

Based on Chang (2015a), I ran an experiment with 500 independent simulations. This means that each simulation generates its own state variables independently—with fresh

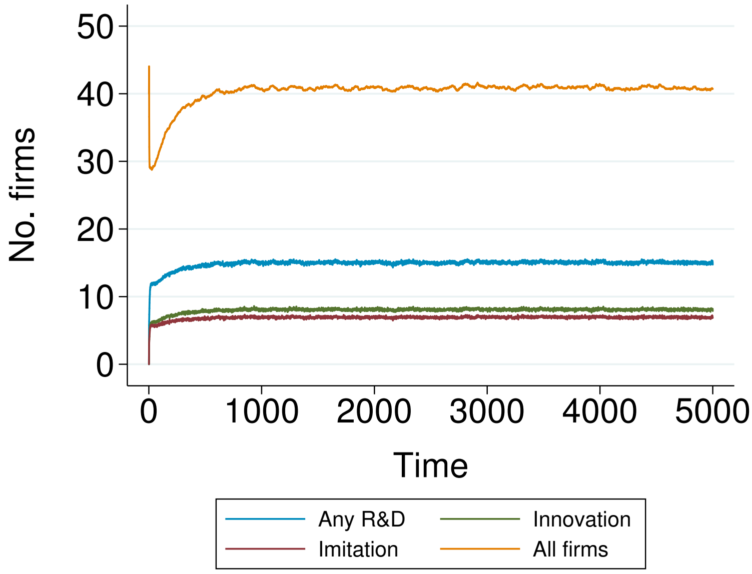



**Figure 2:** Simulation using baseline parameters. number of firms in the market (orange), number of firms pursuing R&D (blue), number of firms innovating (green), number of firms imitating (red).

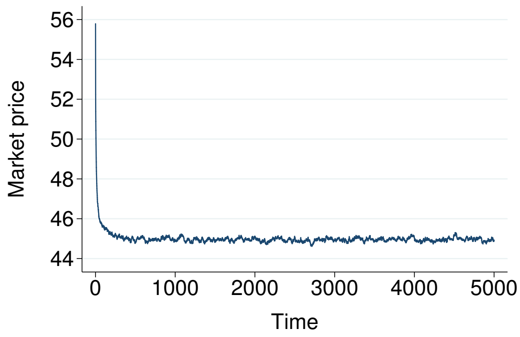
random numbers—, starting with the same initial parameters. The resulting data is an aggregate time series for each variable averaged over all the 500 executions.

**Averaging multiple simulations** Consider $X_s^t$ to be the value of a random variable $X$ collected in period $t$ for each independent simulation $s$. The values plotted in the time series for each period $t$ are $\bar{X}^t = \frac{1}{500}\sum_{s=1}^{500} X_s^t$.

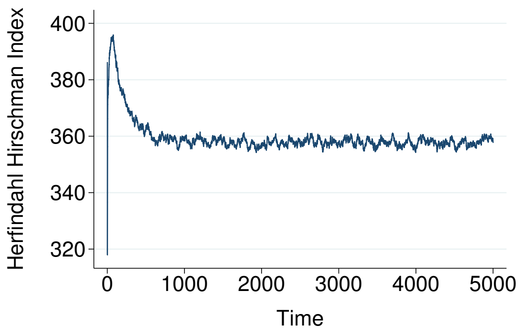
Figure 2 shows the evolution of the number of incumbent firms in the market. It also plots the number of firms pursuing R&D and how many chose either *innovation* or *imitation*. As in Chang (2015a), the number of incumbent firms reaches a steady-state after period $t \simeq 1000$ and fluctuates around 41. Furthermore, we observe an steady-state of 15 firms pursuing R&D, with *innovation* slightly higher than *imitation*.

Figure 3a presents the short-term equilibrium price $\bar{P}^t$ for each period. Figure 3b shows the market concentration defined by the Herfindahl-Hirschman index (HHI) scaled by 10.000. From Chang (2015a), the index is calculated as follows:

$$H^t = \sum_{\forall i \in M^t} \left( \frac{q_i^t}{Q^t} \cdot 100 \right)^2 \tag{3.4.1}$$




(a) $\bar{P}^t$, short-term equilibrium market price.




(b) $H^t$, market concentration as measured by the Herfindahl-Hirschman index (HHI).

**Figure 3:** Simulation using baseline parameters

Figure 4a shows how close the industry is to the optimal technology on average. I defined this index as:

$$PRO^t = \frac{1}{M^t}\left[\sum_{\forall i \in M^t}\left(1 - \frac{D\left(\underline{z}_i^t, \hat{\underline{z}}^t\right)}{N}\right)\right] \tag{3.4.2}$$

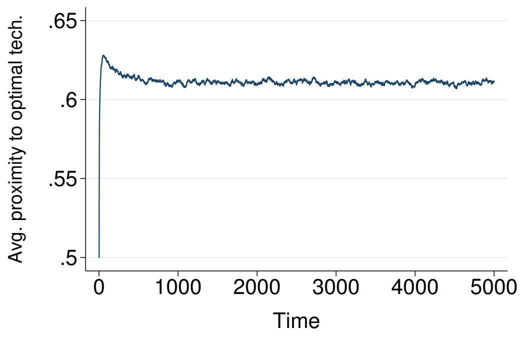
Note that $PRO^t$ has a steady-state around 61% in a scale from 0 to 100%. This value decreases with an increase in $y$ or $g$ because technological shocks are an obstacle for firms to approach the optimal. In section 3.4.3 we will see that $PRO^t$ tends to 100% in absence of shocks.

