



**Tennessee**  
**TECH**

# **Time Value of Money Overview**





Accept today?  
OR  
Accept in 1 year?

Guaranteed \$2,000!!





# The Time Value of Money

- ❖ Money has time value due to the interest it can earn
- ❖ Compound interest: Interest earned on principal AND on prior periods' interest



# Simple versus Compound interest

What is the difference between the future value of \$100 after 2 years at 10% simple interest and 10% compound interest?

Time	Beginning Balance	Simple Interest	Ending Balance
1	\$100	\$10	\$110
2	\$110	\$10	\$120

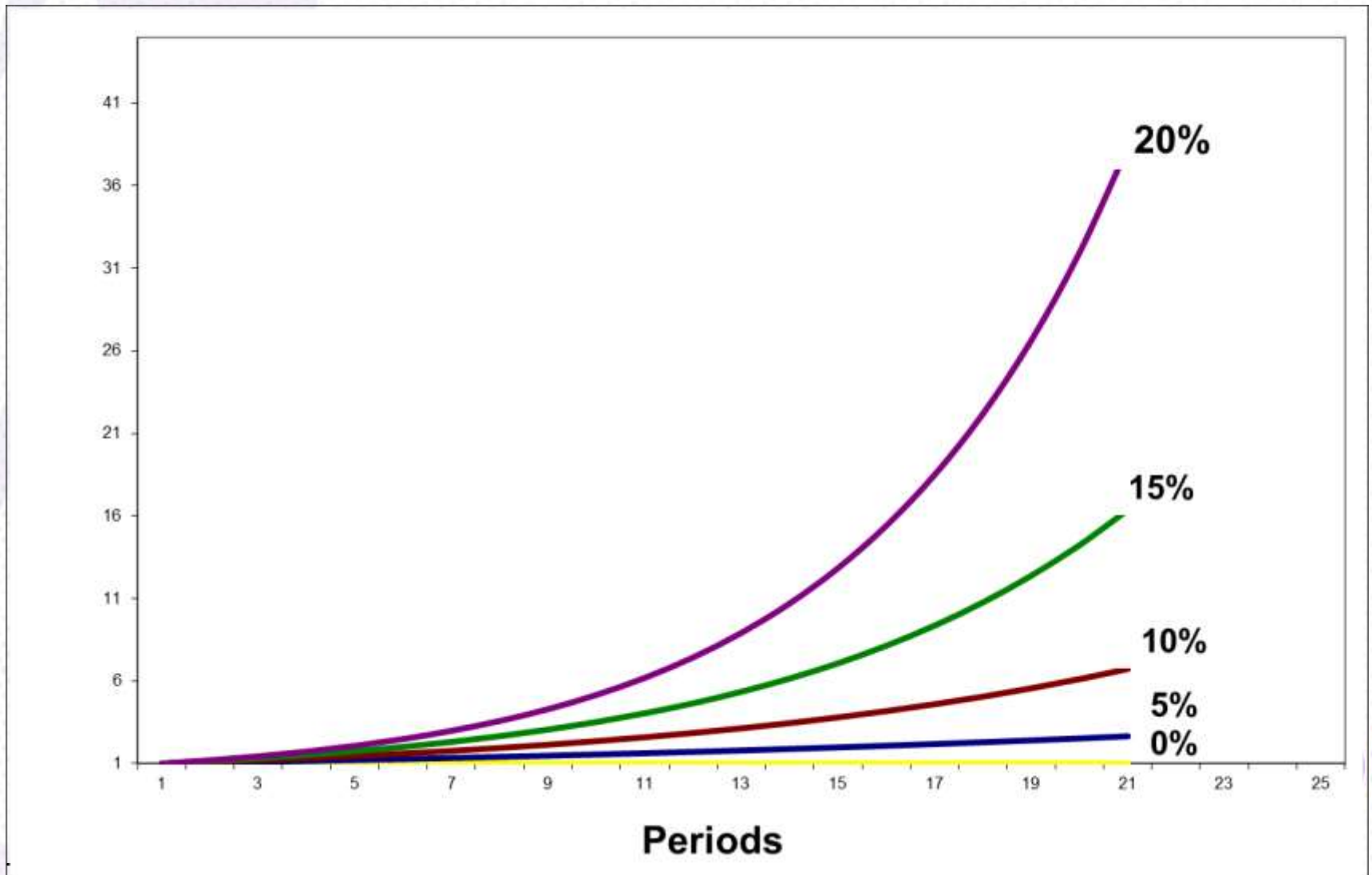
Time	Beginning Balance	Compound Interest	Ending Balance
1	\$100	\$10	\$110
2	\$110	\$11	\$121



