Aula 12 – Morfologia matemática II

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Roteiro

- Abertura e Fechamento morfológico
- Transformada Hit or Miss

ABERTURA E FECHAMENTO MORFOLÓGICO

Abertura morfológica

- Recordando:
  - A erosão reduz/diminui os componentes em uma imagem
  - A dilatação aumenta/expande os componentes em uma imagem

- A **abertura** suaviza o contorno de um objeto, rompe istmos e elimina saliências finas

- A abertura do conjunto A pelo EE B é:
  - $A \circ B = (A \ominus B) \oplus B$
  - A **abertura** de *A* por *B* é a erosão de *A* por *B* seguida de uma dilatação por *B*

Fechamento morfológico

- O fechamento também suaviza contornos, porém, diferente da abertura:
  - funde descontinuidades estreitas
  - elimina pequenos buracos e
  - preenche lacunas (baias) no contorno

- O fechamento do conjunto *A* pelo EE B é:
  - $A \cdot B = (A \oplus B) \ominus B$

- O fechamento de *A* por *B* é a dilatação de *A* por *B* seguida da erosão por *B*

Abertura e Fechamento morfológico

A

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

Abertura morfológica
$(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

Abertura e Fechamento morfológico

Abertura morfológica
$(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Abertura e Fechamento morfológico

Abertura morfológica
$(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0

Abertura e Fechamento morfológico

Abertura morfológica
$(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

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Abertura morfológica
$(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Fechamento morfológico
$(A \oplus B) \ominus B$

Abertura e Fechamento morfológico

Abertura morfológica
$(A \ominus B) \oplus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$A \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	0	0	0	0	0
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1
4	0	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	0
7	1	1	1	1	1	1	1	1	0
8	0	1	1	1	0	0	1	0	0

Fechamento morfológico
$(A \oplus B) \ominus B$

Abertura e Fechamento morfológico

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A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

Abertura morfológica
$(A \ominus B) \oplus B$

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

Fechamento morfológico
$(A \oplus B) \ominus B$

$A \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	0	0	0	0	0
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1
4	0	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	0
7	1	1	1	1	1	1	1	1	0
8	0	1	1	1	0	0	1	0	0

$(A \oplus B) \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	1	0	0	0	0
3	0	1	1	1	1	1	1	1	0
4	0	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

Abertura e Fechamento morfológico

Abertura morfológica
$(A \ominus B) \oplus B$

Fechamento morfológico
$(A \oplus B) \ominus B$

A

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	1	1	1	0	1	1	1	0
4	0	0	0	0	0	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	0	0	0	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

B

0	1	0
1	1	1
0	1	0

$A \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0

$(A \ominus B) \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
2	0	1	1	1	0	0	0	0	0
3	0	0	1	0	0	0	1	0	0
4	0	0	0	0	0	1	1	1	0
5	0	0	1	0	0	0	1	0	0
6	0	1	1	1	0	0	0	0	0
7	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0

$A \oplus B$

	0	1	2	3	4	5	6	7	8
0	0	1	1	1	0	0	0	0	0
1	1	1	1	1	1	0	0	0	0
2	1	1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1
4	0	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	0
7	1	1	1	1	1	1	1	1	0
8	0	1	1	1	0	0	1	0	0

$(A \oplus B) \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	1	0	0	0	0
3	0	1	1	1	1	1	1	1	0
4	0	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

$(A \oplus B) \ominus B$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	0	0	0	0	0
2	0	1	1	1	1	0	0	0	0
3	0	1	1	1	1	1	1	1	0
4	0	0	1	1	1	1	1	1	0
5	0	1	1	1	1	1	1	1	0
6	0	1	1	1	1	1	1	0	0
7	0	1	1	1	0	0	1	0	0
8	0	0	0	0	0	0	0	0	0

TRANSFORMADA HIT OR MISS

Transformada Hit or Miss

- A **transformada hit-or-miss** é uma ferramenta básica para a detecção de formas:
  - Utiliza dois elementos estruturantes para especificar o padrão a ser detectado na imagem.
    - B_1: verifica (testa) os pixels de objetos (1's)
    - B_2: verifica (testa) os pixels de fundo (0's)
  - A transformada hit-or-miss é definida como:
    - $A \circledast B = (A \ominus B_1) \cap (A^c \ominus (B_2))$

0	0	0
0	1	1
0	1	0

B_1

1	1	0
1	0	0
0	0	0

B_2

ou

0	0	×
0	1	1
×	1	×

B

← 0 em B_1 e 0 em B_2.

