

MORTGAGE BORROWING CAPS: DEBT, DEFAULT AND WELFARE

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Abstract

Since 1990, more than 60 countries have introduced caps on household borrowing aiming to curb systemic risk to the financial sector. However, there is very little work evaluating the impact of these policies. I build an overlapping generations, incomplete markets model with housing and defaultable recourse mortgages to (i) measure the impact of leverage and debt payment-to-income (PTI) requirements at loan origination on debt and default, and (ii) assess their welfare impact. I calibrate the model to Portugal, which enacted borrowing caps in 2018, and find that (i) the caps set reduce debt and default significantly (28% drop in total mortgage debt and virtual elimination of the default rate) and are welfare improving (1% in consumption equivalent variation), and (ii) the PTI cap in isolation improves welfare by more for the same reduction of debt and default. The key mechanism is that a lower default rate reduces spreads and frees up resources for consumption, which offsets the costs associated with constraining access to mortgages. PTI caps are more efficient as they are less restrictive than leverage caps when agents are young, but prevent mortgage refinancing for those experiencing a string of negative income shocks. The role of the rental market is central to these findings, as it allows credit-constrained agents to adjust their housing consumption without buying a house. This indicates that the extent of rental market regulation in each country is key to determine the impact of these policies on credit, housing markets, and welfare.

VERY PRELIMINARY - DO NOT CITE

Keywords: Mortgage, macroprudential policy, default, recourse, LTV, PTI, house prices.

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1 Introduction

Rapid household debt growth in advanced economies in the 1990s and in the 2000s was followed by a surge in default in the wake of the Global Financial Crisis, particularly in mortgage loan contracts. In response, policymakers introduced new financial sector regulations, which in many cases include caps on household leverage or debt service at loan origination. This tool of macro-prudential policy aims to contain household credit growth and default, prevent a feedback loop between credit provision and house prices, or curb the banking sector's exposure to real estate and credit risks, for example. By 2018, roughly 3/4 of European Union Member States had enacted some form of caps on consumer loan contracts ([ESRB, 2019](#)). In the U.S., the Dodd-Frank act established that lenders must be able to provide evidence that borrowers will be able to meet their scheduled debt payments when generating a high leverage loans, which [Defusco et al. \(2020\)](#) argue amounts to a tax on leverage.

Despite the prevalence in the adoption of these policies, which around 60 countries have introduced since 1990 ([Acharya et al., 2020](#)), there is little research on their effects. In the empirical literature, to take advantage of the recently implemented policies and the availability of loan-level data in some countries, a number of papers have been produced: [van Bakkum et al. \(2019\)](#) study the implementation of a loan-to-value (LTV) limit in the Netherlands in 2011; [Acharya et al. \(2020\)](#) investigate the LTV and loan-to-income caps (LTI) in Ireland in 2015; [Defusco et al. \(2020\)](#) study the "ability-to-pay" rule, enshrined in the Dodd-Frank act and which took effect in 2014. These papers broadly conclude that introducing borrowing caps in their various forms reduces household leverage and default, and lowers house price growth in the short run.

However, they have no means of answering a number of additional questions. First, what are the long-run effects of mortgage caps on debt and default? Second, how do they impact household welfare?

In order to answer these questions, a theoretical model is required. To the best of my knowledge, only the work by [Hatchondo et al. \(2015\)](#) discusses the issue of borrowing caps in a model with incomplete markets and detailed household mortgage decisions. However, their paper presents

a number of limitations: house prices are exogenous, rental markets are limited, and there is no study of borrowing caps other than LTV limits. This is the case because their focus is on the effects of introducing recourse mortgages in the U.S., rather than on household borrowing caps per se. While acknowledging that there might be good reasons to impose an LTV cap they find that doing so is welfare reducing in their model. Furthermore, the study of LTV limits separately from other caps may omit complementarity effects and the existence of an optimal combination of these policies. In practice, many countries that have indeed imposed a combination of caps rather than one in isolation, which raises the issue of whether there is an interior solution to the optimal policy mix.

Using a structural model of household mortgage default, I study the introduction of caps on residential mortgage origination in Portugal in 2018. The goals of this policy were (i) to reduce the stock of household debt, and (ii) to reduce default risk. I find that this will result in a reduction of 28% in mortgage debt, and virtual elimination of the mortgage default rate in the long run. This follows from the fact that the caps are a binding constraint for young agents, which form a large percentage of new mortgage loans. Thus, this paper contributes to the literature by providing a quantitative assessment of the long-run effectiveness of borrowing caps in achieving the goals set by policymakers.

I also estimate that the introduction of this policy improves welfare by 1% in consumption equivalent variation, which is generated by a reduction of interest rates due to a lower incentive to default. This is in contrast to the results obtained by [Hatchondo et al. \(2015\)](#). I show that a key modelling feature behind this difference in results is the ability to rent any house size that one can also buy. This draws on the insight by [Kaplan et al. \(2020\)](#) in the case of the relationship between credit supply and house prices. In a model where only the smallest house size is available for rental, as in the former framework, I find a small reduction of welfare, which would certainly be amplified once the transition to the new steady state were to be taken into account.

The structural model built to obtain these answers stems from the work of [Hatchondo et al. \(2015\)](#), [Favilukis et al. \(2017\)](#), and [Kaplan et al. \(2020\)](#) on housing, long-term mortgages, default,

and rental markets. The setting is an open economy with overlapping generations of households subject to uninsurable idiosyncratic risk in labour income. They consume non-durable goods and housing services, and save in a risk-free bond which pays a fixed interest rate set by international markets. Housing services may be obtained by either renting a house or purchasing one. Owning a house is attractive because it provides the owner with extra utility with respect to renting a house of the same size. Households may finance their house purchase using long-term defaultable recourse mortgages provided by competitive, financial intermediaries owned by risk-neutral foreigners. Mortgage origination caps prevent lenders from providing a given contract to a borrower if he does not satisfy either down payment requirements (the LTV cap) or a ratio of schedule debt payments to current after-tax labour income (the PTI cap). If a household chooses to default on its mortgage payments, the financial intermediary repossesses the house in order to pay for the outstanding debt. If the foreclosure is not sufficient to fully repay, the bank is able to extract recourse payments from the defaulting household each period until the debt is paid or the household obtains debt relief.

The rental sector is operated by competitive and risk-neutral foreigners. To show the importance of this market, in the baseline version of the model there are no *a priori* restrictions on the ability of the rental sector to provide any level of house quality also available for purchase by households. Finally, house prices are endogenous and are determined by supply and demand from households, the rental sector, and the addition of new units by a competitive construction sector.

The substantial drop in household debt follows from the fact that a large fraction of the aggregate mortgage credit flow is concentrated on young agents who start their life with meagre savings and therefore choose loan contracts with low down payments. Setting an LTV limit amounts to requiring a minimum down payment on new loans, forcing agents to either save up in order to meet the new requirements, to buy a smaller house or to rent one instead of becoming homeowners, thus restricting the flow of high leverage loans and the future stock of debt. The PTI limit also targets similar borrowers by restricting job market entrants, which have lower

wages on average, from choosing loans with high debt payments, which are correlated with high LTV.

The collapse of mortgage default is the result of architecture that leads to the default decision: It is only optimal for a household to default in order to obtain debt relief, i.e., to reduce the overall amount it must repay the bank. Otherwise, it is always better to sell the house, collect the equity, and rent. By requiring a sufficiently high minimum down payment, in the case of the LTV cap, the policy ensures that default is never optimal in the steady state, given that the bank is always able to recover its investment from selling the house, and that default is a costly decision for the homeowner. The PTI cap is less effective at reducing debt and default, but its positive impact on welfare is double that of the baseline policy. This is because PTI caps are less restrictive at the initial loan origination, but are binding when agents attempt to refinance their mortgages after a series of negative income shocks. Thus, agents which are likely to default are not able to increase their leverage, limiting the probability of default and, therefore, spreads.

The reduction in spreads is at the heart of the policy trade-off: on the one hand, restricting access to mortgages prevents a subset of agents from purchasing a house at a young age; on the other hand, by increasing down payments it reduces interest rates and creates savings for households down the road. I show that the latter effect predominates in a steady state comparison and that the rental market is key for this result, as it allows agents to adjust their consumption of housing services when prevented from accessing the housing market. If the rental market were modelled in the same way as in [Hatchondo et al. \(2015\)](#) or in [Corbae and Quintin \(2015\)](#), the conclusion would be reversed.

The rest of the paper is organized as follows. Section [2](#) describes the evolution of the Portuguese economy and, in particular, of household mortgage debt leading up to 2018, when borrowing caps were introduced by the central bank. Section [3](#) provides detail on the actual policy and looks at credit markets before and after the policy was announced and implemented. Section [4](#) describes the model. Section [5](#) describes the calibration. Section [6](#) presents the main exercises and discusses the results. Section [7](#) concludes.

