

Investment-Specific Technological Change and Earnings Inequality in the U.S.*

Pedro Brinca[†] João B. Duarte[‡] Hans H. Holter[§] João G. Oliveira[¶]

February 14, 2021

[Download here the latest version](#)

Abstract

Since 1980 the U.S. economy has experienced a large increase in income inequality. To explain this phenomenon we develop a life-cycle, overlapping generations model with uninsurable labor market risk, a detailed tax system, investment-specific technological change (ISTC) and a set of distinct labor varieties. We calibrate our model to match key characteristics of the U.S. economy and study how ISTC, shifts in taxation, government debt and employment have contributed to the rise in income inequality. We find that these structural changes can account for close to one third of the observed increase in the post-tax income Gini. The main mechanisms at play are the rise in the wage premium of non-routine workers, resulting from capital-non-routine complementarity, as well as a reduction of the progressivity of the labor income tax schedule, which increases post-tax inequality. We show that ISTC alone accounts for roughly 15% of the change observed in post-tax income Gini, while the reduction in progressivity accounts for 16%.

Keywords: Income Inequality, Taxation, Automation

JEL Classification: E21; H21; J31;

*We thank Árpád Ábrahám, Daron Acemoglu, David Autor, Vasco Botelho, Juan Dolado, Loukas Karabarbounis, Nick Kozeniauskas, Musa Orak, Lee Ohanian, Cezar Santos, Pedro Teles, Gianluca Violante, seminar participants at the Lisbon Macro Group, the EUI Economics Department and participants at the PEJ 2018 and the 2019 Winter Meeting of the North-American Econometric society for their helpful comments and suggestions. Pedro Brinca is grateful for financial support from the Portuguese Science and Technology Foundation, grants number SFRH/BPD/99758/2014, UID/ECO/00124/2013 and UID/ECO/00145/2013, POR Lisboa (LISBOA-01-0145-FEDER-007722 and Social Sciences Data Lab, Project 22209), and POR Norte (Social Sciences Data Lab, Project 22209). João Oliveira is grateful for financial support from the Portuguese Science and Technology foundation, grant number SFRH/BD/138631/2018.

[†]Center for Economics and Finance at Universidade of Porto and Nova School of Business and Economics.

[‡]Nova School of Business and Economics.

[§]Department of Economics, University of Oslo

[¶]Nova School of Business and Economics.

1 Introduction

There has been a steady rise in income inequality in the U.S. since 1980. Figure 1 shows that this phenomenon occurred in tandem with a fall in the relative price of investment, which can be viewed as reflecting investment-specific technological change (Krusell et al., 2000; Karabarbounis and Neiman, 2014), and a sharp reduction in tax progressivity (Ferriere and Navarro, 2018). In this paper, we use an incomplete markets model calibrated to the U.S. economy to study how each of these factors has influenced the rise in income inequality.

We design an overlapping generations model featuring investment-specific technological change, a detailed tax schedule, uninsurable idiosyncratic earnings risk, and incomplete markets. To generate factor-biased technological change, we assume some agents are born with abilities that are complement to capital while others are born with abilities that are substitute to capital. These distinct labor varieties are called non-routine and routine, respectively. We incorporate an additional dimension to the analysis of labor varieties by dividing workers into the skilled/unskilled categories. This allows us to take into account the rise in the skill premium (Krusell et al., 2000), and to analyze how the different categories of workers are affected by the selected structural changes. This is close to the spirit of Acemoglu and Restrepo (2017a), although, to our knowledge, we are the first to simultaneously include these two dimensions in standard incomplete markets model.

Aside from the direct impact of a fall in tax progressivity on income dispersion, the main mechanism at work in our model is the rise in the wage premium of non-routine workers, following a drop in investment prices. This is the result of the complementarity between capital and non-routine labor. As investment prices fall, capital accumulation becomes cheaper and firm demand for routine labor drops, along with wages. In contrast, non-routine labor becomes a more productive input, raising wages and labor demand for those workers.

We find that ISTC and the fall in tax progressivity jointly account for at least one third of the increase in the income Gini coefficient. In particular, by means of counterfactual exercises we show that ISTC alone accounts for roughly 15% of the change in model post-tax income Gini,

