

Aula 08 – Redes Neurais Convolucionais

Prof. João Fernando Mari

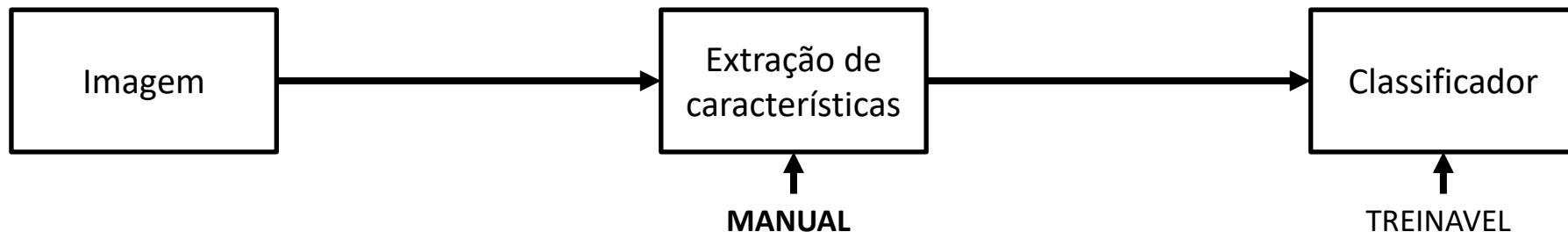
joaofmari.github.io

joaof.mari@ufv.br

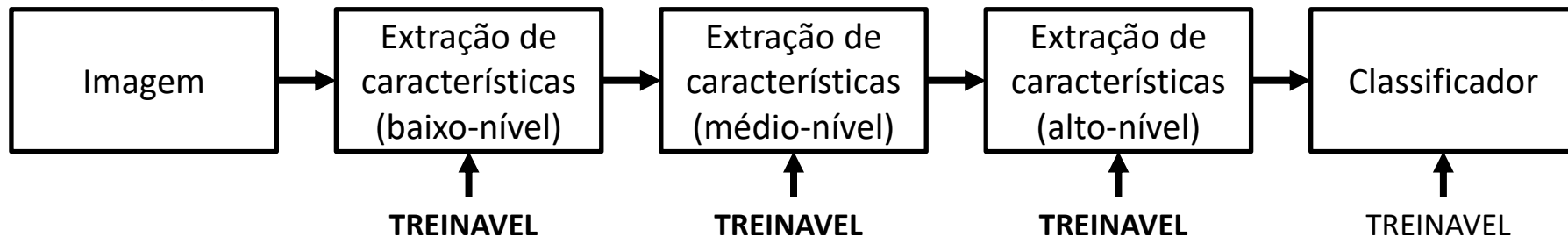
- Pipelines de classificação
- Perceptron de múltiplas camadas (MLP)
- Redes Neurais Convolucionais (CNNs)
- Camada convolucional
- Camada de pooling
- Função de ativação
- Camada completamente conectada
- Camada de saída – softmax
- Função de perda (loss)
- Otimizadores
- Arquiteturas
- Bibliotecas e desenvolvimento
- Conjuntos de imagens

Pipelines de classificação

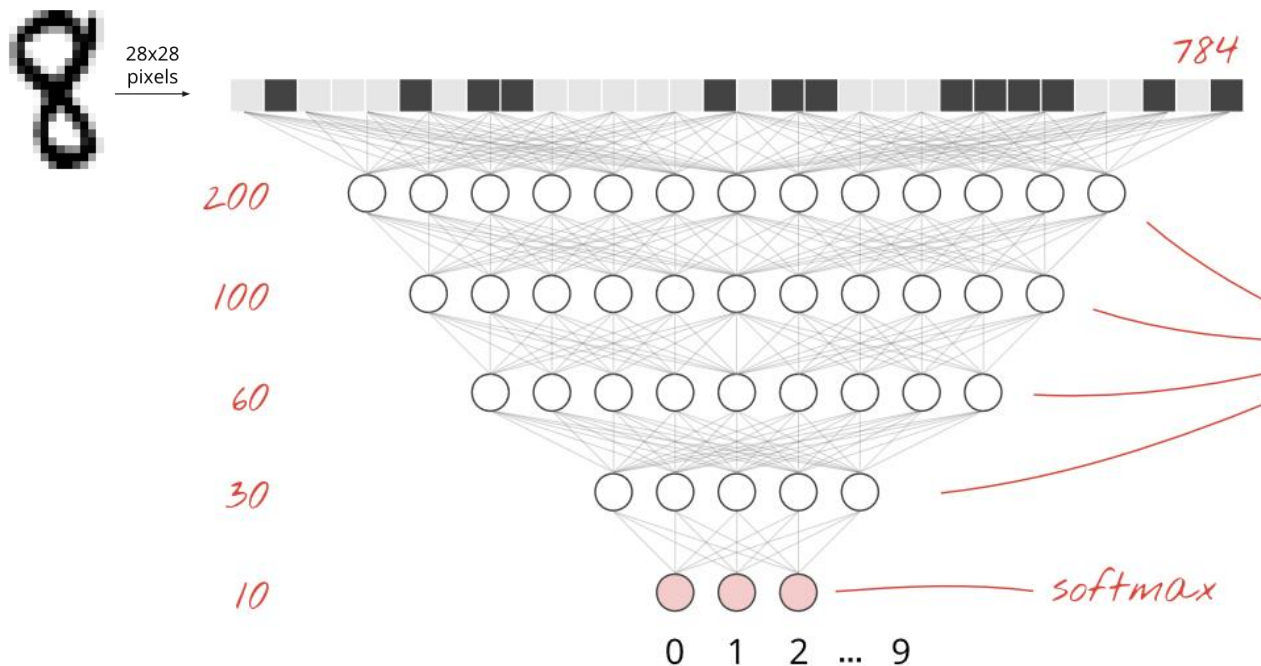
Pipeline clássico de classificação de imagens



Deep Learning

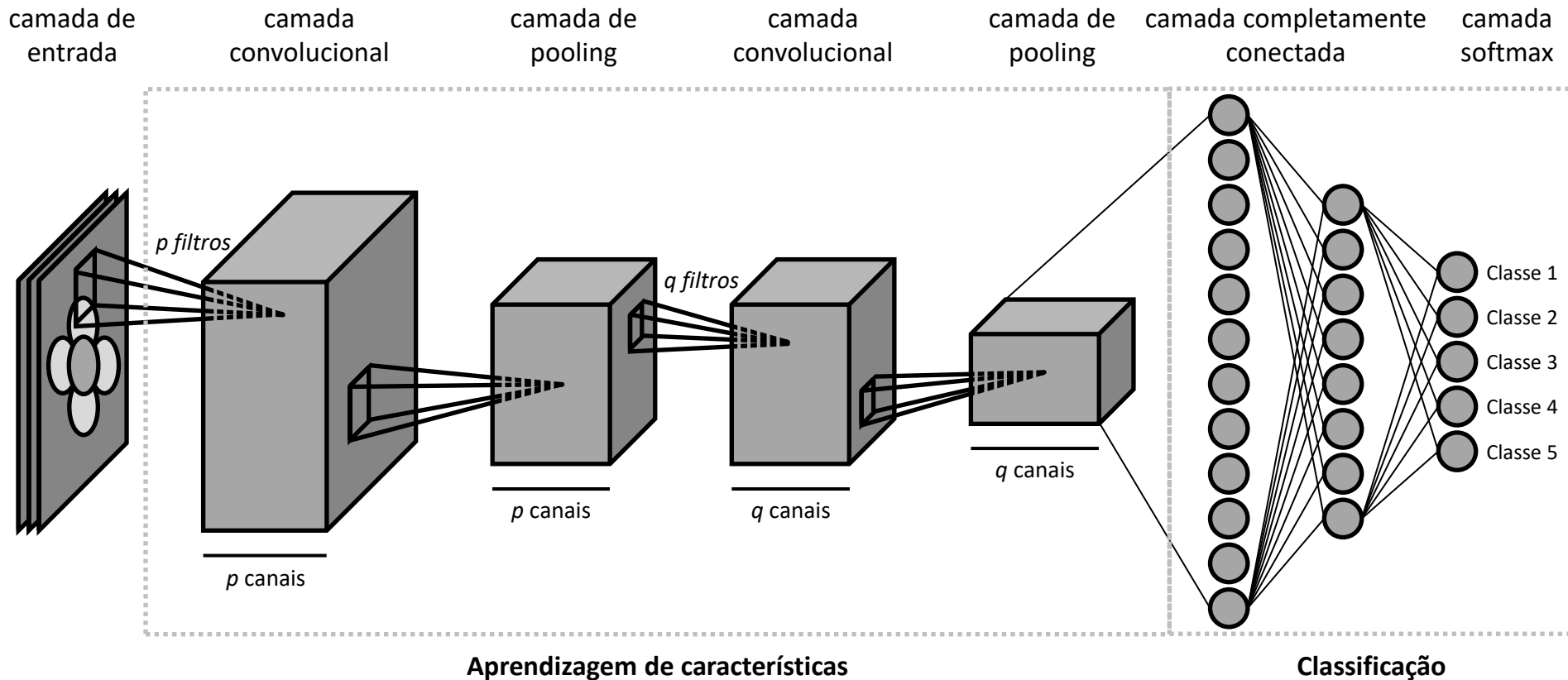


Perceptron de múltiplas camadas (MLP)

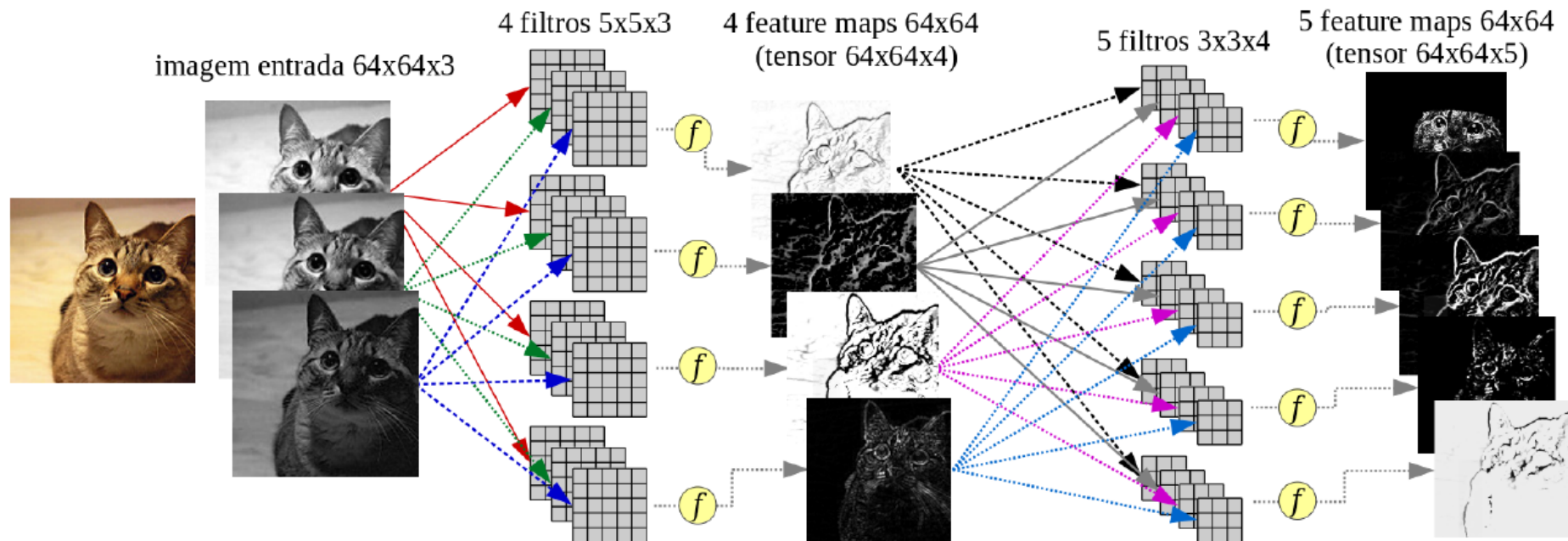


Learn TensorFlow and deep learning, without a Ph.D.

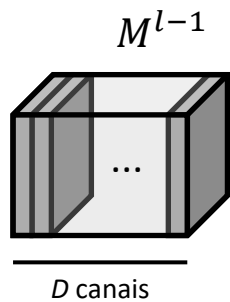
Redes Neurais Convolucionais (CNNs)

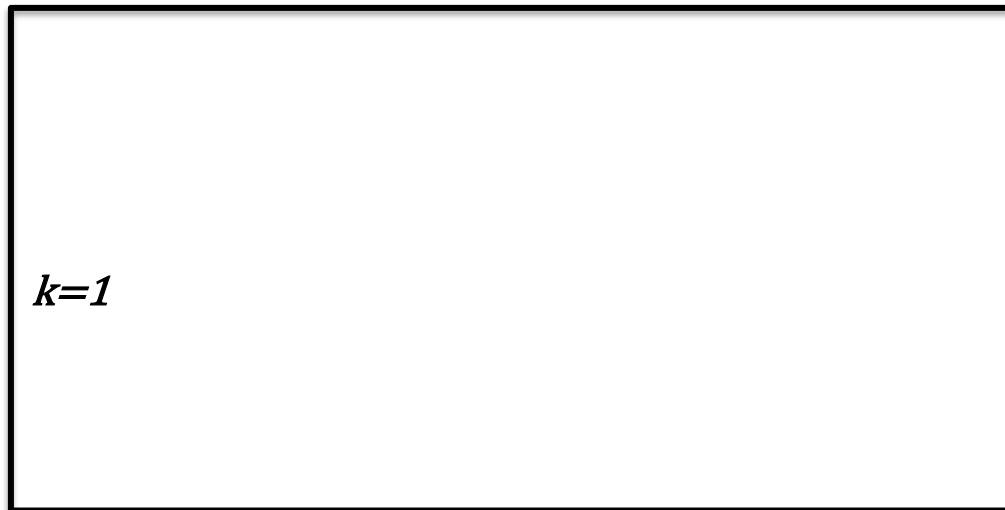
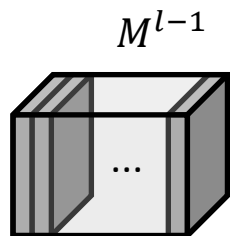


CAMADA CONVOLUCIONAL



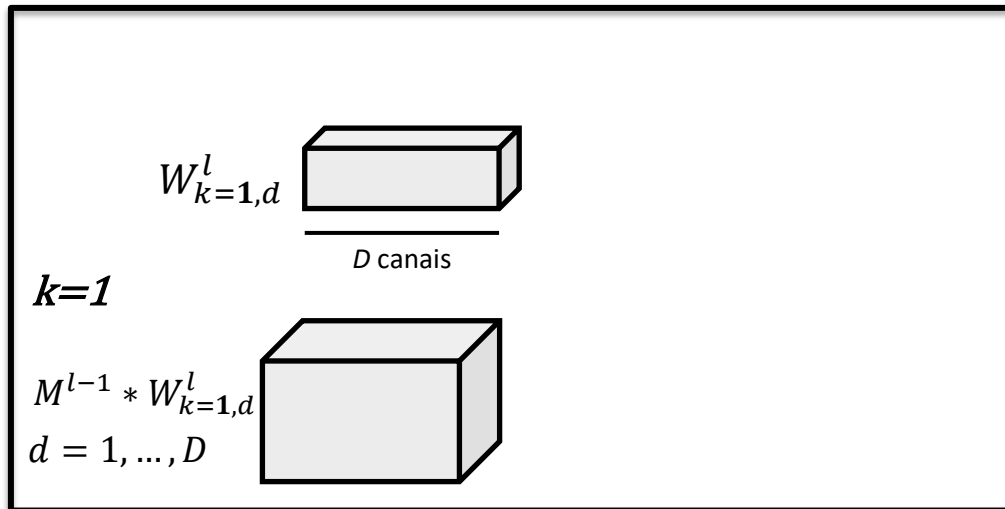
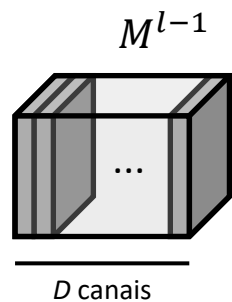
Moacir Ponti. <http://conteudo.icmc.usp.br/pessoas/moacir/p17sibgrapi-tutorial/>