Figure 4b shows the industry marginal cost calculated by an weighted average as in Chang (2015a):

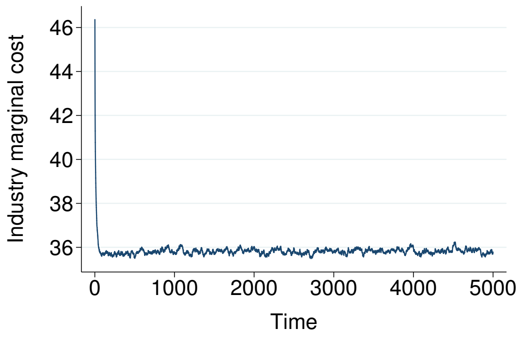
$$WMC^t = \sum_{\forall i \in M^t}\left[\left(\frac{q_i^t}{Q^t}\right) \cdot c_i^t\right] \tag{3.4.3}$$

Figure 5a shows the industry price-cost margin weighted by market share defined in Chang (2015a) as:

$$PCM^t = \sum_{\forall i \in M^t}\left[\left(\frac{q_i^t}{Q^t}\right) \cdot \left(\frac{P^t - c_i^t}{P^t}\right)\right] \tag{3.4.4}$$

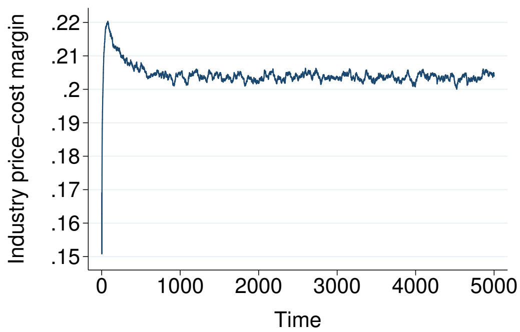



**(a)** $PRO^t$, Average proximity to optimal technology.

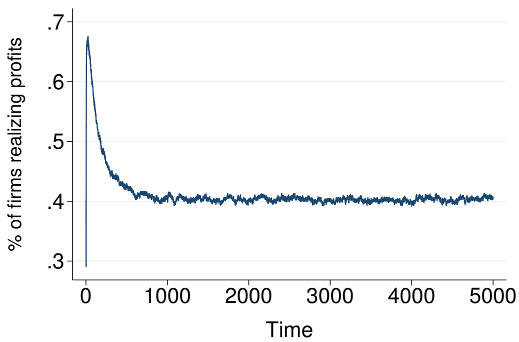



**(b)** $WMC^t$, Weighted marginal cost.

**Figure 4:** Simulation using baseline parameters




**(a)** $PCM^t$, Weighted price-cost margin.



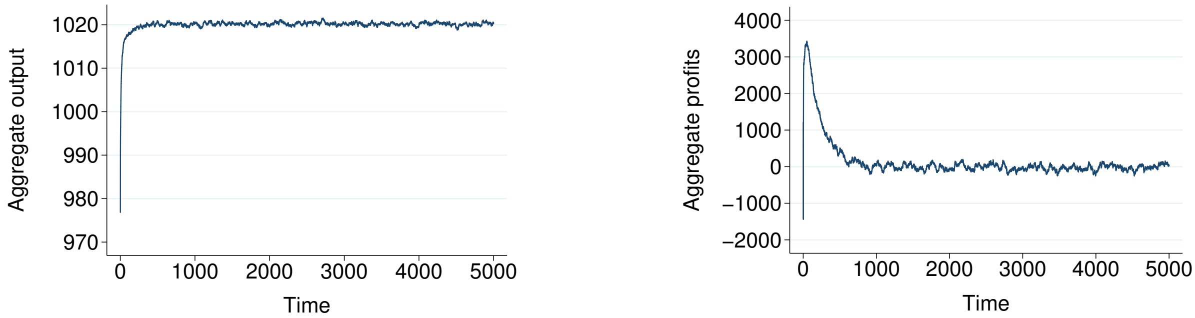

**(b)** Ratio of profitable firms.

**Figure 5:** Simulation using baseline parameters

Figure 5b exhibits the percentage of firms that earned positive profits in period $t$. Figure 6a shows the total output and Figure 6b shows total profits. As expected in a market with free-entry, total profits has a steady-state around 0 after accounting for fixed and variable costs and investment in R&D.

### 3.4.2 Business cycles

In Chang (2015a), chapter 8, there is an experiment with business cycles. This is implemented by a variation in parameter $s^t$ (market size) from one period to another. The simulation can generate either *deterministic* or *stochastic* cycles according to the choice of the researcher. In both cases, the market size is kept constant for the first 2000 periods to allow the market to reach a steady-state.




**(a)** Industry total output. **(b)** Industry total profits.

**Figure 6:** Simulation using baseline parameters

In case of a *deterministic* cycle, $s^t$ is guided by a sinusoidal function:

$$s^t = \begin{cases} \hat{s} & \text{for } 1 \leq t \leq 2000; \\ \hat{s} + \sigma \sin\left[\frac{\pi}{\tau} \cdot t\right] & \text{for } t \geq 2001. \end{cases} \tag{3.4.5}$$

where $\hat{s}$ is the parameter defining a mean (remember that in Table 1 $\hat{s}$ was used to define a constant $s^t$) around which the market size cycles. $\pi$ and parameters $\sigma$ and $\tau$ define the shape of the cycle.