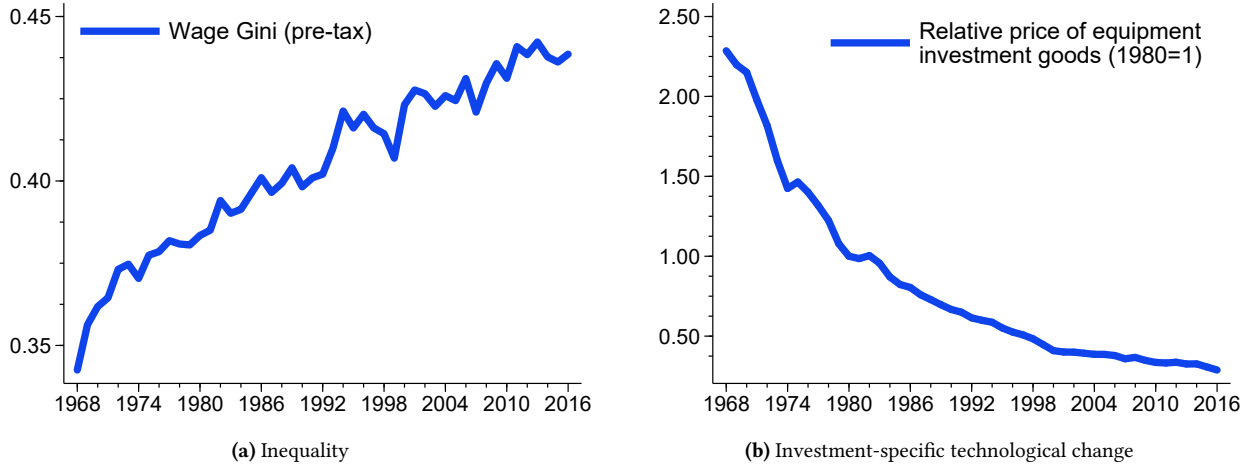


Figure 1: Inequality and ISTC. Notes: the earnings Gini is computed from the CPS for employed workers. Description of the sample is provided on section 3. The relative price of investment is computed as the ratio between equipment investment prices from DiCecio (2009) and the BLS urban consumer price index.

while the reduction in progressivity accounts for 16%. Other structural changes, such as the increase in non-routine relative employment, and social security taxes dampen these mechanisms by reducing the marginal productivity of non-routine labor and increasing the progressivity of the tax system.

The rest of the paper is organized as follows. Section 2 surveys the streams of literature to which this paper is related. In Section 3, we discuss the stylized facts that underlie our modeling choice. In Section 4, we describe the model. In Section 6, we show the calibration strategy. In Section 7, results are presented. Section 8 concludes.

2 Related Literature

This paper is related to the literature which documents a reduction in the labor share of income as a result of a fall in investment prices. Karabarbounis and Neiman (2014) show that the labor share has been declining across countries at least since the early 1980s. Using a general equilibrium model to obtain an expression for the labor share as a function of the price of investment goods, they are able to account for half of the observed decline in the labor share.

[Eden and Gaggl \(2018\)](#) estimate that the labor share in the U.S. has dropped by 6.8 percentage points since 1950. This drop was concurrent to a reduction in routine occupations, and of the price of information and communication technology (ICT) capital goods. Using a mechanism similar to [Karabarbounis and Neiman \(2014\)](#), they estimate that the drop in ICT capital prices accounts for half of the drop in the labour share.

The key ingredient that generates a contraction of the labour share in these models is the substitutability between capital and unskilled or routine labour in the production function, in the tradition of [Krusell et al. \(2000\)](#). This feature, coupled with a reduction in investment good prices, leads firms to substitute away from labour and towards capital. Our main mechanism is similar to that of [Karabarbounis and Neiman \(2014\)](#) and [Eden and Gaggl \(2018\)](#), but our focus are the distributional aspects which they abstract from in analyzing the effects of the drop in investment prices.

This paper is related to the heterogeneous agents literature which quantifies the effect of structural changes in the income and wealth distributions. In particular, [Hubmer et al. \(2017\)](#), which investigate the change in the wealth distribution in the U.S. as a result of changes in taxation. Similarly, [Civale \(2016\)](#) quantifies the impact that ISTC has produced on the wealth distribution. In contrast to these recent contributions, our focus is on the income distribution and on nesting these competing mechanisms on a single model, in order to evaluate their relative contributions.

Our paper is also related to the literature on the effects of automation ([Autor et al., 2003](#); [Michaels et al., 2010](#); [Acemoglu and Restrepo, 2017a](#); [Acemoglu and Restrepo, 2017b](#); [Guerreiro et al. \(2017\)](#); [Acemoglu and Restrepo, 2018](#)). [Autor et al. \(2003\)](#) document that computer capital substitutes for workers in performing cognitive and manual tasks, and complements workers in performing non-routine problems-solving. They argue that these features can account for a substantial fraction of the resulting shift in demand toward college-educated labor. [Michaels et al. \(2010\)](#) use cross-country industry level data to analyze the demand across skill levels and conclude that industries with faster ICT growth had greater increases in relative demand for high

skill workers and larger falls in relative demand for middle skill workers. They find that there is little effect on low-skilled workers mainly performing routine tasks.