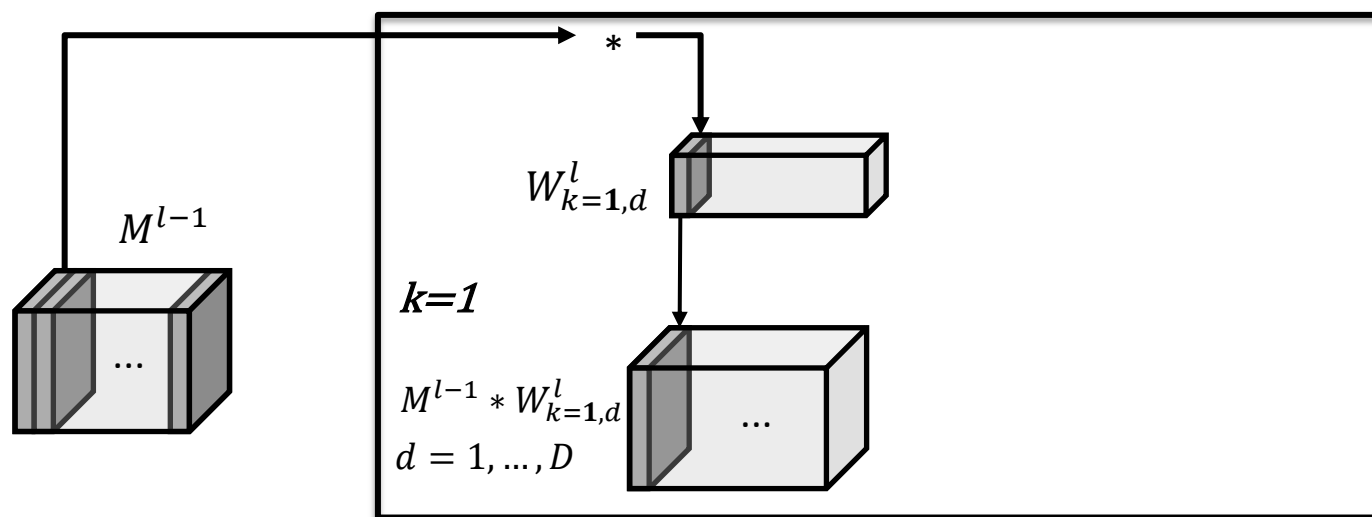
Camada convolucional C^l

Camada convolucional



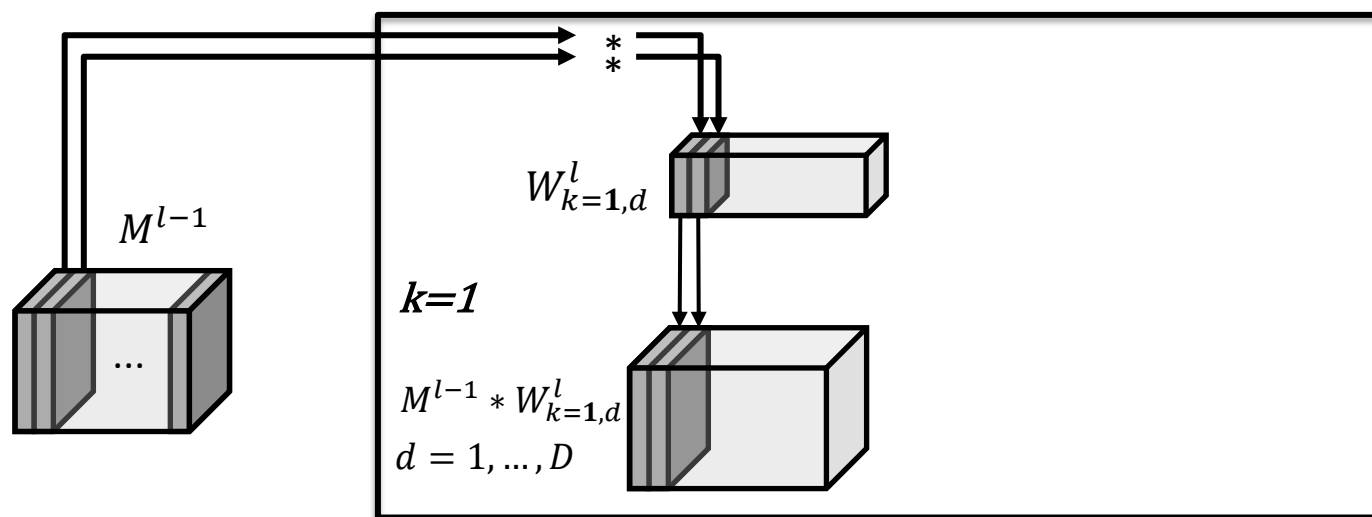
Camada convolucional C^l

Camada convolucional



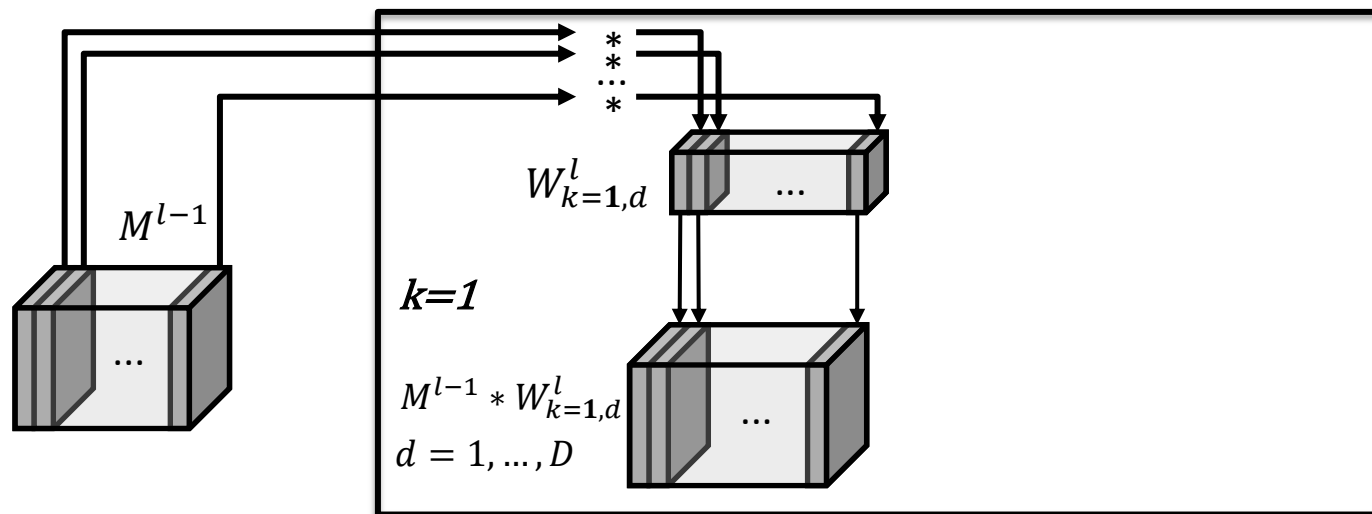
Camada convolucional C^l

Camada convolucional



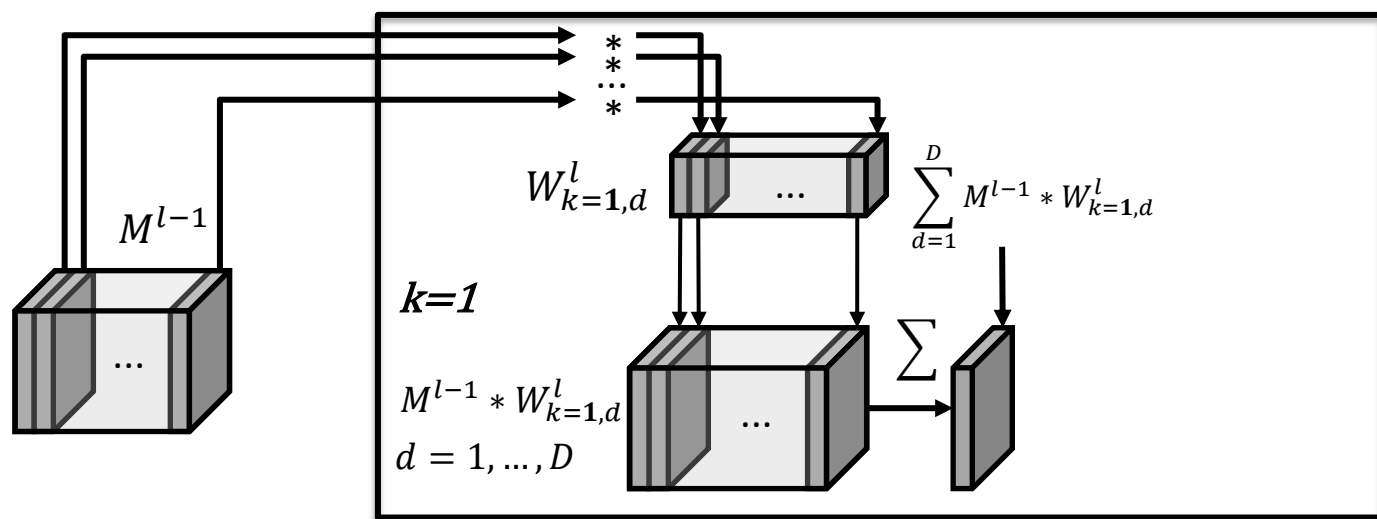
Camada convolucional C^l

Camada convolucional



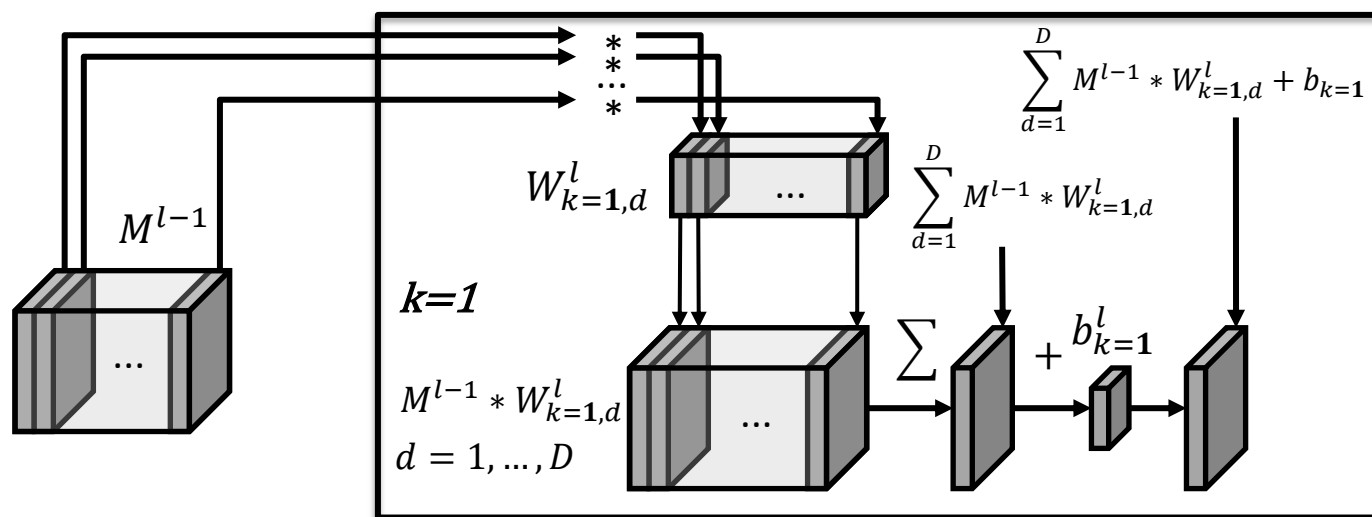
Camada convolucional C^l

Camada convolucional



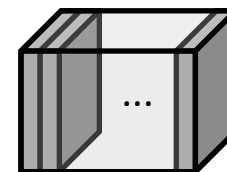
Camada convolucional C^l

Camada convolucional

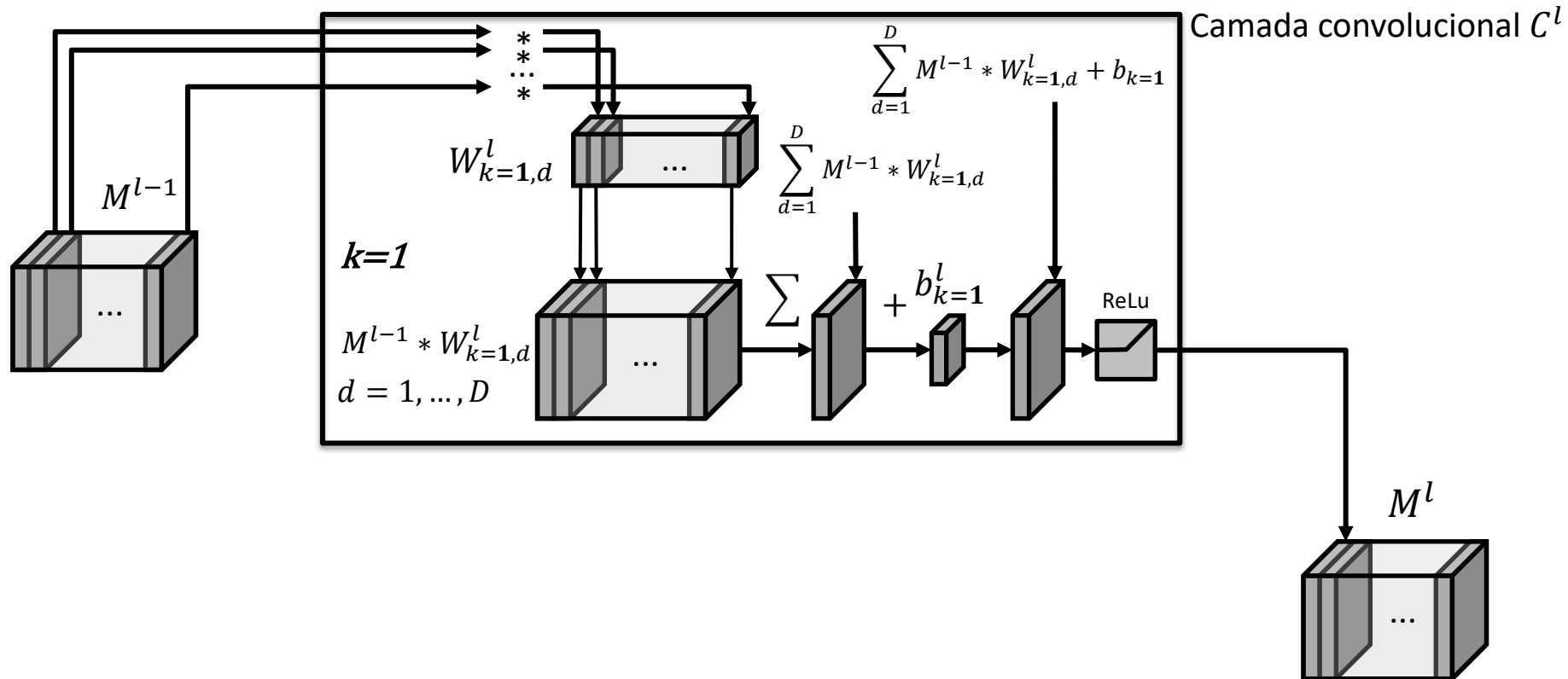


Camada convolucional C^l

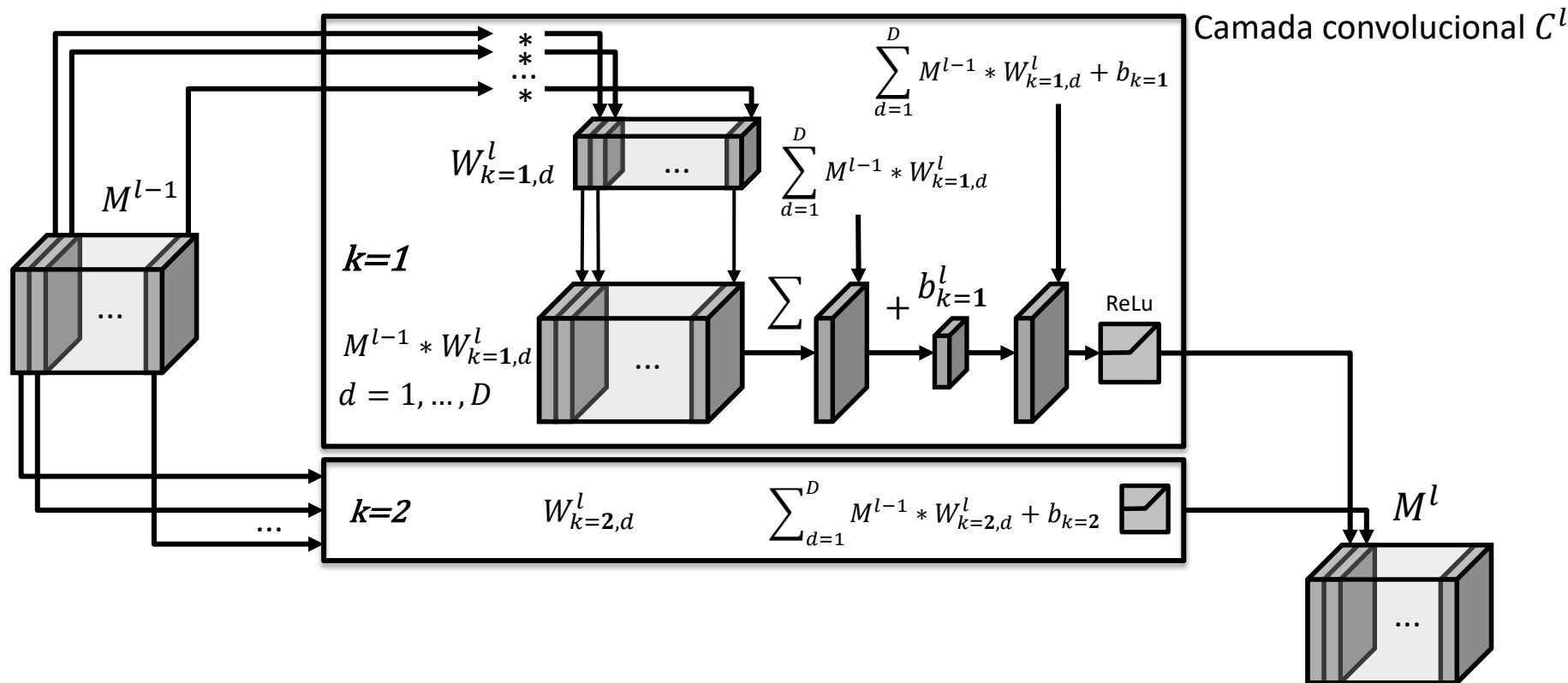
M^l



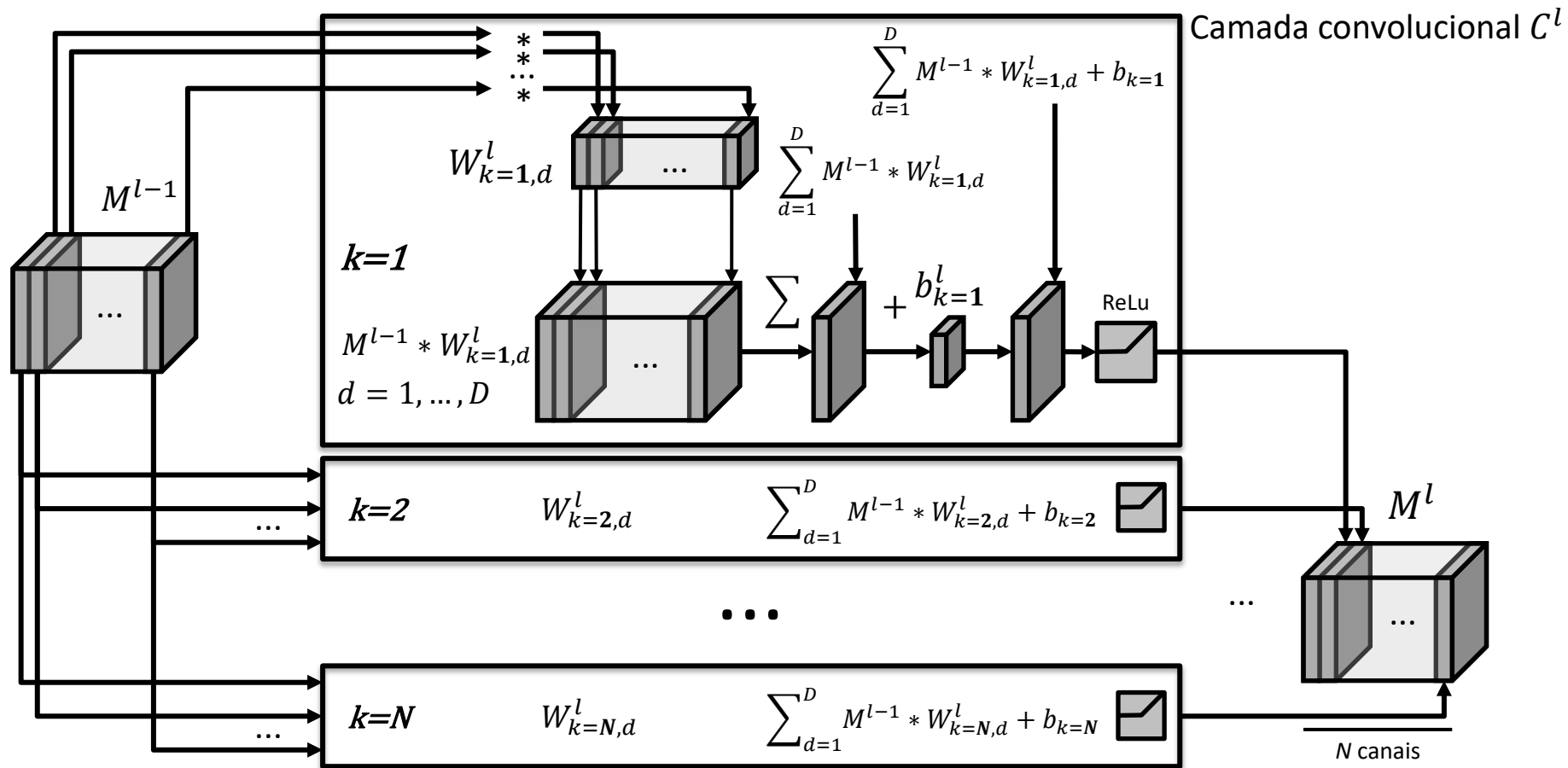
Camada convolucional



Camada convolucional



Camada convolucional



$x[:, :, 0]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 | 0 |
| 0 | 7 | 4 | 4 | 6 | 1 | 0 |
| 0 | 2 | 6 | 2 | 2 | 7 | 0 |
| 0 | 4 | 3 | 7 | 7 | 2 | 0 |
| 0 | 5 | 4 | 1 | 7 | 3 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 | 0 |
| 0 | 4 | 0 | 3 | 1 | 5 | 0 |
| 0 | 4 | 3 | 0 | 0 | 2 | 0 |
| 0 | 2 | 6 | 1 | 7 | 3 | 0 |
| 0 | 3 | 7 | 6 | 5 | 5 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 | 0 |
| 0 | 3 | 7 | 0 | 2 | 4 | 0 |
| 0 | 2 | 6 | 4 | 0 | 6 | 0 |
| 0 | 1 | 3 | 0 | 3 | 5 | 0 |
| 0 | 1 | 1 | 0 | 1 | 4 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$w0[:, :, 1]$

| | | |
|----|----|----|
| -1 | 1 | -1 |
| -1 | 1 | 2 |
| -1 | -1 | 1 |

$w0[:, :, 2]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$x[:, :, 0] * w0[:, :, 0]$

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$x[:, :, 1] * w0[:, :, 1]$

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$b0$

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| 1 |
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$x[:, :, 2] * w0[:, :, 2]$

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$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$x[:, :, 0] * w1[:, :, 0]$

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$x[:, :, 1] * w1[:, :, 1]$

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$b1$

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| 0 |
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$x[:, :, 2] * w1[:, :, 2]$

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Camada convolucional

$x[:, :, 0]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 | 0 |
| 0 | 7 | 4 | 4 | 6 | 1 | 0 |
| 0 | 2 | 6 | 2 | 2 | 7 | 0 |
| 0 | 4 | 3 | 7 | 7 | 2 | 0 |
| 0 | 5 | 4 | 1 | 7 | 3 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 | 0 |
| 0 | 4 | 0 | 3 | 1 | 5 | 0 |
| 0 | 4 | 3 | 0 | 0 | 2 | 0 |
| 0 | 2 | 6 | 1 | 7 | 3 | 0 |
| 0 | 3 | 7 | 6 | 5 | 5 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 | 0 |
| 0 | 3 | 7 | 0 | 2 | 4 | 0 |
| 0 | 2 | 6 | 4 | 0 | 6 | 0 |
| 0 | 1 | 3 | 0 | 3 | 5 | 0 |
| 0 | 1 | 1 | 0 | 1 | 4 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$x[:, :, 0] * w0[:, :, 0]$

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$w0[:, :, 1]$

| | | |
|----|----|----|
| -1 | 1 | -1 |
| -1 | 1 | 2 |
| -1 | -1 | 1 |

$x[:, :, 1] * w0[:, :, 1]$

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$b0$

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| 1 |
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$w0[:, :, 2]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$x[:, :, 2] * w0[:, :, 2]$

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$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$x[:, :, 0] * w1[:, :, 0]$

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$w1[:, :, 1]$

| | | |
|----|----|----|
| -2 | -2 | -2 |
| -2 | 1 | 0 |
| 0 | -2 | 0 |

$x[:, :, 1] * w1[:, :, 1]$

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$b1$

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|---|
| 0 |
|---|

$w1[:, :, 2]$

| | | |
|----|----|----|
| 0 | -2 | 0 |
| 2 | -1 | -1 |
| -2 | 1 | -2 |

$x[:, :, 2] * w1[:, :, 2]$

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|--|--|--|--|--|--|--|
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Camada convolucional

$x[:, :, 0]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 | 0 |
| 0 | 7 | 4 | 4 | 6 | 1 | 0 |
| 0 | 2 | 6 | 2 | 2 | 7 | 0 |
| 0 | 4 | 3 | 7 | 7 | 2 | 0 |
| 0 | 5 | 4 | 1 | 7 | 3 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 | 0 |
| 0 | 4 | 0 | 3 | 1 | 5 | 0 |
| 0 | 4 | 3 | 0 | 0 | 2 | 0 |
| 0 | 2 | 6 | 1 | 7 | 3 | 0 |
| 0 | 3 | 7 | 6 | 5 | 5 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 | 0 |
| 0 | 3 | 7 | 0 | 2 | 4 | 0 |
| 0 | 2 | 6 | 4 | 0 | 6 | 0 |
| 0 | 1 | 3 | 0 | 3 | 5 | 0 |
| 0 | 1 | 1 | 0 | 1 | 4 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$w0[:, :, 1]$

| | | |
|----|----|----|
| -1 | 1 | -1 |
| -1 | 1 | 2 |
| -1 | -1 | 1 |

$w0[:, :, 2]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$w1[:, :, 1]$

| | | |
|----|----|----|
| -2 | -2 | -2 |
| -2 | 1 | 0 |
| 0 | -2 | 0 |

$w1[:, :, 2]$

| | | |
|----|----|----|
| 0 | -2 | 0 |
| 2 | -1 | -1 |
| -2 | 1 | -2 |

$x[:, :, 0] * w0[:, :, 0]$

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$x[:, :, 1] * w0[:, :, 1]$

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$x[:, :, 2] * w0[:, :, 2]$

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$x[:, :, 0] * w1[:, :, 0]$

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$x[:, :, 1] * w1[:, :, 1]$

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$b1$

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| 0 |
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$x[:, :, 2] * w1[:, :, 2]$

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Camada convolucional

$x[:, :, 0]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 |
| 0 | 7 | 4 | 4 | 6 | 1 |
| 0 | 2 | 6 | 2 | 2 | 7 |
| 0 | 4 | 3 | 7 | 7 | 2 |
| 0 | 5 | 4 | 1 | 7 | 3 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 |
| 0 | 4 | 0 | 3 | 1 | 5 |
| 0 | 4 | 3 | 0 | 0 | 2 |
| 0 | 2 | 6 | 1 | 7 | 3 |
| 0 | 3 | 7 | 6 | 5 | 5 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 |
| 0 | 3 | 7 | 0 | 2 | 4 |
| 0 | 2 | 6 | 4 | 0 | 6 |
| 0 | 1 | 3 | 0 | 3 | 5 |
| 0 | 1 | 1 | 0 | 1 | 4 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$w0[:, :, 1]$

| | | |
|----|----|----|
| -1 | 1 | -1 |
| -1 | 1 | 2 |
| -1 | -1 | 1 |

$w0[:, :, 2]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$w1[:, :, 1]$

| | | |
|----|----|----|
| -2 | -2 | -2 |
| -2 | 1 | 0 |
| 0 | -2 | 0 |

$w1[:, :, 2]$

| | | |
|----|----|----|
| 0 | -2 | 0 |
| 2 | -1 | -1 |
| -2 | 1 | -2 |

$x[:, :, 0] * w0[:, :, 0]$

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|----|--|--|--|--|--|
| 12 | | | | | |
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$x[:, :, 1] * w0[:, :, 1]$

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$b0$

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$x[:, :, 2] * w0[:, :, 2]$

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$x[:, :, 0] * w1[:, :, 0]$

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$x[:, :, 1] * w1[:, :, 1]$

