

João Felipe Turcher Rondon da Femea / Q. 1

$$S = \frac{k(k+1)}{2} \text{ e já que se } k \text{ for ímpar}$$

$k+1$ é Par e se $k+1$ for ímpar k é Par

Logo sempre teremos um número ímpar multiplicando, o que implicaria em um fator primo de $S \neq 2$ já que Para $k \neq 1$

Todo número ímpar possui fator primo ímpar.

João Felipe + murilo Rendon da Fonseca / Q. 3

$$\sum_{n=1}^{2015} n(n+1-u)(n+1-u^2) = \sum_{n=1}^{2015} n \left[(n+1)^2 - (n+1)(u+u^2) + 1 \right]$$

$$\sum_{n=1}^{2015} n[(n+1)^2 + n] = \sum_{n=1}^{2015} n^3 + 3n^2 + n$$

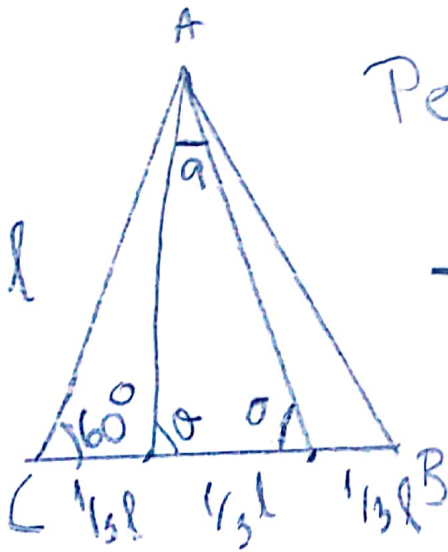
$$S = \frac{n(n+1)}{2} + \frac{3}{6} \frac{n(n+1)(2n+1)}{6} + \left(\frac{n \cdot (n+1)}{2} \right)^2$$

$$S = 2015 \cdot \frac{2016}{2} + \frac{1}{2} \cdot 2016 \cdot 2015 \cdot 3031 + \left(2015 \cdot 1008 \right)^2$$

João Felipe Thuler Rendon da Fonseca / Q. 4

\hat{DAE}

Pela relação de Stewart:



$$-\frac{x^2}{\frac{2}{3}l \cdot \frac{1}{3}l} = 1 - \frac{l^2}{l \cdot \frac{1}{3}l} + \frac{l^2}{l \cdot \frac{2}{3}l}$$

$$\frac{x^2}{\frac{2}{9}l^2} = 1 - 3 - \frac{3}{2} \Rightarrow x^2 = \frac{1}{2}l^2$$

$$x = \frac{\sqrt{2}}{2}l$$

$$\frac{\sin 60^\circ}{\frac{\sqrt{7}}{3}l} = \frac{\sin \alpha}{l} \Rightarrow \sin \alpha = \frac{3\sqrt{3}}{2\sqrt{7}}$$

$$\frac{\sin \alpha}{\frac{\sqrt{7}}{3}l} = \frac{\sin \alpha}{\frac{1}{3}l} \Rightarrow \frac{3\sqrt{3} \cdot \frac{1}{3}}{\frac{2\sqrt{7}}{\sqrt{7}/3}} = \sin \alpha = \frac{3\sqrt{3}}{2}$$

João Felipe Thuler Rendon da Fonseca/Questão 6

$$f(0) = 1; \quad x f(1-x) = 1 - f(x) \stackrel{x \neq 0}{\iff} f(-x) = \frac{1 - f(x)}{x}$$

$$x f(1-x) = 1 - f(-x) \Rightarrow f(-x) = 1 - x f(x)$$

$$\frac{1 - f(x)}{x} = 1 - x f(x) \Rightarrow x - x^2 f(x) = 1 - f(x)$$

$$x - 1 = f(x) [x^2 - 1] \Rightarrow f(x) = 1/x + 1 //$$

$f(0) = 1 // \checkmark$; Logo $f(x) = 1/x + 1$ Para qualquer x Real.

