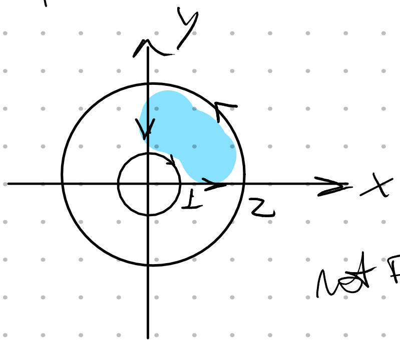
Exercícios parte 1

$\vec{F} = \left(\frac{x^2-y^2}{2}, \frac{x^2}{2}+y^4\right)$

1) $\oint_C \frac{x^2-y^2}{2}\,dx + \left(\frac{x^2}{2}+y^4\right)dy$

$rot = x - y$

$C = \partial D \quad D = \{ 1 \leq x^2+y^2 \leq 4 \quad x \geq 0 \quad y \geq 0 \}$



$rot\,F = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \Rightarrow x - (-y)$

$(x+y) \neq 0$

NÃO conservativo

$\int_C F\,dr = \iint_D rot\,F$

$\iint_D (x+y)\,dA \qquad \begin{cases} x = r\cos\theta \\ y = r\,sen\,\theta \end{cases}$

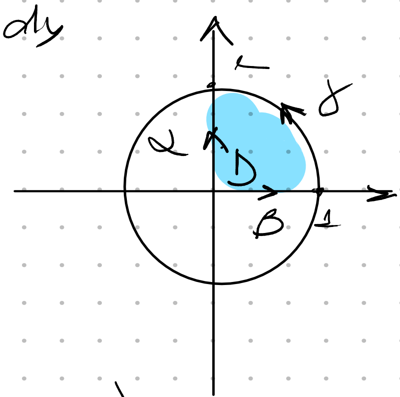
$\int_0^{\pi/2}\int_1^2 r^2\cos\theta + r^2 sen\,\theta \,dr\,d\theta \Rightarrow \frac{r^3}{3}\cos\theta\Big|_1^2 + \frac{r^3}{3} sen\,\theta\Big|_1^2$

$\int_0^{\pi/2} (\cos\theta + sen\,\theta)\frac{7}{3} \Rightarrow \frac{7}{3}\int_0^{\pi/2}(\cos\theta + sen\,\theta)$

$\Rightarrow \frac{7}{3}\left[sen\Big|_0^{\pi/2} - \cos\Big|_0^{\pi/2} \right] = \frac{7}{3}[1 - (-1)] = 14/3$

2) $\displaystyle\int_C e^x \sin y\,dx + (e^x \cos y + x)\,dy$

$\partial Q/\partial x \qquad \partial P/\partial y$



$\text{rot}\,F = 1 + e^x \cos y - e^x \cos y$

$= 1$

$\displaystyle\int_\gamma F + \int_{-\alpha} F + \int_\beta F = \iint_{\text{AZUL}} \text{rot}\,F$

④ ① ② ③

$\alpha = (0, t)\ 0 \leq t \leq 1$

$\alpha' = (0, 1)$

$\beta = (t, 0)\ 0 \leq t \leq 1$

$\beta' = (1, 0)$

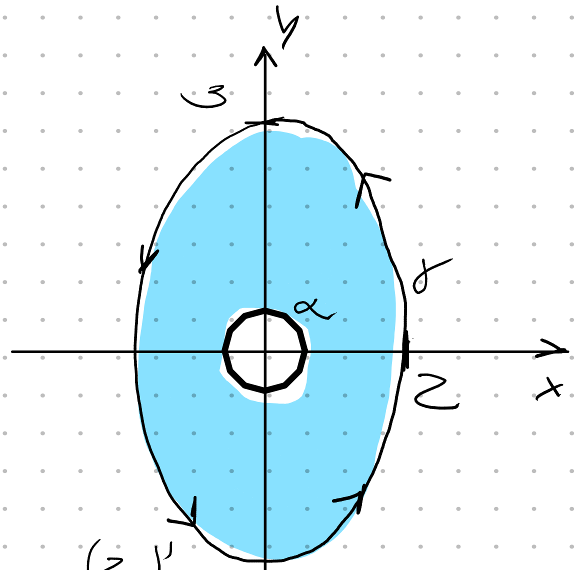
① $\displaystyle\int_\alpha F\,dr = \int_0^1 \cos t = \text{sen}\Big|_0^1 = \text{sen}\,1$