In string of recent papers Daron Acemoglu and Pascual Restrepo have both contributed to measuring the effects of automation and formalized them into a task-based model of the labor market. [Acemoglu and Restrepo \(2017b\)](#) investigate the impact of a greater robot usage in the US local labor markets. Their findings indicate large and robust negative effects on employment and wages. [Acemoglu and Restrepo \(2018\)](#) develop a theoretical framework where automation produces two competing effects on wages and labor demand: a displacement effect resulting from the substitution of labor for machines, reducing the demand for labor and wages; a productivity effect which is the product of cost-savings generated by automation, which increases the demand for labor in the remaining tasks.

In the model of [Guerreiro et al. \(2017\)](#) routine jobs performed by low skill agents can be taken over by automation units. As the marginal cost of producing robots changes across steady states, routine labor wages and employment change in the same direction, given the assumption of substitutability. They study the problem of optimal taxation and find that it is optimal for the government to provide a lump sum rebate financed by taxes on automation units. This result follows from an information asymmetry problem (in the spirit of [Mirrlees, 1971](#)), whereby the social planner cannot distinguish between routine or cognitive workers and is thus unable to condition transfers on individuals' types. Our paper contributes to this literature by analyzing the macroeconomic impact of these mechanisms and quantitatively accounting for their effects on income inequality in the US economy.

To dilute the perceived social cost of these trends several policies have been suggested, including a proposal famously put forth by Bill Gates to tax robots and “even slow down the speed [of automation]” ([Delaney, 2017](#)). The issue of optimal capital taxation has been discussed extensively since the seminal papers by [Chamley \(1986\)](#) and [Lucas \(1990\)](#), who find that the optimal rate of capital taxation is zero in the steady state. In contrast, [Aiyagari \(1995\)](#), using an incomplete markets model with borrowing constraints, determines that the optimal income tax

on capital income is positive, even in the long run.

3 Data

Our analysis of the U.S. labor market is carried out along the two dimensions proposed by [Autor et al. \(2003\)](#) to classify occupations: (i) whether the main tasks are susceptible to automation (routine) or not (non-routine); and (ii) the nature of the tasks involved, i.e., whether they are predominantly cognitive or manual. This classification system yields four mutually exclusive occupation groups: non-routine cognitive (NRC), non-routine manual (NRC), routine cognitive (RC) workers and routine manual (RM). We use data from the Census Bureau Current Population Survey (CPS), spanning the period from 1968 to 2016, to study how quantities and prices have changed since the late 1960s for each of these groups.

We used the Annual Social and Economic Supplement (ASEC) from the March CPS survey available from [Flood et al. \(2018\)](#), which contains data on yearly earnings and hours worked in the previous calendar year. The CPS employs the US Census Bureau 2010 occupation classification system, and we use the cross-walk table of [Cortes et al. \(2016\)](#) to categorize each worker into one of the aforementioned classes. This cross-walk is based on the so-called “consensus” classification scheme of [Autor and Acemoglu \(2011\)](#). The population of interest is the set of non-military, non-institutionalized individuals aged 16 to 70, working full time, full year in the previous year, excluding the self-employed and farm sector workers. See Appendix A for additional details on data treatment.

These data are used to construct time series on group employment, wage differentials, and wage premia associated with skill or . To calculate the wage premia we use the method of [Autor and Acemoglu \(2011\)](#). Concretely, we run yearly cross sectional regressions of hourly log wages on occupation type, race (black, non-white other), potential experience in years, education, and interactions between education, experience and occupation type up to the forth order separately for each gender. We then construct a set of bins for every combination of gender/race/work