Transformada Hit or Miss

$A = C \cup D \cup E$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

D

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

D’

1	1	1	1	1
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1

Transformada Hit or Miss

$A = C \cup D \cup E$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

D

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

A^c

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

D’

1	1	1	1	1
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1

Transformada Hit or Miss

$A = C \cup D \cup E$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

D

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

$A \ominus D$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

A^c

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

D’

1	1	1	1	1
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1

Transformada Hit or Miss

$A = C \cup D \cup E$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

D

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

$A \ominus D$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

A^c

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

D’

1	1	1	1	1
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1

$A^c \ominus (D')$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Transformada Hit or Miss

$A = C \cup D \cup E$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

D

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

$A \ominus D$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$\cap$

$(A \ominus D) \cap (A^c \ominus (D'))$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

A^c

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

D’

1	1	1	1	1
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1

$A^c \ominus (D')$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Transformada Hit or Miss

$A = C \cup D \cup E$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

D

0	0	0	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

$A \ominus D$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$\cap$

$(A \ominus D) \cap (A^c \ominus (D'))$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

A^c

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	0	0	1	1	1
1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

D'

1	1	1	1	1
1	0	0	0	1
1	0	0	0	1
1	0	0	0	1
1	1	1	1	1

$A^c \ominus (D')$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$(A \ominus D) \cap (A^c \ominus (D'))$

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Detecção de cantos usando transformada *hit-or-miss*

	1	1		1	1		
	1	1	1	1	1	1	
			1	1			
	1		1	1	1		
	1	1	1	1	1		
	1	1	1	1	1		

A

	1	1		1	1		
	1	1	1	1	1	1	
			1	1			
	1		1	1	1		
	1	1	1	1	1		
	1	1	1	1	1		

A

$$\bigcup_{k=1}^{4} \left(A \circledast B^k \right)$$

Obs. 1: 4-conectividade.
Obs. 2: × = não importa se 0 ou 1.

Detecção de cantos usando transformada *hit-or-miss*

0	0	×
0	1	1
×	1	×

B^1

	1	1		1	1		
	1	1	1	1	1	1	
			1	1			
	1		1	1	1		
	1	1	1	1	1		
	1	1	1	1	1		

A

	1_{B^1}			1_{B^1}			

$$\bigcup_{k=1}^{4} \left(A \circledast B^k\right)$$

Obs. 1: 4-conectividade.
Obs. 2: × = não importa se 0 ou 1.