In case of a *stochastic* cycle, Chang (2015a) sets $s^t$ as follows:

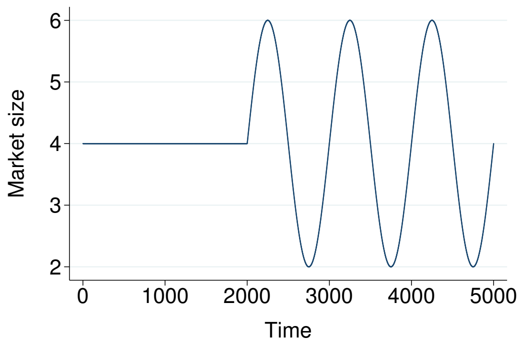
$$s^t = \begin{cases} \hat{s} & \text{for } 1 \leq t \leq 2000; \\ \max\{0.1, (1-\theta)\hat{s} + \theta s^{t-1} + \epsilon^t\} & \text{for } t \geq 2001. \end{cases} \tag{3.4.6}$$

where $\theta$ is "rate of persistence in demand" and $\epsilon^t$ is the "random noise" uniformly distributed between $-0.5$ and $+0.5$. (Chang, 2015a)

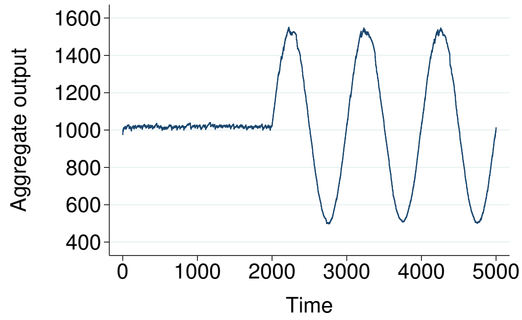
In this section I make a superficial analysis of the results. Chang (2015a) studies these and other aspects thoroughly.

### Results with deterministic cycles

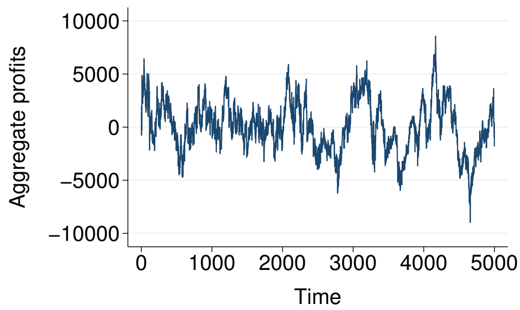
I executed one single simulation with the following specific parameters—all the others




**(a)** $s^t$, market size.




**(b)** Industry total output.




**(c)** Industry total profits.

**Figure 7:** Simulation with deterministic business cycles

were set as in Table 1:

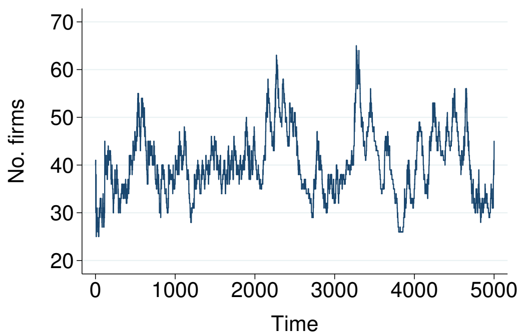
| Notation | Description | Value |
|---|---|---|
| $\hat{s}$ | Mean market size | 4 |
| $\sigma$ | Wave amplitude | 2 |
| $\tau$ | Wave period of half turn | 500 |

Table 2: parameters in deterministic cycles.

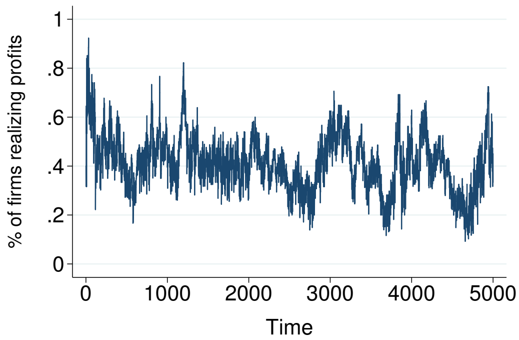
Figure 7a exhibits the market size through time. As we can observe, each cycle has a deterministic form. From figure 7b it is clear that the industry output moves with the cycle as firms adjust their quantities to the new demand. In a similar result, figure 7c exhibits total profits increasing during a boom and decreasing during a bust—a clearer effect could be observed by plotting the averages of 500 replications.

From figure 8a we observe that the number of firms moves with the cycle. This is explained by the free-entry aspect of the model, which allows the number of new entrants to adjust to the demand. Furthermore, figure 8b shows that a higher percentage of firms earns positive profits during a boom than during a bust.

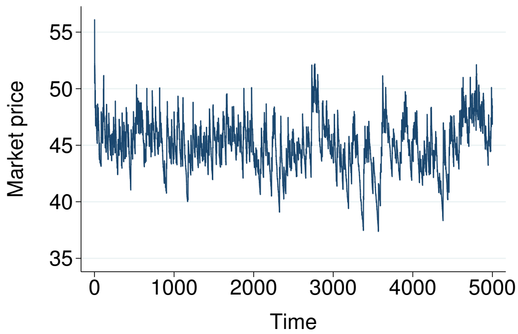
With respect to market price $\bar{P}^t$, figure 8c exhibits an inverse pattern. Price goes down during a boom, as a result of a higher number of competitors, and goes up during a bust. Once again, an unequivocal negative relation could be observed from the averages of 500 independent replications.