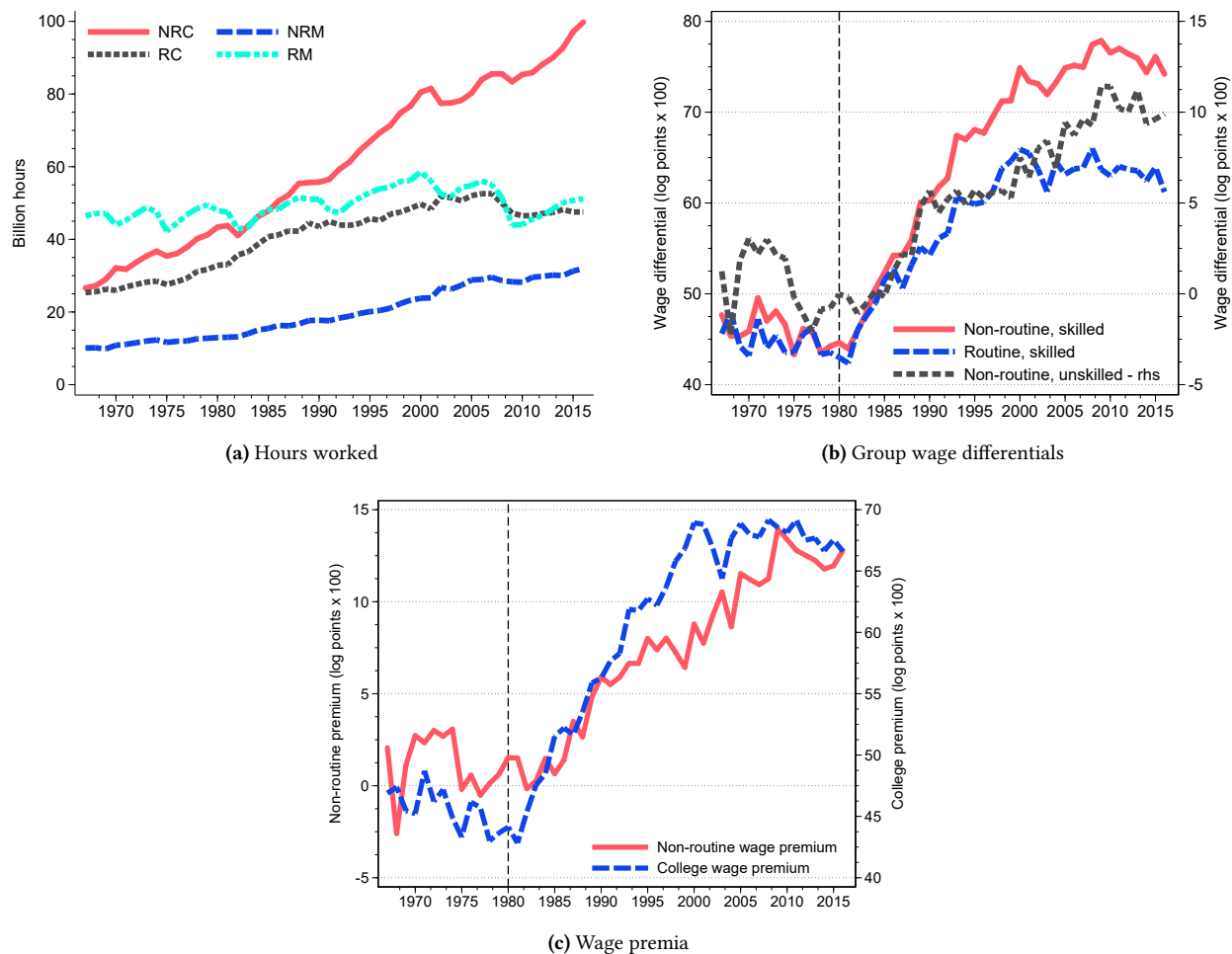


Figure 2: The rise of non-routine skilled labor. Notes: Wage differentials are obtained as the log difference between the average wage of each group. Groups for wages are constructed with using a constant composition of individual observable characteristics (experience, education, etc). Wage statistics are for males.

experience/task type/education level with constant weights across periods, defined as the weight of a given group on total employment in 1980. The regression is used to predict the log-wage for each group in each year. Wage premia are defined as the log-difference in predicted wages between two groups whose only difference is in task type. The variables used are described on Appendix C.

Figure 2 shows the evolution of employment and wages for the selected groups. We can discern four main stylized facts: (i) the strong performance of NRC workers compared to other groups and, in particular, RM workers; (ii) the strong growth of skilled worker groups relative

to unskilled; (iii) the rise of the skilled and non-routine skilled premia. Our estimate of the skill premium has leveled off since 2000, informing our decision to focus our modeling efforts on the routine/non-routine dimension. To the best of our knowledge, we are the first to present estimates of both the skill and the non-routine wage premia which are orthogonal to each other.¹

The central hypothesis in this paper is that one of the main drivers of the increase in inequality since the 1980s has been the discriminating effect that investment specific technological change has had on these four groups due to its diverse interaction with each labor variety. This reasoning is similar to that of [Krusell et al. \(2000\)](#), [Karabarbounis and Neiman \(2014\)](#), [Acemoglu and Restrepo \(2017b\)](#), and [Eden and Gaggl \(2018\)](#).

This choice was made due to the quantitative importance of ISTC for the long-run growth of output per hours worked in the U.S. economy, originally estimated to be 60% in [Greenwood et al. \(1997\)](#), as well as its potential to disrupt labor market conditions. Indeed, [Krusell et al. \(2000\)](#) used a model of capital-skill complementarity and ISTC to study the increased wage dispersion in the U.S economy and are able to track the progress of the skill premium. Similarly to [Acemoglu and Restrepo \(2017b\)](#) and [Eden and Gaggl \(2018\)](#), we view the process of ISTC as akin to increased automation of routine tasks in the economy. However, we focus on the wage premium rather than on worker displacement in this paper.