From here on, compound  
interest only



# The Power of Compound Interest





# **Time Value of Money**

## **FV-Lump Sum**





# Future value (FV) of a **lump sum**

- ❖ Lump Sum: investment only happens once in one “big” lump sum
- ❖ Future Value: Have some amount today, trying to figure out how much you’ll have in the future.

$$FV = PV(1 + I)^N$$

Where:

FV=Future Value

PV=Present Value

I=Interest rate, in decimal form

N=number of years



# FV of a Lump sum: Example

Example: Sue now has \$125. How much would she have after 8 years if she deposits it in her **commercial bank** which pays 7% interest with annual compounding?

\***Commercial bank:** Traditional department store of finance serving a variety of savers and borrowers. Ex. First Tennessee Bank



# Independent Practice

You receive an inheritance of \$100,000 and decide to save it for retirement. If you can deposit the inheritance in a **money market mutual fund** that 1% interest, how much will you have when you retire in 25 years?

\***Money market mutual fund:** is a diversified fund that collects small amounts from many individual investors and invests only in **money market** instruments (those with less than 1 year until maturity).



# Independent Practice Solution





# **Time Value of Money**

## **PV-Lump Sum**



# Present value (PV) of a Lump Sum

- ❖ Present Value: Know the amount in the future (perhaps a target) and want to know how much you should invest today. Again, investment today is a one time deal.

$$PV = \frac{FV}{(1 + I)^N}$$

Where:

FV=Future Value

PV=Present Value

I=Interest rate, in decimal form

N=number of years



# PV of a Lump sum: Example

A friend asks to borrow money and promises to pay you \$5,000 after 5 years. Assuming annual compounding and interest rates of 8%, what is the maximum amount you would lend your friend **today** to ensure you do not lose money on the deal?



# Independent practice

Suppose you are offered an investment that pays \$5,000 in 3 years. If interest rates are 4%, what is the maximum amount you should be willing to pay for this investment? Assume annual compounding.





# Independent Practice Solution



# **Time Value of Money**

## **Lump Sum: Finding N or I**



# Finding the interest rate & the number of years

- ❖ Sometimes we may know the present and future values of an amount but we'll want to find what interest rate we are earning. Other times we may want to find how long it takes to accumulate a certain amount of money.
- ❖ Finding  $I$  and  $N$  by hand takes more complex algebra. To find these variables, we will use the financial calculator or Excel.



# Lump Sum: Finding I or N

## Example

- ❖ You are offered an opportunity to invest in a **hedge fund** that will double your investment in 5 years. What interest rate does this hedge fund earn?

\***Hedge fund:** Organization that pools investor funds to purchase financial instruments. Very high risk and requires large minimum investments (usually greater than \$1 million)





# Independent Practice

- ❖ Your parents currently have \$500,000 and will retire as soon as they hit their \$3,000,000 goal. If they invest in an account that pays 9.37% interest, how long before they can retire?