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| -13 | | | | | |
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$b1$

| |
|---|
| 0 |
|---|

$x[:, :, 2] * w1[:, :, 2]$

| | | | | | |
|----|--|--|--|--|--|
| -2 | | | | | |
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| | | | | | |
| | | | | | |

$x[:, :, 0]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 |
| 0 | 7 | 4 | 4 | 6 | 1 |
| 0 | 2 | 6 | 2 | 2 | 7 |
| 0 | 4 | 3 | 7 | 7 | 2 |
| 0 | 5 | 4 | 1 | 7 | 3 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$x[:, :, 0] * w0[:, :, 0]$

| | | | | |
|----|----|--|--|--|
| 12 | 26 | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$w0[:, :, 1]$

| | | |
|----|----|----|
| -1 | 1 | -1 |
| -1 | 1 | 2 |
| -1 | -1 | 1 |

$x[:, :, 1] * w0[:, :, 1]$

| | | | | |
|---|---|--|--|--|
| 4 | 7 | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$b0$

| |
|---|
| 1 |
|---|

$x[:, :, 2] * w0[:, :, 2]$

| | | | | |
|-----|---|--|--|--|
| -17 | 0 | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 |
| 0 | 4 | 0 | 3 | 1 | 5 |
| 0 | 4 | 3 | 0 | 0 | 2 |
| 0 | 2 | 6 | 1 | 7 | 3 |
| 0 | 3 | 7 | 6 | 5 | 5 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 2]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$x[:, :, 0] * w1[:, :, 0]$

| | | | | |
|----|----|--|--|--|
| -9 | 14 | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 1] * w1[:, :, 1]$

| | | | | |
|-----|-----|--|--|--|
| -13 | -11 | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$b1$

| |
|---|
| 0 |
|---|

$x[:, :, 2] * w1[:, :, 2]$

| | | | | |
|----|-----|--|--|--|
| -2 | -21 | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 |
| 0 | 3 | 7 | 0 | 2 | 4 |
| 0 | 2 | 6 | 4 | 0 | 6 |
| 0 | 1 | 3 | 0 | 3 | 5 |
| 0 | 1 | 1 | 0 | 1 | 4 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$w1[:, :, 1]$

| | | |
|----|----|----|
| -2 | -2 | -2 |
| -2 | 1 | 0 |
| 0 | -2 | 0 |

$w1[:, :, 2]$

| | | |
|----|----|----|
| 0 | -2 | 0 |
| 2 | -1 | -1 |
| -2 | 1 | -2 |

Camada convolucional

$x[:, :, 0]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 |
| 0 | 7 | 4 | 4 | 6 | 1 |
| 0 | 2 | 6 | 2 | 2 | 7 |
| 0 | 4 | 3 | 7 | 7 | 2 |
| 0 | 5 | 4 | 1 | 7 | 3 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$x[:, :, 0] * w0[:, :, 0]$

| | | | | |
|----|----|----|--|--|
| 12 | 26 | 18 | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$w0[:, :, 1]$

| | | |
|----|----|----|
| -1 | 1 | -1 |
| -1 | 1 | 2 |
| -1 | -1 | 1 |

$x[:, :, 1] * w0[:, :, 1]$

| | | | | |
|---|---|---|--|--|
| 4 | 7 | 6 | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$b0$

| |
|---|
| 1 |
|---|

$x[:, :, 2] * w0[:, :, 2]$

| | | | | |
|-----|---|----|--|--|
| -17 | 0 | 14 | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 |
| 0 | 4 | 0 | 3 | 1 | 5 |
| 0 | 4 | 3 | 0 | 0 | 2 |
| 0 | 2 | 6 | 1 | 7 | 3 |
| 0 | 3 | 7 | 6 | 5 | 5 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 2]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$x[:, :, 0] * w1[:, :, 0]$

| | | | | |
|----|----|---|--|--|
| -9 | 14 | 6 | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 1] * w1[:, :, 1]$

| | | | | |
|-----|-----|-----|--|--|
| -13 | -11 | -21 | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$b1$

| |
|---|
| 0 |
|---|

$x[:, :, 2] * w1[:, :, 2]$

| | | | | |
|----|-----|----|--|--|
| -2 | -21 | -1 | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 |
| 0 | 3 | 7 | 0 | 2 | 4 |
| 0 | 2 | 6 | 4 | 0 | 6 |
| 0 | 1 | 3 | 0 | 3 | 5 |
| 0 | 1 | 1 | 0 | 1 | 4 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$w1[:, :, 1]$

| | | |
|----|----|----|
| -2 | -2 | -2 |
| -2 | 1 | 0 |
| 0 | -2 | 0 |

$w1[:, :, 2]$

| | | |
|----|----|----|
| 0 | -2 | 0 |
| 2 | -1 | -1 |
| -2 | 1 | -2 |

$x[:, :, 0]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 |
| 0 | 7 | 4 | 4 | 6 | 1 |
| 0 | 2 | 6 | 2 | 2 | 7 |
| 0 | 4 | 3 | 7 | 7 | 2 |
| 0 | 5 | 4 | 1 | 7 | 3 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$x[:, :, 0] * w0[:, :, 0]$

| | | | | |
|----|----|----|----|--|
| 12 | 26 | 18 | 25 | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$w0[:, :, 1]$

| | | |
|----|----|----|
| -1 | 1 | -1 |
| -1 | 1 | 2 |
| -1 | -1 | 1 |

$x[:, :, 1] * w0[:, :, 1]$

| | | | | |
|---|---|---|----|--|
| 4 | 7 | 6 | -1 | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$b0$

| |
|---|
| 1 |
|---|

$x[:, :, 2] * w0[:, :, 2]$

| | | | | |
|-----|---|----|----|--|
| -17 | 0 | 14 | -2 | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 |
| 0 | 4 | 0 | 3 | 1 | 5 |
| 0 | 4 | 3 | 0 | 0 | 2 |
| 0 | 2 | 6 | 1 | 7 | 3 |
| 0 | 3 | 7 | 6 | 5 | 5 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 2]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$x[:, :, 0] * w1[:, :, 0]$

| | | | | |
|----|----|---|---|--|
| -9 | 14 | 6 | 7 | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 1] * w1[:, :, 1]$

| | | | | |
|-----|-----|-----|-----|--|
| -13 | -11 | -21 | -17 | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$b1$

| |
|---|
| 0 |
|---|

$x[:, :, 2] * w1[:, :, 2]$

| | | | | |
|----|-----|----|---|--|
| -2 | -21 | -1 | 3 | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 |
| 0 | 3 | 7 | 0 | 2 | 4 |
| 0 | 2 | 6 | 4 | 0 | 6 |
| 0 | 1 | 3 | 0 | 3 | 5 |
| 0 | 1 | 1 | 0 | 1 | 4 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$w1[:, :, 1]$

| | | |
|----|----|----|
| -2 | -2 | -2 |
| -2 | 1 | 0 |
| 0 | -2 | 0 |

$w1[:, :, 2]$

| | | |
|----|----|----|
| 0 | -2 | 0 |
| 2 | -1 | -1 |
| -2 | 1 | -2 |

$x[:, :, 0]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 |
| 0 | 7 | 4 | 4 | 6 | 1 |
| 0 | 2 | 6 | 2 | 2 | 7 |
| 0 | 4 | 3 | 7 | 7 | 2 |
| 0 | 5 | 4 | 1 | 7 | 3 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$x[:, :, 0] * w0[:, :, 0]$

| | | | | |
|----|----|----|----|----|
| 12 | 26 | 18 | 25 | 21 |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$w0[:, :, 1]$

| | | |
|----|----|----|
| -1 | 1 | -1 |
| -1 | 1 | 2 |
| -1 | -1 | 1 |

$x[:, :, 1] * w0[:, :, 1]$

| | | | | |
|---|---|---|----|----|
| 4 | 7 | 6 | -1 | 12 |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$b0$

| |
|---|
| 1 |
|---|

$x[:, :, 2] * w0[:, :, 2]$

| | | | | |
|-----|---|----|----|----|
| -17 | 0 | 14 | -2 | -8 |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 |
| 0 | 4 | 0 | 3 | 1 | 5 |
| 0 | 4 | 3 | 0 | 0 | 2 |
| 0 | 2 | 6 | 1 | 7 | 3 |
| 0 | 3 | 7 | 6 | 5 | 5 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 2]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$x[:, :, 0] * w1[:, :, 0]$

| | | | | |
|----|----|---|---|----|
| -9 | 14 | 6 | 7 | 18 |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 1] * w1[:, :, 1]$

| | | | | |
|-----|-----|-----|-----|----|
| -13 | -11 | -21 | -17 | -9 |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$b1$

| |
|---|
| 0 |
|---|

$x[:, :, 2] * w1[:, :, 2]$

| | | | | |
|----|-----|----|---|-----|
| -2 | -21 | -1 | 3 | -17 |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 |
| 0 | 3 | 7 | 0 | 2 | 4 |
| 0 | 2 | 6 | 4 | 0 | 6 |
| 0 | 1 | 3 | 0 | 3 | 5 |
| 0 | 1 | 1 | 0 | 1 | 4 |
| 0 | 0 | 0 | 0 | 0 | 0 |