Central to investment-specific technological change are the falling prices of capital goods, which can be interpreted as evidence of increasing productivity in the investment goods sector. As an illustration of this interpretation, consider that in the 1950s a computer was leased for 200,000 per month in inflation-adjusted 2010 dollars, plus the costs of the staff and energy required to operate it.² Today, any computer or smartphone equipped with microprocessors costs a fraction of that price and is able to deliver a processing speed which is many million times that of a large-scale computer in the 1950s.³ To get a sense of the scale of technological change, the CPU of a Play Station 2 is 1,500 times faster than the guidance computer on Apollo 11, while the

¹[Eden and Gaggl \(2018\)](#) do not show their estimate of the skill wage premium.

²Source: http://ethw.org/Early_Popular_Computers,_1950_-_1970.

³Not to mention holding a much larger quantity of information: in 1956, IBM's 305 RAMAC disk could hold 5 MB of information, while the computer on which this paper was written has a total of 4.78 TB in hard drive memory.

Apple iPhone4 is 4,000 times faster.

Is there reason to believe that this source of growth has a uniform impact across labor markets? [Krusell et al. \(2000\)](#) argue that this is not the case. Using aggregate U.S. data they estimate the parameters for a CES production function where capital, skilled and unskilled labor are embedded. They find that capital is a gross complement with skilled labor and a gross substitute for unskilled labor. Therefore, secular growth is skill-biased and is able to reproduce the rise in the skill premium observed in the U.S. since the start of the 1980s, highlighting the importance of worker training for productivity and inequality. Both [Karabarbounis and Neiman \(2014\)](#) and [Eden and Gaggl \(2018\)](#) depart from similar hypotheses in building their frameworks.

4 Model

Our model is of the Bewley-Aiyagari-Hugget⁴ variety: an incomplete markets economy with overlapping generations of heterogeneous agents and partial uninsurable idiosyncratic risk that generates both income and wealth distributions. The basic setup is that of [Brinca et al. \(2016\)](#), with a more detailed production function.

There are four types of households – non-routine cognitive (NRC), non-routine manual (NRM), routine cognitive (RC) and routine manual (RM) – that derive utility from non-durable consumption and leisure. Non-routine households are born with certain abilities, such as creativity, that allow them to perform tasks that are complement to capital. Routine households, on the other hand, are born with abilities that allow them to perform tasks that are substitute to capital.⁵ Each type of households face an idiosyncratic uninsurable stream of earnings in the form of wages, and make joint decisions about consumption, savings and hours worked.

For the production side of the economy, we draw heavily on the modeling strategy in [Krusell et al. \(2000\)](#) and [Karabarbounis and Neiman \(2014\)](#). There are three final goods sectors in the

⁴See [Bewley \(2000\)](#), [Aiyagari \(1994\)](#), and [Hugget \(1993\)](#).

⁵In the specification of the production function, we actually nest routine cognitive households with capital, to allow for the possibility that their occupation type is complementary to capital, an hypothesis which is supported by the fact that their wage premium with respect to routine manual workers has been increasing over time

economy: consumption goods, non-equipment capital good, and an equipment capital good sector. This formulation allows us to express the price of equipment goods as a function of the level of technology in that sector relative to the consumption goods sector. The production function is extended in order to encompass a greater variety of labor types. The asset structure used follows the same framework of [Krusell et al. \(2010\)](#) to include investment prices in the formulation of the household decision.

Demographics

We assume the economy is populated by a set of $J - 1$ overlapping generations, as in [Brinca et al. \(2016\)](#). A household starts his life at age 20 and after retiring at age 65 households face an age-dependent probability of dying, $\pi(j)$, dying with certainty at age 100. We define a period in the model to correspond to one year. Thus, j , the household's age, varies between 0 (for age 20 households) and 80 (for age 100 households). $\omega(j) = 1 - \pi(j)$ defines the age-dependent probability of surviving, and so, at any given period, using a law of large numbers, the mass of retired agents of age $j \geq 45$ is equal to $\Omega_j = \prod_{i=45}^{j-1} \omega(i)$. There are no annuity markets, so that a fraction of households leave unintended bequests which are redistributed in a lump-sum manner between the households that are currently alive, denoted by Γ . We include a bequest motive in this framework to make sure that the age distribution of wealth is empirically plausible, as in [Brinca et al. \(2018\)](#) and [Brinca et al. \(2019\)](#).

Households also differ across persistent idiosyncratic productivity shocks, permanent ability, asset holdings, and a discount factor assuming four distinct values $\beta \in \{\beta_1, \beta_2, \beta_3, \beta_4\}$, which are uniformly distributed across agents. Working age agents have to choose how much to work, n_t , how much to consume, c_t , and how much to save, k_{t+1} , to maximize their utility. Retired households have consumption and saving decisions and receive a retirement benefit, Ψ_t .