# Independent Practice Solution



# **Time Value of Money**

## **FV: Annuities**



# Terminology: annuities

Term	Definition
Annuity	Stream of equal periodic cash flows over a stated period of time
Ordinary (deferred) annuity	annuity whose payments occur at the end of each period
Annuity due	annuity whose payments occur at the beginning of each period





# FV of an ordinary annuity ( $FVA_{ORD}$ )

- ❖ You are trying to find the future value of a series of equal payments that occur at regular frequency for a finite period of time. You can find the future value of each individual payment and add them up as shown below:

$$FVA_{ORD} = PMT(1+I)^{N-1} + PMT(1+I)^{N-2} + PMT(1+I)^{N-3} + \dots + PMT(1+I)^0$$

- ❖ This process can get cumbersome when you think about long annuities—i.e. mortgages or retirement savings. Alternatively, the long formula above is the mathematical equivalent of the shorter formula below:

$$FVA_{ORD} = PMT \left[ \frac{(1+I)^N - 1}{I} \right]$$



# FVA<sub>ORD</sub> Example

Sarah wants to invest in a **mutual fund** to save for her daughter's college. She can save \$6,000 per year and her first payment will come one year from now. The mutual fund offers an expected return of 8% in the future. Under these circumstances, how much money will Sarah have for her daughter's college in 3 years?

**\*Mutual fund:** Organization that pools investor funds to purchase financial instruments and reduce risk through diversification





# Independent Practice

- ❖ You want to go to Europe 5 years from now, and you can save \$3,100 per year, beginning one year from today. You plan to deposit the funds in a mutual fund that you think will return 8.5% per year. Under these conditions, approximately how much would you have 5 years from now?





# Independent Practice Solution



# **Time Value of Money**

## **PV: Annuities**



# PV of an ordinary annuity ( $PVA_{ORD}$ )

- ❖ You are trying to find the present value of a series of equal payments that occur at regular frequency for a finite period of time. You can find the present value of each individual payment and add them up as shown below:

$$PVA_N = \frac{PMT}{(1+I)} + \frac{PMT}{(1+I)^2} + \frac{PMT}{(1+I)^3} + \dots + \frac{PMT}{(1+I)^N}$$

- ❖ This process can get cumbersome when you think about long annuities—i.e. mortgages or retirement savings. Alternatively, the long formula above is the mathematical equivalent of the shorter formula below:

$$PVA_{ORD} = PMT \left[ \frac{1 - \frac{1}{(1+I)^N}}{I} \right]$$



# $PVA_{ORD}$ example

- ❖ You are planning for retirement. You expect to live for 20 years after you retire and you would like to be able to withdraw \$100,000 at the end of each year of retirement. If you can invest your money in account that pays 5% interest, how much money should you have saved up when you reach retirement age?







# Independent Practice

- ❖ Your parents won the lottery and have agreed to set aside part of their winnings for you to pay for college. You will need \$4,000 at the end of each year for the next 3 years. How much money should your parents set aside for you if you can invest your savings in an account that earns 8.5% per year?



# Independent Practice Solution



# **Time Value of Money**

## **Ordinary Annuity v. Annuity Due**





# Ordinary Annuity v. Annuity Due

- ❖ ORDINARY: You are planning for retirement. You expect to live for 2 years after you retire and you would like to be able to withdraw \$100,000 **at the end of each year** of retirement. If you can invest your money in account that pays 5% interest, how much money should you have saved up when you reach retirement age?
- ❖ DUE: You are planning for retirement. You expect to live for 2 years after you retire and you would like to be able to withdraw \$100,000 **at the beginning of each year of retirement**. If you can invest your money in account that pays 5% interest, how much money should you have saved up when you reach retirement age?





# Adjusting for annuity due

- ❖ Sometimes, the payments occur at the beginning of the period, instead of at the end. This requires making an adjustment to our previous formulas:

$$FVA_{DUE} = FVA_{ORD}(1 + I)$$

$$PVA_{DUE} = PVA_{ORD}(1 + I)$$



# Example

- ❖ You want to quit your job and go back to school for a law degree 4 years from now, and you plan to save \$3,500 per year, *beginning immediately*. You will make 4 deposits in an account that pays 5.7% interest. Under these assumptions, how much money will you have saved when you start law school?





# Independent Practice

- ❖ You are offered an investment that pays \$2,000 per year at the beginning of each year for the next 5 years. If interest rates are 3%, what is maximum amount you should pay for this investment?



