Teorema di Stokes II

Exemplo 1:

$$\int_C -y \frac{x^2+y^2+1}{x^2+y^2}\,dx + x\,\frac{x^2+y^2+1}{x^2+y^2} + z-1\cos^7(z)\,dz$$

$$\text{rot } F = \begin{vmatrix} i & j & k \\ \partial x & \partial y & \partial z \\ -y\,\dfrac{x^2+y^2+1}{x^2+y^2} & \dfrac{x(x^2+y^2+1)}{x^2+y^2} & z-1\cos^7(z) \end{vmatrix}$$

$(x^2+y^2)(2x) + 2x$

$$= \left(0,\ 0,\ \frac{(x^2+y^2+1)}{x^2+y^2} + x\cdot\left(\frac{2x(x^2+y^2) - (x^2+y^2+1)(2x)}{(x^2+y^2)^2}\right)\right.$$

$$\left.\frac{(x^2+y^2+1)}{x^2+y^2} + y\cdot\left(\frac{2y(\cancel{x^2+y^2}) - (\cancel{x^2+y^2}+1)\,\cancel{(2y)}}{(x^2+y^2)^2}\right)\right)$$

$$= \left(0,0,\ \frac{2(x^2+y^2+1)}{x^2+y^2}\right. - \frac{(x^2+y^2+1)\cdot 2x^2}{(x^2+y^2)^2} - \frac{(x^2+y^2+1)2y^2}{\cancel{(x^2+y^2)^2}}$$

$$\frac{-(x^2+y^2+1)(2x^2+y^2) - 2(x^2+y^2+1)\cancel{(x^2+y^2)}}{(x^2+y^2)^{\cancel{2}}}$$

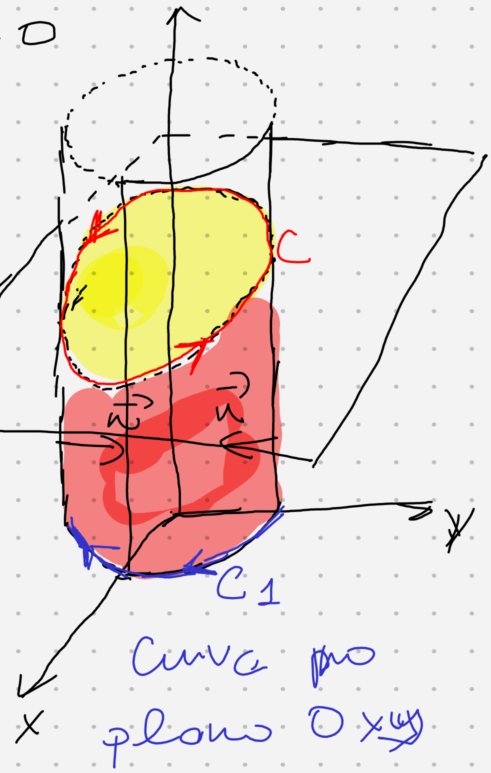
$$= \underbrace{\left(0,0,\ \frac{0}{(x^2+y^2)}\right)}$$

$$\text{rot } F = (0,0,2)$$

$\text{Dom}(F) = \mathbb{R} \setminus \{\text{eixo } z\} \qquad x^2+y^2 \neq 0$

$$\text{Curva } C: \begin{cases} z = y+3 & [\text{plano}] \\ 4x^2+y^2 = 4 & [\text{cilindro}] \end{cases}$$

"normal interior"



$$\partial \downarrow$$

$$\int_{C \cup C_1} \vec{F}\, dr = \iint_{\text{vermelho}} \text{rot} F \cdot \vec{n}\, dS$$

$$\int_C F\, dr = \int_{C_1} F\, dr$$

(anti-horário)

Parametrização C_1:

$$4x^2+y^2 = 4 \Rightarrow x^2 + \frac{y^2}{4} = 1$$

$$(1\cos\theta, 2\,\text{sen}\,\theta, 0) \qquad 0 \leq \theta \leq 2\pi$$

$C_1'(z) = 0$

$$\boxed{\int_{C_1} -y\,\frac{x^2+y^2+1}{x^2+y^2}\,dx + x\,\frac{x^2+y^2+1}{x^2+y^2} + \cancel{z-1\cos^2(z)}\,dz}$$

$$C_1' = (-\text{sen}\,\theta, 2\cos\theta, 0)$$

$$\int_0^{2\pi} -2\,\text{sen}\,\theta \cdot \frac{\cos^2\theta + 4\,\text{sen}^2\theta + 1}{\cos^2\theta + 4\,\text{sen}^2\theta} \cdot (-\text{sen}\,\theta)$$

$$+ \cos\theta \cdot \frac{\cos^2\theta + 4\,\text{sen}^2\theta + 1}{\cos^2\theta + 4\,\text{sen}^2\theta} \cdot (2\cos\theta)$$

$$\int_0^{2\pi} 2\left[\frac{\cos^2\theta + 4\,\text{sen}^2\theta + 1}{\cos^2\theta + 4\,\text{sen}^2\theta}\right] d\theta = 2\cdot 3\pi = 6\pi$$