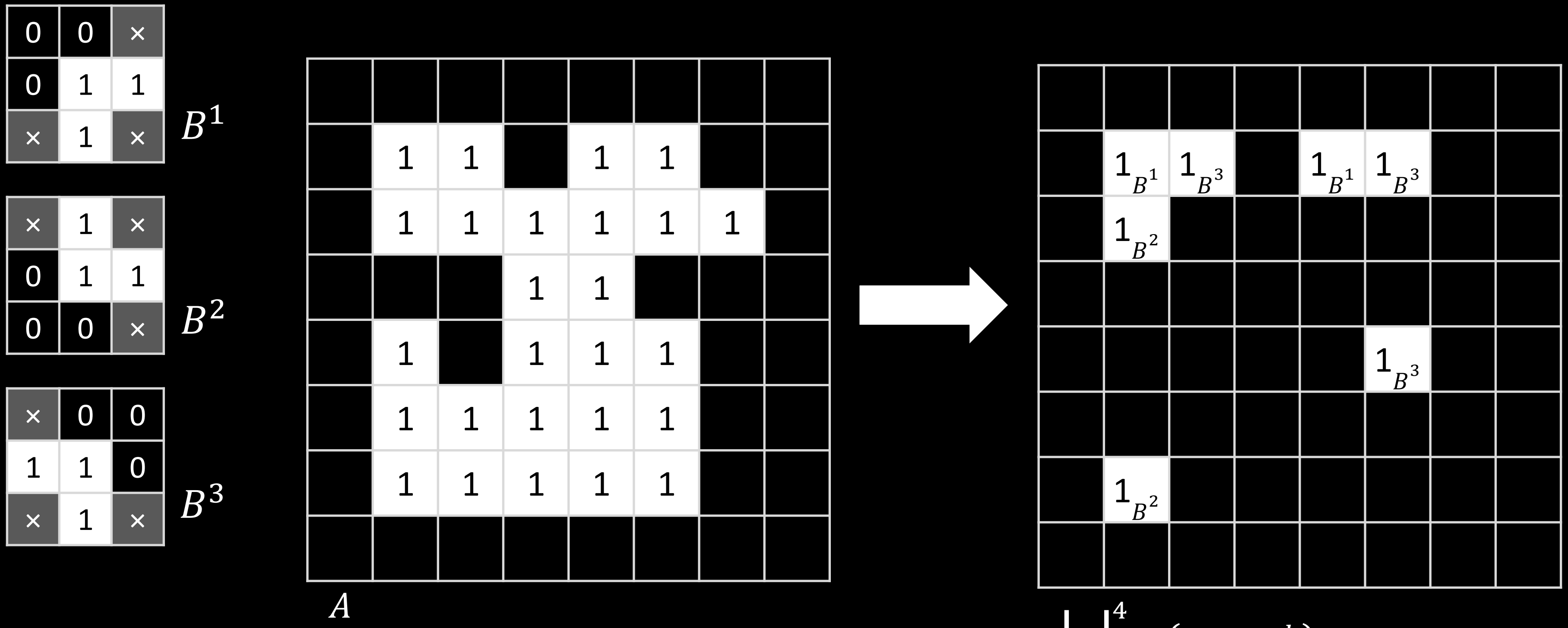
Detecção de cantos usando transformada *hit-or-miss*

$$B^1 = \begin{bmatrix} 0 & 0 & \times \\ 0 & 1 & 1 \\ \times & 1 & \times \end{bmatrix} \qquad B^2 = \begin{bmatrix} \times & 1 & \times \\ 0 & 1 & 1 \\ 0 & 0 & \times \end{bmatrix}$$

$$A = \begin{bmatrix} & & & & & & & \\ & 1 & 1 & & 1 & 1 & & \\ & 1 & 1 & 1 & 1 & 1 & 1 & \\ & & & 1 & 1 & & & \\ & 1 & & 1 & 1 & 1 & & \\ & 1 & 1 & 1 & 1 & 1 & & \\ & 1 & 1 & 1 & 1 & 1 & & \\ & & & & & & & \end{bmatrix} \Rightarrow \begin{bmatrix} & & & & & & & \\ & 1_{B^1} & & & 1_{B^1} & & & \\ & 1_{B^2} & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & 1_{B^2} & & & & & & \\ & & & & & & & \end{bmatrix} = \bigcup_{k=1}^{4} \left(A \circledast B^k\right)$$

Obs. 1: 4-conectividade.
Obs. 2: × = não importa se 0 ou 1.

Detecção de cantos usando transformada *hit-or-miss*



$$\bigcup_{k=1}^{4} (A \circledast B^k)$$

Obs. 1: 4-conectividade.
Obs. 2: × = não importa se 0 ou 1.

Detecção de cantos usando transformada *hit-or-miss*

0	0	×
0	1	1
×	1	×

B^1

×	1	×
0	1	1
0	0	×

B^2

×	0	0
1	1	0
×	1	×

B^3

×	1	×
1	1	0
×	0	0

B^4

	1	1		1	1		
	1	1	1	1	1	1	
			1	1			
	1		1	1	1		
	1	1	1	1	1		
	1	1	1	1	1		

A

	1_{B^1}	1_{B^3}		1_{B^1}	1_{B^3}		
	1_{B^2}						
					1_{B^3}		
	1_{B^2}				1_{B^4}		

$$\bigcup_{k=1}^{4}\left(A \circledast B^k\right)$$

Obs. 1: 4-conectividade.
Obs. 2: × = não importa se 0 ou 1.

Detecção de pontos extremos usando *hit-or-miss*

	1	1			1		
	1		1		1	1	
			1		1		
	1		1	1	1		
	1	1	1	1	1		

A

Detecção de pontos extremos usando *hit-or-miss*

	1	1			1		
	1		1		1	1	
			1		1		
	1		1	1	1		
	1	1	1	1	1		

A

$$\bigcup_{k=1}^{4}\left(A \circledast B^{k}\right)$$

Obs. 1: 4-conectividade.
Obs. 2: × = não importa se 0 ou 1.