# Independent Practice Solution



# **Time Value of Money**

## **Annuities: Solving for I, N or PMT**



# Finding N, I, OR PMT annuities

- ❖ Just like when we dealt with lump sums, to find N and I for an annuity we'll use the financial calculator. In addition, we'll use the calculator if we need to compute the payment amount of an annuity.





# Annuities: Finding N, I, OR PMT

## Example

- ❖ You turned 21 today and decide to start saving to take a Caribbean vacation for your 25<sup>th</sup> birthday. You can invest in an account that offers to pay 4% interest and you expect the trip to cost \$10,000. If you make equal payments at the end of each year, how large should your deposits be to reach your goal?



# Independent Practice

Suppose you inherited \$275,000 and invested it at 8.25% per year. How much could you withdraw at the end of each of the next 20 years?



# Independent Practice Solution





# Additional Practice

1. Suppose you are offered an opportunity to invest in Axe Cap, an incredibly successful hedge fund. You are guaranteed the ability to quadruple your investment after 3 years. What interest rate is the hedge fund offering you?
2. Suppose your grandmother has promised you \$10,000 as a present when you graduate in 2 years. If she can invest her money in an account that pays 4% compounded annually, how much does she need to set aside today to ensure she can deliver on her promise?
3. Julian turned 10 today. His favorite uncle, Vince, promised him a deposit of \$3,000 every year, on his birthday, starting today. On Julian's 18<sup>th</sup> birthday, no deposit will be made but Uncle Vince will give Julian the proceeds to cover a trip and college expenses. If Uncle Vince invests Julian's gift in an account that offers 8% compounded annually, how much does Julian receive on his 18<sup>th</sup> birthday?
4. Imagine you won \$50,000 in the lottery, after-taxes. If you invest the money in account that offers an average of 6%, compounded annually, and you make no additional contributions, how much would you have when you retire in 40 years?
5. You recently visited your local community bank and took out a 30-year mortgage with annual payments and an annual interest rate of 2.75%. If your payments are for \$29,630.66 per year, how much money did you borrow?
6. Paul, a successful PR executive, recently decided to change career paths and begin saving money to attend medical school. He can save \$35,000 per year, at the end of every year, for the next 3 years. If he can invest the funds in an account that pays 7% interest, compounded annually, what will be the value of his savings when he starts med school?
7. Lisa is planning her retirement. She imagines she'll live for 20 years after-retirement and she thinks her ideal retirement would be financed with \$95,000 at the beginning of every retirement year. If her savings are in an account that is expected to generate 5% compounded annually, how much money does Lisa need to accumulate during her working life?
8. Trey received an inheritance of \$350,000. He decides to save it for retirement. The awesome inheritance makes Trey want to become a millionaire in 15 years. How much does he need to contribute to his savings at the end of every year to reach his goal if his account earns an average annual return of 5% per year?





# Additional Practice Solutions



# Additional Practice Solutions



# Additional Practice Solutions



# Additional Practice Solutions





# **Time Value of Money**

## **FV: Mixed Streams**



# Mixed streams

- ❖ Series of unequal cash flows reflecting no particular pattern
- ❖ Future Value: Will be making several, unequal payments over time and you want to know their value at some future point. You must find the FV of each and add them all up.

$$FV = CF_1 * (1 + I)^{N-1} + CF_2 * (1 + I)^{N-2} + ... + CF_N * (1 + I)^{N-N}$$



# EXAMPLE

- ❖ Suppose you inherit \$20,000 and you decide to invest the money for a vacation in 3 years. You decide to contribute \$3,000 at year end, and \$5,000 at the end of the following year. If you invest the money in a mutual fund that offers a 4% annual return, compounded annually, how much do you have when you go on your vacation?



# Independent practice

- ❖ Suppose you begin saving today for your child's college. She will begin college in 4 years. You start by depositing \$12,000 today into an account that pays 5% interest with annual compounding. After two years, you find yourself unable to contribute any more funds but your parents gift her \$15,000 for college. If no other deposits are made, how much is available for your daughter when she begins college?