Prior to joining the labor market, agents draw a permanent ability level, a , from a uniform distribution with thresholds p_{RM} , p_{RC} , p_{NRM} , such that group employment weights match the data. Therefore, an agent is able to supply labor type RM if its draw is no greater than p_{RM} , and so forth.

Labor income

Labor productivity depends on three distinct elements which determine the number of efficiency units each household is endowed with in each period: age j , labor variety group a , and an idiosyncratic productivity shock, u , which we assume follows an AR(1) process:

$$u_{it} = \rho_u u_{it-1} + \varepsilon_{it}, \quad \varepsilon_{it} \sim N(0, \sigma_\varepsilon^2). \quad (1)$$

Thus, household i 's wage is given by:

$$w_{it}(j, a_i, u_{it}) = w_t^s e^{\gamma_1 j + \gamma_2 j^2 + \gamma_3 j^3 + a_i + u_{it}}, \quad (2)$$

where γ_1 , γ_2 and γ_3 are calibrated directly from the data to capture the age profile of wages. Households' labor income depends on the wage per efficiency unit of labor w_t^s , $s \in S \equiv \{NRC, NRM, RC, RM\}$, where s is the labor variety supplied by the household.

Apart from determining the type of labor which can be supplied by the household, permanent ability is also used to accommodate labor income inequality from other sources: (i) each group has an assigned wage differential over the routine manual (a_{NRC} , a_{NRM} , a_{RC}) so as to account for the share of between-group inequality which does not result from the value of the wage premium as determined by relative productivities (such as differences in gender and race composition of groups); (ii) within-group earnings inequality is modeled by constructing a wage distribution within each group, calibrated to match the inequality in the data. Appendix E describes how this is implemented in the algorithm.

Preferences

The utility of households, $U(c_{it}, n_{it})$, is increasing in consumption and decreasing in work hours, $n_{it} \in (0, 1]$, and is defined as:⁶

$$U(c_{it}, n_{it}) = \frac{c_{it}^{1-\lambda}}{1-\lambda} - \chi \frac{n_{it}^{1+\eta}}{1+\eta}. \quad (3)$$

⁶We assume that labor disutility depends only on the level of supply, not on occupation type.

Retired households' utility function has one extra term, as they gain utility from the bequest they leave to living generations:

$$D(h'_{it}) = \varphi \log(h'_{it}). \quad (4)$$

Technology

In this economy, three competitive final goods sectors exist: consumption, structure investment goods, and equipment investment goods. These are produced by transforming a single intermediate input using a linear production technology. All payments are made in the consumption good, which is the numeraire.

The consumption good is obtained by transforming a quantity Z_t^c of intermediate input into output, which is then sold at price p_t^c to both households and the government. The transformation technology is:

$$C_t + G_t = Z_t^c, \quad (5)$$

where Z_t^c is the quantity of input, purchased at p_t^z from a representative intermediate goods firm. Given that the consumption good is competitively produced, its price equals the marginal cost of production:

$$p_t^c = 1 = p_t^z. \quad (6)$$

Likewise, non-equipment investment good firms produce output with a similar technology:

$$X_{st} = Z_t^s, \quad (7)$$

and therefore we have that $p_t^s = 1$. The production of X_{et} , the equipment investment good, uses the transformation technology:

$$X_{et} = \frac{Z_t^e}{\xi_t}, \quad (8)$$

where Z_t^e is the quantity of input z used in the production of the final equipment good. $1/\xi_t$ is the level of technology used in the production of X_{et} relative to the final consumption good. As ξ_t declines, the relative productivity in assembling the equipment good increases. We assume that

ξ_t evolves exogenously in this economy. We obtain the price of the equipment good from the zero profit condition:

$$p_t^e = \xi_t p_t^z = \xi_t, \quad (9)$$

where $\xi_t = p_t^e/p_t^c$ is interpreted as the relative price of the equipment good.

A representative intermediate goods firm produces $Z_t^c + Z_t^s + Z_t^e$ using a constant returns to scale technology in capital and labor inputs, $y_t = F(K_{st}, K_{et}, N_{NRCt}, N_{NRMt}, N_{RCt}, N_{RMt})$, where K_{st} is non-equipment capital and K_{et} is capital equipment. The firm rents non-equipment capital at rate r_t^s , equipment at r_t^e and each labor variety at w_t^s , $s \in S$. Aggregate demand measured in terms of the consumption good, $Y_t = C_t + G_t + X_{st} + \xi_t X_{et}$, factor prices and the price of the intermediate good p_t^z are taken as given. The firm chooses capital and labor inputs each period in order to maximize profits:

$$\Pi_t^z = p_t^z y_t - r_t^s K_{st} - r_t^e K_{et} - \sum_{s \in S} w_t^s N_{st}, \quad (10)$$

subject to:

$$y_t = Z_t^c + Z_t^s + Z_t^e = C_t + G_t + X_{st} + \xi_t X_{et} = Y_t. \quad (11)$$

