Aula 13 – Transformada de Fourier

Prof. João Fernando Mari
joaofmari.github.io
joaof.mari@ufv.br

Roteiro

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- Transformada de Fourier
- O par de transformadas de Fourier
- Analisando a equação da Transforma de Fourier
- A Transformada Discreta de Fourier
- Calculo da DFT

Transformada de Fourier

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- A transformada de Fourier de uma função continua f(t) é definida como:

$$\Im\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu t}dt$$

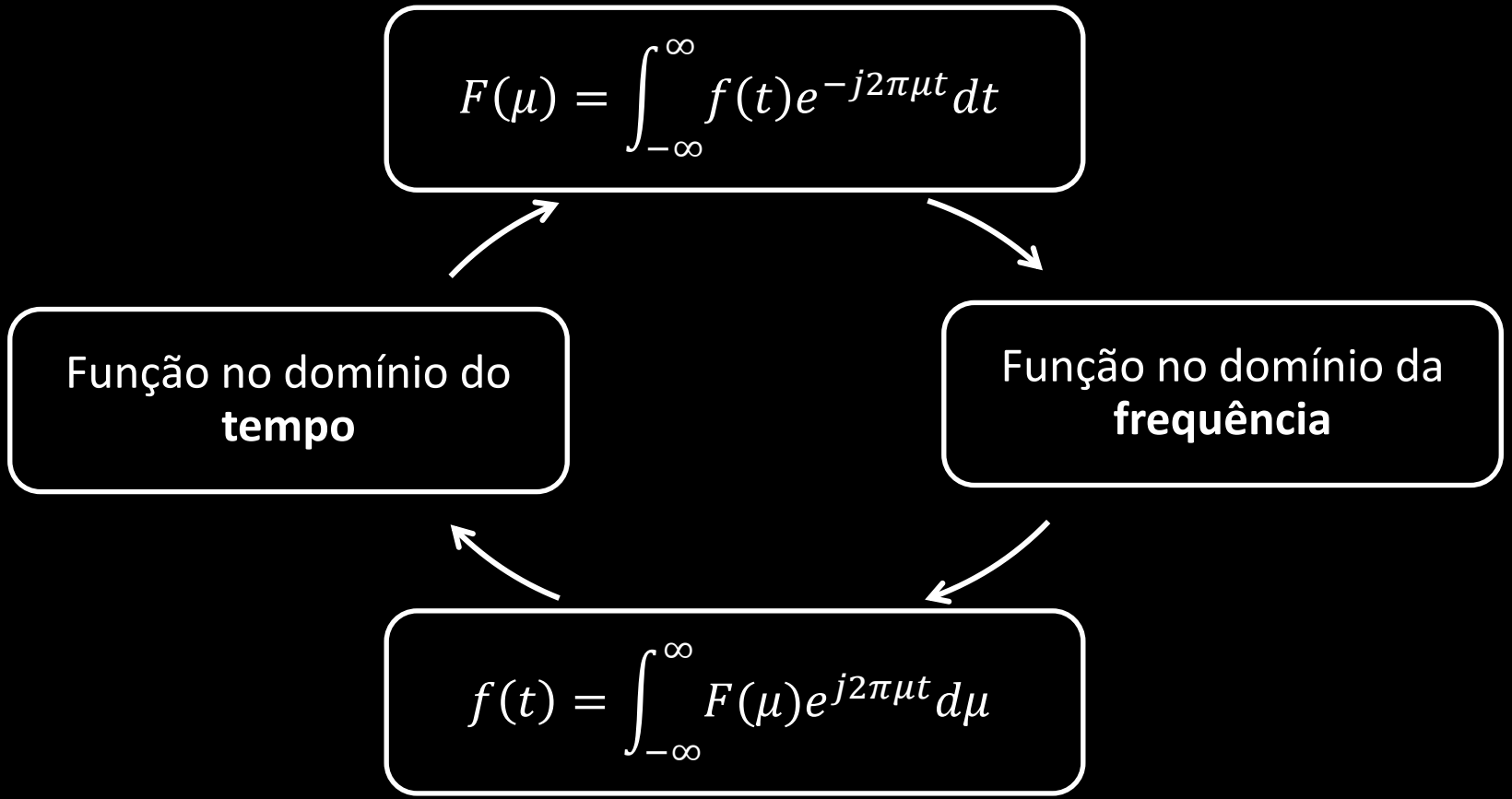
- Como $\Im\{f(t)\}$ é uma função de apenas μ, pois t é eliminada pela integração, a transformada de Fourier de f(t) pode ser expressa como:

$$\mathrm{F}(\mu) = \int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu t}dt$$