# Independent practice solution



# **Time Value of Money**

## **PV: Mixed Streams**



# Mixed streams

- ❖ Present Value: Several, unequal amounts are coming to you at various points in the future and you want to know how much their value is today. You must find the PV of each and add them all up.

$$PV = \frac{CF_1}{(1+I)^1} + \frac{CF_2}{(1+I)^2} + \dots + \frac{CF_N}{(1+I)^N}$$



# Example

Suppose you won the lottery and were given the option for the following payoffs.

Ignoring tax implications, assuming payments are guaranteed, and that the prevailing interest rate is 5%, which option is the most valuable?

Option 1: Lump Sum of \$8 million.

Option 2: \$7 million at the end of year 1 and \$1 million at the end of each of the next 2 years.





# Independent Practice

- ❖ What is the present value of the following cash flow stream if interest rates are 6%?

Year	Cash Flow
1	\$300,000
2	\$250,000
3	\$200,000



# Independent Practice Solution



# **Time Value of Money**

## **PV: Perpetuities**



# Perpetuities

- ❖ Level cash flow stream that continues forever
  - The idea of a payment that goes on forever is hard to imagine but it does exist: after WWII, the British government issued consols (bonds that pay a fixed coupon every year forever) to pay for war reconstruction.
  - Also, preferred stock issued by companies usually pays a fixed amount per quarter as long as the company is in existence. Since corporations theoretically have an eternal life, we use the perpetuity formula to find the value/price of preferred stock.





# Perpetuities

- ❖ Present value of a perpetuity

$$PVofaPerpetuity = \frac{PMT}{I}$$



# Perpetuity Example

What is the present value of a \$100 perpetuity if the interest rate is 7%?



# Independent Practice

You have an opportunity to invest in an asset that offers to make annual payments of \$12 infinitely. If interest rates are 1%, how much should you pay for the investment?



# Independent Practice Solution





# **Time Value of Money**

## **Different Compounding Frequencies**



# Compounding more frequently than annually

❖ To account for frequent compounding you must adjust 2 variables: the interest rate (I) and the number of years (N).

1. Find the # of compounding periods in the year.
2. Divide the I by the # of compounding periods
3. Multiply the N by the # of compounding periods

Compounding Frequency	Adjustment to I	Adjustment to N
Semiannually	Divide by 2	Multiply by 2
Quarterly	Divide by 4	Multiply by 4
Monthly	Divide by 12	Multiply by 12
Daily	Divide by 365	Multiply by 365



# Example

You are considering investing in a Third World bank account that pays a nominal annual rate of 18%, compounded **monthly**. How much do you need to deposit at the end of each month to accumulate \$250,000 in 5 years?



# Independent Practice

- ❖ What is the value of a \$10,000 payment to be received in 10 years if interest rates are 3% compounded quarterly?





# Independent practice solution



# **Time Value of Money**

## **Comparing Interest Rates**



# Comparing Interest Rates

Nominal (quoted or stated) interest rate or Annual Percentage Rate (APR)-the contracted interest rate.

Effective (Equivalent) Annual Rate (EFF or EAR): the annual rate of interest actually being earned, as opposed to the quoted rate.

$$\text{EffectiveAnnualRate}(EFF\%) = \left(1 + \frac{I}{M}\right)^M - 1$$



# Effective Annual Rate Example

Bank A pays 4% interest compounded annually on deposits, while Bank B pays 3.5% compounded daily.

- a. Based on the EAR (or EFF%), which bank should you use?







# Independent Practice

- ❖ Your bank account pays a 6% nominal rate of interest. The interest is compounded quarterly. What is the (a) periodic rate, (b) effective annual rate?



# Independent Practice Solution







# **Time Value of Money**

## **More Challenging TVM**



# SAVING TOWARDS A GOAL

John and Daphne just received a large insurance settlement and would like to ensure that their daughter Ellen's college education is paid for. Ellen just turned 10 and she will be entering college 8 years from now. College tuition and expenses at State University are currently \$14,500 per year, and Ellen is expected to graduate in 4 years. Tuition and other costs will be due at the end of each school year. How much money do John and Daphne need to deposit today in an account that pays 9%, to ensure Ellen's education is paid for if they do not intend to make any more payments?





