⑤ Demonstrate that a OL Least Square fit is equivalent of having a Maximum Likelihood of a gaussian variable with a linear model.

gaussian variable of a linear model: $\mathcal{L}(a,b) = N \prod_i^n \exp\left\{-\frac{1}{2}\left[\frac{(y_i^d - y_i^m(a,b))^2}{\sigma_i^2}\right]\right\}$,

where $x_i^d, y_i^d =$ data variable, $y_i^m(a,b) = a x_i^d + b =$ model variable and $N = \frac{1}{\sqrt{2\pi\sigma}}$

thus, $\mathcal{L}(a,b) = N \exp\left\{-\frac{1}{2}\left[\sum_i^n \frac{(y_i^d - y_i^m(a,b))^2}{\sigma_i^2}\right]\right\}$

$$\Rightarrow \underbrace{-2\ln\{\mathcal{L}(a,b)\}}_{=\bar{\mathcal{L}}} = \sum_i \frac{[y_i^d - y_i^m(a,b)]^2}{\sigma_i^2} - 2\ln[N \cdot e^{-1/2}]$$

$$\bar{\mathcal{L}} = \sum_i \frac{[y_i^d - y_i^m(a,b)]^2}{\sigma_i^2} - 2\ln N$$

thus, $\operatorname{argmax}(\bar{\mathcal{L}}) = \operatorname{argmin}\left\{\sum_i \frac{[y_i^d - y_i^m(a,b)]^2}{\sigma_i^2}\right\}$

by propagation of uncertainty, $\sigma_i^2 = \left(\frac{\partial y_i^d}{\partial x_i^d}\right)^2 \sigma_{x_i}^d \Rightarrow$ does not depends on the model

$$\operatorname{argmax}(\bar{\mathcal{L}}) = \operatorname{argmin}\left\{\sum_i^n [y_i^d - y_i^m(a,b)]^2\right\}$$

which corresponds to minimize the residuals $y_i^d - y_i^m(a,b) := r_i$

thus $\operatorname{argmax}(\bar{\mathcal{L}}) = \operatorname{argmin}\left(\sum_i^n r_i^2\right)$