- Dada F(μ), podemos obter novamente f(t) utilizando a transformada inversa de Fourier, f(t) = $\Im^{-1}\{F(\mu)\}$, expressa como:

$$f(t) = \int_{-\infty}^{\infty} F(\mu)e^{j2\pi\mu t}d\mu$$

O par de transformadas de Fourier



Analisando a equação da Transforma de Fourier

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- Utilizando a fórmula de Euler podemos reescrever...

$$\mathrm{F}(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$

- como:

$$\mathrm{F}(\mu) = \int_{-\infty}^{\infty} f(t) \cos(2\pi\mu t) - j\, sen(2\pi\mu t)\, dt$$

- F(μ) é a própria função f(t) multiplicada por termos senoidais com frequências definidas pelos valores de μ.
  - A variável t (tempo) é eliminada pela integração.
  - Na verdade t pode representar qualquer variável continua: tempo, espaço, etc.
    - As unidades da variável de frequência dependem da unidade definida para t:
      - Se t representa o tempo e está em segundos: μ representa ciclos/s (Hz)
      - Se t representa o espaço e está em metros: μ representa ciclos/metro

A Transformada Discreta de Fourier

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- Dada a natureza continua da transformada de Fourier, ela não pode ser implementada em um computador.

- A transformada discreta de Fourier é:

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M}, \qquad u = 0,1,2,\dots,M-1$$

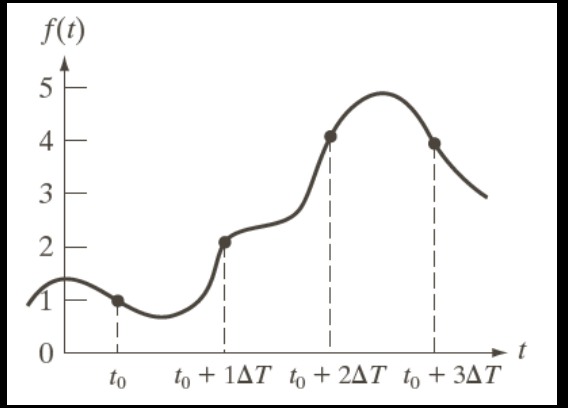
- A transformada inversa discreta de Fourier é :

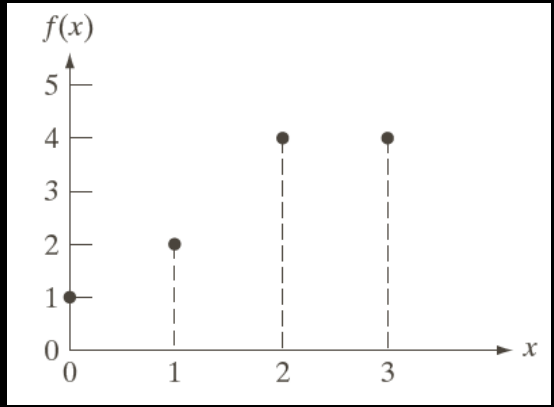
$$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M}, \qquad x = 0,1,2,\dots,M-1$$

Calculo da DFT

- DFT: $F(u) = \sum_{x=0}^{M-1} f(x) e^{-\frac{j2\pi ux}{M}}$

- $F(0) = \sum_{x=0}^{3} f(x) = [f(0) + f(1) + f(2) + f(3)]$
- $F(0) = 1 + 2 + 4 + 4 = 11$
- $|F(0)| = \sqrt{(11)^2 + (0)^2} = 11{,}0$