**(a)** $M^t$, number of firms in the market.




**(b)** Ratio of profitable firms.




**(c)** $\bar{P}^t$, market price.

**Figure 8:** Simulation with deterministic business cycles

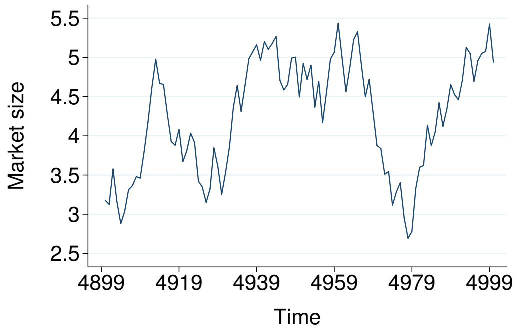
### Results with stochastic cycles

Since stochastic cycles have an unpredictable path that can change direction abruptly, it is more difficult to observe the effects on endogenous variables.

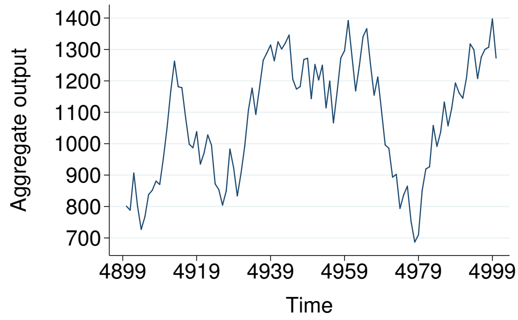
To make an attempt, I generated graphs with the last 100 periods in a single simulation. The specific parameters were set as:

| Notation | Description | Value |
|---|---|---|
| $\hat{s}$ | Mean market size | 4 |
| $\theta$ | Rate of persistence in demand | 0.95 |

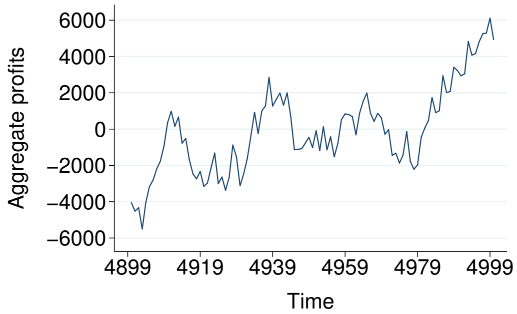
Table 3: parameters in stochastic cycles.



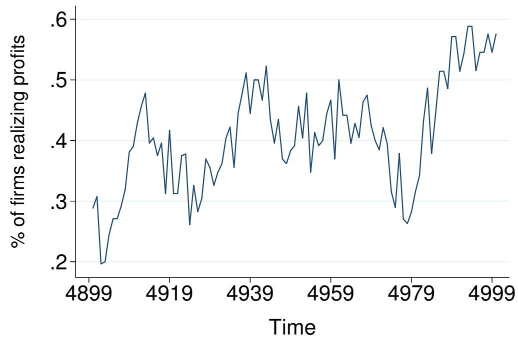

**(a)** $s^t$, market size.




**(b)** Industry total output.




**(c)** Industry total profits.




**(d)** Ratio of profitable firms.

**Figure 9:** Simulation with stochastic business cycles

Figure 9a plots the market size. It exhibits an unpredictable path. Figures 9b, 9c, 9d show a positive relation between market size, output and profits as with *deterministic* cycles. However, we cannot establish a relationship between market size, number of firms and price by looking at the graph of a single replication because of the large short-term variation. In this case, the only way to observe a causal link is by averaging 500 simulations as done by Chang (2015a).

### 3.4.3 No turbulence in technological environment

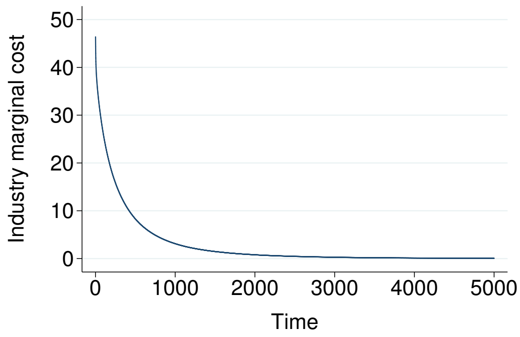
A different experiment can be realized by using the same baseline parameters (see Table 1) except for the technological shocks. Setting $\gamma = 0$ generates a single optimal technology, which is maintained through all periods.

As expected, figure 10a demonstrates that, in the absence of technological shocks, firms approach the optimal through R&D and bring marginal cost to 0. As a consequence, competition reduces the price until it stabilizes (figure 10b). The reason why price does not converge to 0 is that a small number of firms achieve a very high degree of efficiency, which makes it impossible for new entrants to compete. Remember that a new entrant can have any technology that gives, uniformly, a marginal cost between 0 and 100. As a consequence, new competitors will have, on average, a marginal cost equal to 50. When the price is as low as in figure 10b, it becomes unlikely that a new entrant will have a technology efficient enough to generate profit.

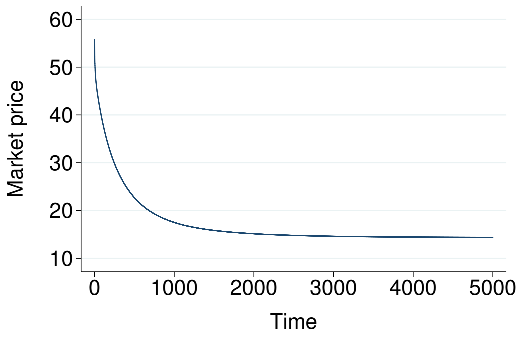
Figure 11 shows the number of firms stabilized in 20, and the number of new entrants dropping to 0 very quickly. A different experiment could show a slower convergence of the number of new entrants by making R&D more likely to fail. This would keep the marginal costs higher for more time and give some space for new competitors. Nevertheless, the eventual convergence to 0 new entrants is inevitable in the absence of turbulence in technological environment.

