

Exercício 5

1) $u = (3, -1, 1)$ $v = (1, 2, 2)$ $w = (2, 0, -3)$

$$a) (u, v, w) = \begin{vmatrix} 3 & -1 & 1 \\ 1 & 2 & 2 \\ 2 & 0 & -3 \end{vmatrix} = \begin{vmatrix} 3 & -1 \\ 1 & 2 \\ 2 & 0 \end{vmatrix}$$

$$-4 - 0 - 3 = -18 - 4 + 0 = -22$$

$$b) (v, u, w) = \begin{vmatrix} 1 & 2 & 2 \\ 3 & -1 & 1 \\ 2 & 0 & -3 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 3 & -1 \\ 2 & 0 \end{vmatrix}$$

$$+4 - 0 + 18 = 22 + 3 + 4 + 0 = 29$$

2) $(u, v, w) = -5$

a) $(-1) - 5 = 5$

d) $(-1)(-1)(-5) = -5$

b) $(-1) - 5 = 5$

c) $(-1)(-1)(-5) = -5$

3) $u = (1, -1, 2)$ $v = (2, 2, 1)$ $w = (-2, 0, -4)$

$$\begin{vmatrix} 1 & -1 & 2 \\ 2 & 2 & 1 \\ -2 & 0 & -4 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 2 & 2 \\ -2 & 0 \end{vmatrix}$$

$$+8 - 0 - 8 = -8 + 2 + 0 = -6$$

Não são coplanares pois o resultado do determinante é diferente de zero.

4) $u = (2, -1, k)$ $v = (1, 0, 2)$ $w = (k, 3, k)$

2	-1	k	2	-1	
1	0	2	1	0	= 0
k	3	k	k	3	

$$-0k^2 - 12 + k + 0k - 2k + 3k = 0$$

$$-12 + k - 2k + 3k = 0$$

$$-12 + 2k = 0$$

$$2k = 12$$

$$k = 6$$

5) $A(1, 1, 0)$ $B(-2, 1, -6)$ $C(-1, 2, -1)$ $D(2, -1, -4)$

$$AB = B - A = (-2, 1, -6) - (1, 1, 0) = (-3, 0, -6)$$

$$AC = C - A = (-1, 2, -1) - (1, 1, 0) = (-2, 1, -1)$$

$$AD = D - A = (2, -1, -4) - (1, 1, 0) = (1, -2, -4)$$

-3	0	-6	-3	0
-2	1	-1	-2	1
1	-2	-4	1	-2

$$+6 + 6 + 0 + 12 - 0 - 24 = 0$$

São coplanares pois o resultado é igual a zero

6) $\vec{a} = (0, -1, 2)$ $\vec{b} = (-4, 2, -1)$ $\vec{c} = (3, m, -2)$

base

$$(c, a, b) = \begin{vmatrix} 3 & m & -2 \\ 0 & -1 & 2 \\ -4 & 2 & -1 \end{vmatrix}$$

$$+8 - 12 - 0m + 3 - 8m + 0 = 33$$

$$8 - 12 + 3 - 8m = 33$$

$$h = \underline{v}$$

$$-8m = 33 - 8 + 12 - 3$$

area base

$$-8m = 34$$

$$m = \frac{-34}{8} = \frac{-17}{4}$$

$$= \underline{33}$$

$$|a \times b| = a = (0, -1, 2)$$

$$b = (-4, 2, -1)$$

$$= \begin{vmatrix} 0 & -1 & 2 & 0 & -1 \\ -4 & 2 & -1 & -4 & 2 \end{vmatrix}$$

$$= (-3, -8, 4)$$

$$|a \times b| = \sqrt{(-3)^2 + (-8)^2 + (4)^2}$$

$$= \sqrt{9 + 64 + 16}$$

$$= \sqrt{89}$$

$$h = \underline{33}$$

$$\sqrt{89}$$

⑦ A(2, 1, 1) B(-1, 0, 1) C(3, 2, -2) D(0, 0, 3)

$$AB = B - A = (-1, 0, 1) - (2, 1, 1) = (-3, -1, 0)$$

$$AC = C - A = (3, 2, -2) - (2, 1, 1) = (1, 1, -3)$$

$$AD = D - A = (-2, -1, 3)$$

$$\begin{vmatrix} -3 & -1 & 0 \\ 1 & 1 & -3 \\ -2 & -1 & 3 \end{vmatrix} = \begin{vmatrix} -3 & -1 \\ 1 & 1 \\ -2 & -1 \end{vmatrix}$$

