Worm Algorithm for the Six-Vertex Model

Computational Methods in Many-Body Physics

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Table of contents

Introduction

Six-Vertex Model

- Model defined on a lattice of coordination number 4
- One arrow per bond connecting neighbouring vertices
- **Ice rule:** At each vertex there are two arrows pointing inwards
- **Ice model:** All configurations are equally likely

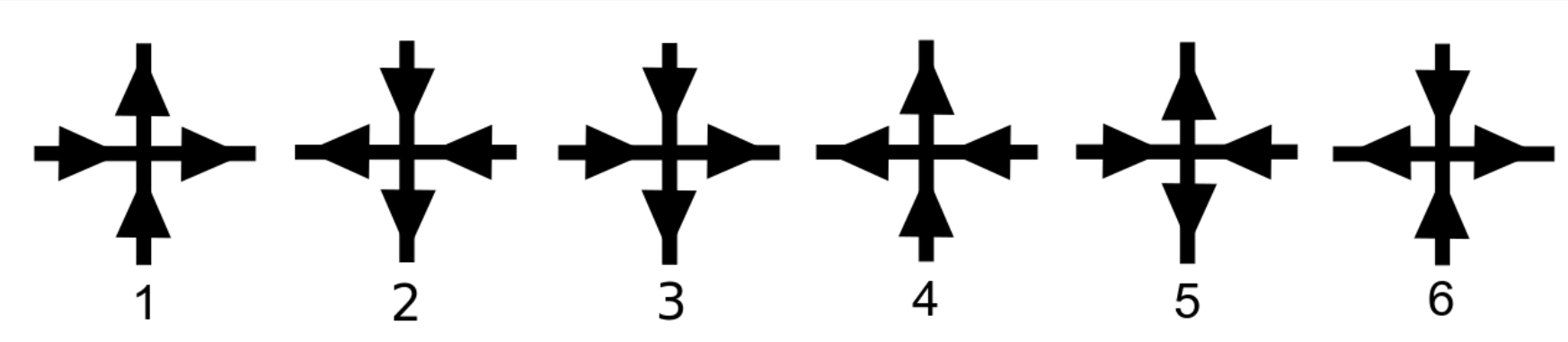

Figure 1: Six valid configurations for each vertex

Worm Algorithm

1. Start from a stable configuration
2. Randomly introduce a defect (i.e. flip a bond arrow)
3. Let the defect propagate until a new stable configuration is reached
   - 3.1 At a vertex, evaluate the nature of the defect (i.e. if there is an extra arrow pointing in or out)
   - 3.2 Randomly choose to flip one of the bonds that will restore the vertex to a valid configuration without going back
   - 3.3 Move on to the vertex connected by the chosen bond

Conventions

Bonds

- Each bond value is stored in a 1D array with an absolute value of ± 1
- When a new stable bond configuration is reached, all the measurements can be made from this bond array.