Substituting $M^t = 20$ in equations 3.2.7 and 3.2.6, and setting $c_i^t = 0 \forall i \in [1, M^t]$ gives the same price and output as shown in the graphs.

Finally, figure 12b shows a progressive decrease in R&D towards zero. Remember that section 3.3.2 describes a process of continuous learning in which successful investments increase the attraction to R&D ($A_i^t$), thus increasing the probability $\alpha_i^t$ of firm $i$ choosing to do research. Conversely, failed investments decrease the probability of pursuing R&D. As a firm approaches the optimal, the likelihood that, by modifying a random task, technology will improve decreases, since there are less inefficient tasks to be chosen. Therefore, research projects tend to fail more often, leading to lower investments as time progresses.




**(a)** $WMC^t$, Weighted marginal cost.




**(b)** $\bar{P}^t$, price.

**Figure 10:** Simulation with no turbulence in technological environment

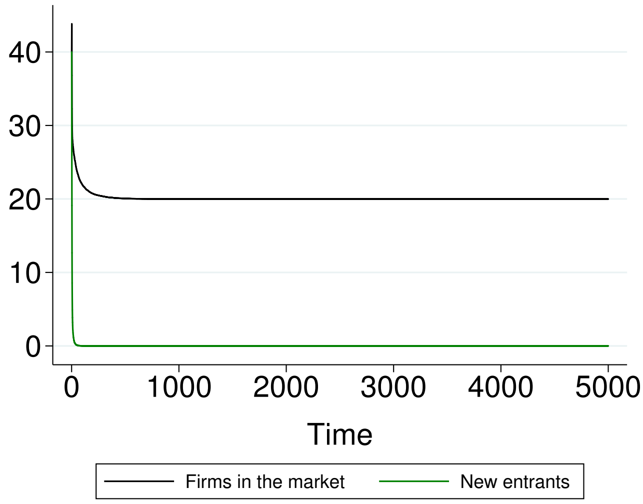
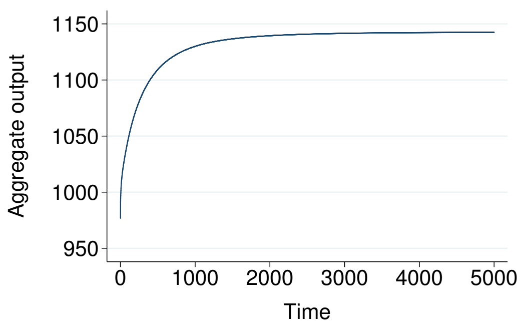
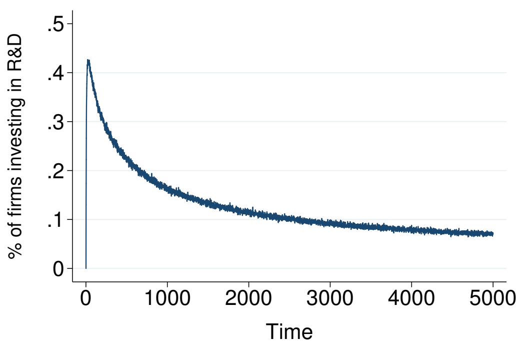



**Figure 11:** Simulation with no turbulence in technological environment. Number of firms in the market (black), number of new entrants (green).




**(a)** Total output




**(b)** Ratio of firms investing in R&D

**Figure 12:** Simulation with no turbulence in technological environment

# 4 Some modifications to the baseline model

In this section I describe two small modifications in the baseline model. The objective is to observe the mechanisms in work by forcing anomalies in the simulation environment.

## 4.1 Occurrence of large shocks

The first modification is about the occurrence of abnormally large shocks in constant intervals. We observe immediate and lagged effects on the market, with firms being directly affected by the shock.

To formalize, starting from the specification in section 3.3.1, consider a new parameter $\lambda$ representing the interval of occurrence of large shocks and another parameter $\sigma$ defining the maximum magnitude of a large shock, which is greater than that of a normal shock ($\sigma > g$). Redefine $\underline{\hat{z}}'$ as being randomly selected from a set—uniform distribution—as follows:

$$\underline{\hat{z}}' \in \begin{cases} \Delta\left(\underline{\hat{z}}^{t-1}, \sigma\right) & \text{if } (t \geq \lambda) \text{ and } (t \bmod \lambda = 1); \\ \Delta\left(\underline{\hat{z}}^{t-1}, g\right) & \text{otherwise.} \end{cases} \tag{4.1.1}$$

where mod is the *modulus* operator. After redefining $\underline{\hat{z}}'$, we maintain the original definition of $\underline{\hat{z}}^t$ as in section 3.3.1:

$$\underline{\hat{z}}^t = \begin{cases} \underline{\hat{z}}' & \text{with probability} \quad \gamma; \\ \underline{\hat{z}}^{t-1} & \text{with probability} \quad 1-\gamma. \end{cases} \tag{4.1.2}$$

### 4.1.1 Results

I executed 500 independent simulations with the following parameters:

| Notation | Description | Value |
|---|---|---|
| $\lambda$ | Interval of occurrence of a large shock | 10 |
| $\sigma$ | Maximum magnitude of a technological shock | 24 |

Table 4: parameters in occurrence of large shocks.