# SAVING TOWARDS A GOAL

John wants to pay for his daughter's 4-year college career, which starts 10 years from today. He wants Mary to attend Stanford and figures that per semester, Stanford will cost \$40,000. Payment is due at the beginning of each semester. How much should he contribute an account at the end of every six months, to finance her college education? Assume that the account pays 5% annual interest rate and that there are only two 6-month semesters in each academic year.





# INDEPENDENT PRACTICE

- ❖ Today is your birthday, and you decide to start saving for your college education. You will begin college on your 18<sup>th</sup> birthday and will need \$4,000 per year at the end of each of the following 4 years. You will make a deposit 1 year from today in an account paying 12% annually and continue to make an identical deposit each year up to and including the year you begin college. If a deposit amount of \$2,542.05 will allow you to reach your goal, what birthday are you celebrating today?



# INDEPENDENT PRACTICE SOLUTION



# INDEPENDENT PRACTICE

You want to begin saving for your retirement in 40 years. You can save \$6000 at the end of each year until you retire and you anticipate that you will live for 20 years after you retire. How much will you be able to withdraw at the end of each year during retirement if you invest your earnings in an account that pays 5% interest?



# INDEPENDENT PRACTICE SOLUTION





# Additional Practice

1. Bryce recently accepted a one-time payment of \$2 million, after taxes, for winning the state lottery. He would like to set aside enough funds for his 3 year old to go to college in 15 years, without making any further contributions. He anticipates college tuition when his daughter starts college will be \$15,000 due at the beginning of each of the 8 academic semesters. If he can invest his winnings in an account that pays, 6%, compounded semiannually, how does Bryce need to set aside today? Assume there are only two 6-month semesters in each academic year.
2. You are offered an investment that pays \$6,000 at the end of the first year, \$4,000 at the end of the next 2 years and \$5,000 at the end of year 4. If interest rates are 4%, with annual compounding, what is the maximum amount you should be willing to pay today for this investment?
3. Harlingen Inc's preferred stock pays an annual, perpetual \$8 dividend . If the annual yield on assets of similar risk is 4%, how much should you pay for each preferred share today?
4. You are planning a trip when you graduate from college in 3 years. You have access to an account that pays 6%, compounded annually. You can contribute \$2,500 now and anticipate to contribute \$3,000 in one year, and \$2,000 in each of the next 2 years. If you manage to make all your planned contributions, how much will you have for your trip?



# Additional Practice

5. You received an inheritance of \$3,000,000 after-taxes. Woo hoo!! You decide to invest it in hopes to retire in 10 years. If you make no more contributions to your savings, you invest your money in an account that pays 8% interest compounded annually and you plan on withdrawing \$550,000 at the end of each year of retirement, how many years could you survive with your inheritance?
6. You recently took out a 30-year, \$300,000 mortgage loan with an APR of 6%, with monthly compounding. Calculate your monthly payment. What is the effective rate of interest you pay on the loan?
7. You are offered a choice between 2 investments. Investment A offers a 20% interest rate compounded quarterly and Investment B offers 19.5% compounded daily. Which do you select?
8. Today is your 25<sup>th</sup> birthday and you are ecstatic because your granny gives you \$8,000 to jumpstart your retirement savings. Given the awesome start, you decide to also contribute part of your paycheck at the end of every month. You open an account with a mutual fund that offers you an APR of 12% with monthly compounding. You plan to retire at age 65 and you expect to live for 20 years, until age 85. You anticipate that \$12,000 at the beginning of every month will make for a comfortable retirement. How much do you need to contribute out of every paycheck to ensure you can meet your retirement goals?



# Additional Practice Solutions

❖ 1.

❖ 2.





# Additional Practice Solutions

❖ 3.

❖ 4.





# Additional Practice Solutions

❖ 5.

❖ 6.



# Additional Practice Solutions

❖ 7.

❖ 8.