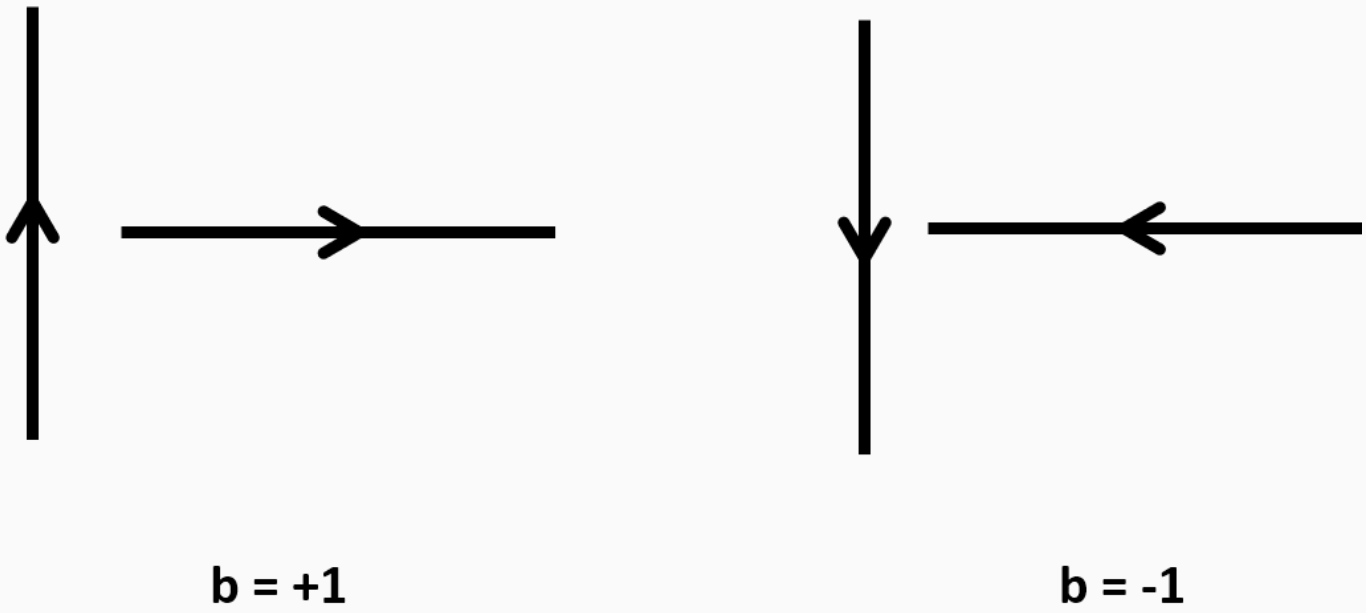

Figure 2: Absolute bond values dependent on orientation.

Vertices

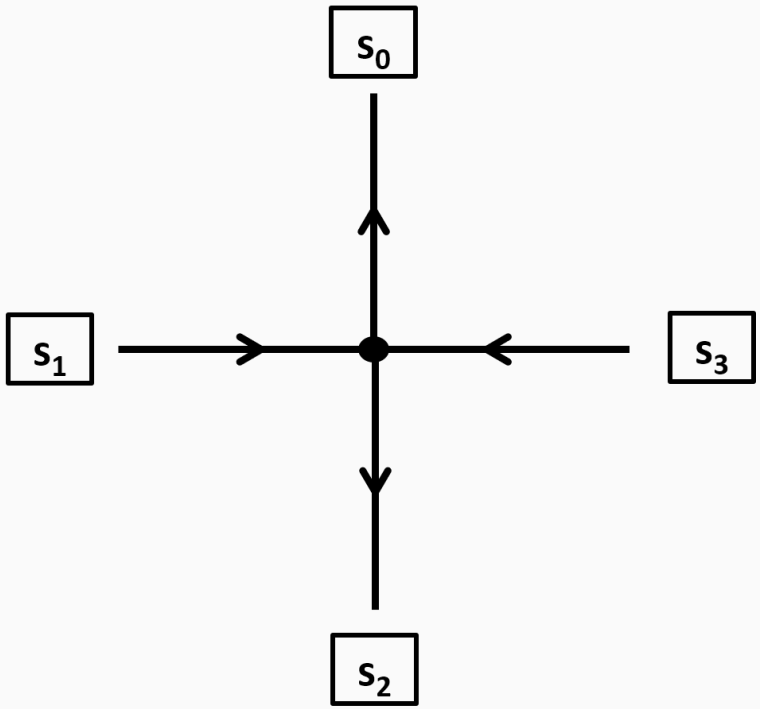

Figure 3: Chosen site indexing.

The absolute
bond values are converted
to values relative to their position
in each vertex with the convention

$$\text{In} = +1, \text{Out} = -1$$

$$\text{Stable configuration} \implies \sum_{i=0}^{3} s_i = 0$$

A First Look

Measuring

- **Aim:** Evaluate how the correlations between bonds of the same nature (i.e. *vertical* or *horizontal*) depend on their distance. Study the influence of:
  - Different system sizes
  - Vertical, horizontal or diagonal displacements
- **Procedure:**
  - Thermalize the system
  - Run the algorithm several times and for each stable configuration reached, record values of $b_0 b_r$ for different values of r
  - Average over all recorded values for each r
  - Plot the results

A priori considerations

- If we restrict the system to a square lattice $L \times L$, it exhibits clear rotational symmetry. Therefore, it is redundant to study correlations between both "vertical" and "horizontal" bonds. We shall stick to "vertical" bonds only.
- We expect different correlations depending on the direction of the displacement.