$w1[:, :, 1]$

| | | |
|----|----|----|
| -2 | -2 | -2 |
| -2 | 1 | 0 |
| 0 | -2 | 0 |

$w1[:, :, 2]$

| | | |
|----|----|----|
| 0 | -2 | 0 |
| 2 | -1 | -1 |
| -2 | 1 | -2 |

$x[:, :, 0]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 | 0 |
| 0 | 7 | 4 | 4 | 6 | 1 | 0 |
| 0 | 2 | 6 | 2 | 2 | 7 | 0 |
| 0 | 4 | 3 | 7 | 7 | 2 | 0 |
| 0 | 5 | 4 | 1 | 7 | 3 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 | 0 |
| 0 | 4 | 0 | 3 | 1 | 5 | 0 |
| 0 | 4 | 3 | 0 | 0 | 2 | 0 |
| 0 | 2 | 6 | 1 | 7 | 3 | 0 |
| 0 | 3 | 7 | 6 | 5 | 5 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 | 0 |
| 0 | 3 | 7 | 0 | 2 | 4 | 0 |
| 0 | 2 | 6 | 4 | 0 | 6 | 0 |
| 0 | 1 | 3 | 0 | 3 | 5 | 0 |
| 0 | 1 | 1 | 0 | 1 | 4 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$w0[:, :, 1]$

| | | |
|----|----|----|
| -1 | 1 | -1 |
| -1 | 1 | 2 |
| -1 | -1 | 1 |

$w0[:, :, 2]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$w1[:, :, 1]$

| | | |
|----|----|----|
| -2 | -2 | -2 |
| -2 | 1 | 0 |
| 0 | -2 | 0 |

$w1[:, :, 2]$

| | | |
|----|----|----|
| 0 | -2 | 0 |
| 2 | -1 | -1 |
| -2 | 1 | -2 |

$x[:, :, 0] * w0[:, :, 0]$

| | | | | |
|----|----|----|----|----|
| 12 | 26 | 18 | 25 | 21 |
| -5 | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 1] * w0[:, :, 1]$

| | | | | |
|----|---|---|----|----|
| 4 | 7 | 6 | -1 | 12 |
| -5 | | | | |
| | | | | |
| | | | | |
| | | | | |

$b0$

| |
|---|
| 1 |
|---|

$x[:, :, 2] * w0[:, :, 2]$

| | | | | |
|-----|---|----|----|----|
| -17 | 0 | 14 | -2 | -8 |
| -3 | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 0] * w1[:, :, 0]$

| | | | | |
|----|----|---|---|----|
| -9 | 14 | 6 | 7 | 18 |
| 7 | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 1] * w1[:, :, 1]$

| | | | | |
|-----|-----|-----|-----|----|
| -13 | -11 | -21 | -17 | -9 |
| -20 | | | | |
| | | | | |
| | | | | |
| | | | | |

$b1$

| |
|---|
| 0 |
|---|

$x[:, :, 2] * w1[:, :, 2]$

| | | | | |
|----|-----|----|---|-----|
| -2 | -21 | -1 | 3 | -17 |
| 3 | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 0]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 | 0 |
| 0 | 7 | 4 | 4 | 6 | 1 | 0 |
| 0 | 2 | 6 | 2 | 2 | 7 | 0 |
| 0 | 4 | 3 | 7 | 7 | 2 | 0 |
| 0 | 5 | 4 | 1 | 7 | 3 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$x[:, :, 0] * w0[:, :, 0]$

| | | | | |
|----|----|----|----|----|
| 12 | 26 | 18 | 25 | 21 |
| 12 | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 1] * w0[:, :, 1]$

| | | | | |
|---|---|---|----|----|
| 4 | 7 | 6 | -1 | 12 |
| 4 | | | | |
| | | | | |
| | | | | |
| | | | | |

$b0$

| |
|---|
| 1 |
|---|

$x[:, :, 2] * w0[:, :, 2]$

| | | | | |
|-----|---|----|----|----|
| -17 | 0 | 14 | -2 | -8 |
| -17 | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 | 0 |
| 0 | 4 | 0 | 3 | 1 | 5 | 0 |
| 0 | 4 | 3 | 0 | 0 | 2 | 0 |
| 0 | 2 | 6 | 1 | 7 | 3 | 0 |
| 0 | 3 | 7 | 6 | 5 | 5 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 1]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$x[:, :, 0] * w1[:, :, 0]$

| | | | | |
|----|----|---|---|----|
| -9 | 14 | 6 | 7 | 18 |
| 7 | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 1] * w1[:, :, 1]$

| | | | | |
|-----|-----|-----|-----|----|
| -13 | -11 | -21 | -17 | -9 |
| -20 | | | | |
| | | | | |
| | | | | |
| | | | | |

$b1$

| |
|---|
| 0 |
|---|

$x[:, :, 2] * w1[:, :, 2]$

| | | | | |
|----|-----|----|---|-----|
| -2 | -21 | -1 | 3 | -17 |
| 3 | | | | |
| | | | | |
| | | | | |
| | | | | |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 | 0 |
| 0 | 3 | 7 | 0 | 2 | 4 | 0 |
| 0 | 2 | 6 | 4 | 0 | 6 | 0 |
| 0 | 1 | 3 | 0 | 3 | 5 | 0 |
| 0 | 1 | 1 | 0 | 1 | 4 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w1[:, :, 1]$

| | | |
|----|----|----|
| -2 | -2 | -2 |
| -2 | 1 | 0 |
| 0 | -2 | 0 |

$w1[:, :, 2]$

| | | |
|----|----|----|
| 0 | -2 | 0 |
| 2 | -1 | -1 |
| -2 | 1 | -2 |

$x[:, :, 0]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 | 0 |
| 0 | 7 | 4 | 4 | 6 | 1 | 0 |
| 0 | 2 | 6 | 2 | 2 | 7 | 0 |
| 0 | 4 | 3 | 7 | 7 | 2 | 0 |
| 0 | 5 | 4 | 1 | 7 | 3 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$x[:, :, 0] * w0[:, :, 0]$

| | | | | |
|----|----|----|----|----|
| 12 | 26 | 18 | 25 | 21 |
| -5 | 28 | 19 | 4 | 24 |
| -5 | 11 | 15 | 17 | 24 |
| 4 | 16 | 20 | 26 | 14 |
| 1 | 16 | 5 | 5 | 20 |

$x[:, :, 1] * w0[:, :, 1]$

| | | | | |
|-----|----|-----|----|----|
| 4 | 7 | 6 | -1 | 12 |
| -5 | 3 | -4 | -9 | 13 |
| -7 | 15 | -10 | -2 | -6 |
| -15 | 8 | 3 | -2 | 15 |
| -12 | 2 | 13 | 3 | 19 |

$b0$

| |
|---|
| 1 |
|---|

$x[:, :, 2] * w0[:, :, 2]$

| | | | | |
|-----|----|----|----|-----|
| -17 | 0 | 14 | -2 | -8 |
| -3 | -5 | 32 | 11 | -10 |
| 9 | -7 | 22 | 12 | -14 |
| 9 | 2 | 17 | 14 | -13 |
| 4 | -1 | 15 | 9 | -5 |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 | 0 |
| 0 | 4 | 0 | 3 | 1 | 5 | 0 |
| 0 | 4 | 3 | 0 | 0 | 2 | 0 |
| 0 | 2 | 6 | 1 | 7 | 3 | 0 |
| 0 | 3 | 7 | 6 | 5 | 5 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 1]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$x[:, :, 0] * w1[:, :, 0]$

| | | | | |
|-----|----|----|----|----|
| -9 | 14 | 6 | 7 | 18 |
| 7 | 20 | 20 | 22 | 17 |
| 3 | 17 | 2 | 22 | 28 |
| -15 | 26 | 27 | 1 | 35 |
| 11 | 15 | 22 | 36 | 35 |

$x[:, :, 1] * w1[:, :, 1]$

| | | | | |
|-----|-----|-----|-----|-----|
| -13 | -11 | -21 | -17 | -9 |
| -20 | -30 | -7 | -27 | -5 |
| -26 | -15 | -34 | -28 | -28 |
| -38 | -34 | -49 | -31 | -21 |
| -15 | -17 | -6 | -19 | -1 |

$b1$

| |
|---|
| 0 |
|---|

$x[:, :, 2] * w1[:, :, 2]$

| | | | | |
|----|-----|-----|-----|-----|
| -2 | -21 | -1 | 3 | -17 |
| 3 | -33 | -25 | -7 | -18 |
| -3 | -5 | -28 | -4 | -16 |
| -7 | -12 | -5 | -15 | -10 |
| -4 | -1 | -11 | 0 | -6 |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 | 0 |
| 0 | 3 | 7 | 0 | 2 | 4 | 0 |
| 0 | 2 | 6 | 4 | 0 | 6 | 0 |
| 0 | 1 | 3 | 0 | 3 | 5 | 0 |
| 0 | 1 | 1 | 0 | 1 | 4 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w1[:, :, 1]$

| | | |
|----|----|----|
| -2 | -2 | -2 |
| -2 | 1 | 0 |
| 0 | -2 | 0 |

$w1[:, :, 2]$

| | | |
|----|----|----|
| 0 | -2 | 0 |
| 2 | -1 | -1 |
| -2 | 1 | -2 |

$x[:, :, 0]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 | 0 |
| 0 | 7 | 4 | 4 | 6 | 1 | 0 |
| 0 | 2 | 6 | 2 | 2 | 7 | 0 |
| 0 | 4 | 3 | 7 | 7 | 2 | 0 |
| 0 | 5 | 4 | 1 | 7 | 3 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 | 0 |
| 0 | 4 | 0 | 3 | 1 | 5 | 0 |
| 0 | 4 | 3 | 0 | 0 | 2 | 0 |
| 0 | 2 | 6 | 1 | 7 | 3 | 0 |
| 0 | 3 | 7 | 6 | 5 | 5 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 | 0 |
| 0 | 3 | 7 | 0 | 2 | 4 | 0 |
| 0 | 2 | 6 | 4 | 0 | 6 | 0 |
| 0 | 1 | 3 | 0 | 3 | 5 | 0 |
| 0 | 1 | 1 | 0 | 1 | 4 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$w0[:, :, 1]$

| | | |
|----|----|----|
| -1 | 1 | -1 |
| -1 | 1 | 2 |
| -1 | -1 | 1 |

$w0[:, :, 2]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$x[:, :, 0] * w0[:, :, 0]$

| | | | | |
|----|----|----|----|----|
| 12 | 26 | 18 | 25 | 21 |
| -5 | 28 | 19 | 4 | 24 |
| -5 | 11 | 15 | 17 | 24 |
| 4 | 16 | 20 | 26 | 14 |
| 1 | 16 | 5 | 5 | 20 |

$x[:, :, 1] * w0[:, :, 1]$

| | | | | |
|-----|----|-----|----|----|
| 4 | 7 | 6 | -1 | 12 |
| -5 | 3 | -4 | -9 | 13 |
| -7 | 15 | -10 | -2 | -6 |
| -15 | 8 | 3 | -2 | 15 |
| -12 | 2 | 13 | 3 | 19 |

$b0$

| |
|---|
| 1 |
|---|

$x[:, :, 2] * w0[:, :, 2]$

| | | | | |
|-----|----|----|----|-----|
| -17 | 0 | 14 | -2 | -8 |
| -3 | -5 | 32 | 11 | -10 |
| 9 | -7 | 22 | 12 | -14 |
| 9 | 2 | 17 | 14 | -13 |
| 4 | -1 | 15 | 9 | -5 |

$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$x[:, :, 0] * w1[:, :, 0]$

| | | | | |
|-----|----|----|----|----|
| -9 | 14 | 6 | 7 | 18 |
| 7 | 20 | 20 | 22 | 17 |
| 3 | 17 | 2 | 22 | 28 |
| -15 | 26 | 27 | 1 | 35 |
| 11 | 15 | 22 | 36 | 35 |

$x[:, :, 1] * w1[:, :, 1]$

| | | | | |
|-----|-----|-----|-----|-----|
| -13 | -11 | -21 | -17 | -9 |
| -20 | -30 | -7 | -27 | -5 |
| -26 | -15 | -34 | -28 | -28 |
| -38 | -34 | -49 | -31 | -21 |
| -15 | -17 | -6 | -19 | -1 |

$b1$

| |
|---|
| 0 |
|---|

$x[:, :, 2] * w1[:, :, 2]$

| | | | | |
|----|-----|-----|-----|-----|
| -2 | -21 | -1 | 3 | -17 |
| 3 | -33 | -25 | -7 | -18 |
| -3 | -5 | -28 | -4 | -16 |
| -7 | -12 | -5 | -15 | -10 |
| -4 | -1 | -11 | 0 | -6 |

Camada convolucional

<https://cs231n.github.io/convolutional-networks/>

$x[:, :, 0]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 | 0 |
| 0 | 7 | 4 | 4 | 6 | 1 | 0 |
| 0 | 2 | 6 | 2 | 2 | 7 | 0 |
| 0 | 4 | 3 | 7 | 7 | 2 | 0 |
| 0 | 5 | 4 | 1 | 7 | 3 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 | 0 |
| 0 | 4 | 0 | 3 | 1 | 5 | 0 |
| 0 | 4 | 3 | 0 | 0 | 2 | 0 |
| 0 | 2 | 6 | 1 | 7 | 3 | 0 |
| 0 | 3 | 7 | 6 | 5 | 5 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 | 0 |
| 0 | 3 | 7 | 0 | 2 | 4 | 0 |
| 0 | 2 | 6 | 4 | 0 | 6 | 0 |
| 0 | 1 | 3 | 0 | 3 | 5 | 0 |
| 0 | 1 | 1 | 0 | 1 | 4 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$w0[:, :, 1]$

| | | |
|----|----|----|
| -1 | 1 | -1 |
| -1 | 1 | 2 |
| -1 | -1 | 1 |

$w0[:, :, 2]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$x[:, :, 0] * w0[:, :, 0]$

| | | | | |
|----|----|----|----|----|
| 12 | 26 | 18 | 25 | 21 |
| -5 | 28 | 19 | 4 | 24 |
| -5 | 11 | 15 | 17 | 24 |
| 4 | 16 | 20 | 26 | 14 |
| 1 | 16 | 5 | 5 | 20 |

$x[:, :, 1] * w0[:, :, 1]$

| | | | | |
|-----|----|-----|----|----|
| 4 | 7 | 6 | -1 | 12 |
| -5 | 3 | -4 | -9 | 13 |
| -7 | 15 | -10 | -2 | -6 |
| -15 | 8 | 3 | -2 | 15 |
| -12 | 2 | 13 | 3 | 19 |

$b0$

| |
|---|
| 1 |
|---|

$x[:, :, 2] * w0[:, :, 2]$

| | | | | |
|-----|----|----|----|-----|
| -17 | 0 | 14 | -2 | -8 |
| -3 | -5 | 32 | 11 | -10 |
| 9 | -7 | 22 | 12 | -14 |
| 9 | 2 | 17 | 14 | -13 |
| 4 | -1 | 15 | 9 | -5 |

Σ

$v[:, :, 0]$

| | | | | |
|--|--|--|--|--|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$w1[:, :, 1]$

| | | |
|----|----|----|
| -2 | -2 | -2 |
| -2 | 1 | 0 |
| 0 | -2 | 0 |

$w1[:, :, 2]$

| | | |
|----|----|----|
| 0 | -2 | 0 |
| 2 | -1 | -1 |
| -2 | 1 | -2 |

$x[:, :, 0] * w1[:, :, 0]$

| | | | | |
|-----|----|----|----|----|
| -9 | 14 | 6 | 7 | 18 |
| 7 | 20 | 20 | 22 | 17 |
| 3 | 17 | 2 | 22 | 28 |
| -15 | 26 | 27 | 1 | 35 |
| 11 | 15 | 22 | 36 | 35 |

$x[:, :, 1] * w1[:, :, 1]$

| | | | | |
|-----|-----|-----|-----|-----|
| -13 | -11 | -21 | -17 | -9 |
| -20 | -30 | -7 | -27 | -5 |
| -26 | -15 | -34 | -28 | -28 |
| -38 | -34 | -49 | -31 | -21 |
| -15 | -17 | -6 | -19 | -1 |

$b1$

| |
|---|
| 0 |
|---|

$x[:, :, 2] * w1[:, :, 2]$

| | | | | |
|----|-----|-----|-----|-----|
| -2 | -21 | -1 | 3 | -17 |
| 3 | -33 | -25 | -7 | -18 |
| -3 | -5 | -28 | -4 | -16 |
| -7 | -12 | -5 | -15 | -10 |
| -4 | -1 | -11 | 0 | -6 |

Σ

$v[:, :, 1]$

| | | | | |
|--|--|--|--|--|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

Camada convolucional

<https://cs231n.github.io/convolutional-networks/>

$x[:, :, 0]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 | 0 |
| 0 | 7 | 4 | 4 | 6 | 1 | 0 |
| 0 | 2 | 6 | 2 | 2 | 7 | 0 |
| 0 | 4 | 3 | 7 | 7 | 2 | 0 |
| 0 | 5 | 4 | 1 | 7 | 3 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 | 0 |
| 0 | 4 | 0 | 3 | 1 | 5 | 0 |
| 0 | 4 | 3 | 0 | 0 | 2 | 0 |
| 0 | 2 | 6 | 1 | 7 | 3 | 0 |
| 0 | 3 | 7 | 6 | 5 | 5 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 | 0 |
| 0 | 3 | 7 | 0 | 2 | 4 | 0 |
| 0 | 2 | 6 | 4 | 0 | 6 | 0 |
| 0 | 1 | 3 | 0 | 3 | 5 | 0 |
| 0 | 1 | 1 | 0 | 1 | 4 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$w0[:, :, 1]$

| | | |
|----|----|----|
| -1 | 1 | -1 |
| -1 | 1 | 2 |
| -1 | -1 | 1 |

$w0[:, :, 2]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$x[:, :, 0] * w0[:, :, 0]$

| | | | | |
|----|----|----|----|----|
| 12 | 26 | 18 | 25 | 21 |
| -5 | 28 | 19 | 4 | 24 |
| -5 | 11 | 15 | 17 | 24 |
| 4 | 16 | 20 | 26 | 14 |
| 1 | 16 | 5 | 5 | 20 |

$x[:, :, 1] * w0[:, :, 1]$

| | | | | |
|-----|----|-----|----|----|
| 4 | 7 | 6 | -1 | 12 |
| -5 | 3 | -4 | -9 | 13 |
| -7 | 15 | -10 | -2 | -6 |
| -15 | 8 | 3 | -2 | 15 |
| -12 | 2 | 13 | 3 | 19 |