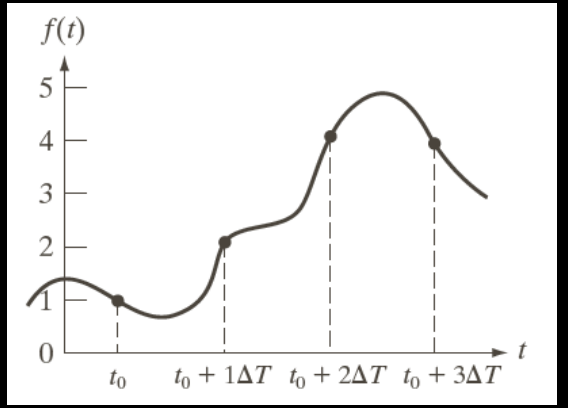


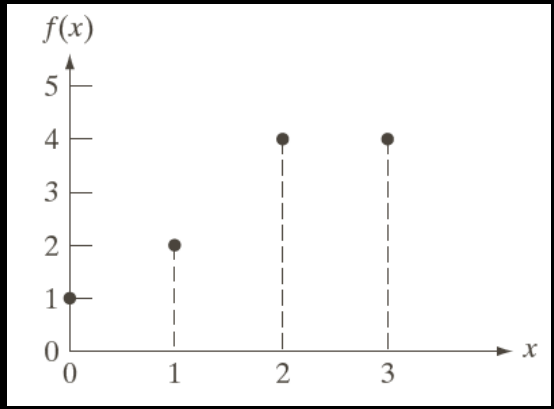


Calculo da DFT

- DFT: $F(u) = \sum_{x=0}^{M-1} f(x) e^{-\frac{j2\pi ux}{M}}$

- $F(1) = \sum_{x=0}^{3} f(x) e^{-j2\pi(1)x/M}$
- $F(1) = 1e^{-j2\pi(1)0/4} + 2e^{-j2\pi(1)1/4} + 4e^{-j2\pi(1)2/4} + 4e^{-j2\pi(1)3/4}$
- $F(1) = 1e^{0} + 2e^{-j\pi/2} + 4e^{-j\pi} + 4e^{-j3\pi/2} = -3 + 2j$
- $|F(1)| = \sqrt{(-3)^2+(2)^2} = 3{,}61$





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- $F(2) = \sum_{x=0}^{3} f(x) e^{-j2\pi(2)x/M}$
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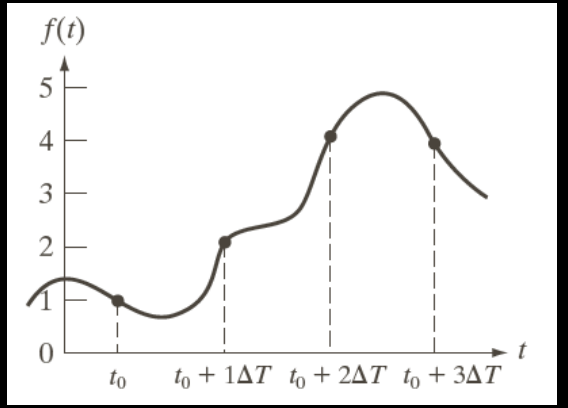

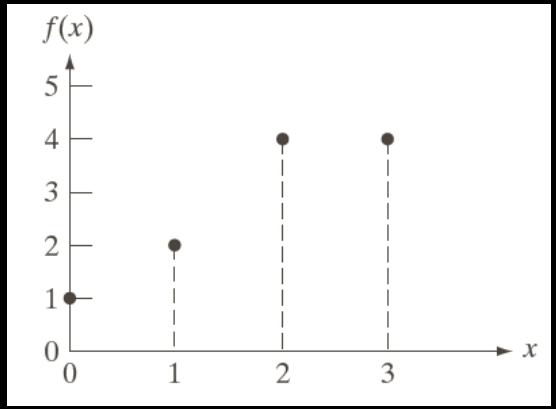

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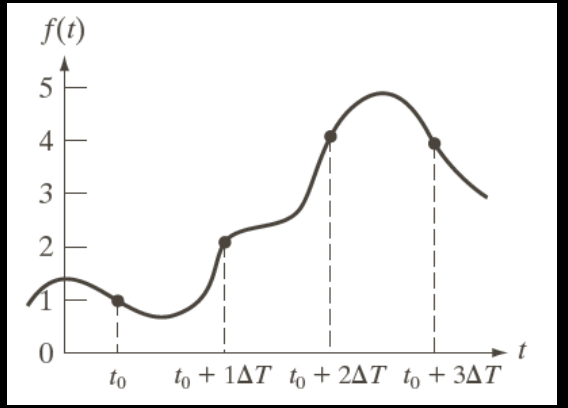

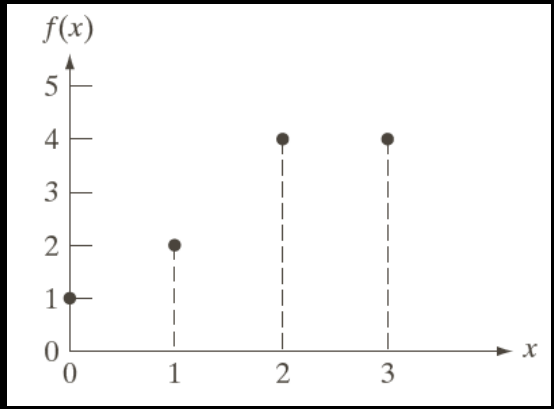

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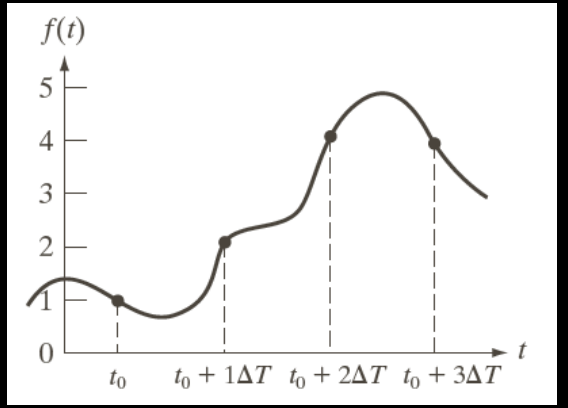