② $\displaystyle\int_\beta F\,dr = \int_0^1 e^t \cancel{\sin 0} \cdot 1 = 0$

③ $\displaystyle\iint_{AZ} \text{rot}\,F = \int_0^{\pi/2}\int_0^1 1\,r = \frac{1}{2}\,\frac{\pi}{2} = \pi/4$

④ = ③ + ① $= \pi/4 + \text{sen}\,1$

3) rotacional $F = \dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y}$



$$F = \left(\frac{-y}{x^2+y^2} \; , \; \frac{x}{x^2+y^2} + 2x \right)$$

$$rot = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} =$$

$$\left[\frac{+(x^2+y^2) - x(2x)}{(x^2+y^2)^2} - \frac{(-1)(x^2+y^2) - (-y)(2y)}{(x^2+y^2)^2} \right] + \overset{(2x)'}{2} = 2$$

$rot\, F = 2$

$$① \int_\alpha F = \int_0^{2\pi} 1 + 2\cos^2 t = 4\pi \qquad \iint_{AZUL} rot\, F = \iint_{elipse} - \iint_{circ} rot$$

$$= [\pi\, 3\, 2 - \pi]\, 2$$

$$= 10\pi$$

$$\int_{\gamma \cup (-\alpha)} F = \iint_{AZUL} rot\, F$$

$$\Rightarrow \int_\gamma F = \iint_{AZUL} rot\, F + \int_\alpha F = 10\pi + 4\pi = \boxed{14\pi}$$

4)

a) $\oint_{\partial D} (a_1 x + a_2 y + a_3)dx + (b_1 x + b_2 y + b_3)dy =$

$(b_1 - a_2)$ Área D

Teo green:

$\oint_{\partial D} F = \iint_D \text{rot} F \qquad \text{rot} F = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = (b_1 - a_2)$

$\iint_D (b_1 - a_2) \iff$ se $(b_1 - a_2)$ cte $\mathbb{R}$

$\Rightarrow (b_1 - a_2) \iint_D 1 \Rightarrow (b_1 - a_2)(\text{area } D)$

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b) i) Para obter Área(D) então $b_1 = 1$
e $a_2 = 0$

portanto $\oint_{\partial D} (a_1 x + \cancel{a_2} y + a_3)dx + (\cancel{b_1} x + b_2 y + b_3)dy$
$\quad a_2 \to 0, \quad b_1 x \to x$

toda a família de integrais resulta na área do domínio D
$\forall$ combinação de a_1, a_3, b_2 e $b_3 \in \mathbb{R}$

ii)

$\oint_{\partial D} = -\text{Área}(D)$ então $b_1 = 0$
$a_2 = 1$

$\forall$ combinação dos demais fatores resulta em uma integral de linha numericamente igual à $-$Área(D)

iii) $\oint_{\partial D} = 2\,\text{área}(D)$ $\boxed{\begin{matrix} b_1 = 1 \\ a_2 = -1 \end{matrix}}$

$\forall$ " " [vale o que já foi dito assim]

c) $\gamma(t) = (\sin t\cos t \;\; i \sin t) \qquad 0 \leq t \leq \pi$

$$\iint_{\text{int}(\gamma)} \text{rot}\, F = \iint_{\text{int}(\gamma)} 1 \qquad \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}$$

$\text{rot}\, F = 1 \Rightarrow$ ex ou $F = (0, x)$ $\quad \text{rot}\, F = \underline{1}$

a área no interior do γ com $F = (0, x)$ é igual à sua integração de linha.

$$\int_\gamma F\,dr = \int_0^\pi 0(\cos^2 t - \text{sen}^2 t) + \overset{\text{sen}\,t\cos t}{x}(\cos t$$

$\gamma'(t) = (\cos^2 t - \text{sen}^2 t, \cos t)$

$$\Rightarrow \int_0^\pi \text{sen}\,t \cos^2 t = -\frac{1}{3}\cos^3(t)\Big|_0^\pi = \frac{-1}{3}\left[-1 - 1\right]$$

$$= \boxed{2/3}$$