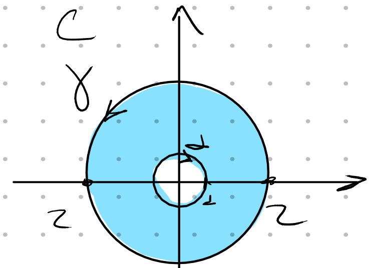
Lista Parte 2 teorema de green

1) $F = \left(\frac{-y}{x^2+y^2} , \frac{x}{x^2+y^2} + 3x \right)$ $\qquad \int_C F = ?$

a) C $\qquad$ circ: $x^2+y^2=4$



$\text{rot}F = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \Rightarrow \cancel{\frac{(x^2+y^2) - x\,2x}{(x^2+y^2)^2}} + 3 - \cancel{\frac{(-1)(x^2+y^2) - (-y)2y}{(x^2+y^2)^2}}$

$\text{rot}F = 3$

$\int_{\gamma \cup \alpha} F = \iint_{AZUL} \text{rot}F \Rightarrow \int_\gamma F = \underbrace{\iint_{AZUL} \text{rot}F}_{1} + \underbrace{\int_{-\alpha} F}_{2}$

2 $\int_{-\alpha} F$

$-\alpha(t) = (\cos t, \text{sen}\, t) \quad 0 \le t \le 2\pi \qquad -\alpha'(t) = (-\text{sen}\, t, \cos t)$

$\int_0^{2\pi} 1 + 3\cos t \cos t = 2\pi + \frac{3}{2}\left(t + \text{sen}\, t \cos t\right)\Big|_0^{2\pi}$

$= 2\pi + 3\pi = \underline{5\pi}$

1 $\iint_{AZUL} 3 = \int_0^{2\pi}\int_1^2 3 = 3\left[\pi R^2 - \pi r^2\right] = 3\left[4\pi - \pi\right]$

$= 9\pi$

$\int_\gamma = 9\pi + 5\pi = 14\pi$

2º Modo de fazer

a) $\int_C F = \underbrace{\int_C d\theta}_{2\pi} + \int_C 3x\,dy \Rightarrow$ {Separando em 2 campos}

$$4\int_0^{2\pi} 3\cos t\cos t = 4\,\frac{3}{2}\left(t + \operatorname{sen} t\cos t\right)\Big|_0^{2\pi} = 6(2\pi) = 12\pi$$

$C = (2\cos t, 2\operatorname{sen} t)$

$C' = (-2\operatorname{sen} t, 2\cos t)$

$$\int_C F = 2\pi + 12\pi = 14\pi$$

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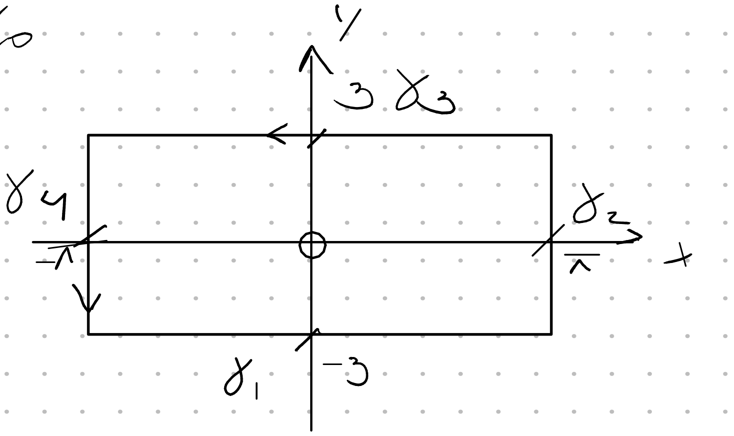
b) C fronteira do retângulo