João Felipe Turmel Bandeira Ferreira/QF

$$P(x) = (x^2 + 2x - 1)(x^2 + x - 1)$$

Achando as raízes: $\frac{-2 \pm \sqrt{8}}{2} = -1 \pm \sqrt{2}$

e $\frac{-1 \pm \sqrt{5}}{2} \Rightarrow \sqrt{5} \approx 2,2; \sqrt{2} \approx 1,4$

$$\begin{aligned} -1 + 1,4 &< \frac{2,2 - 1}{2} \Rightarrow -1 - \sqrt{2} < \frac{-1 - \sqrt{5}}{2} < \frac{-1 + \sqrt{2}}{2} \\ -1 - 1,4 &< \frac{-2,2 - 1}{2} < \frac{-1 + \sqrt{5}}{2} \end{aligned}$$

$$b^2 + c^2 = (a+b)^2 - 2ab = 4 - 2(-1) = 6$$

$$S = 4 \cdot 6 + (a+1)^4 + (a+1)^6 = \left(\frac{1-\sqrt{5}}{2}\right)^4 + \left(\frac{1+\sqrt{5}}{2}\right)^6 + 24$$

$$1/16 - 1/4 \cdot \sqrt{5} + 1/16 \cdot 5 - \frac{4}{16} \cdot \sqrt{55} + 25$$

João Felipe Thurley Rondon da Fonseca / Q.8

$$R: ax + b = y; \quad X_0 = -b/a \quad \text{e} \quad y = b$$

$$b - b/a = 3a; \quad 3a + b = -2; \quad b = -2 - 3a$$

$$-2 - 3a + \frac{2 + 3a}{a} = 3a; \quad 3a^2 = 3a + 2 - 2a - 3a^2$$

$$6a^2 - a - 2 = 0; \quad \text{Como } b = -2 - 3a$$

Para cada coef angular temos 1 reta apenas

Logo a soma dos coef angulares é:

$$a_1 + a_2 = 1/6$$

João Felipe Thuler Roldão Fonseca / 2.9

$$\log a + \log z + p = r + \log y + \log c$$

$$\log b + \log y + q = \log x + \log c + r = \log z + q + r$$

$$q - r = \log x + \log c - \log b - \log y$$

$$r = \log b + \log y - \log z, q = \log x + \log c - \log z$$

$$\log x + \log y + \log z = \log x + \log c - \log z + \log b + \log y$$
$$yz = ab$$
$$\log a + \log z$$

$$\cancel{\log x} + \cancel{\log y} = -2\log z + \cancel{\log y} + \cancel{\log x} + \log c + \log b$$
$$\log z = \frac{\log c + \log b}{2}$$

$$w^k = \cos\left(\frac{2k\pi}{n}\right) + i \sin\left(\frac{2k\pi}{n}\right)$$

$$w^{-k} = \cos\left(-\frac{2k\pi}{n}\right) + i \sin\left(-\frac{2k\pi}{n}\right)$$

$$w^k + w^{-k} = 2 \cos\left(\frac{2k\pi}{n}\right) \Rightarrow \cos\left(\frac{2k\pi}{n}\right) = \frac{w^k + w^{-k}}{2}$$

B) $Z = 1$ $\sqrt[n]{Z} = \cos\left(\frac{2k\pi}{n}\right)$; Para $k=1$ até n

Como: $Z^n - 1 = 0 \Rightarrow (Z-1)(1+X_1+X_2+X_3+\dots+X_{n-1})$

Queremos: $X_1+X_2+X_3+\dots+X_{n-1} = -1$; Como os cosenos não sempre a Parte real: $\sum_{k=1}^{n-1} \cos\left(\frac{2k\pi}{n}\right) = -1$

C) $\cos^4 \theta = \cos^2 \theta \cdot \cos^2 \theta = \left(\frac{\cos 2\theta + 1}{2}\right) \left(\frac{\cos 2\theta + 1}{2}\right)$

$= \frac{1}{4} (\cos^2 2\theta + 2\cos 2\theta + 1)$

$(\cos 2\theta)^2 = \frac{\cos^4 \theta + 1}{2} \Rightarrow \cos^4 \theta = \frac{\cos^4 \theta}{2} + \frac{1}{2} (\cos 2\theta + 1)$

$S = \sum_{k=1}^{n-1} \frac{\cos\left(\frac{4k\pi}{2n+1}\right)}{8} + \sum_{k=1}^n \frac{\cos\left(\frac{2k\pi}{2n+1}\right)}{2} + \sum_{k=1}^n \frac{3}{8}$

Caindo na mesma situação do item B)

$S = -1/8 - 1/2 + \frac{3}{8} \cdot n = \frac{3}{8}n - \frac{5}{8}$