2 The Portuguese financial crisis and mortgage debt

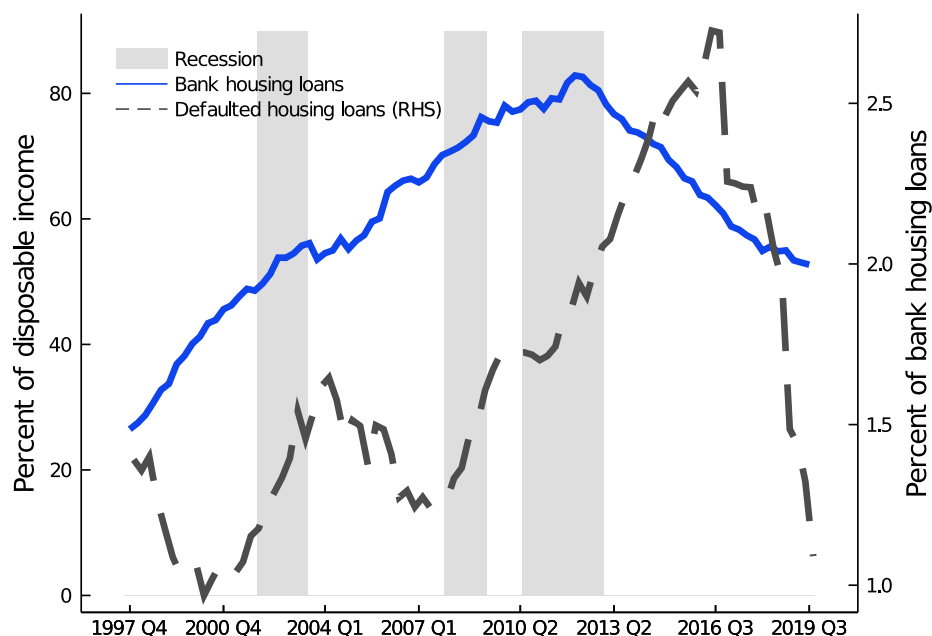
Following the pattern in many advanced economies, especially in the euro area, the Portuguese economy underwent a period characterized by soaring debt accumulation, from the 1990s up until the early 2010s.

Roughly two years after the onset of the Global Financial Crisis, [Banco de Portugal \(2010\)](#) observed that the ratio of total household debt to disposable income in Portugal had increased from 22% in 1990 to 134% in 2009, a “relatively high level, by international standards”. It attributed this development partly to greater access to credit markets, following deregulation, rapid technological change, and declining interest rates resulting from Portugal’s membership in the euro area. It also noted that, like other euro area economies, the country was currently facing “significant and unforeseen increases in deficits and public debt”. By this time, Portuguese 10 year government bonds yields had already breached a 2 percentage point premium with respect to German 10 year government bonds, prefacing a rating cut by Moody’s during that summer.

One year later, facing rising risk premia on its debt and the prospect of bankruptcy, the Portuguese government requested financial assistance from the International Monetary Fund (IMF) and the European Financial Stability Facility. The main features of this period for euro area economies are amply discussed and detailed in previous works such as [Lane \(2012\)](#), for the euro area in general, and [Reis \(2013\)](#), for the case of the Portuguese economy. One salient pattern is the capital inflow to economies like Portugal and Ireland to finance household debt growth and, in particular, mortgage debt, which forms the bedrock of the analysis conducted in this paper.

Figure 1 plots the boom and bust of credit for house purchase in Portugal in the last two decades. The solid line indicates the stock of loans for house purchase granted by banks as a percentage of disposable income, while the dashed line corresponds to the fraction of the stock of loans for house purchase which is overdue. Between 1997 Q4 and 2012 Q1, the stock of loans for house purchased more than trebled, surging from 27% to a peak 83% of disposable income.

The upshot of this mortgage debt accumulation was a persistent growth in rate of defaulted loans. After a relatively modest increase from around 1% of the stock of housing loans in 2000



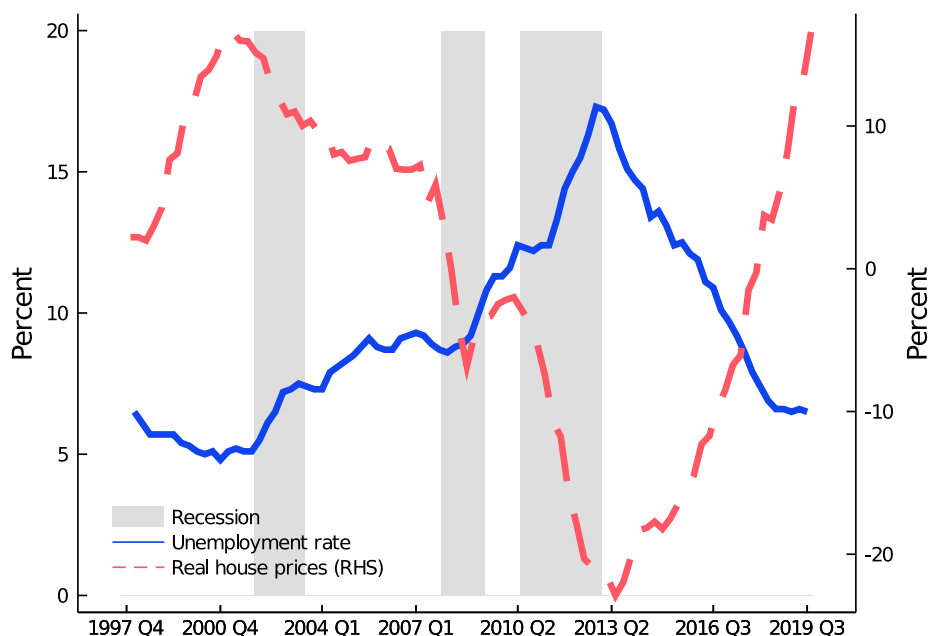
Note: Recessions dates for the Portuguese economy at a quarterly frequency are identified by [Rua \(2017\)](#). Housing debt and default data are from the Bank of Portugal. Disposable income data are from Statistics Portugal and are annualized. Data sources are provided in Appendix [A](#).

Figure 1: Mortgage debt and default

Q1 to 1.5% in 2004 Q1, it declined briefly until early 2007. However, starting in 2008 Q1, the Portuguese economy was hit by two contractions in economic activity in quick succession. The rate of defaulted loans climbed rapidly from 1.3% in 2008 Q1 and reached a peak of 2.7% in 2016 Q3, dropping sharply with the rebound in house prices and economic activity.

The surge in household default rates, which affected all types of consumer loans, contributed to the growing woes in the banking system, which was heavily exposed to the household sector, as recognized by [Banco de Portugal \(2017\)](#). By June 2012, three of the largest banks in the system had to be recapitalized with funds from both the financial assistance package and the general budget ([Eichenbaum et al., 2016](#)). In 2014 and 2015, two additional banks were intervened, including the now defunct BES. [Tribunal de Contas \(2018\)](#), a report by the Portuguese Court of Auditors, estimated that between 2008 and 2017 net public expenses with the financial sector amounted to €16.8 billion, or 8.6% of GDP in 2017.

This evolution in credit markets and the financial sector was mirrored by developments in



Note: The real house prices series is the ratio between the index of residential house prices and a consumer price index expressed as percentage deviations from its full-sample historical mean. The unemployment rate is from Statistics Portugal. The consumer price index and the house price data are from the Organization for Economic Cooperation and Development (OECD). Data sources are provided in Appendix A.

Figure 2: Unemployment and house prices

the labour and housing markets. Figure 2 plots labour market and house price dynamics in the Portuguese economy. The solid line indicates the unemployment rate, while the dashed line corresponds to the deviation of real house prices from their historical mean. Unemployment rose from roughly 5% in 2000 Q1, spurred by multiple recessions, to more than 17% in 2012 Q4, tapering off afterwards as GDP growth picked up. In the residential housing market, prices rose sharply between 1997 Q4 and 2000 Q4, and slowly declined until the end of 2007. Between 2007 Q4 and 2013 Q2, real house prices tumbled by almost 30 percentage points in relative deviations from the mean, in tandem with the rising unemployment rate and the mortgage default rate.

The mechanisms underlying these dynamics have been explored in the works of Corbae and Quintin (2015), Mian and Sufi (2015), and Kaplan et al. (2020) for the U.S., for example. While there are significant disagreements regarding the linkage between credit supply and house prices, the role of high leverage roles and debt service burdens generated by loose credit standards are deemed to be at the heart of foreclosure crises, such as the one that occurred in the Portuguese

economy. Among other reasons, the desire to prevent another such episode and protect the financial system from a wave of defaults in the household sector in the future prompted the macroprudential authority to introduce the borrower-based policy described in the next section.

3 Policy overview

On February 1st 2018, the Bank of Portugal, as Portugal's macroprudential authority, issued a recommendation to all financial institutions granting consumer credit in Portugal regarding the minimum criteria that should be used in assessing borrower solvency at loan origination.¹ These criteria should be applied to all new loan contracts for house purchase, mortgages, and loans for consumption purposes signed by financial institutions operating in national territory. A short summary of the main elements of these criteria follows:

1. **Loan-to-value cap.** For all consumer loan contracts with a real estate guarantee whose purpose is to acquire the borrower's main residence, the ratio between the value of all of the borrower's outstanding loans and the value of the collateral should not exceed 90%. When the loan is granted for purposes other than acquiring a main residence, this cap drops to 80%. If the dwelling being used as collateral has been sold by a financial institution, the LTV cap is 100%. The value of the collateral is calculated as the minimum between the appraisal value and the transaction price.
2. **Payment-to-income cap.** For all consumer loan contracts, the ratio between all of the borrower's monthly debt payments and labor income net of taxes and social security contributions should not exceed 50%. For floating or mixed rate contracts, an interest rate increase of 3 p.p. should be assumed in the monthly payments when calculating this measure. Each financial institution may grant up to 10% of new loans with a PTI from 50% to 60%, and up to 5% higher than 60%.

