Lista 07

a) $2^x = 64$

$2^x = 2^6$

$x = 6$

64	2
32	2
16	2
8	2
4	2
2	2
1	

2^6

b) $8^x = \frac{1}{32}$

$(2^3)^x = \frac{1}{2^5}$ $\quad 2^{3x} = \frac{1}{2^5} \Rightarrow 2^{3x} = 2^{-5} \Rightarrow 3x = -5 \Rightarrow x = -\frac{5}{3}$

c) $4^x = \frac{1}{8} \Rightarrow (2^2)^x = \frac{1}{2^3} \Rightarrow 2^{2x} = 2^{-3} \Rightarrow 2x = -3 \Rightarrow x = -\frac{3}{2}$

d) $2^x = \frac{1}{16} \Rightarrow 2^x = \frac{1}{2^4} \Rightarrow 2^x = 2^{-4} \Rightarrow x = -4$

e) $(\sqrt[3]{2})^x = 8 \Rightarrow 2^{\frac{1}{3}\cdot x} = 2^3 \Rightarrow \frac{1}{3}x = 3 \Rightarrow x = 3.3 \Rightarrow x = 9$

f) $2^{3x-1} = 32 \Rightarrow 2^{3x-1} = 2^5 \Rightarrow 3x - 1 = 5 \Rightarrow 3x = 6 \Rightarrow x = 2$

g) $7^{4x+3} = 49 \Rightarrow 7^{4x+3} = 7^2 \Rightarrow 4x + 3 = 2 \Rightarrow 4x = -1 \Rightarrow x = -\frac{1}{4}$

h) $2^{x^2-x-16} = 16 \Rightarrow 2^{x^2-x-16} = 2^4 \Rightarrow x^2 - x - 16 = 4 \Rightarrow x^2 - x - 16 - 4 =$

$x^2 - x - 20 = 0$

$\Delta = (-1)^2 - 4.1.(-20)$

$\Delta = 1 + 80$

$\Delta = 81$

$x = -(-1) \pm \sqrt{81}$ / 2

$x = \frac{1 \pm 9}{2}$

$x' = \frac{1+9}{2} = 5$

$x'' = \frac{1-9}{2} = -4$

$\{-4, 5\}$

i) $(2^x)^{x-1} = 4 \Rightarrow 2^{x^2-x} = 2^2 \Rightarrow x^2 - x = 2 \Rightarrow x^2 - x - 2 = 0$

$\Delta = (-1)^2 - 4.1.(-2)$

$\Delta = 1 + 8$

$\Delta = 9$

$x = \frac{1 \pm 3}{2}$

$x' = \frac{1+3}{2} = 2$

$x'' = \frac{1-3}{2} = -1$

$\{-1, 2\}$

j) $(9^{x+1})^{x-1} = 3^{x^2+x+4} \rightarrow ((3^2)^{x+1})^{x-1} = 3^{x^2+x+4}$

$(3^{2x+2})^{x-1} = 3^{x^2+x+4}$

$3^{2x^2-2x+2x-2} = 3^{x^2+x+4}$

$2x^2-2x+2x-2 = x^2+x+4$

$2x^2-x^2-2x+2x-x-2-4=0$

$x^2-x-6=0$

$\Delta = (-1)^2 - 4 \cdot 1 \cdot (-6)$

$\Delta = 1+24$

$\Delta = 25$

$X = \frac{1 \pm 5}{2}$

$x' = \frac{1+5}{2} = 3$

$x'' = \frac{1-5}{2} = -2$

$\{-2, 3\}$

k) $3 \cdot 4^{x+1} = 96$

$4^{x+1} = \frac{96}{3} \rightarrow 4^{x+1} = 32 \rightarrow (2^2)^{x+1} = 2^5 \rightarrow 2^{2x+2} = 2^5$

$2x+2 = 5 \rightarrow 2x = 3 \rightarrow x = \frac{3}{2}$

l) $5 \cdot 2^{x^2-4} = 160 \rightarrow 2^{x^2-4} = \frac{160}{5} \rightarrow 2^{x^2-4} = 32 \rightarrow 2^{x^2-4} = 2^5$