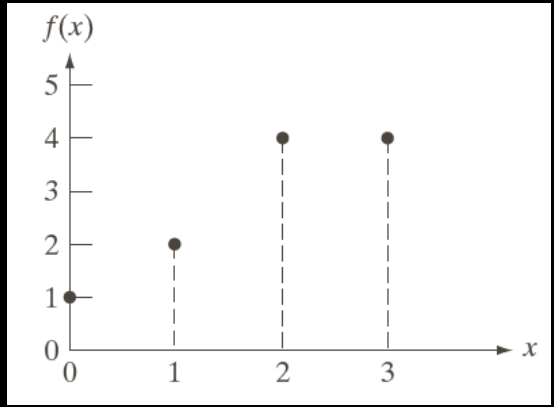

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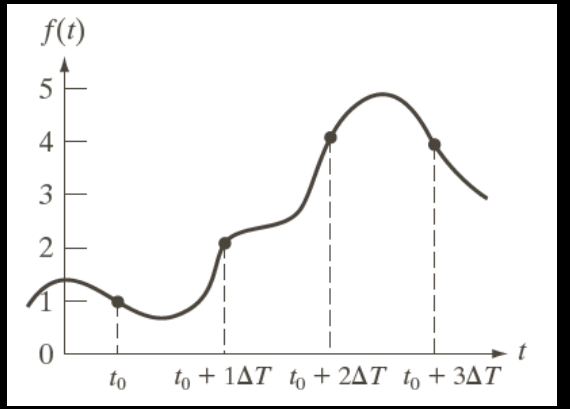

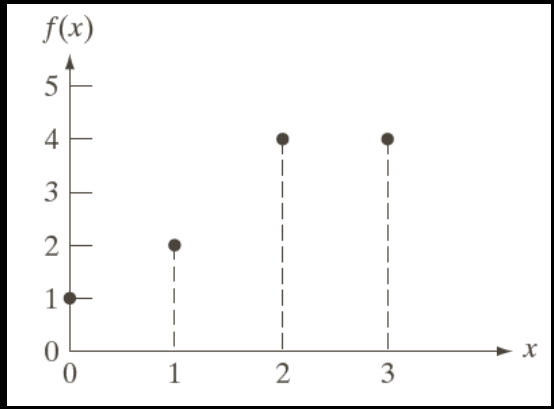

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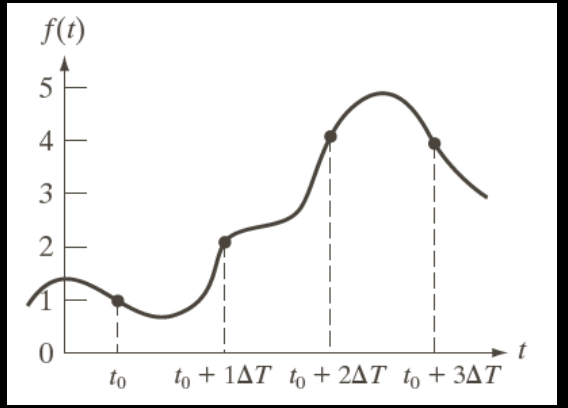