¹The summary, in English, of the policy measure is provided by the Bank of Portugal [here](#). The legal text, in English, is available [here](#).

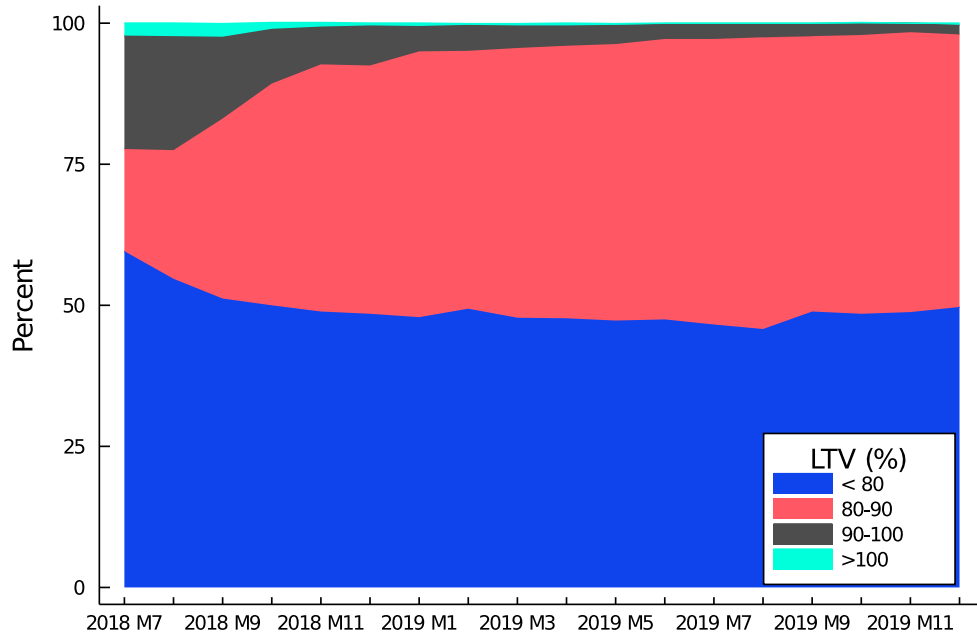
3. **Maturity cap.** The maturity on loans for house purchase and mortgages should not exceed 40 years. The average maturity of new credit agreements should gradually converge to 30 years until the end of 2022.
4. **Regular payments requirement.** New credit agreements should be granted with regular payments of interest and principal.

The regulation is not intended to be binding in the sense that the limits described are not legally enforceable. However, any financial institution not complying with these limits must provide an explanation to the central bank, detailing its reasons for not doing so. The supervisor then evaluates the justification provided by the institution and may take further action if it deems it inadequate.

The caps went into effect on July 1st 2018. The goals of this policy, as spelled out in its summary, are to “*enhance the resilience of the financial sector and the sustainability of households’ financing, thereby, minimizing defaults.*” I interpret this to mean two things in practice: (i) to reduce the stock of high leverage, high debt service loans, and (ii) to reduce default rates.

Approach The level of detail involved in the policy poses a number of tractability challenges. In order to allow for its analysis in a theoretical model, I will need to reduce the scope of the evaluation and make a number of simplifying assumptions, for the moment. First, I will narrow the focus on the cost/benefit analysis for consumers, without taking into account the effects of the policy on bank balance sheets, except by discussing the effects of the policy on the stock of high leverage loans or their default rate. Second, I will assume the existence of a single type of mortgage debt contract available to consumers, i.e., a 40 year contract with regular interest and principal payments. As there are no changes in the level of reference rates in the environment, there is no difference between fixed or floating rate mortgages.² Third, I will not model loans for consumption or other purposes, and focus instead on the effects that the new criteria have on

²This assumption appears less problematic if one takes into account that euro area yield curves indicate that 3-month interest rates are expected to rise from -0.74% to -0.15% in 30 years. Data with reference date of December 29th 2020 from the European Central Bank (ECB) euro area yield curve statistics, available [here](#).



Note: Percentage of new loans for house purchase by LTV interval. LTV is the ratio of total consumer loans to the value of collateral at origination. Data are from the Bank of Portugal. Data sources are provided in Appendix A.

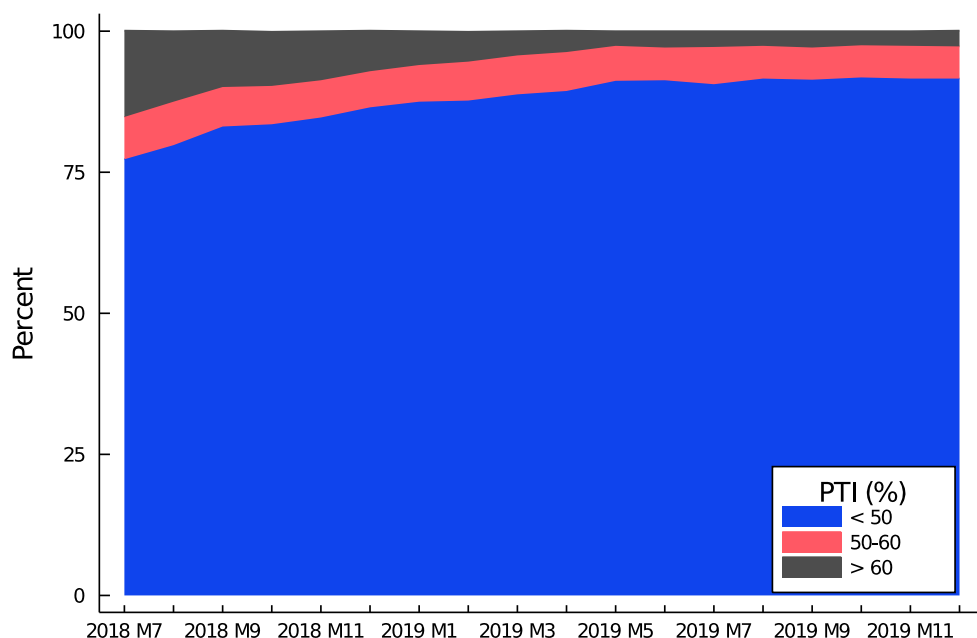
Figure 3: LTV distribution of new loans for house purchase

mortgage loans exclusively. Fourth, I will ignore the exceptions to the PTI cap. Finally, I will not yet take into account the maturity caps.

Impact Figure 3 plots monthly data on the distribution of LTVs of new mortgage loans from the implementation month, July 2018, onward. For each month, the figure indicates the fraction of loans in four different bins, where the bottom bin indicates loans with an LTV lower than 80%, and proceeds in ascending order.

Although the policy had been announced four months earlier, at the end of the first month after implementation 22% of the volume of new loans for house purchase had LTVs above the main cap.³ Despite this, that fraction steadily converged to 2% by the start of 2019. Loans with an LTV between 80% and 90% fully soaked up the weight previously taken up by loans with LTV greater than 90%, indicating a bunching up against the cap, as well as a further 10% from

³Banco de Portugal (2019) attributes this observation to the delay between the reference date of the credit risk analysis performed by the institution, months before the implementation date, and closing the contract, which is the reference date for regulatory reporting purposes.



Note: Percentage of new loans for house purchase by PTI interval. PTI is the ratio of total debt payments to after tax labour income at origination. Data are from the Bank of Portugal. Data sources are provided in Appendix A.

Figure 4: PTI distribution of new loans for house purchase

the lower than 80% category. In the final tally, the 80-90 category more than doubled in relative terms, rising from 18% in July 2018 to 48% by December 2019.

Figure 4 plots monthly data on the distribution of PTIs of new mortgage loans from the implementation month onward. For each month, the figure indicates the fraction of loans in four different bins, where the bottom bin indicates loans with a PTI lower than 50%, and proceeds in ascending order.

A similar pattern to the LTV distribution is observed in this figure, with 22% of the volume of new loans with PTI values above the reference cap in the first month of implementation. The share with a PTI below 50% gradually increased from 77% in July 2018 to 92% by December 2019.

In summary, financial institutions appear to be broadly complying with the recommended limits to mortgage lending standards, albeit with a period of adjustment. This suggests that the caps are binding for a significant fraction of borrowers. The next section proposes a framework to analyze the impact of this policy in terms of its goals, as I have interpreted them, as well as its

implications for consumer welfare.

4 Model

Overview

I build an incomplete markets model with overlapping generations of agents with partially uninsurable idiosyncratic risk to their labor efficiency units endowment. Households consume non-durable goods and housing. They can save in a risk-free annuity with exogenous return, and in housing, an illiquid asset requiring periodic maintenance which also provides the owner with housing services. After retirement, endowment shocks cease, individuals have a positive probability of dying, and earn income from social security and from their financial savings.