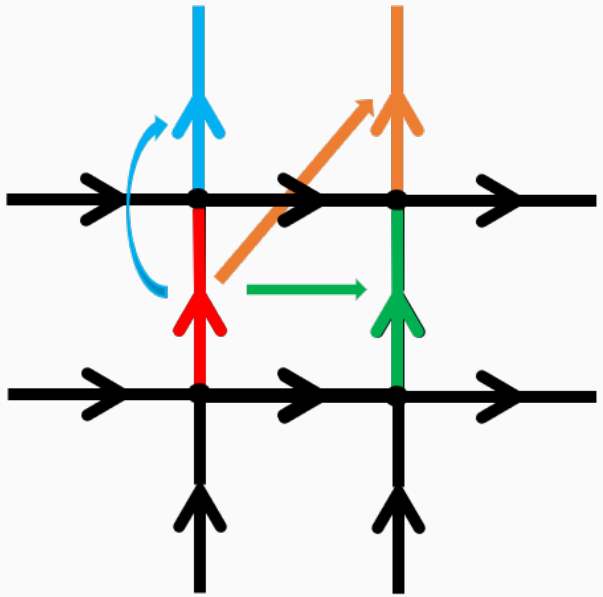

Figure 4: Vertical, diagonal and horizontal displacements.

A priori considerations II

- This system will be subject to periodic boundary conditions.
- We expect the correlations to decay as a power law

$$< b_0 b_r > \propto \frac{1}{r^{\alpha}} \tag{1}$$

  where b_0 is the bond value at an arbitrary origin.
- The translation invariance of the system yields that the correlations should be independent of the choice of origin. Therefore, we can reduce the error by making

$$< b_0 b_r > = \frac{1}{\sharp \mathbf{a}} \sum_{\mathbf{a}} < b_{\mathbf{a}} b_{\mathbf{r}+\mathbf{a}} > \tag{2}$$

  for a given set of lattice vectors $\mathbf{a}$.