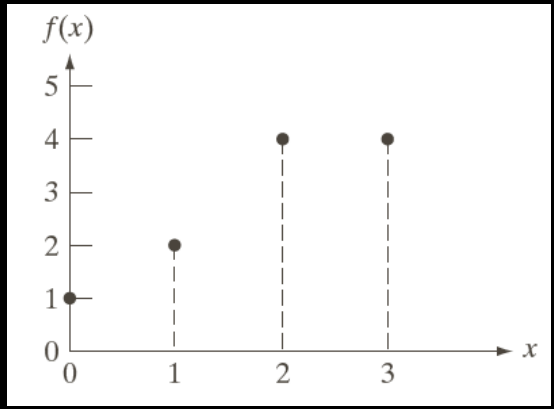

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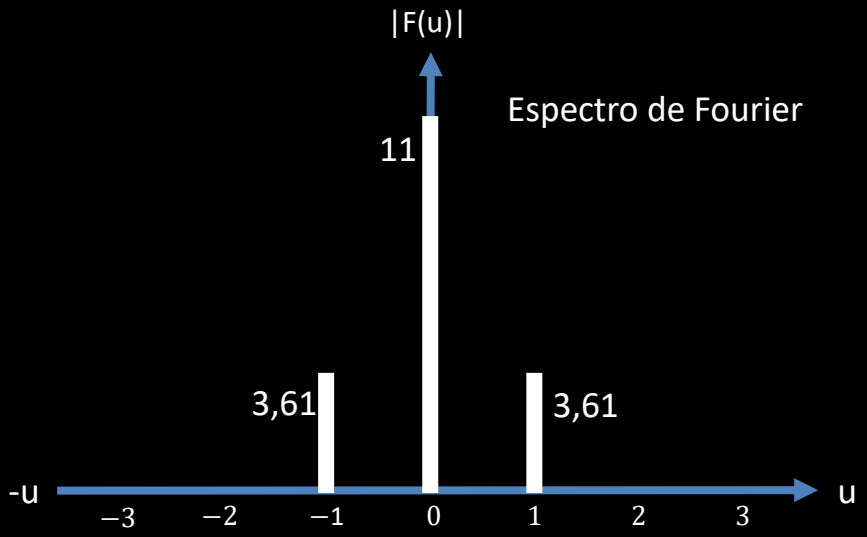
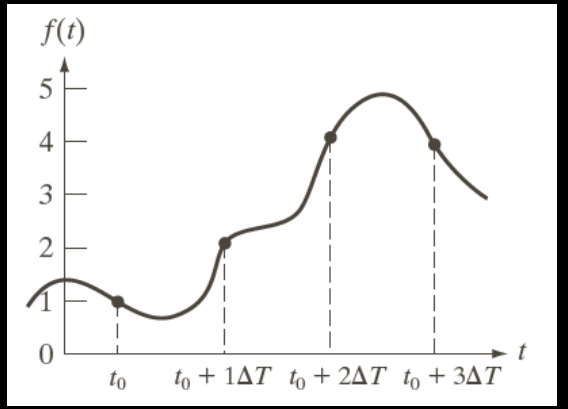
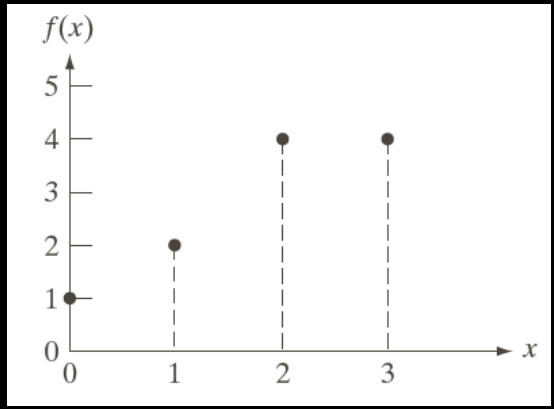


Espectro de Fourier





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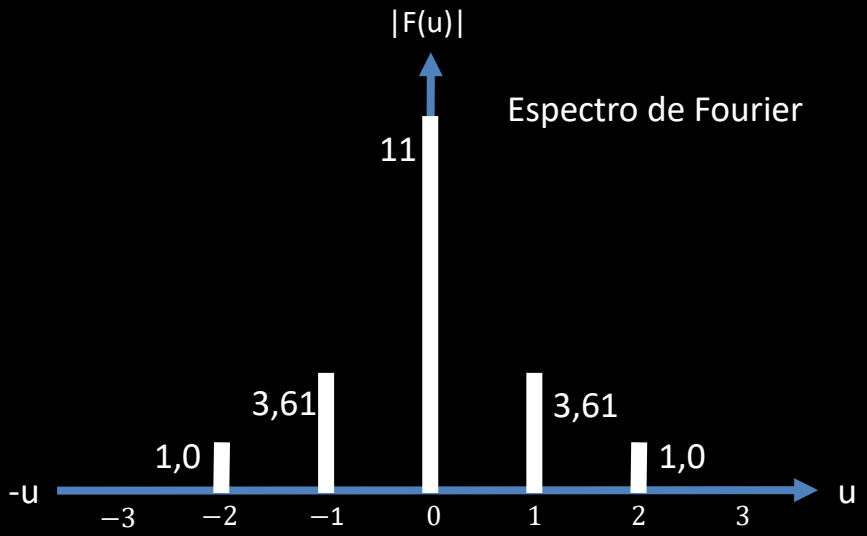