Housing services can be obtained by either buying or renting a house. Homeownership is attractive compared to renting because it provides extra utility to homeowners for the same house size. This is a reduced form method to encompass the many incentives to homeownership relative to renting.⁴ The illiquidity of houses is modelled by adding a transaction cost proportional to the sale value.

To finance their house purchase, agents may borrow using defaultable mortgages with a long-term maturity and with recourse in case of default. These contracts are signed with competitive financial intermediaries which aim to break-even, in expectation, on each contract.

Houses are transacted in a market which includes households, the rental sector, financial intermediaries selling foreclosed properties and a construction sector. Thus, house prices are endogenous, as in [Favilukis et al. \(2017\)](#) and [Kaplan et al. \(2020\)](#). A key ingredient in this model is the fact that I put no *a priori* restrictions on the behaviour of the rental sector, in the spirit of

⁴Homeownership in Portugal in 2017, the year before the implementation of the policy, was 75% compared to a 66% average in the euro area (data from Eurostat, see [Appendix A](#)) and to a value of 65% in 1991 ([Cardoso et al., 2019](#)). In Portugal, measures fostering homeownership have changed across time and included: (i) mortgage payment tax deductions for contracts signed before 2012; (ii) a temporary exemption from property tax in the three years after buying the household permanent residence and a permanently one for low income families; (iii) grants for purchase or construction of houses by low income, low asset families. In addition, the rental market in Portugal is often subject to suite of regulations such as rent freezes and eviction protection for individuals over 65 and low income families.

Kaplan et al. (2020). The implication of this is that any house which can be owned, can also be rented. This is in contrast to the work of Corbae and Quintin (2015) and Hatchondo et al. (2015), and both the predictions in terms of whether the policy achieves its goals and the welfare analysis will be greatly affected by this assumption.

In this section, I will present the main characteristics of the problems of each sector. A detailed description of the household problem, the equilibrium definition, and the algorithm to compute it are presented in Appendix B, C, and D, respectively.

4.1 Households

Demographics Time is discrete. The economy is populated by a continuum of finitely-lived households of measure one. Age is given by $j = 1, \dots, J$, and households have a positive probability S_j of dying after retirement at J_{ret} . Households will die with certainty after age J . In the description below, I omit the subscript j as a general rule, except in cases where the meaning of an expression might become unclear from doing so.

Preferences Expected lifetime utility of the households is given by:

$$\mathbb{E}_0 \left[\sum_{j=1}^J \beta^{j-1} [S_j u_j(c_j, s_j) + (1 - S_j) v(b)] \right], \quad (1)$$

where β is the discount factor, c is consumption of non-durable goods and services, s is the consumption of housing services, and $S_j = 1$ for $j < J_{\text{ret}}$. The expectation is taken over a sequence of idiosyncratic shocks to the labor productivity endowment. v measures the utility from leaving a bequest b .

The utility function, u_j , is given by:

$$u_j = \frac{e_j [(1 - \phi) c_j^{1-\gamma} + \phi s_j^{1-\gamma}]^{\frac{1-\vartheta}{1-\gamma}} - 1}{1 - \vartheta}, \quad (2)$$

where ϕ measures the relative taste for housing services, $1/\gamma$ is the elasticity of substitution be-

tween housing services and non-durable consumption, and $1/\vartheta$ is the intertemporal elasticity of substitution. e_j is an exogenous consumption equivalence scale which captures changes in household size and composition over time. I assume that the utility derived from bequests is given by:

$$v(b) = v \frac{(b - \underline{b})^{1-\vartheta} - 1}{1 - \vartheta}, \quad (3)$$

where v measures the strength of the bequest motive, and \underline{b} reflects the extent to which bequests are luxury goods as in [De Nardi \(2004\)](#).

Endowment Active households have a labor productivity endowment y_j given by:

$$\ln y_j^w = \ln \Theta + a + \chi_j + \epsilon_j, \quad (4)$$

where Θ is aggregate productivity, a is an individual-specific permanent income component, χ_j is the deterministic age profile of productivity, and ϵ_j is a persistent component following a first-order Markov process. The amount received by households after social security payments by the employer, τ_{ss} , and by the employee, τ_{sse} , is given by:

$$y_j = y_j^w \left(\frac{1}{1 + \tau_{ss}} - \tau_{sse} \right)$$

Liquid bond Households can save in a one-period risk-free bond, $b > 0$, with an exogenous fixed price q_b and implied interest rate $r_b = 1/q_b - 1$. The implicit assumption is that the agents operate in a small open economy, and risk-free interest rates are determined in the world market.

Housing A house is an asset which can be bought or rented by households in order to consume housing services. Housing characteristics are summarized by a single index, which can be thought of as quality, and encompasses a number of characteristics ranging from location, to size. Implicitly, I assume households use a common valuation scale of these characteristics.

Owner-occupied housing quality is given by h , where $h \in \mathcal{H} = \{h^0, \dots, h^N\}$ and $h^0 < h^1, \dots, h^{N-1} < h^N$. Rentals are denoted by $\tilde{h} \in \tilde{\mathcal{H}} = \{\tilde{h}^0, \dots, \tilde{h}^{\tilde{N}}\}$ and $\tilde{h}^0 < \tilde{h}^1, \dots, \tilde{h}^{\tilde{N}-1} < \tilde{h}^{\tilde{N}}$. The unit price of housing is denoted by p_h , and the rental rate by ρ . I assume that the housing market is frictionless and competitive. Implicitly, I am making two important assumptions: (i) for housing markets to clear, it is enough that demand and supply of housing units be equal, e.g., higher quality houses can be subdivided and converted into smaller quality houses at no cost and then sold; (ii) both the rental sector and the construction sector will have no cost of adjusting supply. Both of these assumptions are less problematic in a setting where I am comparing steady state outcomes.

Housing services are directly proportional to housing quality, i.e., $s = h$. If an agent owns a house, his housing consumption is given by $s = \omega h$. Every period, homeowners will have to pay a maintenance and taxes on property given by:

$$(\delta_h + \tau_h)p_h h, \quad (5)$$

where δ_h is the depreciation rate, τ_h is the property tax. To model housing as an illiquid asset, I assume that owners selling their house incur a transaction cost $\kappa_h p_h h$.

Mortgages Households can take out mortgages to finance house purchase. Mortgages are liabilities with maturity until J and with the option to default. To obtain a mortgage, agents must pay a fixed origination fee κ^m , after which they receive a transfer of funds qm' , where q is the individual-specific price of the mortgage and m' is the mortgage balance, which includes both the principal and the implicit spreads. Financial intermediaries lend funds at reference rate $r_m = r_b(1 + \iota)$, where ι is an intermediation wedge. Since there are no aggregate shocks to the interest rate in this model, there is no distinction between fixed and adjustable rate mortgages. The mortgage pricing function, $q \leq 1$, is the instrument that financial intermediaries use to set spreads. It depends on all available characteristics of the borrower: age, j , next period assets and liabilities, $\mathbf{x}' := (b', h', m')$, and the elements of the labor productivity endowment process, $\mathbf{y} = (a, \epsilon)$. The down payment made by the households in terms of non-durable consumption

goods is thus $p_h h' - m' q_j(\mathbf{x}', y)$.

At origination, loan contracts must comply with two constraints: (i) a cap on LTV and (ii) a cap on PTI. In the case of the LTV, the funds transferred must not exceed a fraction λ^m of the value of the collateral:

$$qm' \leq \lambda^m p_h h'. \quad (6)$$

In the case of the PTI cap, the scheduled mortgage payment must not exceed a fraction λ^π of after tax labor income $y_j^w - \mathcal{T}_j$:

$$\pi_j^{min}(m) \leq \lambda^\pi [y_j^w - \mathcal{T}_j], \quad (7)$$

where \mathcal{T} denotes tax liabilities and where $\pi_j^{min}(m)$ is given by:

$$\pi_j^{min}(m) = m \frac{r_m(1 + r_m)^{J-j}}{(1 + r_m)^{J-j} - 1}, \quad (8)$$

the formula for the constant payment of an annuity with outstanding balance m and where the first payment is due next period. The borrower may choose to pre-pay his mortgage by choosing $\pi > \pi_j^{min}(m)$. The outstanding balance evolves according to the motion equation $m' = m(1 + r_m) - \pi$. At any moment, the borrower may choose to refinance his mortgage, subject to paying the constant origination cost κ_m and complying with the LTV and PTI caps. He may also choose to sell the house, at which point he must pay the outstanding principal and interest $m(1 + r_m)$.

If the agent defaults, the financial intermediary repossesses the house and sells it at $(1 - \delta_h^d - \tau_h - \kappa_h)p_h h$, where $\delta_h^d > \delta_h$. The depreciation rate of the sale by the financial intermediary is higher to account for expenses associated with foreclosure and eviction processes. If the residual equity is positive after the sale, the household is paid back the difference. If the proceeds from the foreclosure are not enough to pay for the outstanding debt, the households is subject to a recourse payment $\Phi = \min(\kappa_d[y_j^w - \mathcal{T} + b], m^D)$, where κ_d is an attachment limit on cash-on-hand

and m^D denotes the residual value of debt after foreclosure. In the subsequent periods, the agent may exit his defaulter status randomly or by means of paying the outstanding mortgage balance. This formulation implies that the reason why agents default in this model is to obtain debt relief.