Detecção de pontos extremos usando *hit-or-miss*

×	0	×
0	1	0
×	×	×

B^1

	1	1			1		
	1		1		1	1	
			1		1		
	1		1	1	1		
	1	1	1	1	1		

A

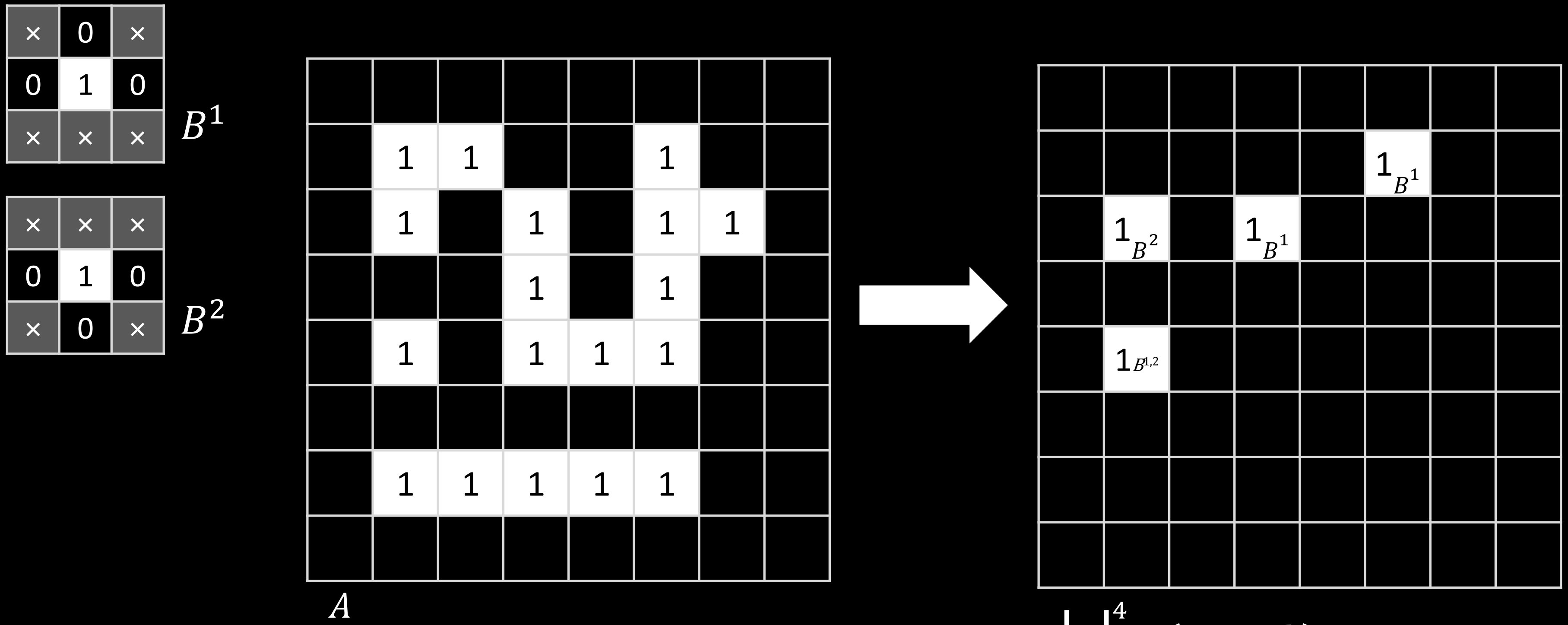
					1_{B^1}		
			1_{B^1}				
	1_{B^1}						

$$\bigcup_{k=1}^{4}\left(A \circledast B^k\right)$$

Obs. 1: 4-conectividade.

Obs. 2: × = não importa se 0 ou 1.