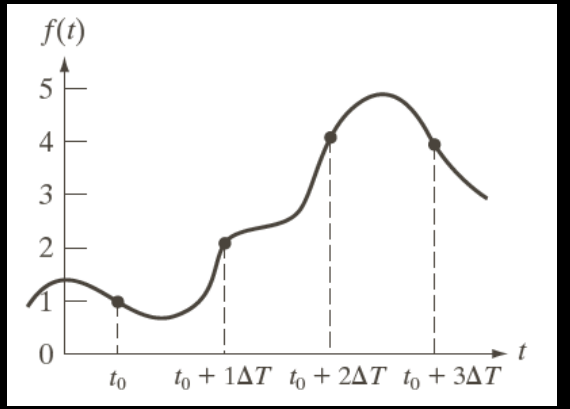

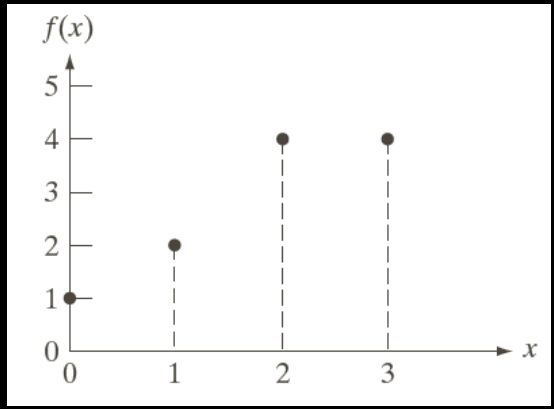

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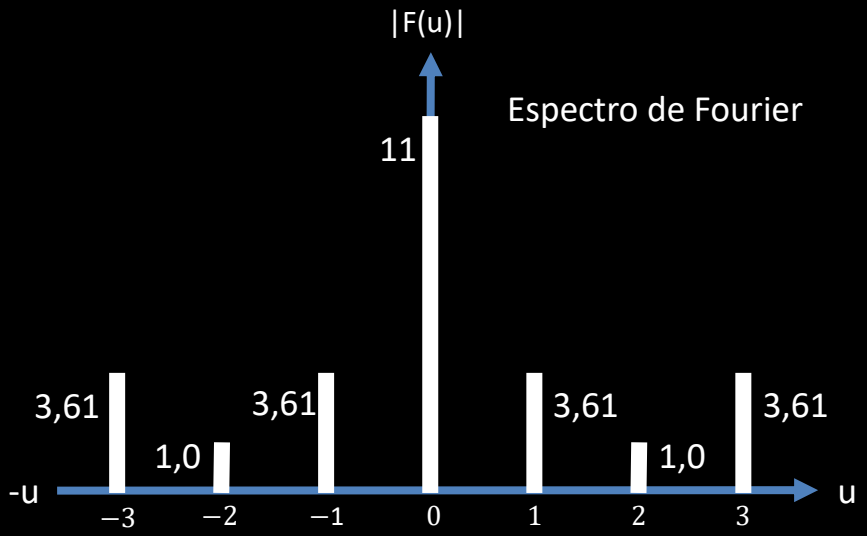