4.2 Financial intermediaries

Financial intermediaries are owned by risk-neutral foreigners and operate in a competitive environment. The risk-neutrality assumptions implies that the condition for pricing mortgage contracts is to break-even in expectation. This allows me to more easily price loan contracts through a zero-profit condition. I am also assuming that the caps imposed on loan contracts cannot be avoided by asking for loans in a different country.

Let $g_j^n(\mathbf{x}, y)$, $g_j^f(\mathbf{x}, y)$, and $g_j^d(\mathbf{x}, y)$ denote mutually exclusive indicators for the decisions to sell, refinance (or change house size), and default, respectively. The price of a mortgage is expressed recursively as:

$$q_j(\mathbf{x}', y) = \frac{1}{(1 + r_m)m'} \mathbb{E}_\epsilon \{ q_{\text{sell}} + q_{\text{default}} + q_{\text{pay}} \}, \quad (9)$$

where

$$q_{\text{sell}} = \left[g_j^n + g_j^f \right] (1 + r_m)m' \quad (10)$$

$$q_{\text{pay}} = \left[1 - g_j^n - g_j^f - g_j^d \right] \cdot (\pi + q_{j+1}(\mathbf{x}'', \mathbf{y}'')[(1 + r_m)m' - \pi]) \quad (11)$$

$$q_{\text{default}} = g_j^d \cdot \min\{(1 - \delta_h^d - \tau_h - \kappa_h)p_h h', m'(1 + r_m)\} + \min\{\kappa_d(b + y_j^w - \mathcal{T}), \Omega_0\}. \quad (12)$$

The mortgage pricing function is solved as in [Chatterjee and Eyigungor \(2012\)](#).

4.3 Rental sector

The housing stock in the rental sector is owned by risk-neutral foreigners, who buy houses in the housing market and rent them to tenants. The reaction function relates the price in the housing market to the rental price:

$$\rho = \psi + p_h - \frac{1 - \delta_h - \tau_h}{1 + r_b} p_h, \quad (13)$$

where ψ are management costs per unit of housing rented, and I assume that the rental sector discounts the future at the exogenous interest rate r_b . I am also assuming that there are no adjustment costs and that the rental sector is able to sell houses without incurring in transaction costs.

4.4 Consumption goods sector

The consumption goods sector is competitive and has a constant returns to scale technology:

$$Y_c = \Theta N_c, \quad (14)$$

where N_c are labor productivity units. The equilibrium wage rate per efficiency unit is therefore equal to Θ .

4.5 Construction sector

The construction sector is competitive and has a Cobb-Douglas production technology:

$$I_h = (\Theta N_h)^\alpha \bar{L}^{(1-\alpha)}, \quad (15)$$

where N_h is the labor input in productivity units, and \bar{L} is a fixed per-period flow of government construction permits, which are sold in a competitive market to developers. The developer

problem is given by:

$$\max_{N_h} p_h I_h - w N_h \quad (16)$$

$$s.t. : I_h = (\Theta N_h)^\alpha \bar{L}^{(1-\alpha)}, \quad (17)$$

where w is the wage rate. I assume that wage is perfectly mobile across sectors, which allows me to write $w = \Theta$ and obtain the following equation for housing investment as a function of house prices only:

$$I_h = [\alpha p_h]^{-\frac{\alpha}{1-\alpha}} \bar{L}.$$

Finally, the total labor productivity input is normalized to unity across production sectors (i.e., $N_c + N_h = 1$).

4.6 Government

The government spends funds on wasteful government consumption, G , and manages a pay-as-you-go social security scheme. It collects revenues by taxing housing at a fixed rate τ_h , taxing labour income using a progressive income tax function and by issuing a period flow of new construction permits. Social security is fully financed through employee and employer taxes on wages.

The functional form for labour income taxation is the one in [Benabou \(2002\)](#), where the tax liability is given by:

$$\mathcal{T}_j = y_j - \tau_y^0 (y_j^w - \rho \tilde{h}_j)^{1-\tau_y^1} \quad (18)$$

where τ_y^0 measures the tax level and τ_y^1 the progressivity of the labour income tax schedule. Rents are tax-deductible and therefore are subject to deduction from the tax base. τ_y^0 is set such that the

government constraint clears in the steady state.

Social security income of retirees is constant for the remainder of their lives and given by:

$$y_{ret} = \rho_{ss} y_{J_{ret}-1}, \quad (19)$$

where ρ_{ss} is the gross average replacement rate, and $y_{J_{ret}-1}$ is labour income before retirement.

5 Calibration

I calibrate the model to the Portuguese economy in 2017, just before the policy is announced. Each model period is two years. Table 1 shows the parameters set externally. Table 2 shows the parameters calibrated by SMM.

Description	Parameter	Value	Source
Demographics			
Survival probability by age	S_j	-	Statistics Portugal
Consumption equivalence scale	e_j	-	HFCS
Preferences			
EOS of housing and non-durable consumption	$1/\gamma$	0.5	Hatchondo et al. (2015)
Risk aversion	ϑ	2	Kaplan et al. (2020)
Productivity			
Aggregate productivity	Θ	1	Assumption
Life cycle profile of earnings	χ_j	-	Brinca et al. (2017)
Auto-correlation of earnings	ρ_ϵ	0.335	Brinca et al. (2017)
Persistence idiosyncratic risk	σ_ϵ	0.439	Brinca et al. (2017)
Housing			
Depreciation rate	δ_h	0.045	Statistics Portugal
Transaction cost	κ_h	0.07	Kaplan et al. (2020)
Elasticity of construction to labor	α	0.6	Kaplan et al. (2020)
Financial instruments			
Risk-free interest rate	r_b	0.007	Bank of Portugal
Origination cost	κ_m	0.21	500€ in the model
Government and SS			
Property tax	τ_h	0.009	Portuguese tax authority
Progressivity parameter	τ_1^y	0.136	Brinca et al (2017)
Government consumption to GDP	G	0.17	Statistics Portugal
SS tax employees	τ_{sse}	0.11	Social Security
SS tax employers	τ_{ss}	0.238	Social Security
Gross average replacement rate	ρ_{ss}	54.7	OECD

Table 1: External calibration summary

Description	Parameter	Value	Target	Model	Data
Discount factor	β	0.99	NW to employee compensation	-0.23	2.9
Bequest motive strength	ν	1.2	Ratio of net worth at 75/50	2	0.914
S.D. of permanent component	σ_a	0.423	S.D. log of household earnings	0.824	0.824
Owner housing grid	\mathcal{H}	-	Housing NW/ NW		
			p10	0.22	0.25
			p50	0.73	0.75
			p90	0.99	0.99
Foreclosure loss	κ_d	0.205	Foreclosure depreciation rate	0.25	0.25
Extra utility	ω	1.06	Homeownership rate	0.63	0.75
Management costs	ψ	0.008	Homeownership < 35	0.10	0.42
Land permits	\bar{L}	0.2	Housing investment / GDP	0.07	0.28
Attachment limit	κ_d	0.3	Foreclosure rate	0.009	0.005
Lowest rental quality	\tilde{h}_1	0.17	Frac. gov housing property	0.05	0.064

Table 2: Parameters calibrated by SMM

6 Quantitative analysis

In this section, I use the model to predict the long-run impact of the policy and to understand the role of LTV and PTI caps individually and in combination. The main experiment conducted in this analysis is a simultaneous tightening of both LTV and PTI caps, corresponding to the levels set in the policy enacted by the Bank of Portugal:

$$\lambda^m : 1.0 \longrightarrow 0.9$$

$$\lambda^\pi : 1.0 \longrightarrow 0.5$$

Results are reported in Table 3. Compared to the baseline, the economy with mortgage borrowing caps has a 30 percent reduction in mortgage debt to GDP, and the foreclosure rate is cut to nearly zero. The enacted policy improves welfare by 1% in consumption equivalent variation for unborn individuals, which results from a reduction of spreads for every chosen level of leverage. The key mechanism at play is that the gains from savings in reduced debt payments more than compensate for the additional down payment requirements imposed on households when financing house purchase.

Homeownership drops by 8 p.p. with respect to the baseline economy, but the existence of

	Baseline (pre-policy)	90% LTV 50% PTI	90% LTV 100% PTI	100% LTV 50% PTI
Mortgage debt to GDP	46	33	35	41
Foreclosure rate	0.94	0.03	0.00	0.70
Homeownership	64	56	55	60
House prices	1.00	1.00	1.00	1.00
Consumption equivalent variation (%)	-	1.0	1.6	1.8

Note: All statistics are in percentage terms, except house prices which are normalized to 1 in the baseline economy.

Table 3: Long-run impact of mortgage borrowing caps.

a rental market for all house sizes allows households to increase their housing consumption as they save up to put afford a down payment.