$x[:, :, 2] * w0[:, :, 2]$

| | | | | |
|-----|----|----|----|-----|
| -17 | 0 | 14 | -2 | -8 |
| -3 | -5 | 32 | 11 | -10 |
| 9 | -7 | 22 | 12 | -14 |
| 9 | 2 | 17 | 14 | -13 |
| 4 | -1 | 15 | 9 | -5 |

$b0$

| |
|---|
| 1 |
|---|

Σ

$v[:, :, 0]$

| | | | | |
|-----|----|----|----|----|
| 0 | 34 | 39 | 23 | 35 |
| -12 | 27 | 48 | 7 | 28 |
| -2 | 20 | 28 | 48 | 5 |
| -1 | 27 | 41 | 39 | 17 |
| -6 | 15 | 34 | 18 | 35 |

$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$w1[:, :, 1]$

| | | |
|----|----|----|
| -2 | -2 | -2 |
| -2 | 1 | 0 |
| 0 | -2 | 0 |

$w1[:, :, 2]$

| | | |
|----|----|----|
| 0 | -2 | 0 |
| 2 | -1 | -1 |
| -2 | 1 | -2 |

$x[:, :, 0] * w1[:, :, 0]$

| | | | | |
|-----|----|----|----|----|
| -9 | 14 | 6 | 7 | 18 |
| 7 | 20 | 20 | 22 | 17 |
| 3 | 17 | 2 | 22 | 28 |
| -15 | 26 | 27 | 1 | 35 |
| 11 | 15 | 22 | 36 | 35 |

$x[:, :, 1] * w1[:, :, 1]$

| | | | | |
|-----|-----|-----|-----|-----|
| -13 | -11 | -21 | -17 | -9 |
| -20 | -30 | -7 | -27 | -5 |
| -26 | -15 | -34 | -28 | -28 |
| -38 | -34 | -49 | -31 | -21 |
| -15 | -17 | -6 | -19 | -1 |

$x[:, :, 2] * w1[:, :, 2]$

| | | | | |
|----|-----|-----|-----|-----|
| -2 | -21 | -1 | 3 | -17 |
| 3 | -33 | -25 | -7 | -18 |
| -3 | -5 | -28 | -4 | -16 |
| -7 | -12 | -5 | -15 | -10 |
| -4 | -1 | -11 | 0 | -6 |

$b1$

| |
|---|
| 0 |
|---|

Σ

$v[:, :, 1]$

| | | | | |
|-----|-----|-----|-----|-----|
| -24 | -18 | -16 | -7 | -8 |
| -10 | -43 | -12 | -12 | -6 |
| -26 | -3 | -60 | -10 | -16 |
| -60 | -20 | -27 | -45 | 4 |
| -8 | -3 | 5 | 17 | 28 |

$x[:, :, 0]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 | 0 |
| 0 | 7 | 4 | 4 | 6 | 1 | 0 |
| 0 | 2 | 6 | 2 | 2 | 7 | 0 |
| 0 | 4 | 3 | 7 | 7 | 2 | 0 |
| 0 | 5 | 4 | 1 | 7 | 3 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 | 0 |
| 0 | 4 | 0 | 3 | 1 | 5 | 0 |
| 0 | 4 | 3 | 0 | 0 | 2 | 0 |
| 0 | 2 | 6 | 1 | 7 | 3 | 0 |
| 0 | 3 | 7 | 6 | 5 | 5 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 | 0 |
| 0 | 3 | 7 | 0 | 2 | 4 | 0 |
| 0 | 2 | 6 | 4 | 0 | 6 | 0 |
| 0 | 1 | 3 | 0 | 3 | 5 | 0 |
| 0 | 1 | 1 | 0 | 1 | 4 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$w0[:, :, 1]$

| | | |
|----|----|----|
| -1 | 1 | -1 |
| -1 | 1 | 2 |
| -1 | -1 | 1 |

$w0[:, :, 2]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$x[:, :, 0] * w0[:, :, 0]$

| | | | | |
|----|----|----|----|----|
| 12 | 26 | 18 | 25 | 21 |
| -5 | 28 | 19 | 4 | 24 |
| -5 | 11 | 15 | 17 | 24 |
| 4 | 16 | 20 | 26 | 14 |
| 1 | 16 | 5 | 5 | 20 |

$x[:, :, 1] * w0[:, :, 1]$

| | | | | |
|-----|----|-----|----|----|
| 4 | 7 | 6 | -1 | 12 |
| -5 | 3 | -4 | -9 | 13 |
| -7 | 15 | -10 | -2 | -6 |
| -15 | 8 | 3 | -2 | 15 |
| -12 | 2 | 13 | 3 | 19 |

$x[:, :, 2] * w0[:, :, 2]$

| | | | | |
|-----|----|----|----|-----|
| -17 | 0 | 14 | -2 | -8 |
| -3 | -5 | 32 | 11 | -10 |
| 9 | -7 | 22 | 12 | -14 |
| 9 | 2 | 17 | 14 | -13 |
| 4 | -1 | 15 | 9 | -5 |

$b0$

| |
|---|
| 1 |
|---|

Σ

$v[:, :, 0]$

| | | | | |
|-----|----|----|----|----|
| 0 | 34 | 39 | 23 | 35 |
| -12 | 27 | 48 | 7 | 28 |
| -2 | 20 | 28 | 48 | 5 |
| -1 | 27 | 41 | 39 | 17 |
| -6 | 15 | 34 | 18 | 35 |

$y[:, :, 0]$

| | | | | |
|--|--|--|--|--|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |



$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$w1[:, :, 1]$

| | | |
|----|----|----|
| -2 | -2 | -2 |
| -2 | 1 | 0 |
| 0 | -2 | 0 |

$w1[:, :, 2]$

| | | |
|----|----|----|
| 0 | -2 | 0 |
| 2 | -1 | -1 |
| -2 | 1 | -2 |

$x[:, :, 0] * w1[:, :, 0]$

| | | | | |
|-----|----|----|----|----|
| -9 | 14 | 6 | 7 | 18 |
| 7 | 20 | 20 | 22 | 17 |
| 3 | 17 | 2 | 22 | 28 |
| -15 | 26 | 27 | 1 | 35 |
| 11 | 15 | 22 | 36 | 35 |

$x[:, :, 1] * w1[:, :, 1]$

| | | | | |
|-----|-----|-----|-----|-----|
| -13 | -11 | -21 | -17 | -9 |
| -20 | -30 | -7 | -27 | -5 |
| -26 | -15 | -34 | -28 | -28 |
| -38 | -34 | -49 | -31 | -21 |
| -15 | -17 | -6 | -19 | -1 |

$x[:, :, 2] * w1[:, :, 2]$

| | | | | |
|----|-----|-----|-----|-----|
| -2 | -21 | -1 | 3 | -17 |
| 3 | -33 | -25 | -7 | -18 |
| -3 | -5 | -28 | -4 | -16 |
| -7 | -12 | -5 | -15 | -10 |
| -4 | -1 | -11 | 0 | -6 |

$b1$

| |
|---|
| 0 |
|---|

Σ

$v[:, :, 1]$

| | | | | |
|-----|-----|-----|-----|-----|
| -24 | -18 | -16 | -7 | -8 |
| -10 | -43 | -12 | -12 | -6 |
| -26 | -3 | -60 | -10 | -16 |
| -60 | -20 | -27 | -45 | 4 |
| -8 | -3 | 5 | 17 | 28 |

$y[:, :, 1]$

| | | | | |
|--|--|--|--|--|
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |



$x[:, :, 0]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 3 | 4 | 6 | 2 | 0 |
| 0 | 7 | 4 | 4 | 6 | 1 | 0 |
| 0 | 2 | 6 | 2 | 2 | 7 | 0 |
| 0 | 4 | 3 | 7 | 7 | 2 | 0 |
| 0 | 5 | 4 | 1 | 7 | 3 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 1]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 5 | 5 | 1 | 7 | 3 | 0 |
| 0 | 4 | 0 | 3 | 1 | 5 | 0 |
| 0 | 4 | 3 | 0 | 0 | 2 | 0 |
| 0 | 2 | 6 | 1 | 7 | 3 | 0 |
| 0 | 3 | 7 | 6 | 5 | 5 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$x[:, :, 2]$ 5×5 + pad 1

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 6 | 5 | 2 | 3 | 6 | 0 |
| 0 | 3 | 7 | 0 | 2 | 4 | 0 |
| 0 | 2 | 6 | 4 | 0 | 6 | 0 |
| 0 | 1 | 3 | 0 | 3 | 5 | 0 |
| 0 | 1 | 1 | 0 | 1 | 4 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$w0[:, :, 0]$

| | | |
|----|----|---|
| -1 | 1 | 1 |
| 1 | 1 | 2 |
| 0 | -2 | 1 |

$w0[:, :, 1]$

| | | |
|----|----|----|
| -1 | 1 | -1 |
| -1 | 1 | 2 |
| -1 | -1 | 1 |

$w0[:, :, 2]$

| | | |
|----|----|---|
| -1 | -1 | 1 |
| 1 | -2 | 2 |
| 2 | -1 | 2 |

$x[:, :, 0] * w0[:, :, 0]$

| | | | | |
|----|----|----|----|----|
| 12 | 26 | 18 | 25 | 21 |
| -5 | 28 | 19 | 4 | 24 |
| -5 | 11 | 15 | 17 | 24 |
| 4 | 16 | 20 | 26 | 14 |
| 1 | 16 | 5 | 5 | 20 |

$x[:, :, 1] * w0[:, :, 1]$

| | | | | |
|-----|----|-----|----|----|
| 4 | 7 | 6 | -1 | 12 |
| -5 | 3 | -4 | -9 | 13 |
| -7 | 15 | -10 | -2 | -6 |
| -15 | 8 | 3 | -2 | 15 |
| -12 | 2 | 13 | 3 | 19 |

$x[:, :, 2] * w0[:, :, 2]$

| | | | | |
|-----|----|----|----|-----|
| -17 | 0 | 14 | -2 | -8 |
| -3 | -5 | 32 | 11 | -10 |
| 9 | -7 | 22 | 12 | -14 |
| 9 | 2 | 17 | 14 | -13 |
| 4 | -1 | 15 | 9 | -5 |

$b0$

| |
|---|
| 1 |
|---|

Σ

$v[:, :, 0]$

| | | | | |
|-----|----|----|----|----|
| 0 | 34 | 39 | 23 | 35 |
| -12 | 27 | 48 | 7 | 28 |
| -2 | 20 | 28 | 48 | 5 |
| -1 | 27 | 41 | 39 | 17 |
| -6 | 15 | 34 | 18 | 35 |

$y[:, :, 0]$

| | | | | |
|---|----|----|----|----|
| 0 | 34 | 39 | 23 | 35 |
| 0 | 27 | 48 | 7 | 28 |
| 0 | 20 | 28 | 48 | 5 |
| 0 | 27 | 41 | 39 | 17 |
| 0 | 15 | 34 | 18 | 35 |



ReLU



$w1[:, :, 0]$

| | | |
|----|----|---|
| -1 | -2 | 1 |
| 1 | 1 | 2 |
| -2 | 2 | 2 |

$w1[:, :, 1]$

| | | |
|----|----|----|
| -2 | -2 | -2 |
| -2 | 1 | 0 |
| 0 | -2 | 0 |

$w1[:, :, 2]$

| | | |
|----|----|----|
| 0 | -2 | 0 |
| 2 | -1 | -1 |
| -2 | 1 | -2 |

$x[:, :, 0] * w1[:, :, 0]$

| | | | | |
|-----|----|----|----|----|
| -9 | 14 | 6 | 7 | 18 |
| 7 | 20 | 20 | 22 | 17 |
| 3 | 17 | 2 | 22 | 28 |
| -15 | 26 | 27 | 1 | 35 |
| 11 | 15 | 22 | 36 | 35 |

$x[:, :, 1] * w1[:, :, 1]$

| | | | | |
|-----|-----|-----|-----|-----|
| -13 | -11 | -21 | -17 | -9 |
| -20 | -30 | -7 | -27 | -5 |
| -26 | -15 | -34 | -28 | -28 |
| -38 | -34 | -49 | -31 | -21 |
| -15 | -17 | -6 | -19 | -1 |

$x[:, :, 2] * w1[:, :, 2]$

| | | | | |
|----|-----|-----|-----|-----|
| -2 | -21 | -1 | 3 | -17 |
| 3 | -33 | -25 | -7 | -18 |
| -3 | -5 | -28 | -4 | -16 |
| -7 | -12 | -5 | -15 | -10 |
| -4 | -1 | -11 | 0 | -6 |

$b1$

| |
|---|
| 0 |
|---|

Σ

$v[:, :, 1]$

| | | | | |
|-----|-----|-----|-----|-----|
| -24 | -18 | -16 | -7 | -8 |
| -10 | -43 | -12 | -12 | -6 |
| -26 | -3 | -60 | -10 | -16 |
| -60 | -20 | -27 | -45 | 4 |
| -8 | -3 | 5 | 17 | 28 |

$y[:, :, 1]$

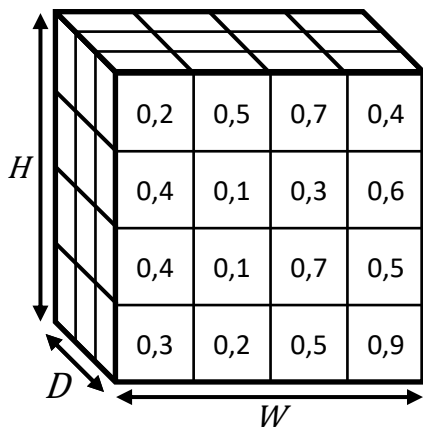
| | | | | |
|---|---|---|----|----|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 4 |
| 0 | 0 | 5 | 17 | 28 |