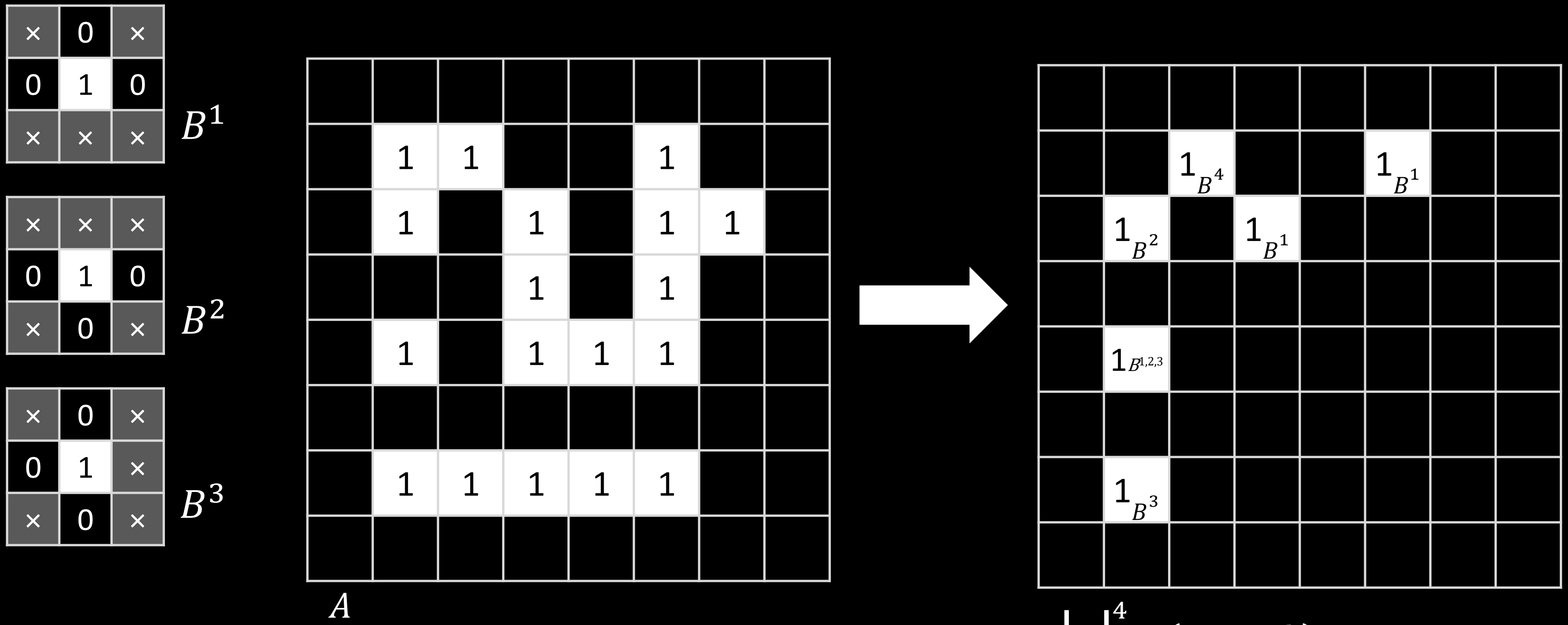
Detecção de pontos extremos usando *hit-or-miss*



$$\bigcup_{k=1}^{4} (A \circledast B^k)$$

Obs. 1: 4-conectividade.
Obs. 2: × = não importa se 0 ou 1.

Detecção de pontos extremos usando *hit-or-miss*



$$\bigcup_{k=1}^{4} \left(A \circledast B^k \right)$$

Obs. 1: 4-conectividade.
Obs. 2: × = não importa se 0 ou 1.

Detecção de pontos extremos usando *hit-or-miss*

×	0	×
0	1	0
×	×	×

B^1

×	×	×
0	1	0
×	0	×

B^2

×	0	×
0	1	×
×	0	×

B^3

×	0	×
×	1	0
×	0	×

B^4

	1	1			1		
	1		1		1	1	
			1		1		
	1		1	1	1		
	1	1	1	1	1		

A

		1_{B^4}			1_{B^1}		
	1_{B^2}		1_{B^1}			1_{B^4}	
	$1_{B^{1,2,3,4}}$						
	1_{B^3}				1_{B^4}		

$$\bigcup_{k=1}^{4}\left(A \circledast B^k\right)$$

Obs. 1: 4-conectividade.
Obs. 2: × = não importa se 0 ou 1.

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