What is the the effect of imposing only the LTV cap? This instrument is able to generate 2/3 of the drop observed in the previous experiment. It also reduces the default rate to zero. The reason why can be understood by examining the options and payoffs available to a homeowner with not enough cash-on-hand to pay the mortgage installment:

1. **Sell house.** The agent can sell his house, collecting the residual equity, given by $(1 - \delta_h - \tau_h - \kappa_h)p_h h - (1 + r_m)m$, and either buying or renting another;
2. **Refinance the mortgage.** The agent can refinance the mortgage, increasing his existing leverage, having to pay the mortgage origination cost and a higher interest rate than before, given the larger exposure and the increased risk of default;
3. **Default.** The agent agent can default, in which case he will receive the residual equity value of the house after the bank repossesses it, $(1 - \delta_h^d - \tau_h)p_h h - (1 + r_m)m$. If equity is negative, i.e., house foreclosure is insufficient to pay the amount owed to the financial intermediary, the agent will be subject to a recourse payment on his cash-on-hand.

Default will only be attractive relative to selling the house if equity is negative and if residual

debt after default is higher than the required recourse payments. In other words, the value of the default option is in obtaining debt relief. Therefore, targeting leverage directly is very effective at curbing future default.

In contrast, setting the PTI in isolation reduces debt by only 1/2 of the drop implied by the main policy. The reduction in default is also much lower. The reason for this is that income at origination is an imperfect measure of defaults that may occur many periods ahead. This results from the specification of the productivity endowment process: in complying with the PTI limit, there is no distinction between the sources of labor income, i.e., whether observed income is the result of a high persistent component or low labor market experience, which are key in determining probability of default. However, the PTI cap is still successful in reducing total debt, mostly due to the fact that income is lower at the start of life, when the agents can only afford a smaller down payment. This prompts them to save up in the mean time and obtain mortgages later with higher down payments than would otherwise be the case. In spite of this, capping the PTI achieves the highest welfare improvement of all three experiments. This indicates that there is an optimal combination of LTV/PTI caps and that the LTV cap, even though it is able to achieve the main goals of the policy to a larger extent, is too restrictive.

The effects of the same policy with a housing market structure à la [Hatchondo et al. \(2015\)](#) are shown in Table 4. The drop in mortgage debt is only half of the drop observed in the unrestricted model, due to the fact that agents must own a house in order to increase their housing consumption services and are, therefore, more willing to take on additional leverage and higher debt payments to do so. In contrast to the unrestricted model, the same policy produces a much lower welfare gain and even a small cost in the case of the main experiment. However, some of the conclusions hold: the LTV cap has a greater impact on debt and default, and the PTI cap in isolation is able to generate a greater welfare improvement.

The conclusion from this exercise is that the modeling of the rental market is key. In its absence, one would have concluded that the impacts on debt would be half of what they are when one does not impose a-priori restrictions on the rental sector. The same is true of the welfare

	Baseline (pre-policy)	90% LTV 50% PTI	90% LTV 100% PTI	100% LTV 50% PTI
Mortgage debt to GDP	46	40	40	45
Foreclosure rate	0.94	0.00	0.00	0.49
Homeownership	68	66	66	67
House prices	1.00	1.00	1.00	1.00
Consumption equivalent variation (%)	-	-0.001	0.04	0.3

Table 4: Long-run impact of mortgage borrowing caps with a restricted rental market. All statistics are in percentage terms, except house prices which are normalized to 1 in the baseline economy.

analysis: welfare gains are significantly lower and even negative. Had I taken into account the transition to the new steady state, these policies would have a greater chance of being welfare reducing.

7 Conclusion and next steps

In this paper, I investigate the impact of mortgage borrowing caps on debt, default, and welfare. To illustrate the effects of this policy, I calibrated a structural model of mortgage debt and default to the Portuguese economy, where the central bank enacted loan-to-value and payment-to-income limits in 2018. I find that in combination, loan-to-value and payment-to-income caps are effective at reducing debt and default, and improve welfare by 1% in consumption equivalent variation. The central trade-off at play in this policy is restricting access to housing services by constraining access to debt contracts while at the same time compressing spreads by reducing the incentive to default.

A key modelling feature is the existence of an unrestricted rental market for housing services, which dampens the impact of the policy on overall mortgage debt and default. It also makes the baseline policy welfare improving when compared to the estimates obtained by using a simpler

structure of the rental market as employed by many previous papers.

The LTV cap is more successful at reducing debt and default because it targets leverage directly and residual home equity is key to the default decision. In contrast, the PTI cap is less restrictive of debt and default, but it leads to almost double the welfare improvement with respect to the baseline policy. This indicates that there is an interior solution to the optimal policy problem.

The next steps in this paper will be to: (i) analyse how households adjust to the new policies; (ii) flesh out the mechanisms by which caps improve welfare; (iii) find the optimal policy combination; (iii) investigate alternative policy instruments, such as LTIs and a tax on leverage; (iv) take into account the effect of transitions; (v) introduce a fragmented housing market, in order to assess if the same conclusions still hold.

Several future strands of research are worth pursuing: First, it would be useful to model the balance sheet of financial intermediaries to understand the effects of their adjustment after the introduction of mortgage borrowing caps. Second, I implicitly assume that all financial intermediaries are alike and owned by foreigners, making no provision for the fact that mortgage default may lead lenders like depository institutions to default and create additional costs for the economy. Finally, households are also able to take out unsecured loans for consumption purposes, which are also affected and targeted by borrowing caps but which are not covered in this work.

APPENDIX

The Appendix is organized as follows. Section [A](#) indicates data definitions and sources. Section [B](#) describes the household problems. Section [C](#) describes the equilibrium concept. Section [D](#) outlines the algorithms for the computation of the equilibrium.

A Data appendix

Bank housing loans: Stock of loans of other monetary financial institutions (OMFIs) to private individuals for housing purposes (all maturities).⁵ *Bank of Portugal, Monetary and Financial Statistics, B.4.1.4, September 2020.*

Disposable income: Seasonally adjusted nominal net disposable income at quarterly frequency, annualized. *Statistics Portugal, National Accounts, Table A.2.2, January 2020.*

Defaulted housing loans: Stock of OMFI loans to private individuals for house purchase which are overdue. *Bank of Portugal, Monetary and Financial Statistics, Table B.4.1.4, September 2020.*

Unemployment rate: Seasonally adjusted unemployment rate for population aged between 15 and 74 years old (2011 series). *Statistics Portugal, Labour Force Survey, January 2020.*

House price index: Seasonally adjusted index of residential house prices. *OECD, Analytical House Price Indicators, variable hpi, February 2021.*

Consumer price index: Seasonally consumer price index (all items). *OECD, consumer price indices, variable cpaltt01.ixob.q, February 2021.*

⁵Monetary financial institutions (MFIs) are resident credit institutions as defined in European Union law, and other resident financial institutions whose business is to receive deposits or close substitutes for deposits from entities other than MFIs and, for their own account (at least in economic terms), to grant credits or make investments in securities. Definition provided by the European Central Bank. The OMFI are all MFI except central banks.

LTV of new loans for house purchase: Percentage of new loans for house purchase by interval of LTV at origination. Data from [Banco de Portugal \(2020\)](#). *Bank of Portugal, August 2020*.

PTI of new loans for house purchase: Percentage of new loans for house purchase by interval of PTI at origination. Data from [Banco de Portugal \(2020\)](#). *Bank of Portugal, August 2020*.

Homeownership rate: Percentage of the population owning their primary residence in 2017. *Eurostat, Statistics on Income and Living Conditions, variable ilc_lvho02, February 2021*.

Financial net worth: Financial of net worth of households and non-profit institutions serving households (NPISH). Data for 2017. Consolidated values. *Banco de Portugal, National Financial Accounts, Table F.2.1.4, August 2020*.

Employee compensation: Annual wage and non-wage compensation of employees. Data for 2017. *Statistics Portugal, National Accounts, Table B.5.1, August 2020*.

Net worth: Household-level net worth calculated as the difference between the value of all assets owned and the value of all outstanding liabilities. Data for 2017. *European Central Bank, Eurosystem Household Finance and Consumption Survey (HFCS), 3rd wave, variable dn3001, May 2020*.

Household earnings: Total annual labor earnings at the household level. Data for 2017. *European Central Bank, Eurosystem HFCS, 3rd wave, variable di1100, May 2020*.

Housing net worth: Household-level difference between reported value of the household main residence and the value of all liabilities used to finance it. Calculated for homeowners only. Data for 2017. *European Central Bank, Eurosystem HFCS, 3rd wave, variables hb1701, hb1702, hb1703, hb2100, hb0900, May 2020*.

Housing net worth: Household-level difference between reported value of the household main residence and the value of all liabilities used to finance it. Calculated for homeowners

only. Data for 2017. *European Central Bank, Eurosystem HFCS, 3rd wave, variables hb1701, hb1702, hb1703, hb2100, hb0900, May 2020.*

Residential housing investment: Annual gross residential housing investment by the private sector measure at current prices. Data for 2017. *Statistics Portugal, Tables B.2.5, B.3.5, B.4.1.10, and B.5.11., May 2020.*

Gross domestic product (GDP): Annual GDP, final consumption expenditure approach measured at current prices. Data for 2017. *Statistics Portugal, Table A.1.1.2, August 2020.*

Foreclosure rate: Data from [Kaplan et al. \(2020\)](#) for the U.S.