$$+ 9 + 3 - 1 - 3 + 3 - 6 - 0 = 25$$

$$+ 3 - 3 + 9 - 1 + 3 - 6 = 25$$

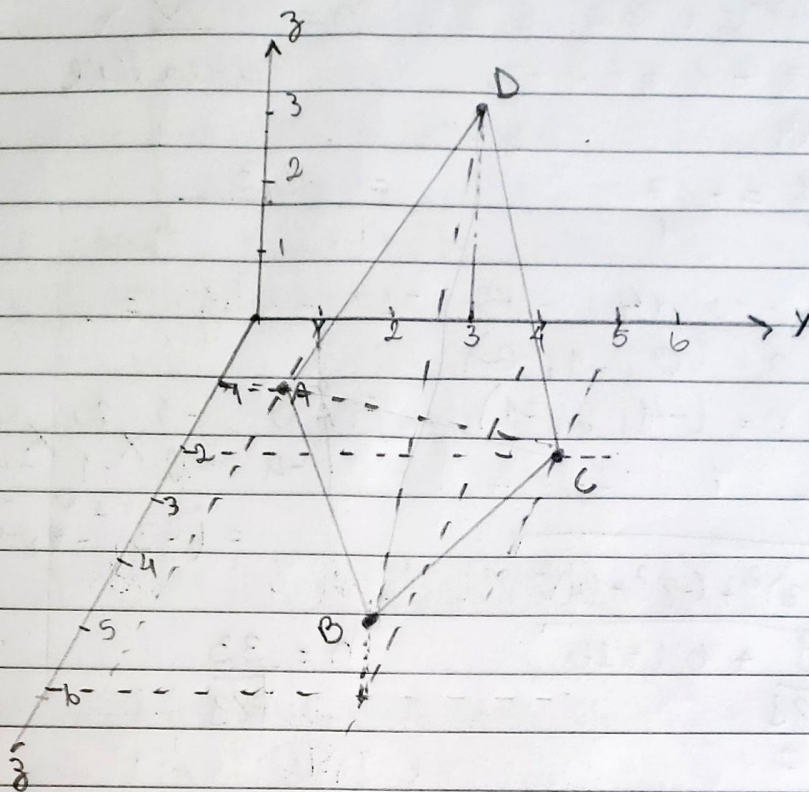
$$23 + 5 = 25$$

$$23 = 20$$

$$3 = 10$$

$$D = (0, 0, -10)$$

8) $A(1,1,0)$ $B(6,4,1)$ $C(2,5,0)$ $D(0,3,3)$



$$AB = B - A = (6,4,1) - (1,1,0) = (5,3,1)$$

$$AC = C - A = (2,5,0) - (1,1,0) = (1,4,0)$$

$$AD = D - A = (0,3,3) - (1,1,0) = (-1,2,3)$$

$$\begin{array}{c|ccc|cc} 1 & 5 & 3 & 1 & 5 & 3 \\ 6 & 1 & 4 & 0 & 1 & 4 \\ & -1 & 2 & 3 & -1 & 2 \\ \hline 1 & (+4) & -0 & -9 & +60 & -0+2 \\ \hline 6 & & & & & \end{array}$$

$$\frac{1}{6} (57) = \frac{57 \div 3}{6 \div 3} = \frac{19}{2} \text{ u.v}$$

lista 5

9) $A(-2, 4, -1)$ $B(-3, 2, 3)$ $C(1, -2, -1)$

$D(0, y, 0)$

$AB = B - A = (-3, 2, 3) - (-2, 4, -1) = (-1, -2, 4)$

$AC = C - A = (1, -2, -1) - (-2, 4, -1) = (3, -6, 0)$

$AD = D - A = (0, y, 0) - (-2, 4, -1) = (2, y-4, 1)$

$$v = \frac{1}{b} \cdot \begin{vmatrix} -1 & -2 & 4 \\ 3 & -6 & 0 \\ 2 & y-4 & 1 \end{vmatrix} = \frac{1}{b} \cdot \begin{vmatrix} -1 & -2 \\ 3 & -6 \\ 2 & y-4 \end{vmatrix} = \frac{1}{b} \cdot (-6 + 12(y-4)) = \frac{1}{b} \cdot (-6 + 12y - 48) = \frac{1}{b} \cdot (12y - 54)$$

$48 + 12 + 12y - 48$

$+ (12 + 12y) = 6$

$12 + 12y = 36$

$12y = 36 - 12$

$12y = 24$

$y = \frac{24}{12} = 2$

$D = (0, 2, 0)$

10) $\cos = \frac{u \cdot v}{|u||v|} = \frac{u \cdot v}{|u||v|} = \cos 120^\circ |u||v|$
 $u \cdot v = -\frac{1}{2} \cdot 3 \cdot 4 = -6$

a) $|u+v|^2 = u^2 + 2uv + v^2 = |u|^2 + 2uv + |v|^2$

$|u+v|^2 = 9 + (-12) + 16 = 13 \Rightarrow \sqrt{13}$

b) $|u \times (v-u)| = |u \times v - u \times u| = |u \times v| = |\vec{u}| |\vec{v}| \sin \theta$
 $= 3 \cdot 4 \cdot \left(-\frac{\sqrt{3}}{2}\right) = -6\sqrt{3}$

$$= 3 \cdot 4 \cdot \left(\frac{\sqrt{3}}{2} \right) = \frac{12\sqrt{3}}{2} = 6\sqrt{3}$$

c)