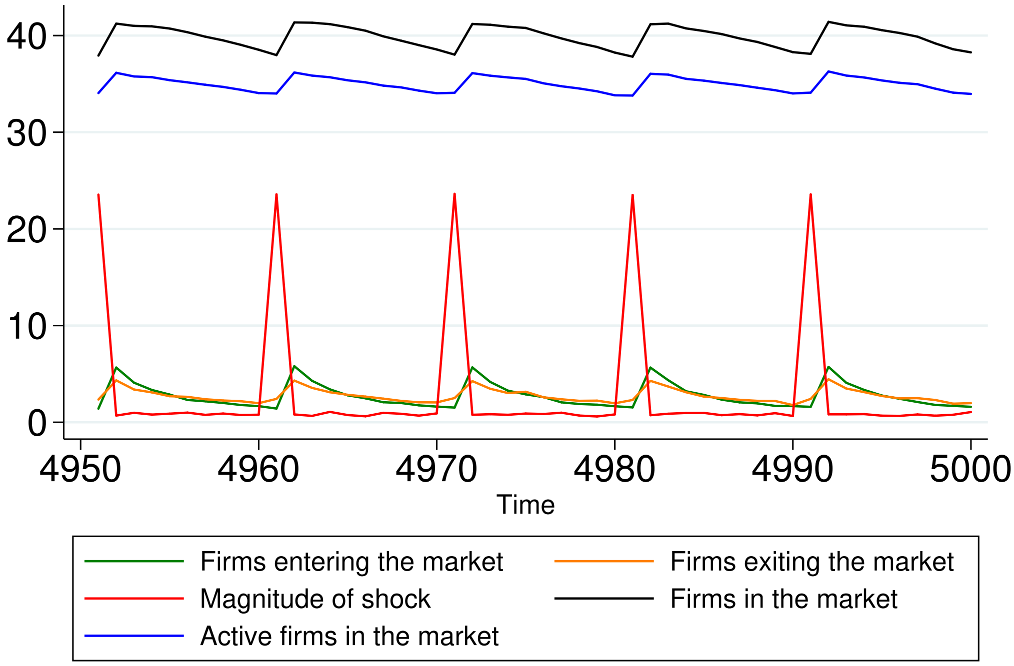



**Figure 13:** Results for 500 replications with occurrence of large shocks every 10 years. Number of incumbent firms (black), number of active incumbents (blue), number of firms entering the market (green), number of firms exiting the market (orange), magnitude of technological shock (red).

All other parameters were set according to the baseline (Table 1). By averaging the variables through all simulations as in 3.4.1, I obtained the following results:

Figure 1 shows a plot for the last 50 years. We observe that between any two large shocks the number of firms that are either entering or exiting decreases slowly, but there is a slightly higher number of exits. This results in a smooth decrease in the number of firms towards some steady-state. This is explained by the fact that some incumbents are progressively forced to exit the market by the preceding shock as their wealth is consumed by continuous losses. This is a result of a sudden increase in marginal costs, which gives no time to adapt through R&D.

However, before a steady-state is reached, a new shock occurs, which causes (i) an instantaneous increase in the number of exits, and (ii) an even higher increase in the number of entries in the subsequent period. The explanation for these phenomena is that, during a large shock, firms that have little wealth cannot survive the sudden increase in costs, therefore they leave the market immediately. However, the shock also causes an increase in prices, which present a good opportunity for potential entrants. Therefore, more firms enter during the subsequent period. This process is repeated every 10 periods.

## 4.2 Multiple optimal technologies

Chang (2009) offered a version of the model with multiple optimal technologies in an environment absent of shocks. In this section, however, I present a small modification of Chang (2015a) in which a new optimal technology is added to the environment at specified intervals (1000 periods) but technological shocks continue to occur. When a new optimal technology arises, preexisting ones continue to be available. There is no special reason for the choice of 1000 periods or for the progressive addition of new optima. However, implemented this way, the simulation allows us to observe the effects of each addition after a steady-state has been reached.

To formalize, remember the definition of marginal cost of firm $i$ at period $t$ defined in equation 3.1.2. In that section, there was an optimal technology $\underline{\hat{z}}^t \in \{0,1\}^N$. Based on this definition, consider a set $\underline{\hat{Z}}^t = \{\underline{\hat{z}}^t \mid \underline{\hat{z}}^t \in \{0,1\}^N\}$ containing all optimal technologies available at time $t$, being $N$ the number of tasks. Given firm $i$'s own technology $\underline{z}_i^t$, its marginal cost will be defined by the Hamming distance to the closest optimal $\underline{\hat{z}}_k^t \in \underline{\hat{Z}}^t$ s.t. $D\left(\underline{z}_i^t, \underline{\hat{z}}_k^t\right) \leq D\left(\underline{z}_i^t, \underline{\hat{z}}_j^t\right) \forall \underline{\hat{z}}_j^t \in \underline{\hat{Z}}^t$. The marginal cost is, therefore:

$$c_i^t\left(\underline{z}_i^t, \underline{\hat{z}}_k^t\right) = 100 \cdot \frac{D\left(\underline{z}_i^t, \underline{\hat{z}}_k^t\right)}{N}. \tag{4.2.1}$$

$\underline{\hat{Z}}^t$ starts with one optimal technology in period $t = 1$. It will receive an addition of an extra optimal at each interval of $\lambda$ periods. Therefore, the number of elements contained in $\underline{\hat{Z}}^t$, in time $t$, is defined by the ceiling operator as $|\underline{\hat{Z}}^t| = \lceil \frac{t}{\lambda} \rceil$. To illustrate, consider $\lambda = 1000$ in a time horizon of 5000 periods:

$$|\hat{\underline{Z}}^t| = \begin{cases} 1 & \text{for } 1 \leq t \leq 1000; \\ 2 & \text{for } 1001 \leq t \leq 2000; \\ 3 & \text{for } 2001 \leq t \leq 3000; \\ 4 & \text{for } 3001 \leq t \leq 4000; \\ 5 & \text{for } 4001 \leq t \leq 5000. \end{cases} \tag{4.2.2}$$

### 4.2.1 Results

I executed 500 independent simulations and averaged the results as in 3.4.1. A new optimal technology was added at each 1000 periods.