Gross replacement rate: Average estimated individual (male) gross pension entitlement divided by gross pre-retirement earnings of an individual who entered the labor market in 2006. *OECD, Pensions at a Glance, variable single.male.20.natretage.3_5.fullnobreak.1.grr, August 2020.*

Government consumption expenditures: Annual government final consumption expenditures. Data for 2017. *Statistics Portugal, Table A.1.2.5.1, August 2020.*

Social Security tax rates: Social Security tax rates for for-profit entities and their employees. *Portuguese Social Security, contribution rate tables, August 2020.*

Housing stock: Housing stock by institutional sector measured at current prices. Data for 2017. *Statistics Portugal, Capital Stock Accounts, Tables B.2.5, B.3.5, B.4.1.10, B.5.11, August 2020.*

Household weights: Sample weights of surveyed households, used to construct the relative weight of the population by age. Data for 2017. *European Central Bank, Eurosystem HFCS, 3rd wave, variable hw0010, May 2020.*

Consumption units: Consumption equivalence scale. Data for 2017. *European Central Bank, Eurosystem HFCS, 3rd wave, variable dh0002, May 2020.*

Survival probabilities: Survival probability by age for both sexes for the years 2016-2018. Data for 2017. *Statistics Portugal, Life Tables, August 2020.*

B Household problems

In this section, I detail the dynamic programs of each household type. To make the notation simpler, I drop the j subscript from both the state and the choice variables, while keeping it in the utility and value functions; $\Xi_j(\mathbf{y})$ denotes the distribution of the labour income variable, y' , conditional on the vector of individual characteristics, \mathbf{y} . There are three types of households: nonhomeowners, homeowners and defaulters. Each household type can be active or retired. The latter bequeath their net wealth upon death, which is equally redistributed among households. To simplify the exposition, I only present the problems of households in active life, with the exception of the last subsection.

B.1 Nonhomeowners

A household that enters the period with no outstanding mortgage debt and no housing has value function \mathbb{V}_j^N . This household type chooses between continuing to rent or buying a house:

$$\mathbb{V}_j^N(b, \mathbf{y}) = \max \left\{ V_j^r(b, \mathbf{y}), V_j^b(b, \mathbf{y}) \right\},$$

where V_j^r is the value of renting and $V_j^b(b, \mathbf{y})$ the value of acquiring a house.

B.1.1 Rent

A household who opts to become a renter will choose: (i) consumption, c ; (ii) the quantity of the liquid bond, b' , and (iii) the size of the rental, \tilde{h} . The problem is given by:

$$V_j^r(b, \mathbf{y}) = \max_{c, \tilde{h}', b'} u_j(c, s) + \beta \mathbb{E}_\epsilon \left[\mathbb{V}_{j+1}^N(b', \mathbf{y}') \right] \quad (\text{A-1})$$

s.t. :

$$c + q_b b' + \rho \tilde{h}' \leq b + y - \mathcal{T}(y, \rho \tilde{h}') + \theta$$

$$c \geq 0, b' \geq 0, s = \tilde{h}' \in \tilde{\mathcal{H}}, y \sim \Xi_j(\mathbf{y}),$$

where θ is the value of per household bequests.

B.1.2 Buy

A household who opts to buy a house and become a homeowner chooses: (i) consumption, c ; (ii) the quantity of the liquid bond, b' ; (iii) housing size, h' , and (iv) mortgage size, m' . The problem is given by:

$$V_j^o(b, \mathbf{y}) = \max_{c, h', b', m'} u_j(c, s) + \beta \mathbb{E}_\epsilon [\mathbb{V}_{j+1}^H(\mathbf{x}', \mathbf{y}')] \quad (\text{A-2})$$

s.t. :

$$c + q_b b' + p_h h' + \kappa_m \mathbb{1}_{m' > 0} \leq b + y - \mathcal{T}(y, \rho \tilde{h}') + q_j(\mathbf{x}', \mathbf{y}) m' + \theta$$

$$q_j(\mathbf{x}', \mathbf{y}) m' \leq \lambda^m p_h h' \quad (\text{A-3})$$

$$\pi_j^{\min}(m') \leq \lambda^\pi (y - \mathcal{T}) \quad (\text{A-4})$$

$$c \geq 0, b' \geq 0, s = \omega h', h' \in \mathcal{H}, y \sim \Xi_j(\mathbf{y}),$$

where the indicator $\mathbb{1}_{m' > 0}$ takes the value 1 if the household takes out a mortgage and zero otherwise, and \mathbb{V}_{j+1}^H is the value function of a household that enters age $j+1$ as a homeowner. Equations [A-3](#) and [A-4](#) are the LTV and the PTI limits, respectively.

B.2 Homeowner

A household that enters the period as a homeowner has value function \mathbb{V}_j^H . This household type chooses between (i) paying its mortgage (if any); (ii) refinancing its current mortgage; (iii) default

on its mortgage; or (iv) sell its house and either buy another or rent:

$$\mathbb{V}_j^H(\mathbf{x}, \mathbf{y}) = \max \left\{ \begin{array}{ll} \text{Pay :} & V_j^P(\mathbf{x}, \mathbf{y}) \\ \text{Refinance :} & V_j^f(\mathbf{x}, \mathbf{y}) \\ \text{Sell :} & \mathbb{V}_j^N(b^N, \mathbf{y}) \\ \text{Default :} & \mathbb{V}_j^D(b^D, m^D, \mathbf{y}) \end{array} \right.$$

Note that in the case of the option to sell the house the homeowner problem collapses to the that of a nonhomeowner where the liquid bond is given by:

$$b^N = b + (1 - \delta_h - \tau_h - \kappa_h)p_h h - (1 + r_m)m, \quad (\text{A-5})$$

i.e., the starting bond holding plus the net-of-costs proceeds from the sale of the house after the outstanding mortgage is paid back.

B.2.1 Pay mortgage

A household who opts to make a mortgage payment (or keep the current house size with no outstanding mortgage balance) chooses: (i) consumption, c' ; (ii) the quantity of the liquid bond, b' and (iii) the value of the mortgage payment. The problem is:

$$V_j^P(\mathbf{x}, \mathbf{y}) = \max_{c, b', \pi} u_j(c, s) + \beta \mathbb{E}_\epsilon [\mathbb{V}_{j+1}^H(\mathbf{x}', \mathbf{y}')] \quad (\text{A-6})$$

s.t. :

$$c + q_b b' + (\delta_h + \tau_h)p_h h + \pi \leq b + y - \mathcal{T}(y, 0) + \theta$$

$$\pi_{j-1}^{min}(m) \leq \pi \leq (1 + r_m)m \quad (\text{A-7})$$

$$m' = (1 + r_m)m - \pi$$

$$c \geq 0, b' \geq 0, s = \omega h', h' = h, y \sim \Xi_j(\mathbf{y}),$$

where equation A-7 indicates that mortgage payments cannot be below the scheduled contract payments.

B.2.2 Refinance mortgage

A household who opts to refinance its mortgage chooses: (i) consumption, c' ; (ii) the value of the liquid bond, b' , and (iii) the new mortgage size, m' . The problem is:

$$V_j^f(\mathbf{x}, \mathbf{y}) = \max_{c, b', m'} u_j(c, s) + \beta \mathbb{E}_\epsilon [\mathbb{V}_{j+1}^H(\mathbf{x}', \mathbf{y}')] \quad (\text{A-8})$$

s.t. :

$$c + q_b b' + (\delta_h + \tau_h) p_h h + (1 + r_m) m + \kappa_m$$

$$\leq b + y - \mathcal{T}(y, 0) + q_j(\mathbf{x}', \mathbf{y}) m' + \theta$$

$$q_j(\mathbf{x}', \mathbf{y}) m' \leq \lambda^m p_h h'$$

$$\pi_j^{\min}(m') \leq \lambda^\pi (y - \mathcal{T})$$

$$c \geq 0, b' \geq 0, s = \omega h', h' = h, y \sim \Xi_j(\mathbf{y}).$$

B.2.3 Default

The value of the option to default collapses to the value function of a defaulter (whose problem is described in the next subsection), where:

$$m^D = - \frac{\min\{(1 - \delta_h^d - \tau_k - \kappa_h) p_h h - (1 + r_m) m, 0\}}{1 + r_m}, \quad (\text{A-9})$$

$$b^D = b + \max\{(1 - \delta_h^d - \tau_k - \kappa_h) p_h h - (1 + r_m) m, 0\}, \quad (\text{A-10})$$

Equation A-9 implies that the default option value is equal to the value function of a defaulter that starts the period with a mortgage balance equal to the residual amount owed after collateral repossession. This value is divided by $1 + r_m$ to make it consistent with the defaulter problem. If the repossession value exceeds the amount owed, the outstanding mortgage balance is zero.

In this case, the household starts with additional bond wealth, given by equation A-10. Note that this formulation implies that either: (i) $m^D > 0$ and $b^D = b$; or (ii) $m^D = 0$ and $b^D > b$. The case where $m^D = 0$ will never occur.