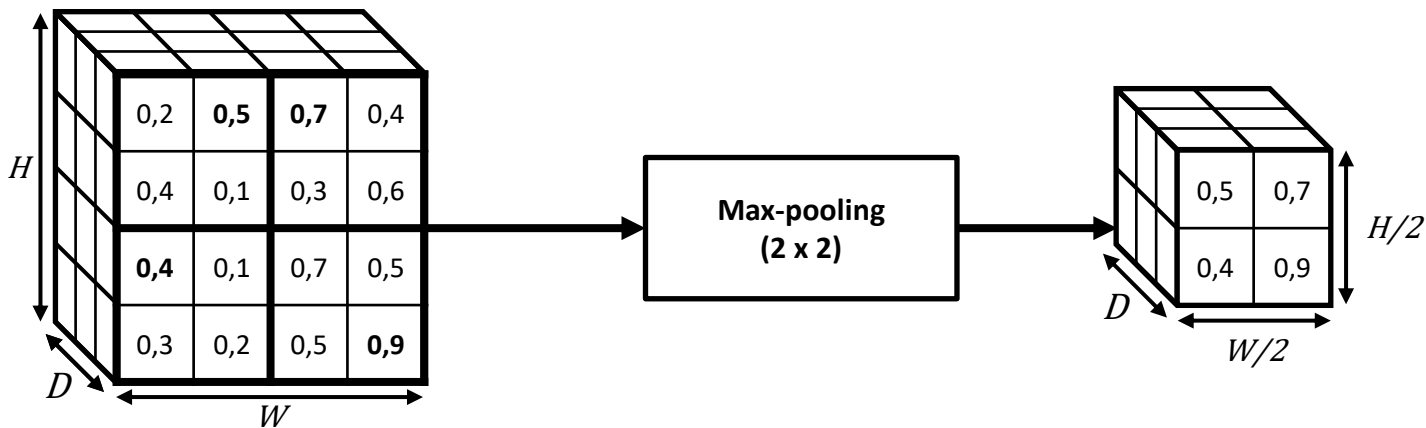
ReLU



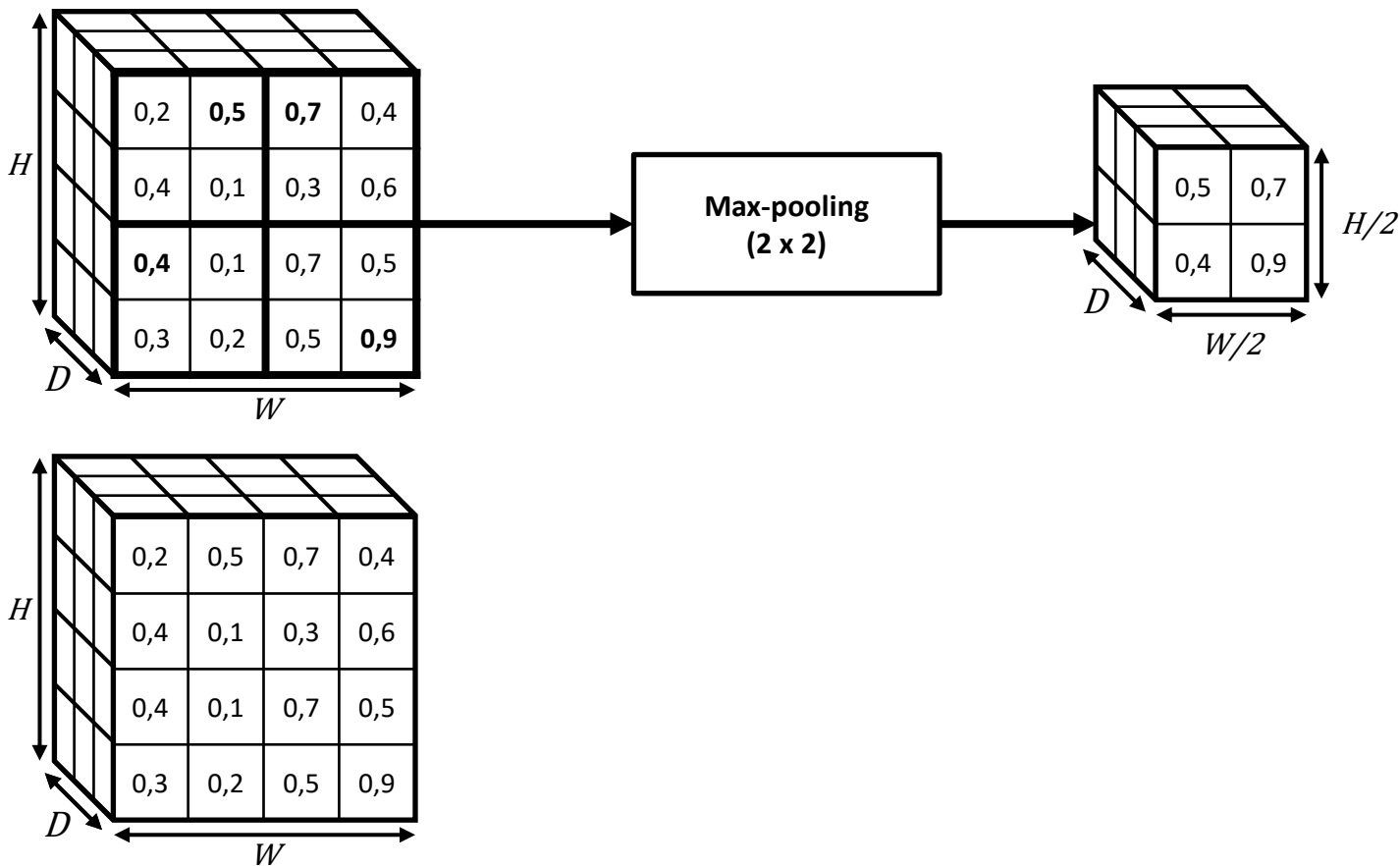
CAMADA DE POOLING



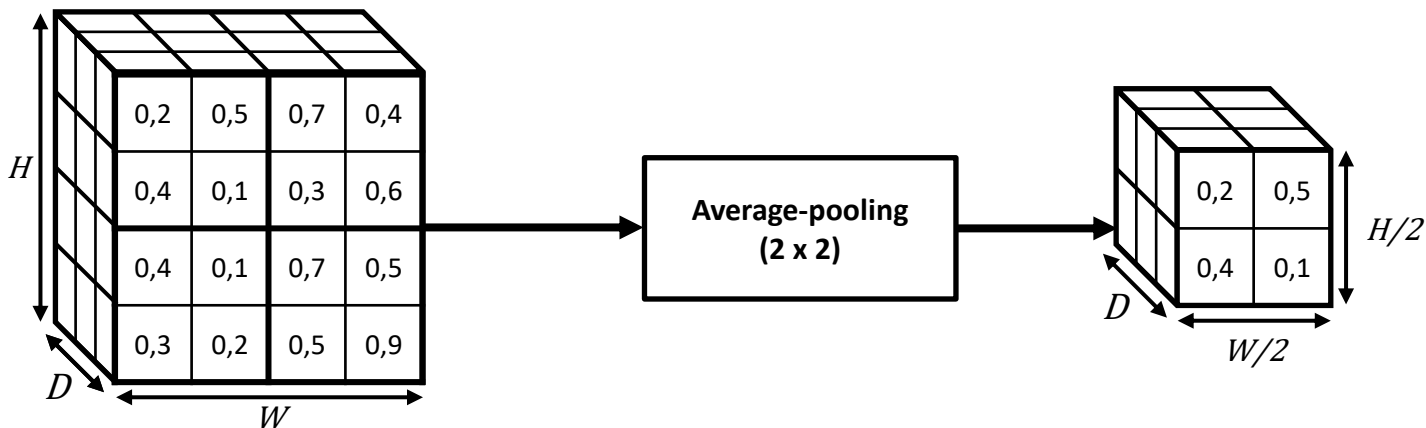
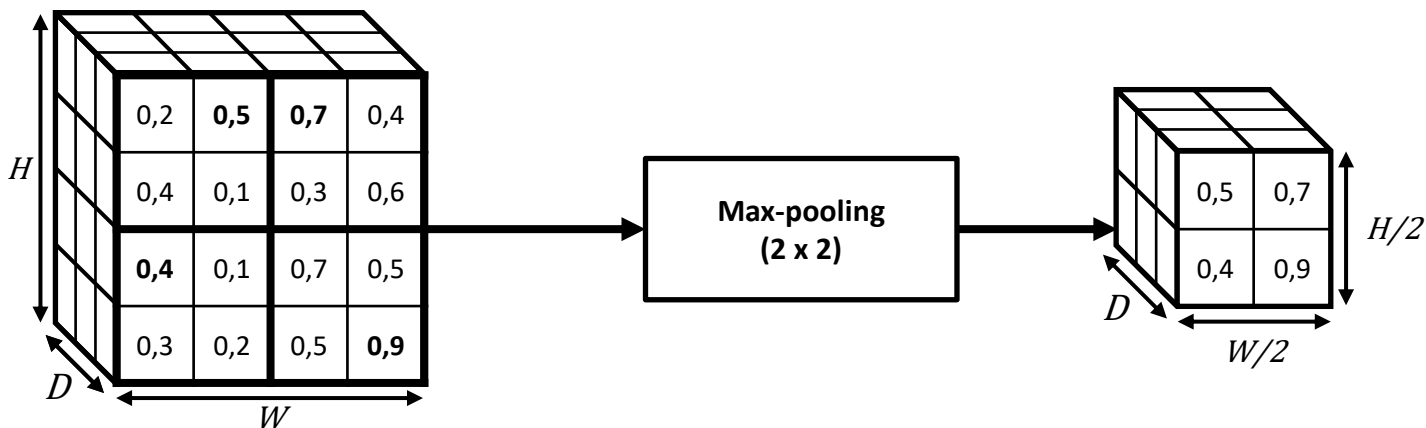
Camada de pooling



Camada de pooling



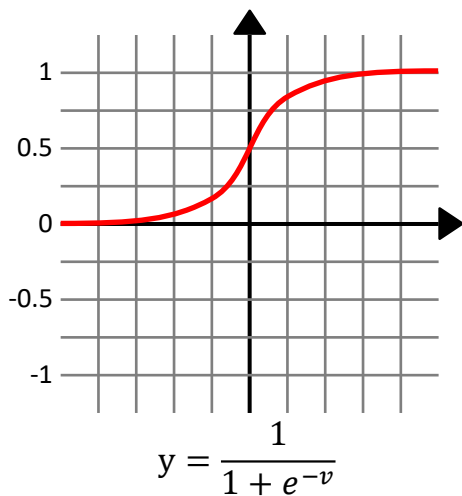
Camada de pooling



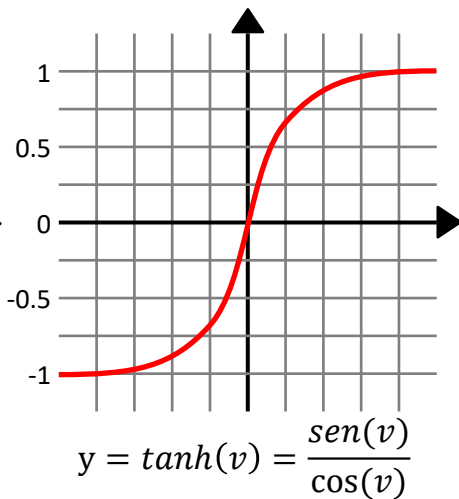
FUNÇÃO DE ATIVAÇÃO

Função de ativação

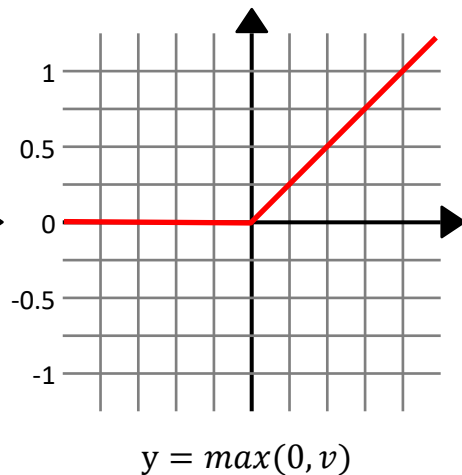
Logística



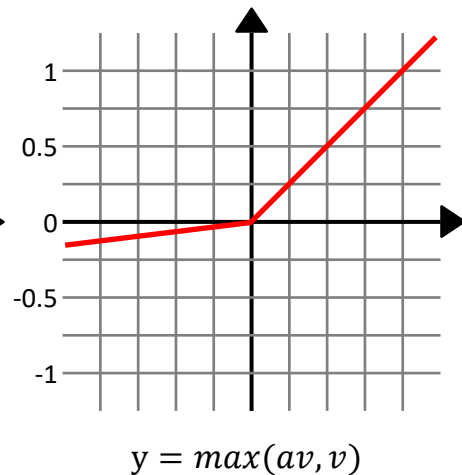
Tangente hiperbólica



ReLu



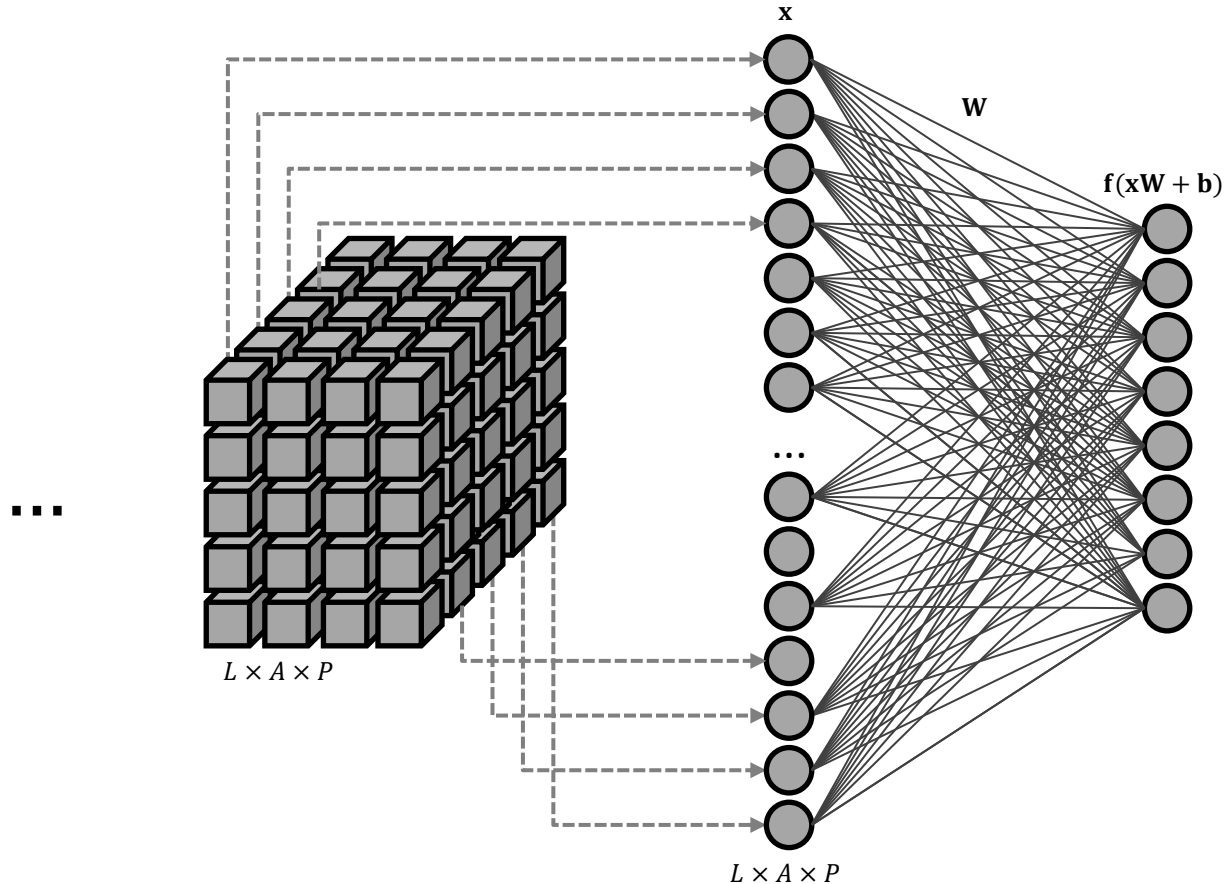
PReLU



- Se $a=0,01 \rightarrow$ Leak ReLu

CAMADA COMPLETAMENTE CONECTADA

Camada completamente conectada



CAMADA DE SAÍDA - SOFTMAX

Camada de saída – softmax

- Função softmax para M classes:

$$- \text{softmax}(x_i) = \frac{e^{x_i}}{\sum_{j=0}^{M-1} e^{x_j}}$$

- **Exemplo:**

- $\mathbf{x} = [-0,8 \quad 2,0 \quad 6,0 \quad -2,7 \quad 0,8]$

- $\sum_{j=0}^{M-1} x_j = 5,3$

- *Soma != de 1,0. Não pode ser interpretado como probabilidades.*

- $\sum_{j=0}^{M-1} e^{x_j} = 0,4493 + 7,3891 + 403,4288 + 0,0672 + 2,2255 = 413,5599$

- $\text{softmax}(x_i) = [0,0011 \quad 0,0179 \quad 0,9755 \quad 0,0002 \quad 0,0054]$

- $\sum_{j=0}^{M-1} \text{softmax}(x_i) = 1,0$

- *Representa a probabilidade da amostra pertencer a cada classe.*

FUNÇÃO DE PERDA (LOSS)

Função de perda (loss)

- Entropia cruzada para mais de 2 classes ($M > 2$):
 - $L(\mathbf{y}, \hat{\mathbf{y}}) = - \sum_{j=0}^{M-1} \mathbf{y}_j \cdot \log(\hat{\mathbf{y}}_j)$
- Entropia cruzada para 2 classes ($M=2$):
 - $L(\mathbf{y}, \hat{\mathbf{y}}) = -(\mathbf{y} \cdot \log(\hat{\mathbf{y}}) + (1 - \mathbf{y}) \log(1 - \hat{\mathbf{y}}))$

Entropia cruzada para $M > 2$

- 5 classes, classificação **correta**, com 72% de probabilidade:
 - $\mathbf{y} = [0 \quad 0 \quad 0 \quad 1 \quad 0]$
 - $\hat{\mathbf{y}} = [0,20 \quad 0,0 \quad 0,05, \quad 0,72 \quad 0,03]$
 - $L(\mathbf{y}, \hat{\mathbf{y}}) = -(0 \times \log 0,2 + 0 \times \log 0,0 + 0 \times \log 0,5 + 1 \times \log 0,72 + 0 \times \log 0,03)$
 - $L(\mathbf{y}, \hat{\mathbf{y}}) = -(\log 0,72) = 0,14267$

Entropia cruzada para $M > 2$

- 5 classes, classificação **correta**, com 72% de probabilidade:
 - $\mathbf{y} = [0 \quad 0 \quad 0 \quad 1 \quad 0]$
 - $\hat{\mathbf{y}} = [0,20 \quad 0,0 \quad 0,05, \quad 0,72 \quad 0,03]$
 - $L(\mathbf{y}, \hat{\mathbf{y}}) = -(0 \times \log 0,2 + 0 \times \log 0,0 + 0 \times \log 0,5 + 1 \times \log 0,72 + 0 \times \log 0,03)$
 - $L(\mathbf{y}, \hat{\mathbf{y}}) = -(\log 0,72) = 0,14267$
- 5 classes, classificação **correta**, com 52% de probabilidade:
 - $\mathbf{y} = [0 \quad 0 \quad 0 \quad 1 \quad 0]$
 - $\hat{\mathbf{y}} = [0,30 \quad 0,0 \quad 0,05, \quad 0,52 \quad 0,13]$
 - $L(\mathbf{y}, \hat{\mathbf{y}}) = -(0 \times \log 0,3 + 0 \times \log 0,0 + 0 \times \log 0,5 + 1 \times \log 0,52 + 0 \times \log 0,13)$
 - $L(\mathbf{y}, \hat{\mathbf{y}}) = -(\log 0,52) = 0,284$

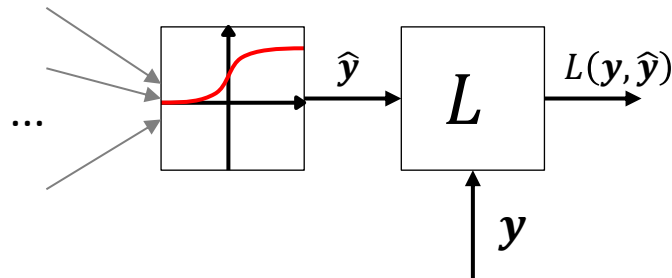
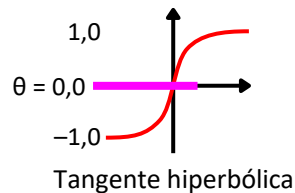
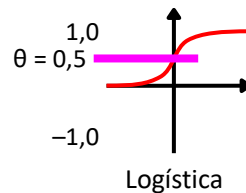
Entropia cruzada para $M > 2$

- 5 classes, classificação **correta**, com 72% de probabilidade:
 - $\mathbf{y} = [0 \quad 0 \quad 0 \quad 1 \quad 0]$
 - $\hat{\mathbf{y}} = [0,20 \quad 0,0 \quad 0,05, \quad 0,72 \quad 0,03]$
 - $L(\mathbf{y}, \hat{\mathbf{y}}) = -(0 \times \log 0,2 + 0 \times \log 0,0 + 0 \times \log 0,5 + 1 \times \log 0,72 + 0 \times \log 0,03)$
 - $L(\mathbf{y}, \hat{\mathbf{y}}) = -(\log 0,72) = 0,14267$
- 5 classes, classificação **correta**, com 52% de probabilidade:
 - $\mathbf{y} = [0 \quad 0 \quad 0 \quad 1 \quad 0]$
 - $\hat{\mathbf{y}} = [0,30 \quad 0,0 \quad 0,05, \quad 0,52 \quad 0,13]$
 - $L(\mathbf{y}, \hat{\mathbf{y}}) = -(0 \times \log 0,3 + 0 \times \log 0,0 + 0 \times \log 0,5 + 1 \times \log 0,52 + 0 \times \log 0,13)$
 - $L(\mathbf{y}, \hat{\mathbf{y}}) = -(\log 0,52) = 0,284$
- 5 classes, classificação **incorreta**:
 - $\mathbf{y} = [0 \quad 0 \quad 0 \quad 1 \quad 0]$
 - $\hat{\mathbf{y}} = [0,60 \quad 0,0 \quad 0,07, \quad 0,30 \quad 0,03]$
 - $L(\mathbf{y}, \hat{\mathbf{y}}) = -(0 \times \log 0,6 + 0 \times \log 0,0 + 0 \times \log 0,07 + 1 \times \log 0,3 + 0 \times \log 0,03)$
 - $L(\mathbf{y}, \hat{\mathbf{y}}) = -(\log 0,3) = 0,5229$

Entropia cruzada para M=2

- 2 classes, classificação correta:

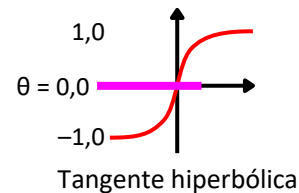
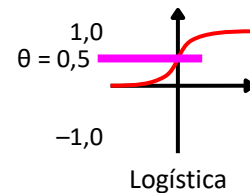
- $\mathbf{y} = [0]$
- $\hat{\mathbf{y}} = [0,20]$
- $L(\mathbf{y}, \hat{\mathbf{y}}) = -(0 \times \log 0,2 + (1 - 0) \times \log(1 - 0,2))$
- $L(\mathbf{y}, \hat{\mathbf{y}}) = -(0 \times \log 0,2 + (1) \times \log(0,8)) = -(\log(0,8)) = 0,09691$



Entropia cruzada para M=2

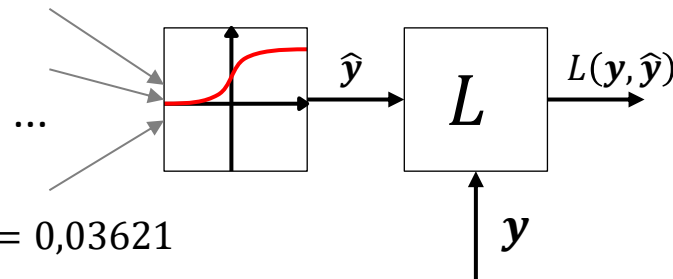
- 2 classes, classificação correta:

- $\mathbf{y} = [0]$
- $\hat{\mathbf{y}} = [0,20]$
- $L(\mathbf{y}, \hat{\mathbf{y}}) = -(0 \times \log 0,2 + (1 - 0) \times \log(1 - 0,2))$
- $L(\mathbf{y}, \hat{\mathbf{y}}) = -(0 \times \log 0,2 + (1) \times \log(0,8)) = -(\log(0,8)) = 0,09691$



- 2 classes, classificação correta:

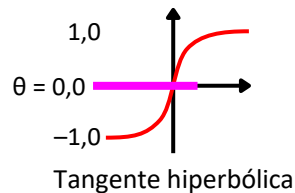
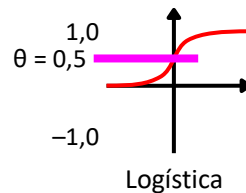
- $\mathbf{y} = [1]$
- $\hat{\mathbf{y}} = [0,92]$
- $L(\mathbf{y}, \hat{\mathbf{y}}) = -(1 \times \log 0,92 + (1 - 1) \times \log(1 - 0,92))$
- $L(\mathbf{y}, \hat{\mathbf{y}}) = -(1 \times \log 0,92 + (0) \times \log(0,08)) = -(\log(0,92)) = 0,03621$



Entropia cruzada para M=2

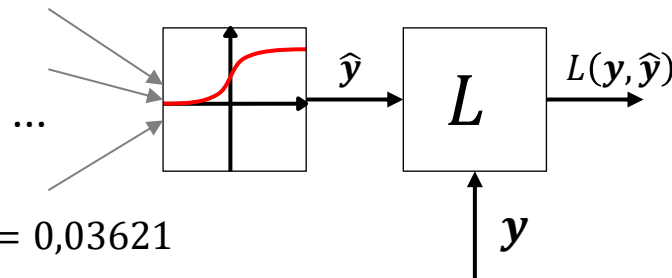
- 2 classes, classificação correta:

- $\mathbf{y} = [0]$
- $\hat{\mathbf{y}} = [0,20]$
- $L(\mathbf{y}, \hat{\mathbf{y}}) = -(0 \times \log 0,2 + (1 - 0) \times \log(1 - 0,2))$
- $L(\mathbf{y}, \hat{\mathbf{y}}) = -(0 \times \log 0,2 + (1) \times \log(0,8)) = -(\log(0,8)) = 0,09691$



- 2 classes, classificação correta:

- $\mathbf{y} = [1]$
- $\hat{\mathbf{y}} = [0,92]$
- $L(\mathbf{y}, \hat{\mathbf{y}}) = -(1 \times \log 0,92 + (1 - 1) \times \log(1 - 0,92))$
- $L(\mathbf{y}, \hat{\mathbf{y}}) = -(1 \times \log 0,92 + (0) \times \log(0,08)) = -(\log(0,92)) = 0,03621$



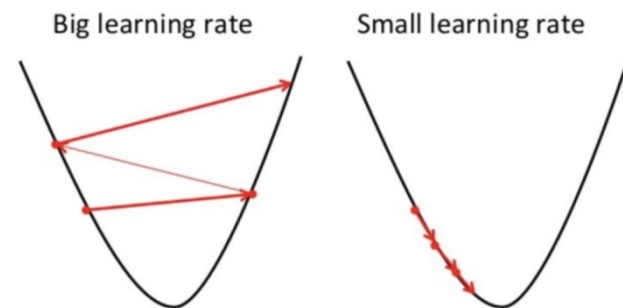
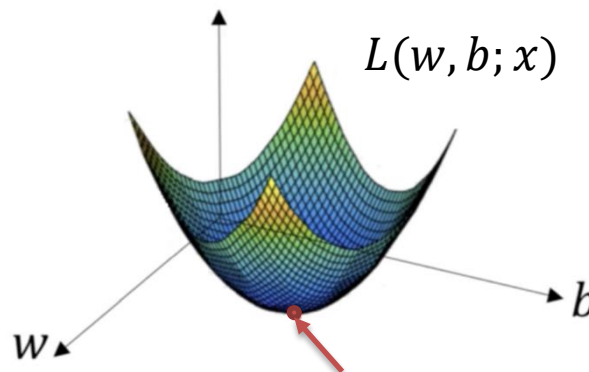
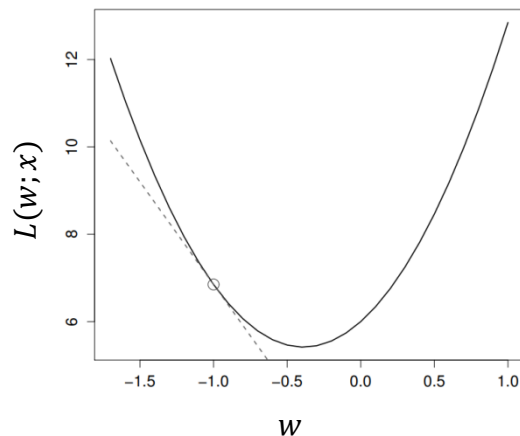
- 2 classes, classificação incorreta:

- $\mathbf{y} = [0]$
- $\hat{\mathbf{y}} = [0,65]$
- $L(\mathbf{y}, \hat{\mathbf{y}}) = -(0 \times \log 0,65 + (1 - 0) \times \log(1 - 0,65))$
- $L(\mathbf{y}, \hat{\mathbf{y}}) = -(0 \times \log 0,65 + (1) \times \log(0,35)) = -(\log(0,35)) = 0,45593$

OTIMIZADORES

Otimizadores

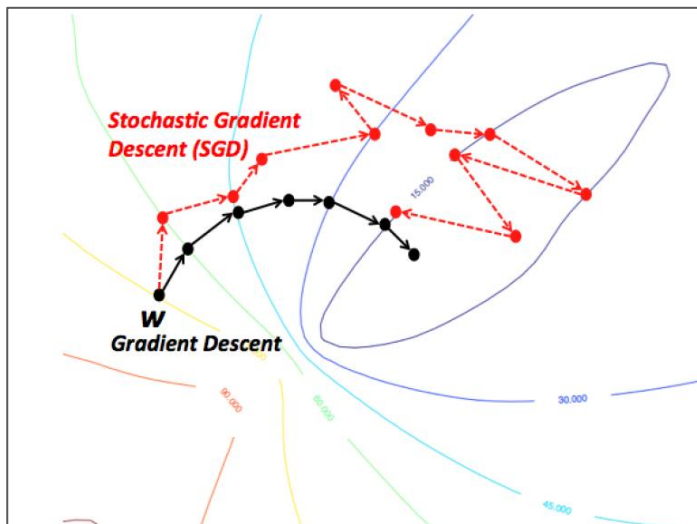
- Gradiente descendente (GD - *Gradient descent*):
 - $W_{t+1} = W_t - \eta \sum_{j=1}^N \nabla L(W; x_j)$
 - N é o tamanho do conjunto de treinamento



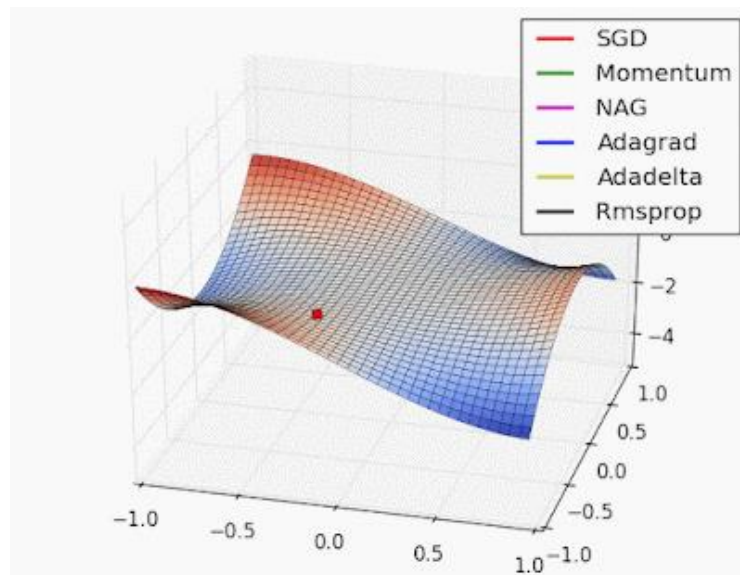
Donges. *Gradient Descent in Machine Learning: A Basic Introduction*.
<https://builtin.com/data-science/gradient-descent>

Otimizadores

- Gradiente descendente estocástico (SGD – *Stochastic gradient descent*):
 - $W_{t+1} = W_t - \eta \sum_{j=1}^B \nabla L(W; x_j^B)$
 - B é o tamanho do mini-lote (*mini-batch*)



- SGD com momentum:
 - $W_{t+1} = W_t - \eta \sum_{j=1}^B \nabla L(W; x_j^B)$
 - B é o tamanho do mini-lote (*mini-batch*)
 - $W_{t+1} = W_t + \alpha(W_t - W_{t-1}) + (1 - \alpha)[- \eta \sum_{j=1}^B \nabla L(W; x_j^B)]$



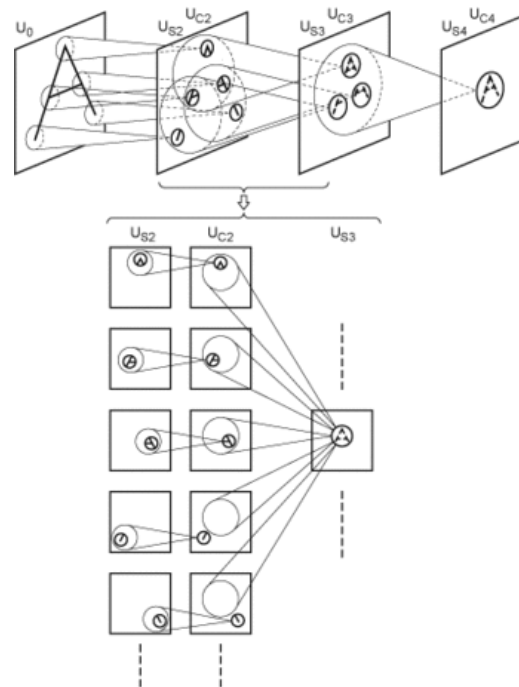
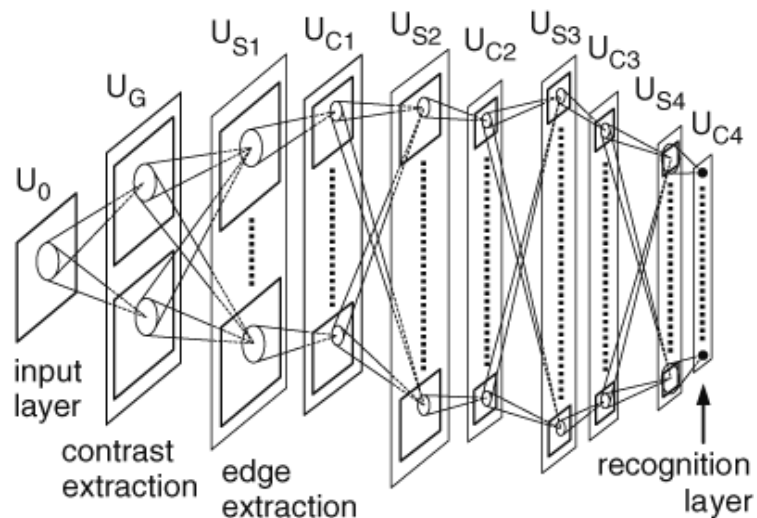
<http://www.denizyuret.com/2015/03/alec-radfords-animations-for.html>

- Outros otimizadores:
 - AdaGrad - *Adaptive Gradient*
 - AdaDelta - *Adaptive learning rate*
 - RMSProp - *Root Mean Squared Propagation*
 - Adam - *Adaptive moment estimation*
 - ...

ARQUITETURAS

Arquiteturas

- Neocognitron (1979)

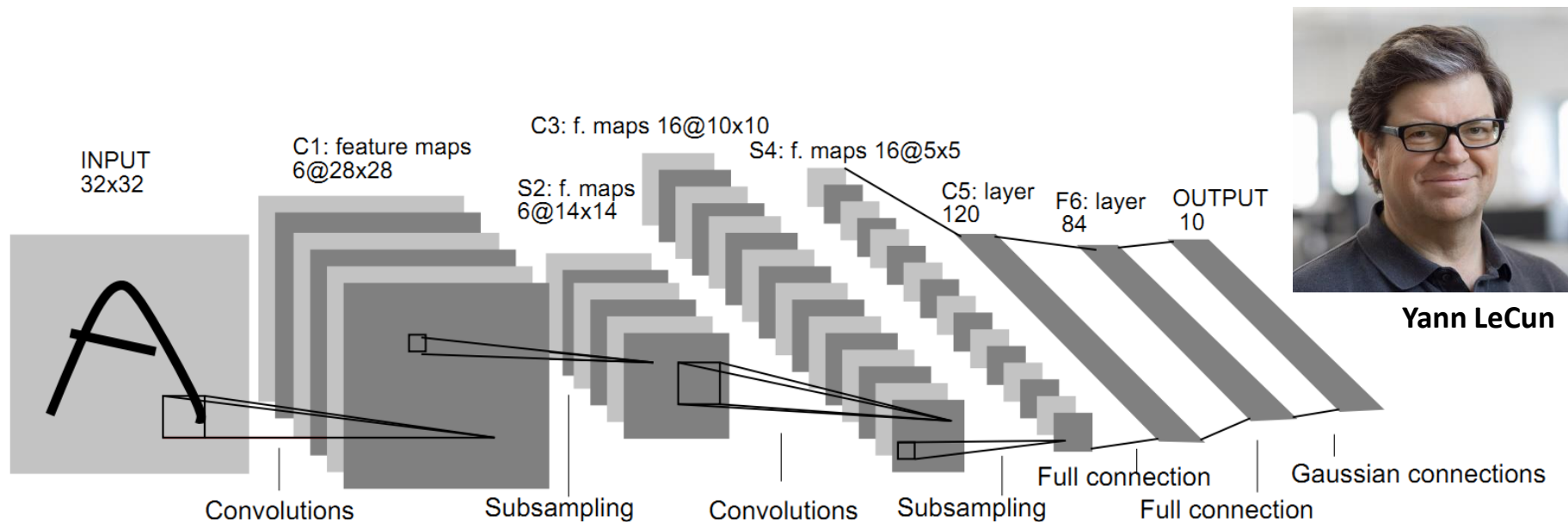


Kunihiro Fukushima

Fukushima, K. (1980). "Neocognitron: A self-organizing neural network model for a mechanism of pattern recognition unaffected by shift in position". *Biological Cybernetics*. 36 (4): 193–202.

Arquiteturas

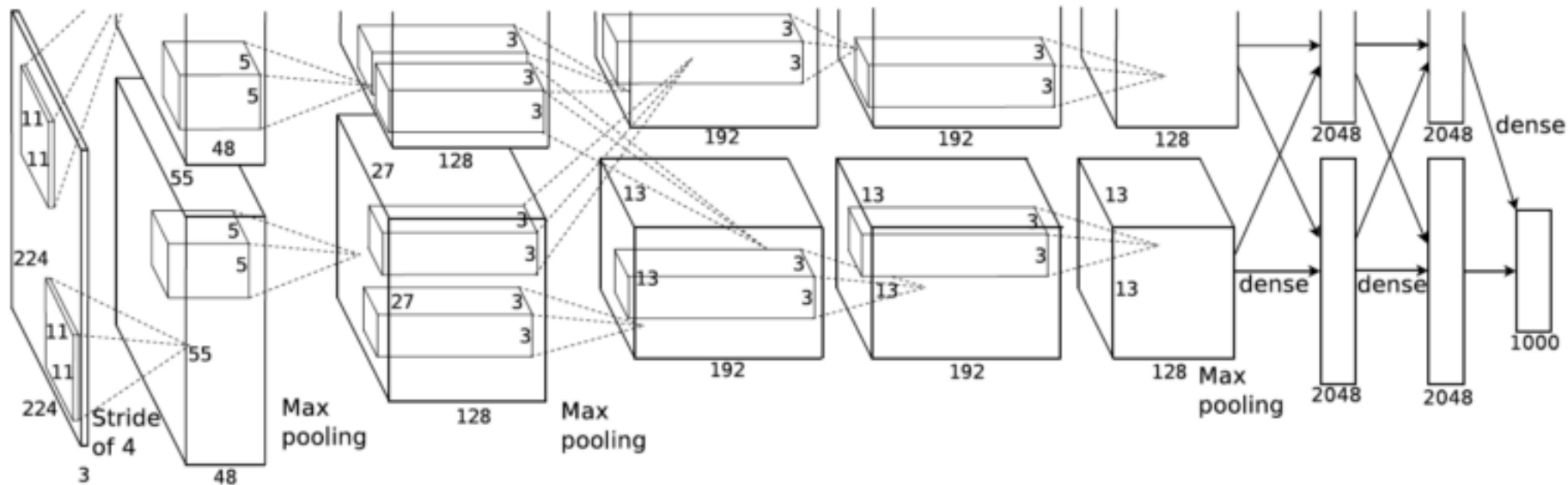
- LeNet-5 (1998)



Lecun, Y. et al. (1998). "Gradient-based learning applied to document recognition". *Proceedings of the IEEE*. 86 (11): 2278–2324.