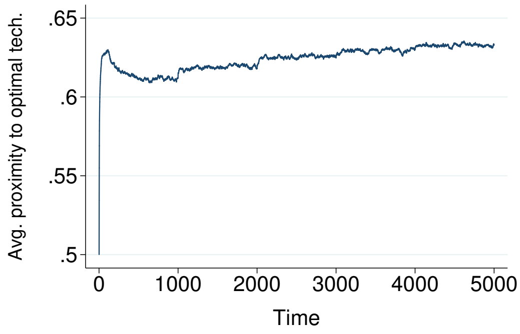
Firms are more likely to obtain a technology similar to optimal when there are multiple possibilities. This happens both naturally during birth, since a firm's initial technology can be closer to one optimum and far from another, and also through R&D, as firms' random walks (innovation) have a greater likelihood to modify a task that then becomes equal to one of the multiple optimal possibilities. As more firms approach one of the multiple optima, R&D by imitation also acquires a higher probability to copy an efficient task.

When an extra optimal technology arises, we observe an increase in the average proximity to any of the possibilities (Figure 14a). This increase in efficiency results in a lower marginal cost (Figure 14b). With lower costs, firms choose a higher output (Figure 14c), causing a decrease in short-term equilibrium price (Figure 14d).

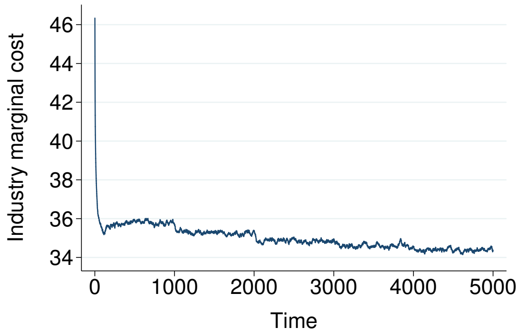
Furthermore, the existence of different paths to efficiency generates a higher technological diversity (Figure 14e) defined as in Chang (2015a):

$$DIV^t = \frac{2}{N|M^t|(|M^t|-1)} \sum_{\forall i,j \in M^t, i \neq j} D\left(\underline{z}_i^t, \underline{z}_j^t\right) \tag{4.2.3}$$

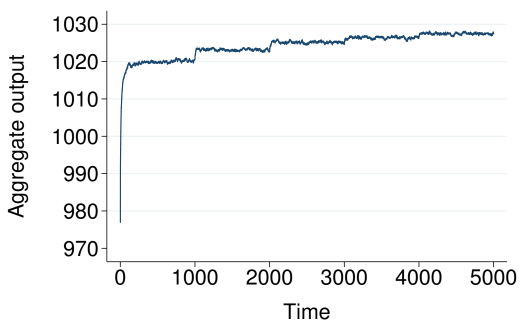
We also observe a decrease in market concentration (Figure 14f). During the simulations, when a new optimum arose, incumbents were taken by surprise. Large firms that had been able to systematically reduce marginal cost and seize market share now observe



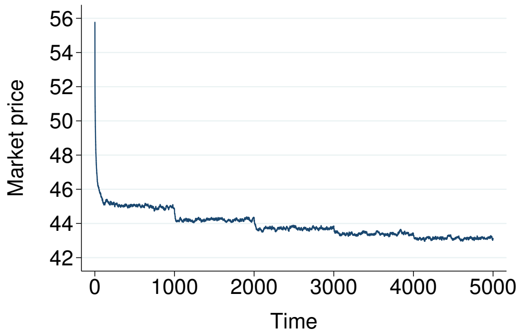

**(a)** $PRO^t$, Average proximity to any of the optimal technologies.



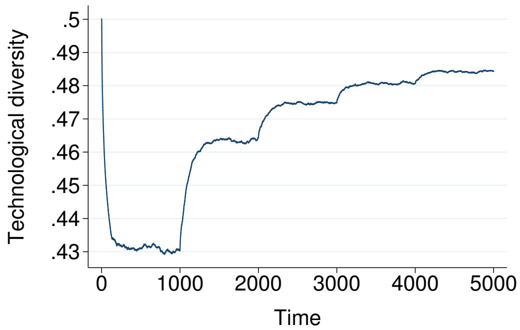

**(b)** $WMC^t$, Weighted marginal cost.



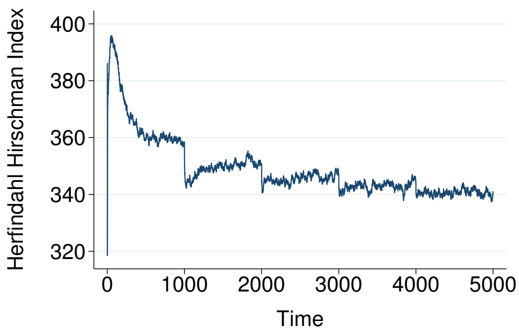

**(c)** Industry total output.




**(d)** $\bar{P}^t$, short-term equilibrium market price.




**(e)** $DIV^t$, Index of technological diversity.




**(f)** $H^t$, market concentration as measured by the Herfindahl-Hirschman index (HHI).

**Figure 14**

some of the inefficient competitors to gain a sudden increase in efficiency. The graph also shows that, after a new optimum is added, concentration starts to increase progressively. This is the effect of R&D competition giving advantage to some firms. In any case, the steady-state HHI is always below the one with less optimal technologies.