Starting small

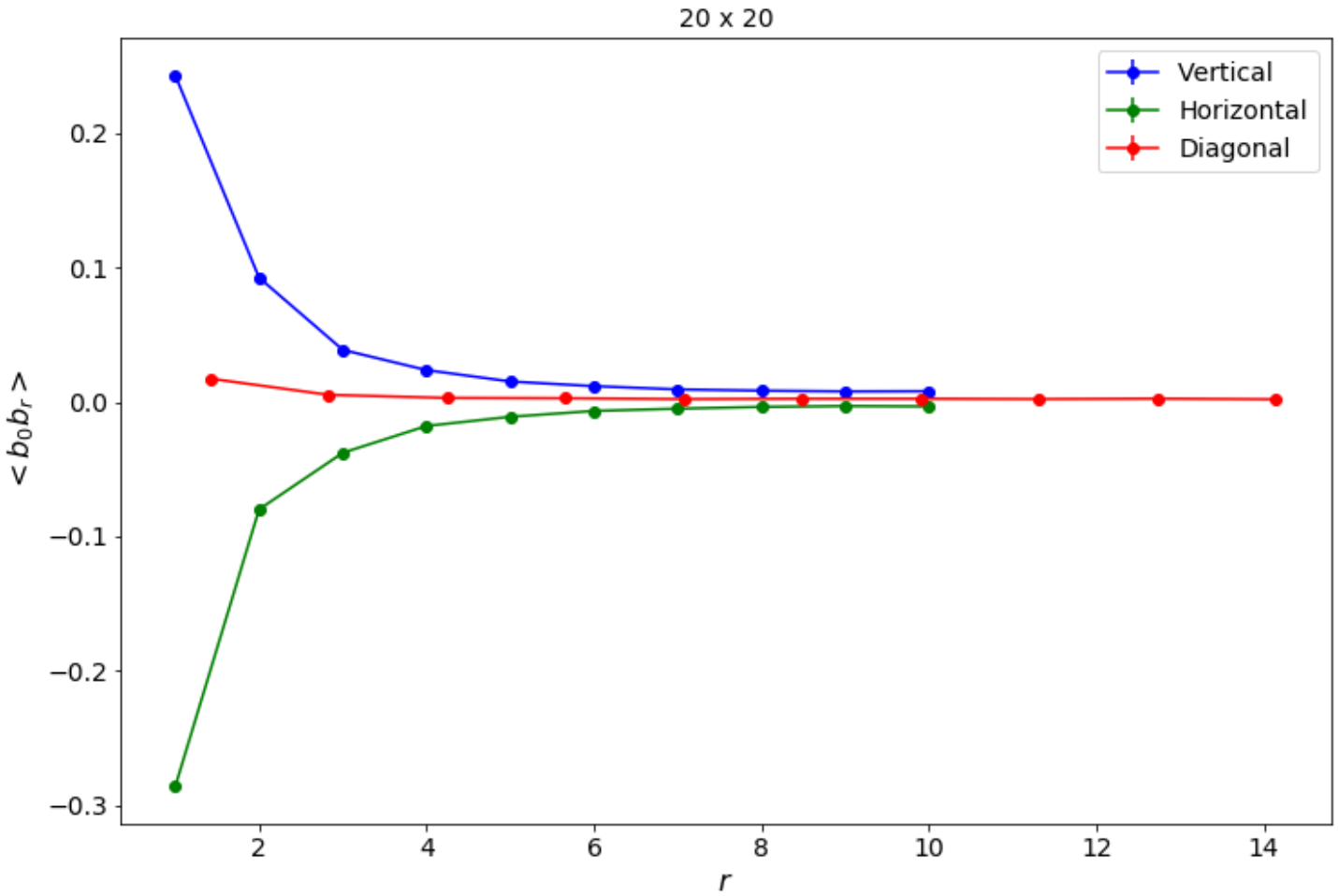

Figure 5: Power law decay of the correlation functions.