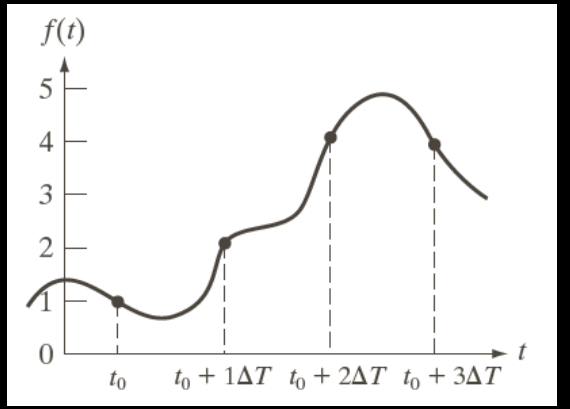

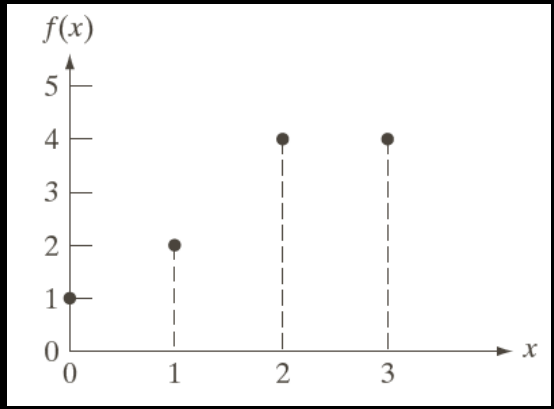

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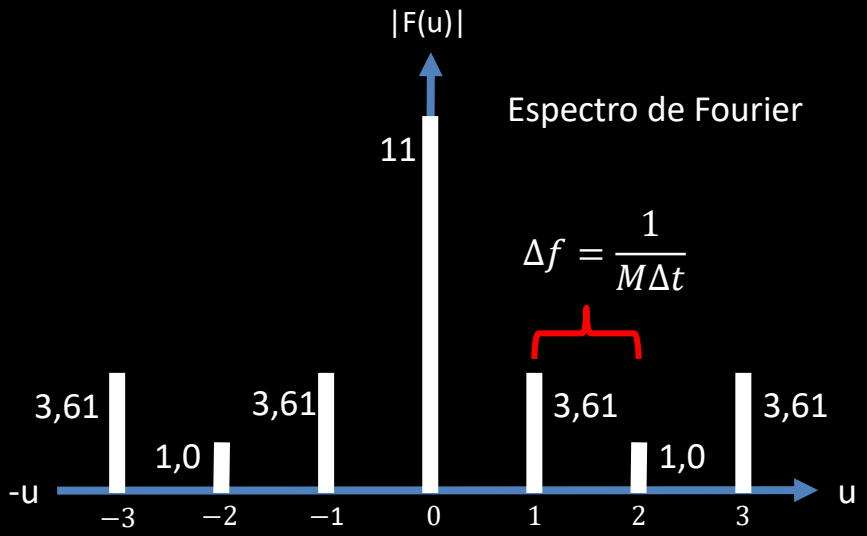

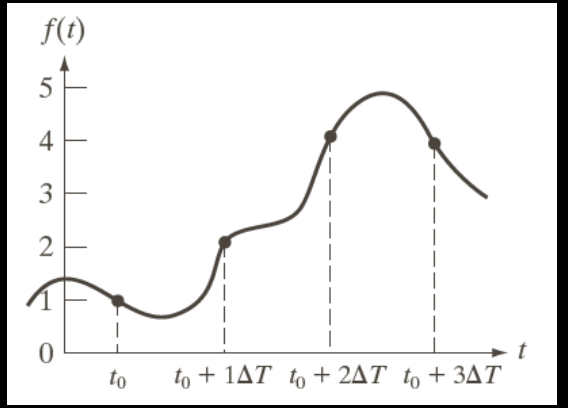

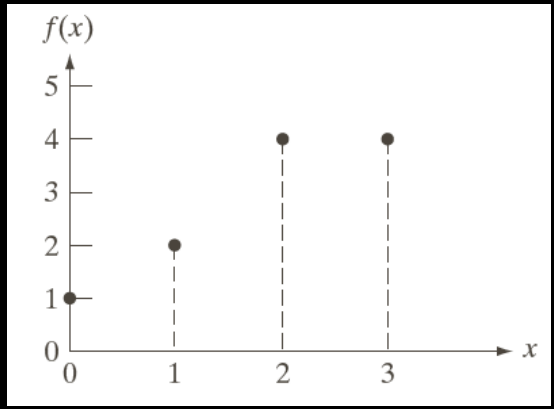

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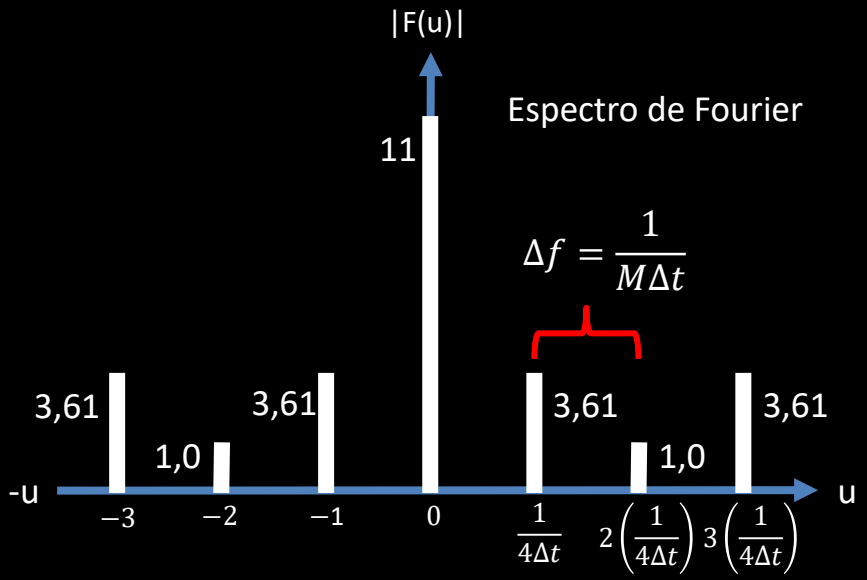