This setup implies that $Z_t^c = C_t + G_t$, $Z_t^s = X_{st}$, $Z_t^e = \xi_t X_{et}$, and $F(.) = Y_t = C_t + G_t + X_{st} + \xi_t X_{et}$. We assume that the production function of intermediate goods is Cobb-Douglas over non-equipment

capital and CES over the remaining inputs:⁷

$$F(.) = A_t G(.) = A_t K_{st}^\alpha \left[\sum_{i=1}^3 \varphi_i Z_{it}^{\frac{\sigma-1}{\sigma}} + \left(1 - \sum_{i=1}^3 \varphi_i \right) N_{RMt}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma(1-\alpha)}{\sigma-1}}, \quad (12)$$

$$Z_{1t} = \left[\phi_1 K_{et}^{\frac{\rho_1-1}{\rho_1}} + (1 - \phi_1) N_{NRCt}^{\frac{\rho_1-1}{\rho_1}} \right]^{\frac{\rho_1}{\rho_1-1}}, \quad Z_{2t} = \left[\phi_2 K_{et}^{\frac{\rho_2-1}{\rho_2}} + (1 - \phi_2) N_{NRMt}^{\frac{\rho_2-1}{\rho_2}} \right]^{\frac{\rho_2}{\rho_2-1}},$$

$$Z_{3t} = \left[\phi_3 K_{et}^{\frac{\rho_3-1}{\rho_3}} + (1 - \phi_3) N_{RCt}^{\frac{\rho_3-1}{\rho_3}} \right]^{\frac{\rho_3}{\rho_3-1}},$$

where A_t is total factor productivity, ϕ and φ are distribution parameters, ρ is the elasticity of substitution between capital and the nested labor variety, and σ is the elasticity of substitution between each composite Z_{it} and routine manual labor. Complementarity between the two inputs in Z_{it} requires that $\rho_i < \sigma$.

Each variety of labor input is measured in efficiency units, $N_{st} \equiv h_{st} \varrho_{st}$, where h_{st} is the raw number of hours worked in the aggregate and ϱ_{st} is an efficiency index representing the latent quality per hour worked of labor of type s at period t . ϱ_{st} can be interpreted as a occupation-specific technology level, due to research and development, or as human capital accumulation.

⁷Krusell et al. (2000), Karabarbounis and Neiman (2014), and Eden and Gaggl (2018) use CES production functions where capital equipment is nested with all labor varieties except for RM, which is isolated. The reason for this setup is the set of symmetry restrictions on substitution elasticities imposed by the CES functional form, as explained in Krusell et al. (2000). In a nutshell, this nesting form allows for complementarity between capital and differentiated labor (NRC NRM, RC) while permitting the elasticities of substitution between routine routine manual labor and other labor varieties to be different. Our version is an extension of this framework with a finer breakdown over labor varieties. In estimating the production function, we use the Simulated pseudo-Maximum Likelihood (SPMLE) method proposed by Ohanian et al. (1997) which was also applied in Krusell et al. (2000). Our application is described in the next section.

Firm maximization implies that marginal products equal factor prices:⁸

$$w_t^{NRC} = \Xi_t \varphi_1 \left[\phi_1 \left(\frac{K_{et}}{N_{NRCt}} \right)^{\frac{\rho_1-1}{\rho_1}} + (1 - \phi_1) \right]^{\frac{\sigma-\rho_1}{(\rho_1-1)\sigma}} [1 - \phi_1] \varrho_{NRCt}, \quad (13)$$

$$w_t^{NRM} = \Xi_t \varphi_2 \left[\phi_2 \left(\frac{K_{et}}{N_{NRCt}} \right)^{\frac{\rho_2-1}{\rho_2}} + (1 - \phi_2) \left(\frac{N_{NRMt}}{N_{NRCt}} \right)^{\frac{\rho_2-1}{\rho_2}} \right]^{\frac{\sigma-\rho_2}{(\rho_2-1)\sigma}} [1 - \phi_2] \left(\frac{N_{NRMt}}{N_{NRCt}} \right)^{-\frac{1}{\rho_2}} \varrho_{NRMt}, \quad (14)$$

$$w_t^{RC} = \Xi_t \varphi_3 \left[\phi_3 \left(\frac{K_{st}}{N_{NRCt}} \right)^{\frac{\rho_3-1}{\rho_3}} + (1 - \phi_3) \left(\frac{N_{RCt}}{N_{NRCt}} \right)^{\frac{\rho_3-1}{\rho_3}} \right]^{\frac{\sigma-\rho_3}{(\rho_3-1)\sigma}} [1 - \phi_3] \left(\frac{N_{RCt}}{N_{NRCt}} \right)^{-\frac{1}{\rho_3}} \varrho_{RCt}, \quad (15)$$