$x^2-4 = 5 \rightarrow x^2 = 9 \rightarrow x = \pm 3$

m) $\left(\frac{1}{25}\right)^x = 125 \rightarrow \left(\frac{1}{5^2}\right)^x = 5^3 \rightarrow (5^{-2})^x = 5^3 \rightarrow 5^{-2x} = 5^3$

$-2x = 3 \rightarrow x = \frac{-3}{2}$

n) $3 \cdot 2^{x^2-4} = 6144 \rightarrow 2^{x^2-4} = \frac{6144}{3} \rightarrow 2^{x^2-4} = 2048$

$2^{x^2-4} = 2^{11} \rightarrow x^2-4-11 = 0 \rightarrow x^2-15 = 0$

$x^2 = 15 \rightarrow x = \pm\sqrt{15}$

(2) a) $\log_4 16 = x \Rightarrow 4^x = 16 \Rightarrow 4^x = 4^2 \Rightarrow x = 2$

b) $\log_{\frac{1}{4}} 32 = x \Rightarrow \left(\frac{1}{4}\right)^x = 32 \Rightarrow \left(\frac{1}{2^2}\right)^x = 2^5 \Rightarrow 2^{-2x} = 2^5$

$\Rightarrow -2x = 5 \Rightarrow x = -\frac{5}{2}$

c) $\log_{0,25} 8 = x \Rightarrow 0,25^x = 8 \Rightarrow \left(\frac{1}{4}\right)^x = 8 \Rightarrow \left(\frac{1}{2^2}\right)^x = 2^3 \Rightarrow (2^{-2})^x = 2^3$

$2^{-2x} = 2^3 \Rightarrow -2x = 3 \Rightarrow x = -\frac{3}{2}$

d) $\log_{25} 0,008 = x \Rightarrow 25^x = 0,008 \Rightarrow 25^x = \frac{8}{1000} \Rightarrow 25^x = \frac{1}{125}$

$(5^2)^x = \frac{1}{5^3} \Rightarrow 5^{2x} = 5^{-3} \Rightarrow 2x = -3 \Rightarrow x = -\frac{3}{2}$

e) $\log_{\sqrt{5}} 5 = x \Rightarrow \sqrt{5}^x = 5 \Rightarrow 5^{\frac{1}{2}x} = 5^1 \Rightarrow \frac{1}{2}x = 1 \Rightarrow x = 2$

f) $\log_2 0,25 = x \Rightarrow 2^x = 0,25 \Rightarrow 2^x = \frac{1}{4} \Rightarrow 2^x = \frac{1}{2^2} \Rightarrow 2^x = 2^{-2}$

$x = -2$

(3) $S = \log_{100} 0,001 + \log_{1,5} \frac{4}{9} - \log_{1,25} 0,64$

① $\log_{100} 0,001 = x \Rightarrow 100^x = 0,001 \Rightarrow 100^x = \frac{1}{1000} \Rightarrow (10^2)^x = \frac{1}{10^3}$

$\Rightarrow 10^{2x} = 10^{-3} \Rightarrow 2x = -3 \Rightarrow x = -\frac{3}{2}$

② $\log_{1,5} \frac{4}{9} = x \Rightarrow 1,5^x = \frac{4}{9} \Rightarrow \left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^{-2} \Rightarrow x = -2$

③ $\log_{1,25} 0,64 = x \Rightarrow 1,25^x = 0,64 \Rightarrow \left(\frac{5}{4}\right)^x = \left(\frac{5}{4}\right)^{-2} \Rightarrow x = -2$

$S = -\frac{3}{2} + (-2) - (-2) = -\frac{3}{2} - 2 + 2 = -\frac{3}{2}$

(4) a) $\log_2 \left(\frac{a^2}{b^5 \sqrt[3]{c}}\right) = x$

$= \log_2 (a^2) - (\log_2 (b^5 \sqrt[3]{c}))$

$= 2 \log_2 a - (\log_2 b^5 + \log_2 \sqrt[3]{c})$

$= 2 \log_2 a - (5 \log_2 b + \log_2 c^{\frac{1}{3}})$

$= 2 \log_2 a - (5 \log_2 b + \frac{1}{3} \log_2 c)$

$= 2 \log_2 a - 5 \log_2 b - \frac{1}{3} \log_2 c$

b) $\log \left(\sqrt{\frac{ab^3}{c^2}}\right) \rightarrow \log \left(\left(\frac{ab^3}{c^2}\right)^{\frac{1}{2}}\right)$