Exercício 1)

$$\int_C \left(\frac{-y}{2x^2+y} + yz\right)dx + \left(\frac{x}{2x^2+y^2} - xy\right)dy +$$
$$\left(e^{z^2} + z^2\right)dz$$

$$F = \left(\frac{-y}{2x^2+y^2} + yz,\ \frac{x}{2x^2+y^2} - xy,\ e^{z^2} + z^2\right)$$

decompondo em F_1 e F_2:

$$F_1 = \left(\frac{-y}{2x^2+y^2},\ \frac{x}{2x^2+y^2},\ 0\right) \leftarrow \text{Dom } F_1 = \mathbb{R} \setminus \{(0,0,z)\}$$

$$F_2 = \left(yz,\ -xy,\ e^{z^2} + z^2\right) \leftarrow \text{Dom } \mathbb{R}$$

(*Sugestão: note que* $\vec{F} = \vec{F}_1 + \vec{F}_2$, *onde* $\vec{F}_1(x,y) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\right)$ *e* $\vec{F}_2(x,y) = (0, 3x)$ *e que* $\int_C \vec{F}\cdot d\vec{r} = \int_C \vec{F}_1 \cdot d\vec{r} + \int_C \vec{F}_2 \cdot d\vec{r}$.)

$$\text{rot } F_1 = \begin{vmatrix} i & j & k \\ \partial x & \partial y & \partial z \\ \frac{-y}{2x^2+y^2} & \frac{x}{2x^2+y^2} & 0 \end{vmatrix} =$$

$$= \left(0, 0, \frac{(2x^2+y^2) - x(4x)}{(2x^2+y^2)^2} + \frac{(2x^2+y^2) - y(2y)}{(2x^2+y^2)}\right)$$

$$\frac{2(2x^2+y^2) - 4x^2 - 2y^2}{(2x^2+y^2)^2} \rightarrow -2(2x^2+y^2)$$

rot $F_1 = \vec{0}$

$$\text{rot } F_2 = \begin{vmatrix} i & j & k \\ \partial x & \partial y & \partial z \\ yz & -xz & e^{z^2}+z^2 \end{vmatrix} = (x, y, -2z)$$

curva $\begin{cases} z = 5 - x^2 - 2y^2 \\ z = 3x^2 + 1 \end{cases}$

• elipse do plano

$Oxy = 3x^2 + 1 = 5 - x^2 - 2y^2$

$4x^2 + 2y^2 = 4$

Elipse: $x^2 + \frac{y^2}{2} = 1$

$$\int_C F\,dr = \underbrace{\int_C F_1\,dr}_{\text{I}} + \underbrace{\int_C F_2\,dr}_{\text{II}}$$

(I) $\int_C F_1\,dr = \iint_{\text{cilindro}} \cancel{\text{rot } F_1}^{\,0} - \iint_{\text{elipse (horário)}} F_1\,dr$ (1)

(II) $\int_C F_2\,dr = \iint_{\text{cilindro (2)}} \text{rot } F_2 \cdot \vec{n} + \iint F_2\,dr$ (3)

sentido anti-horário

(1) elipse $\gamma = (\cos\theta, \sqrt{2}\,\text{sen}\,\theta, 0)$ $0 \le \theta \le 2\pi$

$\gamma' = (-\operatorname{sen}\theta, \sqrt{2}\cos\theta, 0)$

$$\int_0^{2\pi} \frac{-\sqrt{2}\operatorname{sen}\theta}{2\cos^2\theta + 2\operatorname{sen}^2\theta} \cdot (-\operatorname{sen}\theta) + \frac{\cos\theta}{2\cos^2\theta + 2\operatorname{sen}^2\theta} (\sqrt{2}\cos\theta)$$

$$\int_0^{2\pi} \frac{\sqrt{2}\operatorname{sen}^2\theta}{2} + \frac{\sqrt{2}\cos^2\theta}{2}\, d\theta = 2\pi \cdot \frac{\sqrt{2}}{2} = \boxed{\sqrt{2}\pi}$$

② cilindro

$$\begin{cases} x = \cos\theta \\ y = \sqrt{2}\operatorname{sen}\theta \\ z = z \end{cases} \Rightarrow$$