Arquiteturas

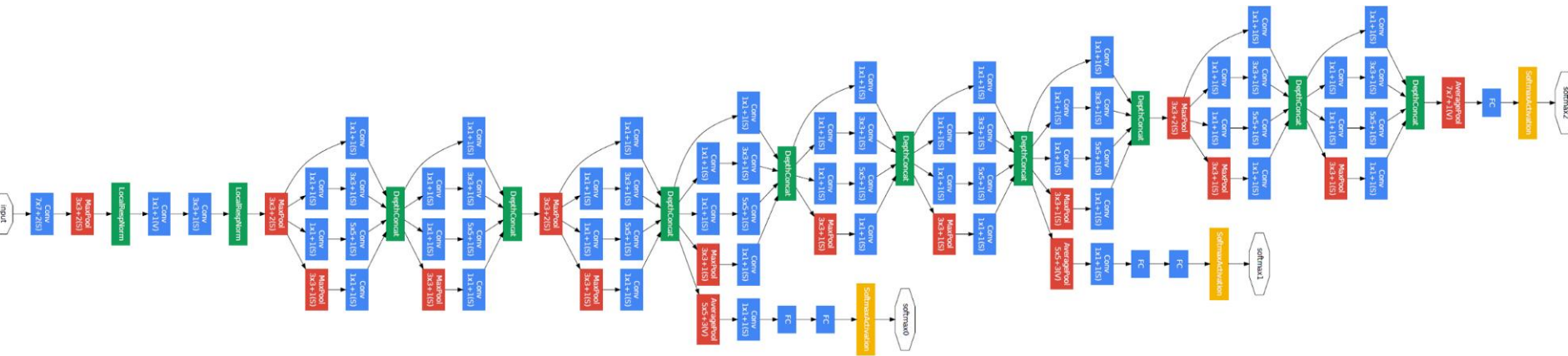
- AlexNet (2012)



Krizhevsky, Sutskever e Hinton. **ImageNet Classification with Deep Convolutional Neural Networks**. NeurIPS 2012

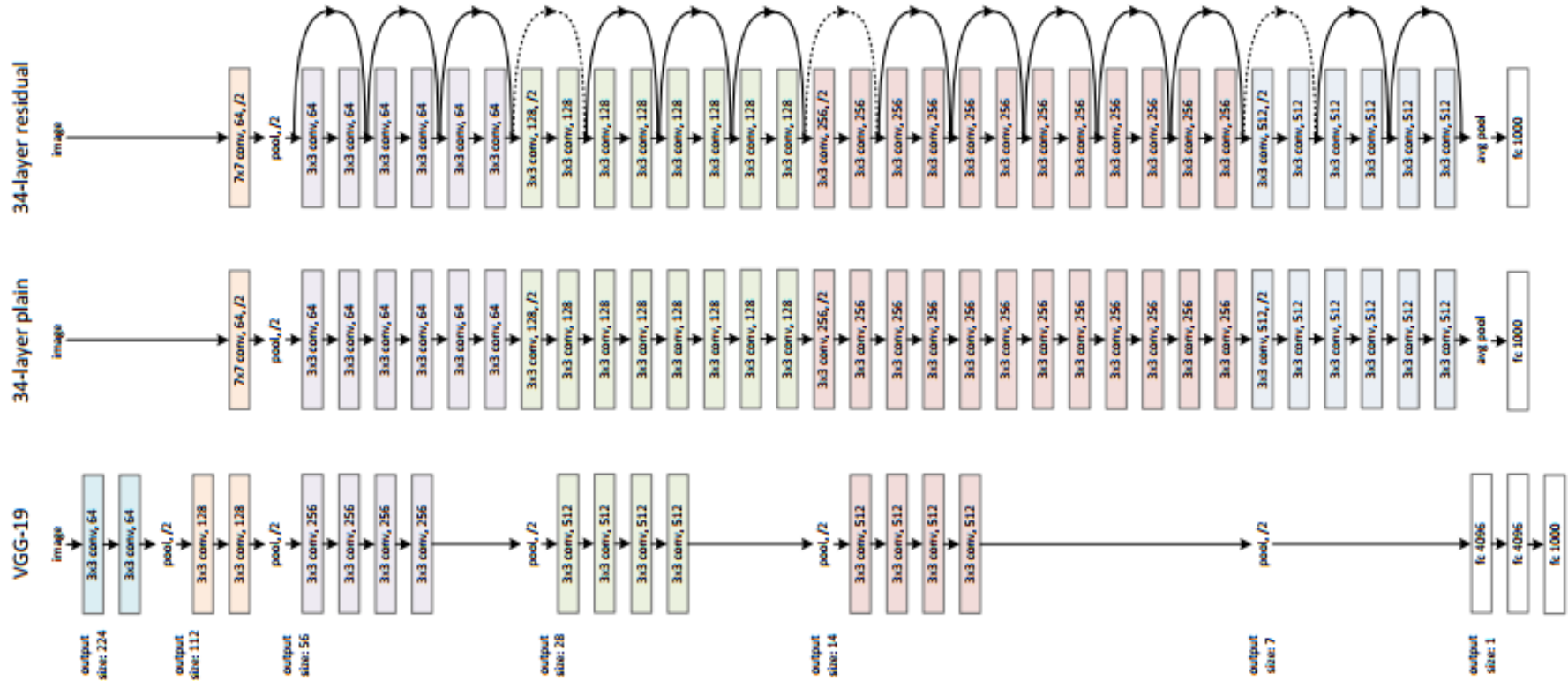
Arquiteturas

- Inception (GoogLeNet) (2015)



Szegedy, Christian (2015). "Going deeper with convolutions". CVPR2015.

- VGG (2014) e ResNet (2015)



Simonyan e Zisserman. Very Deep Convolutional Networks for Large-Scale Image Recognition. 2014

He et al. Deep Residual Learning for Image Recognition. 2015.

Arquiteturas

- DenseNet (2017)

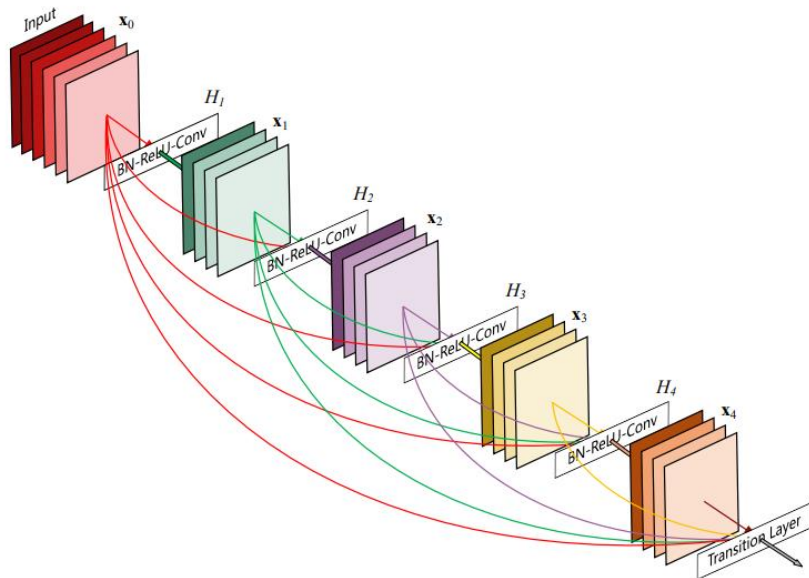


Figure 1: A 5-layer dense block with a growth rate of $k = 4$. Each layer takes all preceding feature-maps as input.

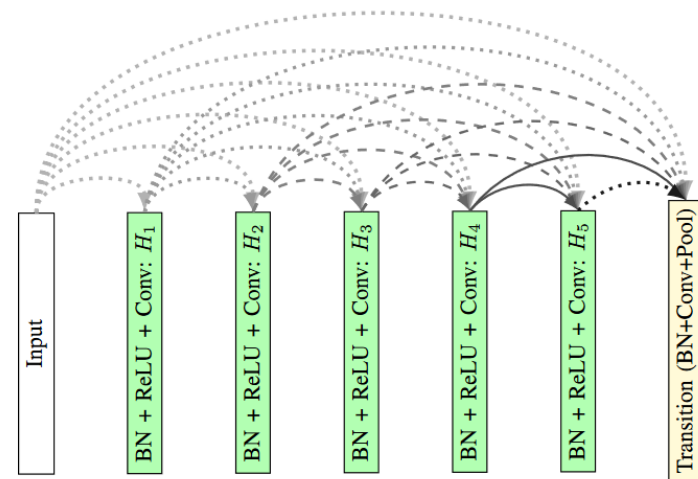
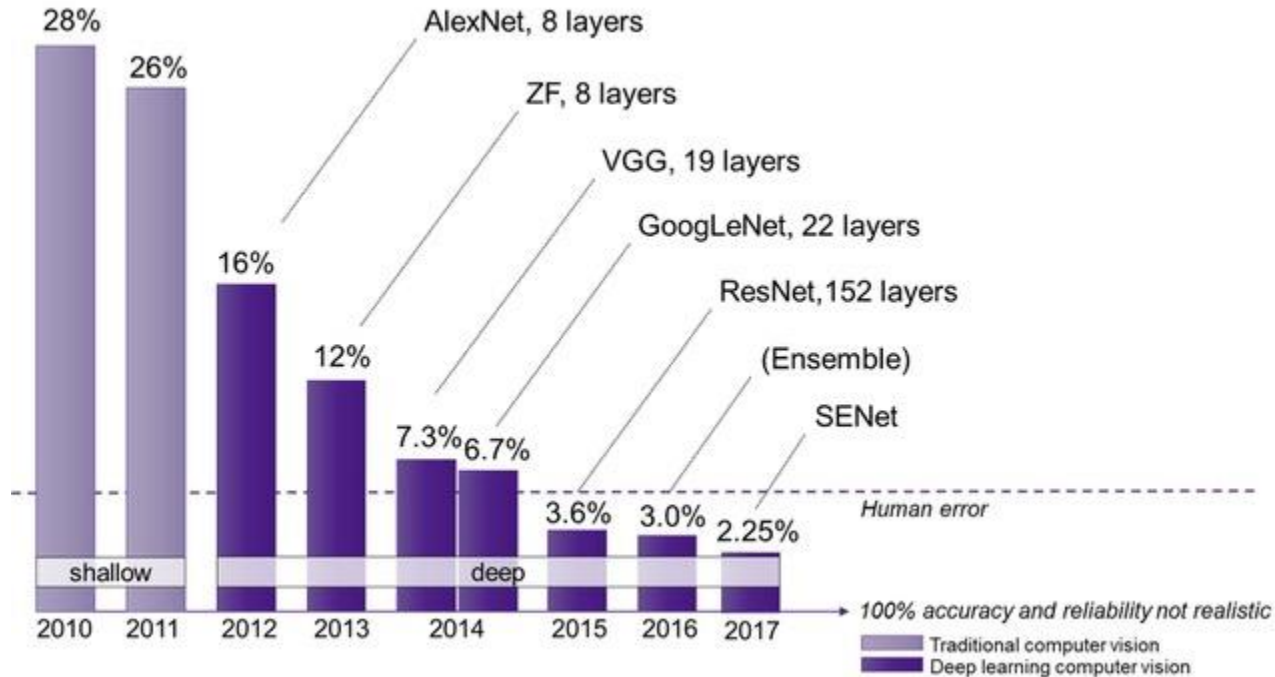


Figure 10. Illustration of a DenseBlock with 5 functions H_l and a Transition Layer.

Arquiteturas

- ImageNet Large Scale Visual Recognition Challenge
 - <https://image-net.org/challenges/LSVRC/>



<https://semiengineering.com/new-vision-technologies-for-real-world-applications/>

BIBLIOTECAS E DESENVOLVIMENTO

- O treinamento de CNNs possui alto custo computacional.
 - Recomenda-se que sejam treinados usando GPUs.
 - O Google Colab fornece acesso à GPUs (com algumas restrições).



Bibliotecas e desenvolvimento

- Principais bibliotecas para Deep Learning e Redes Neurais Convolucionais
 - PyTorch
 - <https://pytorch.org/>
 - Tensorflow
 - <https://www.tensorflow.org/>



Bibliotecas e desenvolvimento

- **Anaconda Distribution:**
 - Distribuição Python com suporte às principais bibliotecas
 - <https://www.anaconda.com/products/distribution>
- **Google Colab:**
 - Ambiente de execução em nuvem com GPUs.
 - <https://colab.research.google.com>



CONJUNTOS DE IMAGENS

Conjuntos de imagens

- MNIST

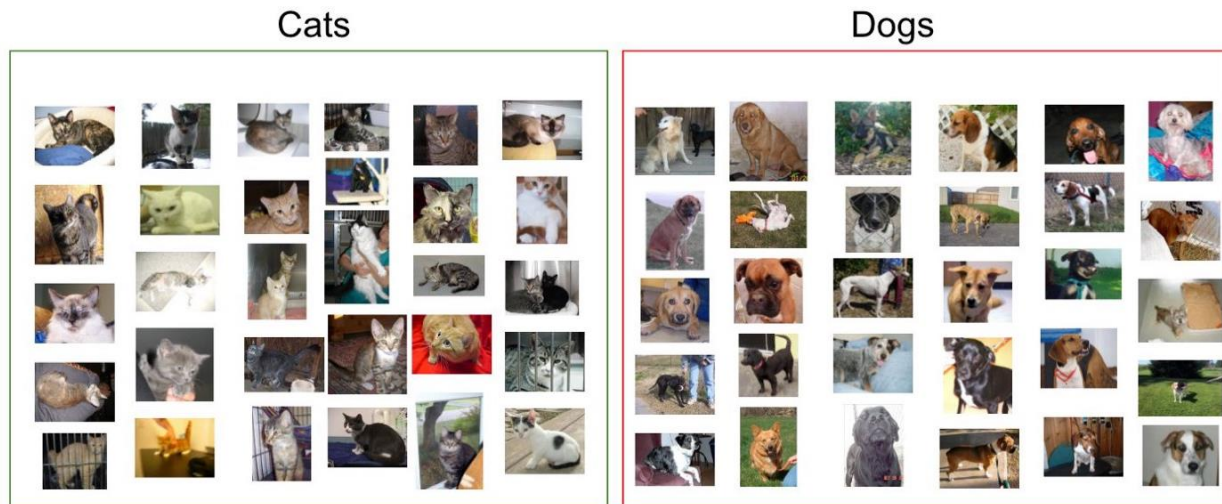
- <http://yann.lecun.com/exdb/mnist/>
- 60,000 training images
- 10,000 testing images
- 28 x 28 pixels
- Níveis de cinza



Conjuntos de imagens

- **Cats vs. Dogs:**

- <https://www.kaggle.com/c/dogs-vs-cats>
- 25,000 images de treinamento
- 12,500 imagens de teste
- 2 classes
- Diversos tamanhos
- RGB



Sample of cats & dogs images from Kaggle Dataset

Conjuntos de imagens

- **CIFAR10:**

- <https://www.cs.toronto.edu/~kriz/cifar.html>
- 50,000 training images
- 10,000 testing images
- 10 classes
- 32 x 32 pixels
- RGB

airplane

automobile

bird

cat

deer

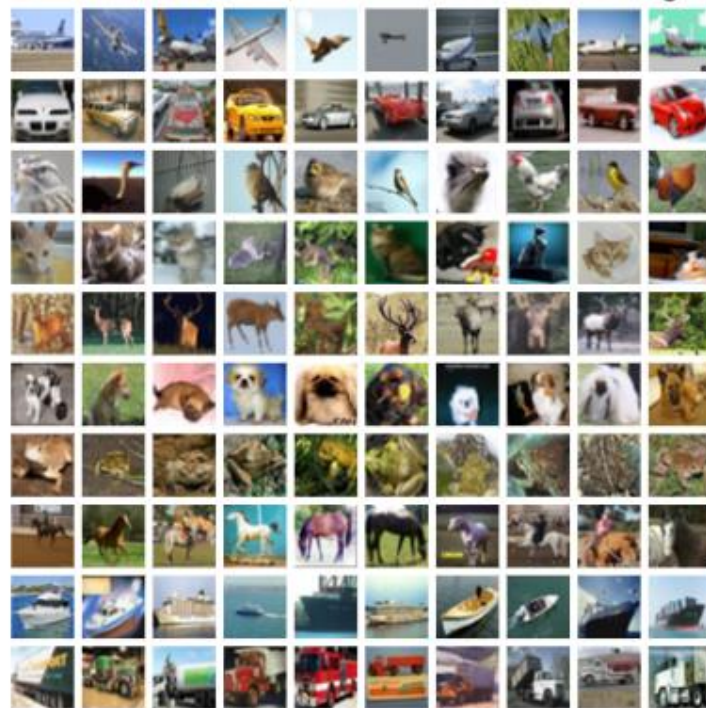
dog

frog

horse

ship

truck



- **ImageNet:**

- <https://www.image-net.org/>
- ~1,000,000 imagens
- 1,000 classes
- RGB

IMGENET



- Ponti et al. **Everything You Wanted to Know about Deep Learning for Computer Vision but Were Afraid to Ask**. Sibgrapi 2017.
- Moacir Ponti (ICMC-USP). **Material para o minicurso *Deep Learning***
 - https://github.com/maponti/deeplearning_intro_datascience
- **Learn TensorFlow and deep learning, without a Ph.D.**
 - <https://cloud.google.com/blog/products/gcp/learn-tensorflow-and-deep-learning-without-a-phd>
- CS231n: Convolutional Neural Networks for Visual Recognition
 - <http://cs231n.github.io/>
- Goodfellow, Bengio e Courville. **Deep Learning**. MIT Press, 2016
 - <https://www.deeplearningbook.org/>
- The MathWorks, Inc. **What is a Convolutional Neural Network? 3 things you need to know.**
 - <https://www.mathworks.com/discovery/convolutional-neural-network-matlab.html>

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- Lecun, Y. et al. (1998). **Gradient-based learning applied to document recognition**. Proceedings of the IEEE. 86 (11): 2278–2324.
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- Krizhevsky, Sutskever e Hinton. **ImageNet Classification with Deep Convolutional Neural Networks**. NeurIPS 2012.
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- Huang et al. **Densely Connected Convolutional Networks**. CVPR 2017.
- Rodrigues, L. F.; Naldi M. C., Mari, J. F. **Comparing convolutional neural networks and preprocessing techniques for HEP-2 cell classification in immunofluorescence images**. Computers in Biology and Medicine, 2019.
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