$= \frac{1}{2} \log \left(\frac{ab^3}{c^2}\right)$

$= \frac{1}{2} (\log (ab^3) - \log (c^2))$

$= \frac{1}{2} (\log a + \log (b^3) - \log (c^2))$

$= \frac{1}{2} (\log a + 3 \log b - 2 \log c)$

$= \frac{1}{2} \log a + \frac{3}{2} \log b - \log c$

(5) a) $5^{\log_5 8} = 8$

b) $7^{\log_7 9} = 9$

c) $e^{\ln 3} = 3$

(b) a) $\log_5 \sqrt[3]{25} = x \rightarrow 5^x = \sqrt[3]{25} \rightarrow 5^x = 2,5^{\frac{1}{3}} \rightarrow 5^x = (5^2)^{\frac{1}{3}}$

$\rightarrow 5^x = 5^{\frac{2}{3}} \rightarrow x = \frac{2}{3}$

b) $\log_{10}\left(\frac{1}{\sqrt{1000}}\right) = x \rightarrow 10^x = \frac{1}{1000^{\frac{1}{2}}} \rightarrow 10^x = 1000^{-\frac{1}{2}} \rightarrow 10^x = (10^3)^{-\frac{1}{2}}$

$\rightarrow 10^x = 10^{-\frac{3}{2}} \rightarrow x = \frac{-3}{2}$

c) $\ln e^{-3} \rightarrow -3 \ln e^1 \rightarrow -3 \cdot 1 = -3$

(7) a) $\log_3 5 \rightarrow \frac{\log 5}{\log 3} \cong 1,465$

b) $\log_2 7 \rightarrow \frac{\log 7}{\log 2} \cong 2,807$

c) $\log_{100} 3 = \frac{\log 3}{\log 100} \cong 0,238$

d) $\log_2 3 = \frac{\log 3}{\log 2} \cong 1585$

e) $\log_5 3 = \frac{\log 3}{\log 5} \cong 0,683$

f) $\log_8 9 = \frac{\log 9}{\log 8} \cong 1,057$

g) $\log_{100} 5 = \frac{\log 5}{\log 100} \cong 0,349$

(8) $\log 2 = 0,301$
$\log 3 = 0,477$

$\log \sqrt[3]{2^2 \cdot 3}$
$= \log (2^2 \cdot 3)^{\frac{1}{3}}$
$= \frac{1}{3} \log (2^2 \cdot 3)$

$= \frac{1}{3} \log 2^2 + \frac{1}{3} \log 3$

$= \frac{2}{3} \cdot 0{,}301 + \frac{1}{3} \cdot 0{,}477 = 0{,}201 + 0{,}159 \cong 0{,}36$

(9) a) $x > 3$

b) $x^2 - 4x - 5$

$\Delta = (-4)^2 - 4 \cdot 1 \cdot (-5)$

$\Delta = 16 + 20$

$\Delta = 36$

$x = \frac{4 \pm \sqrt{36}}{2}$

$x = \frac{4 \pm 6}{2}$ $\begin{cases} x' = \frac{4+6}{2} = 5 \\ x'' = \frac{4-6}{2} = -1 \end{cases}$

$\rightarrow x > 5$

c) $-x^2 + 5x - 4$

$\Delta = (5)^2 - 4 \cdot (-1) \cdot (-4)$

$\Delta = 25 - 16$

$\Delta = 9$

$x = \frac{-5 \pm \sqrt{9}}{-2}$

$x = \frac{-5 \pm 3}{-2}$ $\begin{cases} x' = \frac{-5+3}{-2} = 1 \\ x'' = \frac{-5-3}{-2} = 4 \end{cases}$

$\rightarrow 1 < x < 4$

d) $x < 2$ e $x \neq 1$

(10) $pH = \log_{10}\left(\frac{1}{H^+}\right)$

$pH = \log\left(\frac{1}{1 \cdot 10^{-8}}\right) \rightarrow pH = \log(100\,000\,000) = 8$