$\vec{F} = \vec{F}_1 + \vec{F}_2$

$$\int_C F = \underbrace{\cancel{\int_C d\theta}}_{2\pi} + \int_C 3x\,dy$$

$-\gamma_4 = (-\pi, t)\ -3 \leq t \leq 3$

$\gamma_2 = (\pi, t)\ -3 \leq t \leq 3$

$\int_C 3x\,dy$ (γ_1 e γ_3 não importam)

$$= \int_{\gamma_4} 3x\,dy + \int_{\gamma_2} 3x\,dy \Rightarrow -\int_{-3}^{3} 3(-\pi)\,1 + \int_{-3}^{3} 3\pi$$

$$2\cdot 3\pi\,(3+3) = 6\pi\cdot 6 = 36\pi$$

$$\int_\gamma F = \int_\gamma F_1 + \int_\gamma F_2 \Rightarrow \int_\gamma F = 2\pi + 36\pi = 38\pi$$

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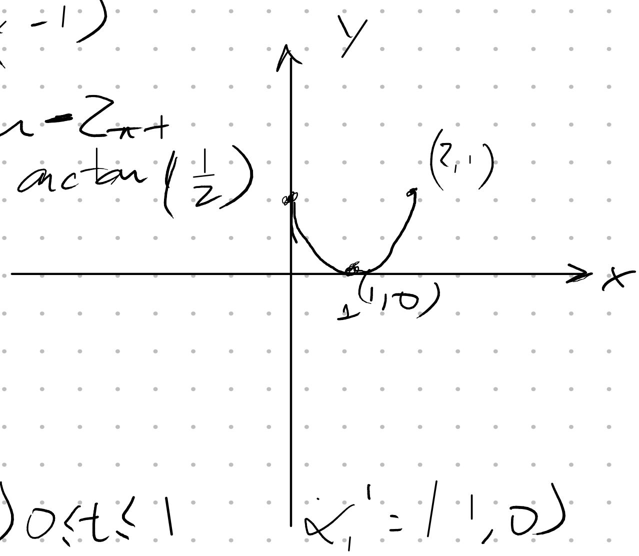
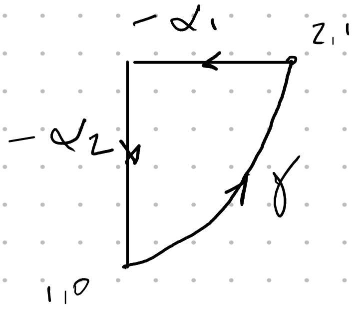
2) $F = d\theta$ $\qquad y = (x-1)^2$



Vai ter que dar $\arctan\left(\frac{1}{2}\right)$ ou $-2\pi + \arctan\left(\frac{1}{2}\right)$

↳ Propriedades do campo $d\theta$

Realizando as contas:



$\alpha_1 = (1+t, 2) \quad 0 \leq t \leq 1 \qquad \alpha_1' = (1,0)$

$\alpha_2 = (1, t) \quad 0 \leq t \leq 2 \qquad \alpha_2' = (0,1)$

$$\int_{\alpha_1} F = \int_0^1 \frac{-y}{x^2+y^2}\,dx + 0\,\cancel{dy} = \int_0^1 \frac{2}{4+(1+t)^2}$$

$$\int_{\alpha_2} F = \int_0^2 \frac{-y}{x^2+y^2}\,\cancel{dx} + \frac{x}{x^2+y^2}\,dy \Rightarrow$$

$$\Rightarrow \int_0^2 \frac{1}{1+t^2}\,\cancel{dy}\,dt \Rightarrow \int_0^2 \frac{1}{1+t^2}\,dt = \arctan(2)$$

b) 0 ou $2k\pi \quad \forall k \in \mathbb{Z}$

3) $\int_C \left[x \ln(x^2+1) + (x^2+1)y\right] dx + \alpha\left[\frac{x^3}{3} + x + \sin y\right] dy$

a) Cte α para que seja independente do caminho

$\text{rot } F = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \Rightarrow \alpha x^2 + \alpha - (x^2+1) = x^2(\alpha - 1) + (\alpha - 1)$

para $\text{rot } F = 0$ então $\alpha = 1$ //

$\gamma = (t, t)$