Starting small

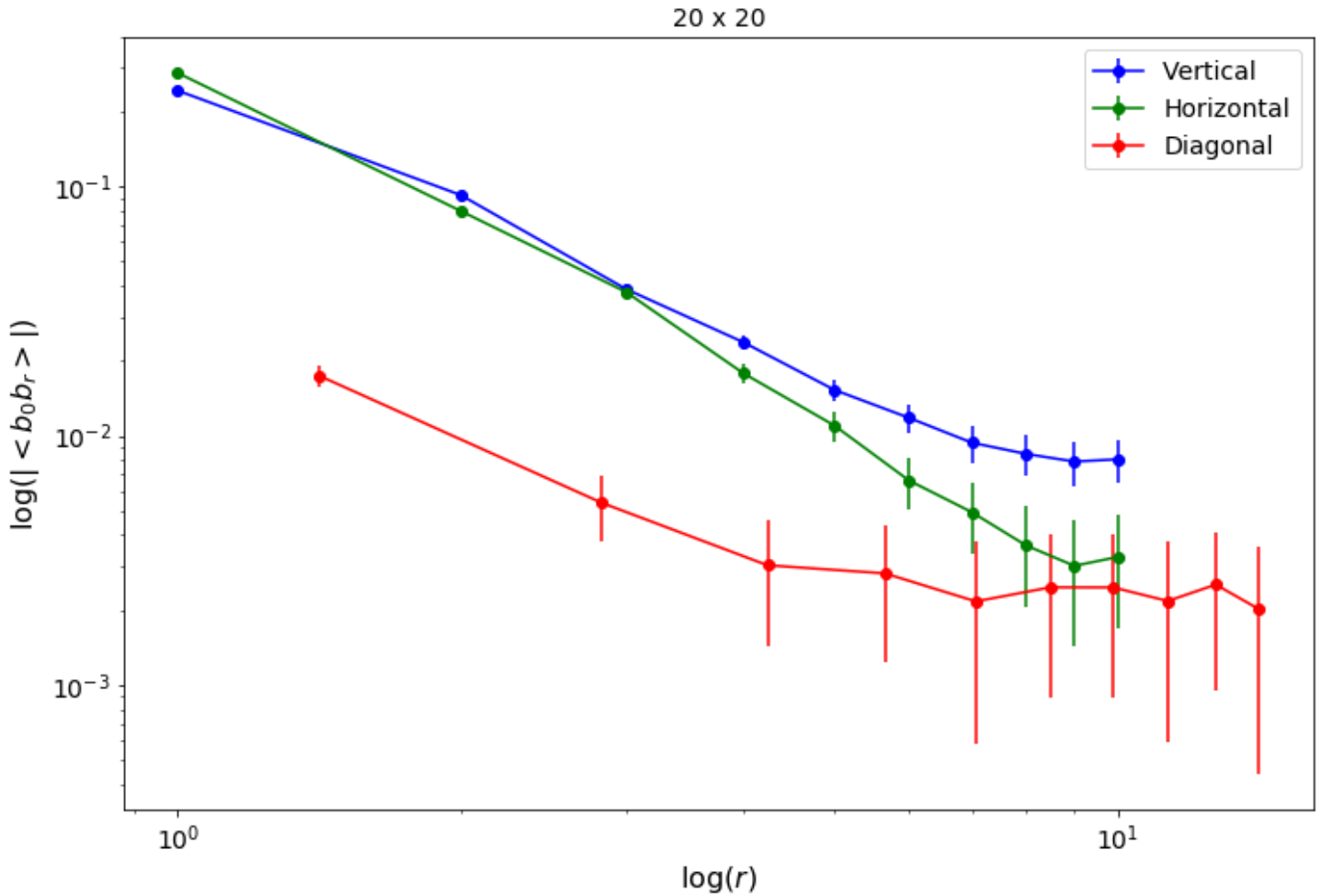

Figure 6: Log-log scale unveils unexpected deviations.

Starting small

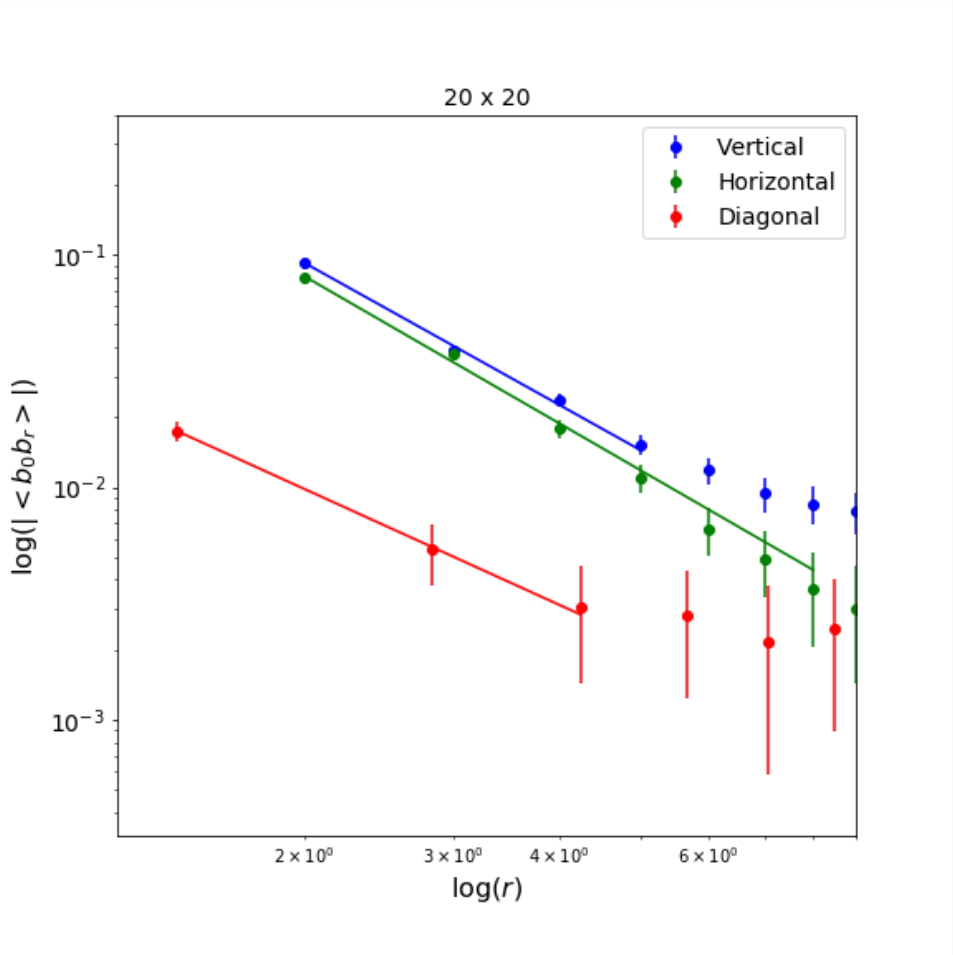

Figure 7: Fit results to the relevant points.

$$< b_0 b_r >= \frac{\beta}{r^{\alpha}} \qquad (3)$$

	α	β
V	2.02 $\pm$ 0.07	0.38 $\pm$ 0.02
H	2.10 $\pm$ 0.08	-0.35 $\pm$ 0.02
D	1.65 $\pm$ 0.05	0.0309 $\pm$ 0.0008