(11) $M = C(1+i)^n$

$(1+i)^n = \frac{M}{C}$

$\log(1+i)^n = \log\frac{M}{C}$

$n = \frac{\log\frac{M}{C}}{\log(1+i)} = \frac{\log\frac{13}{8}}{\log(1{,}12)} = \frac{0{,}2730}{0{,}0492} = 5{,}54$

(12) $M = C(1+i)^n$

$177{,}15 = 150(1+0{,}05)^n$

$(1{,}05)^n = \frac{177{,}15}{150}$

$(1{,}05)^n = 1{,}183$

$n \cdot \log(1{,}05) = \log 1{,}183$

$n = \frac{\log 1{,}183}{\log 1{,}05} \cong 3{,}44$

(13) $3Q = Q(1+0{,}02)^n$

$(1{,}02)^n = \frac{3\cancel{Q}}{\cancel{Q}}$

$n \log(1{,}02) = \log 3$

$n = \frac{\log 3}{\log(1{,}02)} \cong 55{,}48$

(14) $48{,}10 = 44(1+0{,}096)^n$

$(1{,}096)^n = \frac{48{,}10}{44}$

$(1{,}096)^n = 1{,}093$

$n \log(1{,}096) = \log 1{,}093$

$n = \frac{\log 1{,}093}{\log(1{,}096)} \cong 0{,}97$ (Quase 1 mês)

(15) a) 100 —(5d)— 200 —(10d)— 400 —(15d)— 800 —(20d)— 1600 —(25d)— 3200

b) $P(t) = 100 \cdot 2^{t/5}$

c) $250\,000 = 100 \cdot 2^{t/5}$

$2^{t/5} = \frac{250\,000}{100}$

$2^{t/5} = 2500$

$\log 2^{t/5} = \log 2500$

$\frac{t}{5} = \frac{\log 2500}{\log 2}$

$\frac{t}{5} = 11{,}29$

$t = 11{,}29 \cdot 5 \cong 56{,}45$

16) $Q(t) = Q_0 e^{-0,0001t}$

$2000 = Q_0 e^{-0,0001 \cdot 5000}$

$2000 = Q_0 \cdot e^{-0,5}$

$2000 = Q_0 \cdot 2,72^{-0,5}$

$2000 = Q_0 \cdot \frac{1}{2,72^{0,5}}$

$2000 = Q_0 \cdot \frac{1}{1,65}$ (arredondamento)