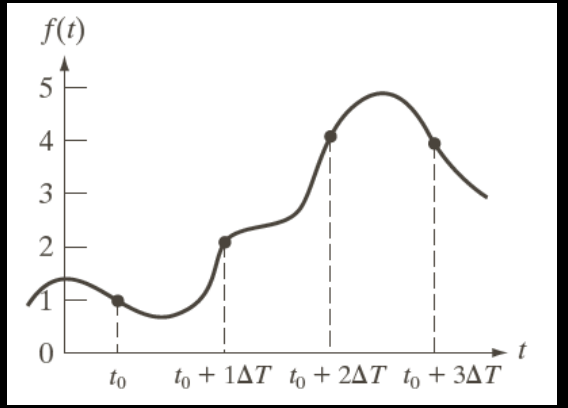

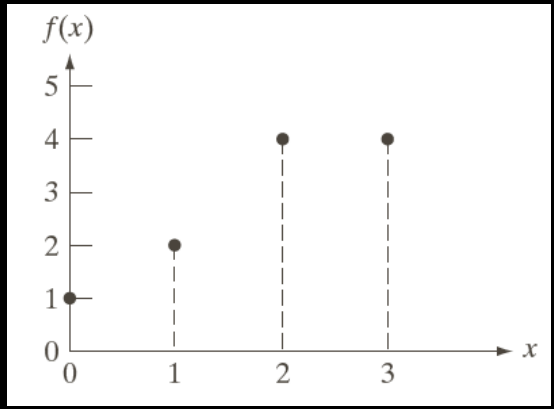

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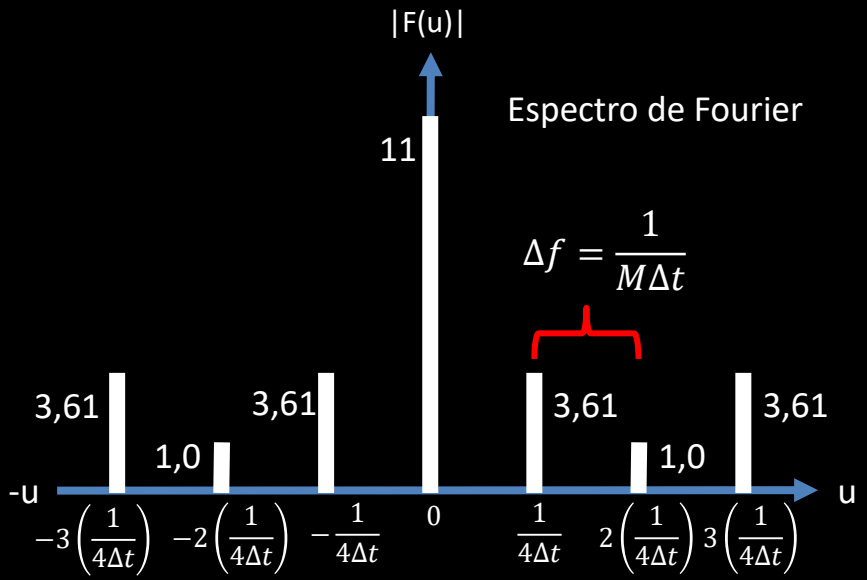

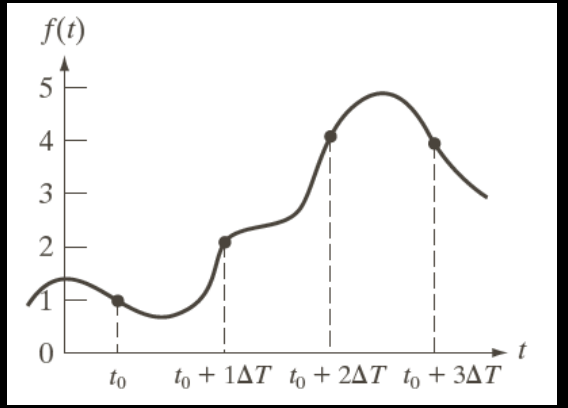

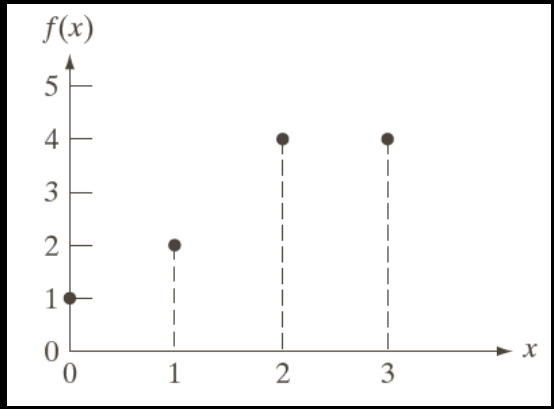

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