Table 1: Fitting parameters

Scaling Up

Vertical Displacements

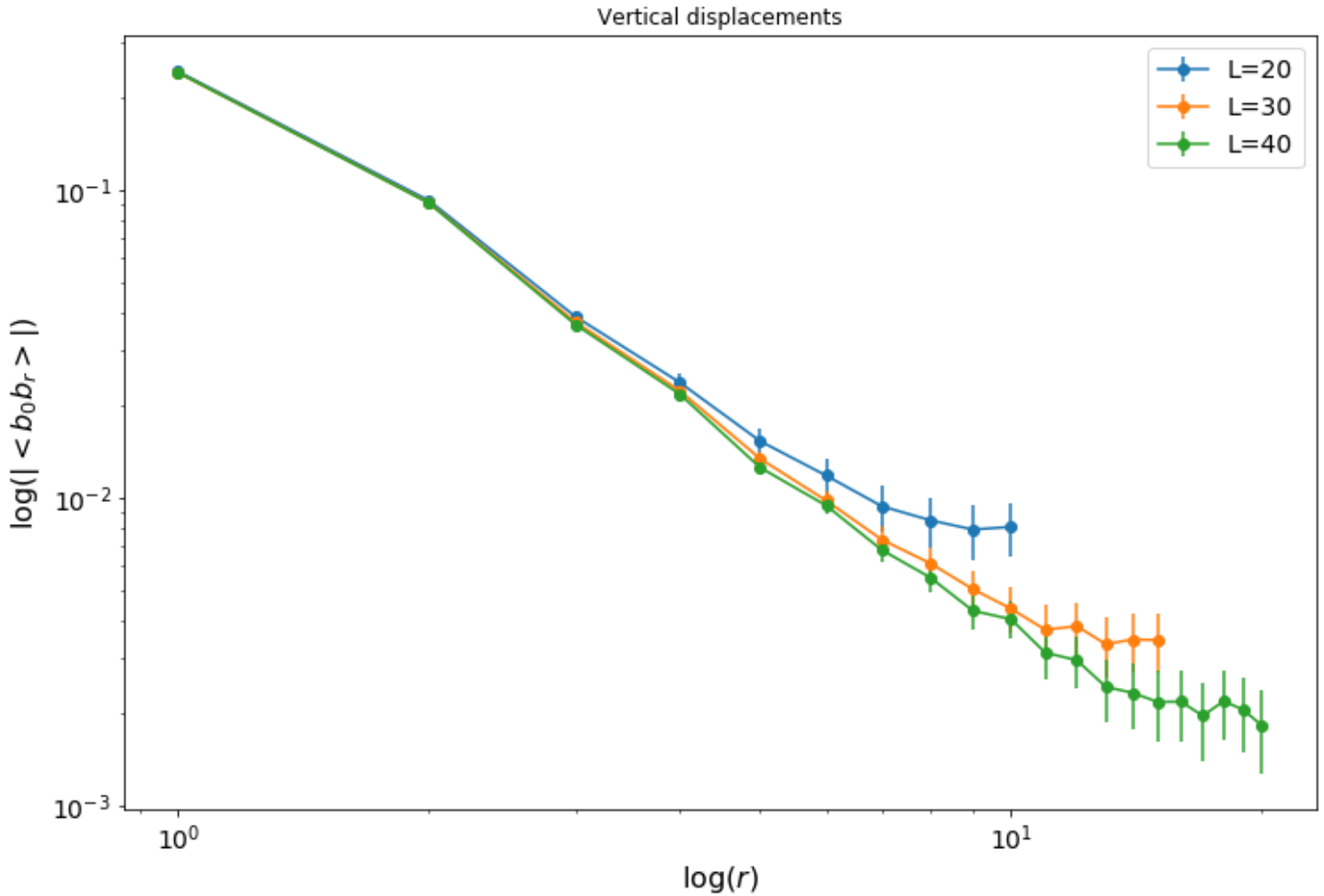

Figure 8: Point of departure from expected power law seems to scale with L.