$1,65 \cdot 2000 = Q_0$

$Q_0 = 3300$

17) $N(t) = 1500 \cdot 2^{0,2t}$

$250000 = 1500 \cdot 2^{0,2t}$

$2^{0,2t} = \frac{250000}{1500}$

$2^{0,2t} = 166,67$

$0,2t \log 2 = \log 166,67$

$0,2t = \frac{\log 166,67}{\log 2}$

$0,2t = 7,38$

$t = \frac{7,38}{0,2}$

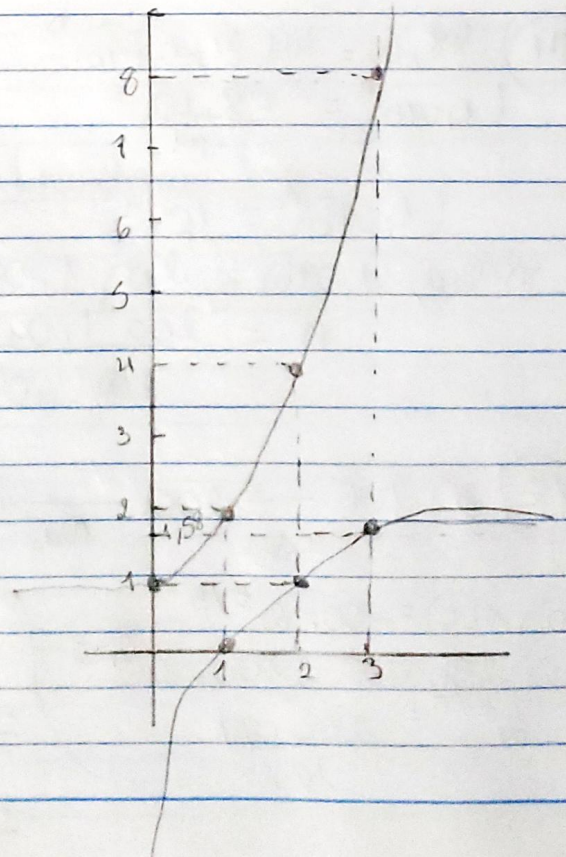
$t \cong 36,9$

18) $y = 2^x$

x	y
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
0	$2^0 = 1$

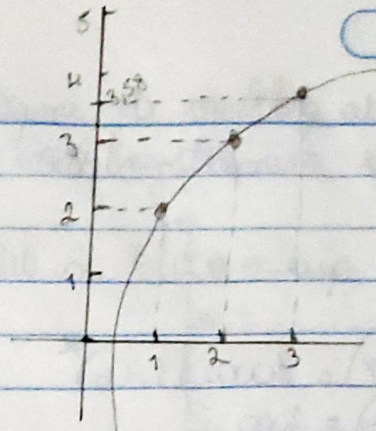
$y = \log_2 x$

x	y
1	$\log_2 \frac{1}{2} = \log 1 = 0$
2	$\log_2 2 = 1$
3	$\log_2 3 \cong 1,58$



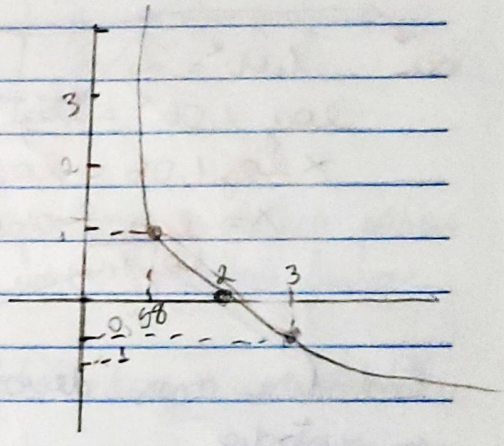
19) a) $2 + \log_{\frac{1}{2}} x$

x	y
1	$0+2=2$
2	$1+2=3$
3	$1,58+2=3,58$



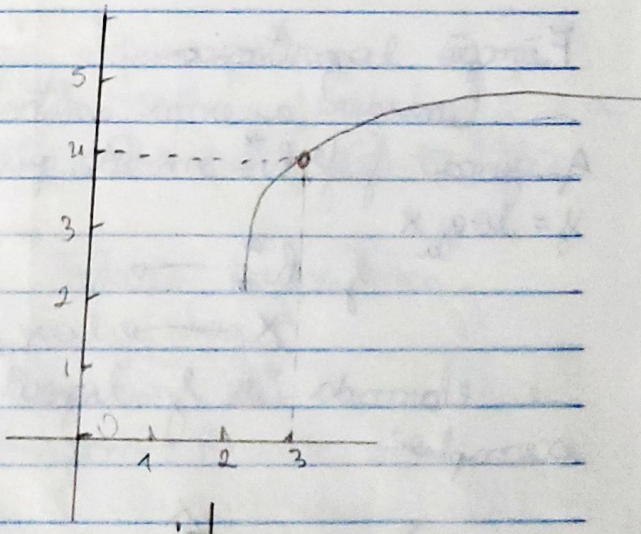
b) $y = 1 + \log_{\frac{1}{2}} x$

x	y
1	$\log_{\frac{1}{2}} 1 = 0+1=1$
2	$\log_{\frac{1}{2}} 2 = -1+1=0$
3	$\log_{\frac{1}{2}} 3 = -1,58+1=-0,58$



c) $y = \log(x-2)+4$

x	y
1	$\log(1-2)+4 =$
2	$\log(2-2)+4 =$
3	$\log(3-2)+4 = 4$



d) $y = \log_2(x+2)$

x	y
1	$\log_2(1+2) \cong 1,58$
2	$\log_2(2+2) \cong 2$
3	$\log_2(3+2) \cong 2,32$