

First written examination of
Algoritmos e Estruturas de Dados

Outubro 12, 2015 Duration: no more than 30 minutes

Name:

Student number:

- 4.0 1: If $f(n) = \Theta(g(n))$, what can we say about how $f(n)$ is upper and lower bounded?

Answer:

- 4.0 2: Join (using lines do connect the dots) the functions with the same computational complexity.

$20n - 3$	•	•	$\log n^3$
$3n + \frac{10^9}{\log n}$	•	•	$\frac{n(n+1)}{2}$
$3n \log n + 17n + 100$	•	•	10^{-5}
$n^2 + 2n \log n + 3$	•	•	$10^6 + \log n$
$n^3 + 2^n$	•	•	$100(2^n + n!)$
2^n	•	•	$n^2 - 2n + 5$

- 4.0 **3:** Sort the following functions in increasing order of complexity (use the function number in your answer):

Function number	The function	Your answer
1	$0.1n^2 + 10n + 7$	
2	$0.001n^3 + 100n^2$	
3	1.5^n	
4	$n!/2^n$	
5	e^n	
6	$23n + 7n^2/\log n$	
7	$n^2 + 10n \log n$	
8	$n^{2.001}$	

- 4.0 **4:** Give an estimate of the computational complexity of the following function:

```
void init(int n,int *a)
{
    for(int i = 1;i <= n;i++)
        for(int j = i;j <= n;j += i)
            a[j] = i;
}
```

Answer:

4.0 **5:** What is the value returned by the call `f(5,7)`?

```
int f(int n,int m)
{
    int i,j,r;

    r = 0;
    for(i = 0;i <= n;i++)
        for(j = i;j <= m;j++)
            r += i;
    return r;
}
```

Answer:

Useful formulas:

- $\sum_{k=1}^n 1 = n$
- $\sum_{k=1}^n k = \frac{n(n+1)}{2}$
- $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2}\right)^2$
- $\sum_{k=1}^n \frac{1}{k} \approx \log n$
- $\sum_{k=n}^m f(k) = \sum_{k=1}^m f(k) - \sum_{k=1}^{n-1} f(k)$