Horizontal Displacements

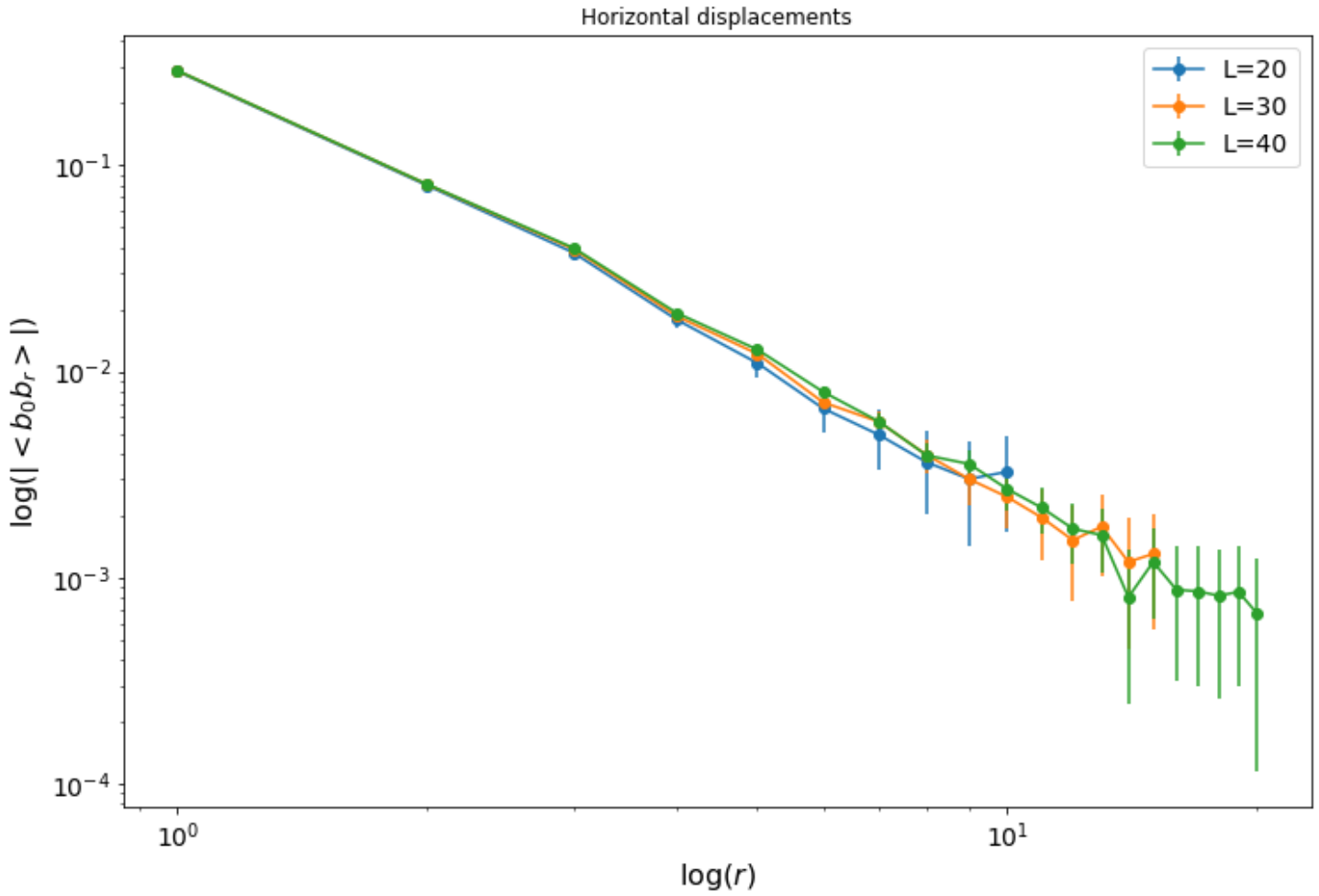

Figure 9: Horizontal displacements hold expected behavior for larger distances.