# 5 Conclusion

This paper replicates the study of Chang (2015a). It is an implementation of an agent-based model with heterogeneous firms that can invest in research to reduce costs. After analysing the results both with the original model and after modifications, the response of endogenous variables to exogenous interferences were compatible with the expected from economic theory.

With the baseline model, we observed an emergence of a steady-state in all cases even in the presence of technological turbulences. The steady-state is not a static equilibrium but a point around which the variable revolves. In the occurrence of business cycles, we observed firms reacting to the new demand size and generating pro-cyclical output, total profits, number of firms, and percentage of firms earning profits. On the contrary, prices were counter-cyclical as in Chang (2015a).

With the occurrence of large shocks, we observed the number of incumbent firms, the number of firms entering, and the number of firms exiting to react both instantly and with a lag. An abnormally large shock causes an instantaneous increase in exits, but an even bigger increase in entries in the next period.

With the existence of multiple optimal technologies, we observed, as expected, a larger heterogeneity in production methods. Furthermore, as a consequence of higher expected pay-off from R&D, firms manage to reduce marginal costs and set a lower equilibrium price. We also observed a decrease in market concentration.

A possible extension of this model regards the financial aspects, with interest rates, credit and long-term investments. It would be interesting to observe decisions of production, investment and borrowing taking into account the value of money through time, liquidity, risk and expected return in an economy with credit institutions. An obvious challenge would be to model the decisions about R&D considering a longer horizon than the current period.

# 6 Appendix

In this highly optional appendix I present a few parts of a record extracted from one simulation with the baseline parameters in period $t = 1000$—a complete record of a single simulation generates approximately 1.5 million lines. At this period, the industry is already consolidated with very efficient firms, and it is very hard for a potential entrant to have a technology capable of competing. The record shows details of each decision made by firms and gives a concrete notion to those unfamiliar with agent-based models. The format is not didactic because records are used for debugging. Remember that the final objective of a simulation is to generate data to be analyzed by a researcher, and records are too technical for any scientific purpose. Nevertheless, the reader might find interesting to see how complex are the interactions of autonomous agents even in the simplest models as the one used in this paper.

In reading the records, have in mind that in the lines beginning with *FIRM*, the number in sequence is the unique identification of that firm. Since we are in period $t = 1000$, many firms have already lived and died, therefore the unique identifications will be big numbers. In some lines we can observe a sequence of 0 and 1. These are technologies—the number of a task counts from right to left.

——————— PROCESSING PERIOD t=1000 ———————

[FIRM 20960] Decided to NOT pursue R&D. Prob of R&D: 0.32.

[FIRM 21350] Decided to INNOVATE. Prob of R&D: 0.35; Prob of Innovation: 0.54.

[FIRM 21350] Modified task 44

[FIRM 21350] Before: 00100110000100010101100100010111010110001001110110010**0**0100010101100110
0101111101000010011100000011

[FIRM 21350] After: 0010011000010001010110010001011101011000100111011001**1**01000101011001100
1011111010000100111000000 11

[FIRM 21350] Lowered marginal cost through INNOVATION. Previous MC: 36.46; New MC: 35.42

[FIRM 13236] Decided to IMITATE. Prob of R&D: 0.36; Prob of Imitation: 0.43.

[FIRM 13236] Selecting a firm to imitate...

[FIRM 13236] Point in CDF: 0.370

[FIRM 13236] Competitor 33957 has a 0.51% probability of being observed.

[FIRM 13236] Competitor 37737 has a 0.51% probability of being observed.

[FIRM 13236] Competitor 38039 has a 0.51% probability of being observed.

[FIRM 13236] Competitor 37116 has a 2.26% probability of being observed.

[FIRM 13236] Competitor 38592 has a 2.26% probability of being observed.

[FIRM 13236] Competitor 08465 has a 1.59% probability of being observed.

[FIRM 13236] Competitor 35595 has a 4.24% probability of being observed.

[FIRM 13236] Competitor 11401 has a 4.24% probability of being observed.

[FIRM 13236] Competitor 36865 has a 2.91% probability of being observed.

[FIRM 13236] Competitor 32323 has a 4.24% probability of being observed.

[FIRM 13236] Competitor 36822 has a 6.45% probability of being observed.

[FIRM 13236] Competitor 26734 has a 8.89% probability of being observed.

[FIRM 13236] Competitor 26734 was observed. Point in CDF: 0.370.

[FIRM 13236] Current technology: 0101001001011010100100110011111001111011001110100000000000110 011101001010100110000111001010**1**00011.

[FIRM 13236] Technology to imitate: 0100001000011100011010100111011110010010101011100001100001 101111000110111101101010001011100**0**00001.

[FIRM 13236] Task to copy: 6.

[FIRM 13236] New technology: 0101001001011010100100110011111001111011001110100000000000110011 1010010101001100001110010100**0**00011.

[FIRM 13236] Lowered marginal cost through IMMITATION. Previous MC: 34.38; New MC: 33.33.

Entry decisions: Processing...

[FIRM 39961] Potential entrant decided to NOT ENTER. Expected price: 40.87; MC: 45.83.

[FIRM 39962] Potential entrant decided to NOT ENTER. Expected price: 41.26; MC: 60.42.

Shutdown decisions: Processing...

Current active incumbents: 37

Checking if the least efficient firm will deactivate.

[FIRM 17993] Decided to DEACTIVATE. Price: 40.68; MC: 41.67.

Shutdown decisions: OK! 1 firm deactivated.

Updating 36 active firms: OK!

Updating 1 inactive firms: OK!

Firms exiting: 0 left the market.

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