THIS SECTION NEEDS TO BE REWRITTEN IN LIGHT OF THE CHANGES I MADE TO THE HOMEOWNER PROBLEM. A household who opts to default chooses: (i) consumption, c' ; (ii) the quantity of the liquid bond, b' , and (iii) the rental size, \tilde{h}' . The problem is:

$$V_j^d(\mathbf{x}, \mathbf{y}) = \max_{c, \tilde{h}', b'} u_j(c, s) + \beta \mathbb{E}_\epsilon \left[\psi_d \mathbf{V}_{j+1}^D(b', m', \mathbf{y}) + (1 - \psi_d) \mathbf{V}_{j+1}^N(b', \mathbf{y}) \right] \quad (\text{A-11})$$

s.t. :

$$c + q_b b' + \rho \tilde{h}' \leq b + y - \mathcal{T}(y, \rho \tilde{h}') + \Phi + \theta$$

$$\Phi = \max\{(1 - \delta_h^d - \tau_k - \kappa_h)p_h h - (1 + r_m)m, -\kappa_d(b + y - \mathcal{T})\}$$

$$c \geq 0, b' \geq 0, s = \tilde{h}' \in \tilde{\mathcal{H}}, y \sim \Xi_j(\mathbf{y}),$$

where Φ is the recourse function. If the outstanding mortgage balance exceeds the net value of the foreclosed house, the household will have to pay the remainder up to a fraction κ_d of cash-on-hand. With probability ψ_d it will start next period as a defaulter, and with probability $(1 - \psi_d)$ as a nonhomeowner.

Note that this formulation implies that a household that ends the period with $m' = 0$ is still a defaulter. In practice, however, this situation never occurs.

Proposition B.1. *A homeowner will not have zero mortgage balance at the end of the period ($m' = 0$) in the event of default.*

Proof. Let $A := (1 - \delta_h^d - \tau_k - \kappa_h)p_h h - (1 + r_m)m$. If $m' = 0$ when the homeowner defaults, one of the following conditions must be true:

1. $A \geq 0$;
2. $A < 0$ and $A + b - y + \mathcal{T} \geq 0$.

In the case of (1), the household is able to pay all of its debt using the collateral. However, because of the assumption that $\delta_h^d > \delta_h$, were it to sell the house instead of defaulting, the household would be able to both pay off its debt and extract positive net proceeds from the house sale. Thus, this condition can never be true if default is chosen.

In the case of (2), the household must use other resources in order to fully repay its debt. The same argument as before is used: because $\delta_h^d > \delta_h$, selling the house would allow the household to increase its available resources to fully pay off the debt and consume, which implies that default is not optimal in this situation. \square

An implication of this proposition is that a necessary condition for default is:

$$A < 0 \quad \text{and} \quad A + \kappa_d(b - y + \mathcal{T}) \leq 0,$$

meaning that leverage must be large enough for the household to find it optimal to default. For low ψ_d and κ_d , for example, there is a larger probability that the household will be able to discharge a greater fraction of its debt, making default a more attractive option.

B.3 Defaulter

A household that enters the period not owning a house and with no access to mortgage or housing markets has value function V_j^D . A household in this status can leave it only by either paying off all of its debt or by randomly exiting it in a given period. Each period it chooses: (i) consumption,

c' ; (ii) the quantity of the liquid bond, b' , and (iii) the rental size, \tilde{h}' . The problem is:

$$\mathbb{V}_j^D(b, m, \mathbf{y}) = \max_{c, \tilde{h}', b'} u_j(c, s) + \beta \begin{cases} \mathbb{E}_\epsilon [\psi_d \mathbb{V}_{j+1}^D(b', m', \mathbf{y}) + (1 - \psi_d) \mathbb{V}_{j+1}^N(b', \mathbf{y})] & \text{if } m' > 0 \\ \mathbb{E}_\epsilon [\mathbb{V}_{j+1}^N(b', \mathbf{y})] & \text{otherwise} \end{cases}$$

s.t. :

$$c + q_b b' + \rho \tilde{h}' \leq b + y - \mathcal{T}(y, \rho \tilde{h}') - \Phi + \theta$$

$$\Phi = \min\{(1 + r_m)m, \kappa_d(b + y - \mathcal{T})\}$$

$$m' = m(1 + r_m) - \Phi$$

$$c \geq 0, b' \geq 0, s = \tilde{h}' \in \tilde{\mathcal{H}}, y \sim \Xi_j(\mathbf{y}).$$

The branch function indicates that household leaves defaulter status with certainty in case all outstanding debt is paid.

B.4 Last period of life

On period $j = J$ the household decides on the size of the bequest it wishes to leave, from which it draws utility. For example, for a household with outstanding debt and housing the problem is:

$$V_j^P(\mathbf{x}, \mathbf{y}) = \max_{c, b', \pi} u_j(c, s) + \beta v(b) \tag{A-12}$$

s.t. :

$$c + q_b b' + (1 + r_m)m \leq b + y - \mathcal{T}(y, 0) + \theta$$

$$b = b' + (1 - \delta_h - \tau_k - \kappa_h)p_h h$$

$$c \geq 0, b' \geq 0, s = \omega h', h' \in \mathcal{H}, y \sim \Xi_j(\mathbf{y}).$$

In this case, the household pays off the remainder of its mortgage and the house is sold in the beginning of the following period and its net proceeds are bequeathed together with bond quantity b' .

C Equilibrium definition

Let $\mathbf{x}^N := (b, \mathbf{y}) \in \mathbb{X}^n$, $\mathbf{x}^D := (b, m, \mathbf{y}) \in \mathbb{X}^d$, and $\mathbf{x}^H := (b, h, m, \mathbf{y}) \in \mathbb{X}^h$ be the state vector for individual non-homeowners, defaulters, and homeowners. Also, let $\mu_j^N, \mu_j^D, \mu_j^H$ be the corresponding measure of the three household types at age j , where $\sum_{j=1}^J \mu_j^N + \mu_j^D + \mu_j^H = 1$.

A recursive competitive equilibrium is a set of value functions $\{ \mathbb{V}_j^N(\mathbf{x}^N), V_j^r(\mathbf{x}^N), V_j^o(\mathbf{x}^N), \mathbb{V}_j^D(\mathbf{x}^D), \mathbb{V}_j^H(\mathbf{x}^H), V_j^p(\mathbf{x}^H) \}_{j=1}^J$; policy functions $g_j^o(\mathbf{x}^N), g_j^n(\mathbf{x}^H), g_j^f(\mathbf{x}^H), g_j^d(\mathbf{x}^H), g_j^n(\mathbf{x}^H)$

D Algorithm

To solve the model, I apply the modified EGM method proposed by [Fella \(2014\)](#).

D.1 Solution to defaulter problem

Solving the defaulter problem presents an additional hurdle when using the modified EGM: To obtain the continuation value, $\tilde{V}_{b'}(b', m', y')$, given a set of values for the state variables (b', m, y) and a value \tilde{h}' for the rental choice, knowledge of m' is necessary.⁶ The following is a procedure to identify m' and solve the problem:

1. Given (b', m, y) , if any of the following conditions are met we know $m' = 0$, which allows us to compute the continuation value and proceed with the algorithm. If not, go to the next point.

$$(a) \quad b' \geq \frac{(1+r_m)m}{\kappa_d q_b}$$

$$(b) \quad y - \mathcal{T} \geq \frac{(1+r_m)m}{\kappa_d}$$

2. Solve the following non-linear equation for b :

$$\begin{aligned} u_c \left(b + y - \mathcal{T} - \rho \tilde{h} - \min\{\kappa_d(b + y - \mathcal{T}), (1 + r_m)m\} - q_b b' \right) = \\ = \tilde{V}_{b'}(b', (1 + r_m)m - \min\{\kappa_d(b + y - \mathcal{T}), (1 + r_m)m\}, y') \end{aligned} \quad (A-13)$$

3. Obtain m' from $m' = m' = (1 + r_m)m - \min\{\kappa_d(b + y - \mathcal{T}), (1 + r_m)m\}$.

4. Compute $\tilde{V}_{b'}(b', m', y')$ and proceed with the algorithm.

Point 1a follows from the fact that it must always be that $q_b b' < b + y - \mathcal{T}$. This implies that $\kappa_d q_b b' < \kappa_d [b + y - \mathcal{T}]$. Therefore, if $\kappa_d q_b b' \geq (1 + r_m)m$ we know that $\kappa_d [b + y - \mathcal{T}] > (1 + r_m)m$.

⁶Except in case $m = 0$.

Point 1b uses a similar logic: Given that $y - \mathcal{T} \leq b + y - \mathcal{T}$ is always true, then $\kappa_d[y - \mathcal{T}] \leq \kappa_d[b + y - \mathcal{T}]$. So if $\kappa_d[y - \mathcal{T}] \geq (1 + r_m)m$, then $\kappa_d[b + y - \mathcal{T}] \geq (1 + r_m)m$.

Finally, equation A-13 is obtained by substitution from the following system of equations:

$$m' = (1 + r_m)m - \min\{\kappa_d(b + y - \mathcal{T}), (1 + r_m)m\}$$

$$u_c(z - q_b b') = \tilde{V}_{b'}(b', m', y')$$

$$z = b + y - \mathcal{T} - \rho \tilde{h} - \min\{\kappa_d(b + y - \mathcal{T}), (1 + r_m)m\},$$

where the first equation is the law of motion for the mortgage balance of a defaulter, the second is the usual first order condition, and the third is the definition of available resources for consumption and saving in bonds.

Note that solving equation A-13 for b does not imply that b' is necessarily the solution to the problem given (b, m, y) . Because of the possibility of non-concavity of the value function, the first order condition merely provides us with a local maximum which must be tested using the procedure described in the previous section.

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