Diagonal Displacements

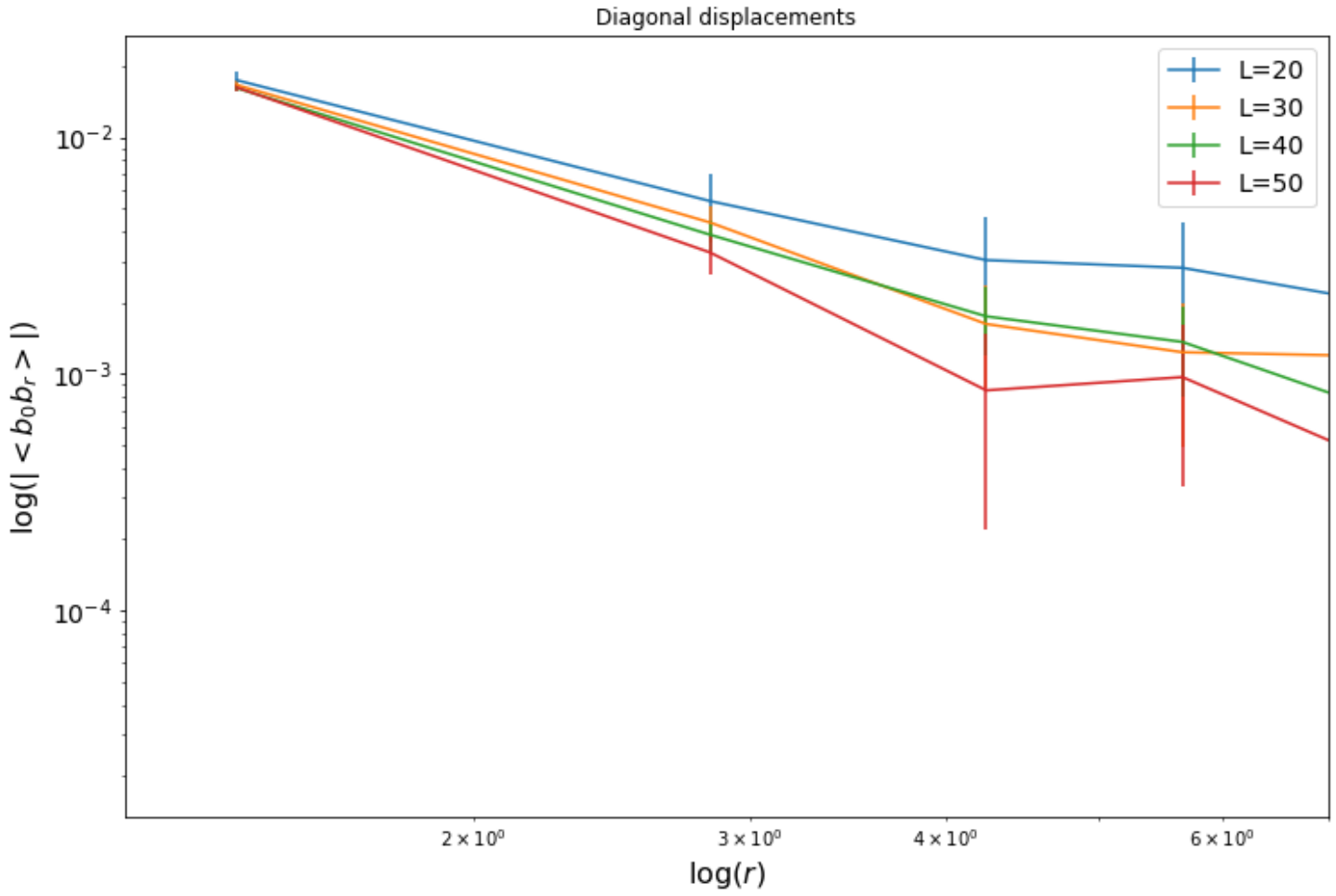

Figure 10: Increasing L leads to slightly higher decay rates.

Fitting results

$$< b_0 b_r >= \frac{\beta}{r^{\alpha}}$$

	Vertical		Horizontal		Diagonal	
L	α	β	α	β	α	β
20	2.02 $\pm$ 0.07	0.38 $\pm$ 0.02	2.10 $\pm$ 0.08	-0.35 $\pm$ 0.02	1.65 $\pm$ 0.05	0.0309 $\pm$ 0.0008
30	2.09 $\pm$ 0.04	0.39 $\pm$ 0.01	2.05 $\pm$ 0.06	-0.34 $\pm$ 0.02	1.97 $\pm$ 0.06	0.033 $\pm$ 0.001
40	2.12 $\pm$ 0.04	0.39 $\pm$ 0.01	2.01 $\pm$ 0.05	-0.33 $\pm$ 0.02	1.99 $\pm$ 0.07	0.032 $\pm$ 0.001

Table 2: Fit results for each type of displacement and diferent system sizes.

Conclusions

Conclusions

- Nearby bonds tend to be aligned in the direction to which they point, but anti-aligned perpendicularly to it;
- Correlations between bonds roughly follow an inverse square law decay;
- Departures from this regime have systematically been observed outside the range $1 << r << L/2$;
- Correlation between bonds is always weaker if displacement is diagonal;
- Increasing system size does not seem to introduce significant or unexpected effects.