$0 \leq z \leq 3\cos^2\theta + 1$

$0 \leq \theta \leq 2\pi$

$X_\theta = (-\operatorname{sen}\theta, \sqrt{2}\cos\theta, 0)$

$X_z = (0, 0, 1)$

$X_\theta \wedge X_z = (\sqrt{2}\cos\theta, \operatorname{sen}\theta, 0)$

Normal interior $= (-\sqrt{2}\cos\theta, -\operatorname{sen}\theta, 0)$

$$\int_0^{2\pi}\int_0^{3\cos^2\theta+1} (x, y, -2z)(-\sqrt{2}\cos\theta, -\operatorname{sen}\theta, 0)$$

$$\int_0^{2\pi}\int_0^{3\cos^2\theta+1} -\sqrt{2}\cos^2\theta - \sqrt{2}\operatorname{sen}^2\theta$$

$$\int_0^{2\pi} -\sqrt{2}(3\cos^2\theta + 1)$$

$$-3\sqrt{2}\int_0^{2\pi} \cancel{\cos^2\theta}^{\pi} - \sqrt{2}\cdot 2\pi$$

$$-3\sqrt{2}\pi \quad -2\sqrt{2}\pi = \boxed{-5\sqrt{2}\pi}$$

$F_2 = (yz, -xz, e^{z^2} + z^2)$

③ elipse $\gamma = (\cos\theta, \sqrt{2}\operatorname{sen}\theta, 0)$ $\quad 0 \leq \theta \leq 2\pi$

$\gamma' = (-\operatorname{sen}\theta, \sqrt{2}\cos\theta, 0)$

$z = 0$ en γ

$$\int_0^{2\pi} \sqrt{2}\operatorname{sen}\theta \cdot 0 \cdot (-\operatorname{sen}\theta) + (-\cos\theta) \cdot 0 \cdot (\sqrt{2}\cos\theta) + (e^0 + 0)(0) = \int_0^{2\pi} 0 = 0$$

$$\int_C F = ① + ② + ③ = \sqrt{2}\pi - 5\sqrt{2}\pi = -4\sqrt{2}\pi$$

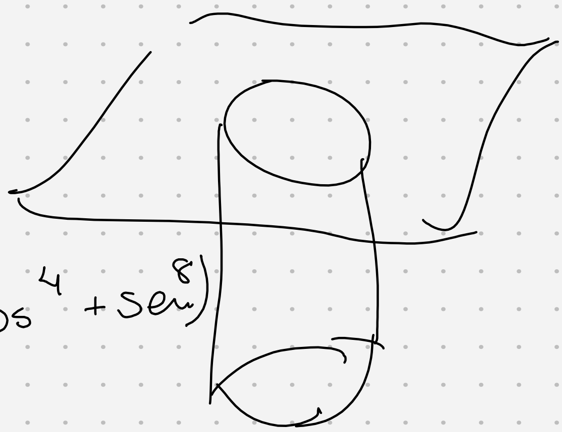
Ex2 $\int_C \left(\frac{xz}{x^2+y^2}+y\right)dx + \left(\frac{yz}{x^2+y^2}\right)dy + e^{z^4}dz$

Dom F = $\mathbb{R} - \{$eje z$\}$

$C: \begin{cases} z = \operatorname{arctg}(5+x^4+y^8) \\ \quad e \\ x^2+y^2=1 \end{cases}$

$\operatorname{rot} F = \begin{vmatrix} i & j & k \\ \partial x & \partial y & \partial z \\ \frac{xz}{x^2+y^2}+y & \frac{yz}{x^2+y^2} & e^{z^4} \end{vmatrix} =$

$\left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}, -1\right) = \operatorname{rot}$



$\begin{cases} x = \cos\theta \\ y = \operatorname{sen}\theta \\ z = z \end{cases}$ $\quad 0 \leq \theta \leq 2\pi$
$\quad 0 \leq z \leq \arctan(5+\cos^4+\operatorname{sen}^8)$

$X_\theta = (-\operatorname{sen}\theta, \cos\theta, 0)$ "normal interno"

$X_z = (0, 0, 1)$

$X_\theta \wedge X_z = \|(\cos\theta, \operatorname{sen}\theta, 0)$ $\quad (-\cos\theta, -\operatorname{sen}\theta, 0)$

$\int_0^{2\pi}\int_0^{\arctan(\ldots)} -\operatorname{sen}\theta(-\cos\theta) + \cos\theta(-\operatorname{sen}\theta) \cdot 0 = 0$

circunferencia

$\gamma = (\cos\theta, \operatorname{sen}\theta, 0) \quad 0 \leq \theta \leq 2\pi$

$\gamma' = (-\operatorname{sen}\theta, \cos\theta, 0)$

$\int_0^{2\pi} (0+\operatorname{sen}\theta)(-\operatorname{sen}\theta) + (0)(\cos\theta) + (e^{0^4})(0)$

$$\int_0^{2\pi} -\operatorname{sen}^2\theta = -\pi \quad //$$