$$w_t^{RM} = \Xi_t (1 - \varphi_1 - \varphi_2 - \varphi_3) \left(\frac{N_{RMt}}{N_{NRCt}} \right)^{-\frac{1}{\sigma}} \varrho_{RMt}, \quad (16)$$

$$r_t^s = A_t \alpha \left[\frac{K_{et}}{N_{NRCt}} \right]^{\alpha-1} \Lambda_t^{\frac{\sigma(1-\alpha)}{\sigma-1}}, \quad (17)$$

$$r_t^e = \Xi_t \left[\varphi_1 \left(\phi_1 \left[\frac{K_{et}}{N_{NRCt}} \right]^{\frac{\rho_1-1}{\rho_1}} + [1 - \phi_1] \right)^{\frac{\sigma-\rho_1}{(\rho_1-1)\sigma}} \phi_1 \left(\frac{K_{et}}{N_{NRCt}} \right)^{-\frac{1}{\rho_1}} + \right. \\ \left. \varphi_2 \left(\phi_2 \left[\frac{K_{et}}{N_{NRCt}} \right]^{\frac{\rho_2-1}{\rho_2}} + [1 - \phi_2] \left[\frac{N_{NRMt}}{N_{NRCt}} \right]^{\frac{\rho_2-1}{\rho_2}} \right)^{\frac{\sigma-\rho_2}{(\rho_2-1)\sigma}} \phi_2 \left(\frac{K_{et}}{N_{NRCt}} \right)^{-\frac{1}{\rho_2}} + \right. \\ \left. \varphi_3 \left(\phi_3 \left[\frac{K_{et}}{N_{NRCt}} \right]^{\frac{\rho_3-1}{\rho_3}} + [1 - \phi_3] \left[\frac{N_{RCt}}{N_{NRCt}} \right]^{\frac{\rho_3-1}{\rho_3}} \right)^{\frac{\sigma-\rho_3}{(\rho_3-1)\sigma}} \phi_3 \left(\frac{K_{et}}{N_{NRCt}} \right)^{-\frac{1}{\rho_3}} \right], \quad (18)$$

⁸Marginal products are expressed as functions of the ratios between each factor and the non-routine cognitive labor for the purpose of constructing the solution algorithm.

where⁹

$$\Xi_t = A_t \left[\frac{K_{st}}{N_{NRCt}} \right]^\alpha [1 - \alpha] \Lambda_t^{\frac{1-\sigma\alpha}{\sigma-1}}.$$

The capital laws of motion are:

$$K_{st+1} = (1 - \delta_s)K_{st} + X_{st}, \quad (19)$$

$$K_{et+1} = (1 - \delta_e)K_{et} + X_{et}, \quad (20)$$

where δ_s and δ_e are the depreciation rates.

Government

The social security system is managed by the government and runs a balanced budget. Revenues are collected from taxes on employees and on the representative firm at rates τ_{ss} and $\tilde{\tau}_{ss}$, respectively, and are used to pay retirement benefits, Ψ_t .

The government taxes consumption, τ_c , and capital income, τ_k , at flat rates. The labor income tax follows a non-linear functional form as in [Heathcote et al. \(2019\)](#) and [Benabou \(2002\)](#):

$$y_a = 1 - \theta_0 y^{-\theta_1}, \quad (21)$$

where θ_0 and θ_1 define the level and progressivity of the tax schedule, respectively. y is the pre-tax labor income and y_a is the after-tax labor income.¹⁰

Tax revenues from consumption, labor and capital income taxes are used to finance public

⁹Variable Λ_t is defined as:

$$\begin{aligned} \Lambda_t = & \varphi_1 \left(\phi_1 \left[\frac{K_{et}}{N_{NRCt}} \right]^{\frac{\rho_1-1}{\rho_1}} + [1 - \phi_1] \right)^{\frac{\rho_1(\sigma-1)}{(\rho_1-1)\sigma}} + \varphi_2 \left(\phi_2 \left[\frac{K_{et}}{N_{NRCt}} \right]^{\frac{\rho_2-1}{\rho_2}} + [1 - \phi_2] \left[\frac{N_{NRMt}}{N_{NRCt}} \right]^{\frac{\rho_2-1}{\rho_2}} \right)^{\frac{\rho_2(\sigma-1)}{(\rho_2-1)\sigma}} \\ & + \varphi_3 \left(\phi_3 \left[\frac{K_{et}}{N_{NRCt}} \right]^{\frac{\rho_3-1}{\rho_3}} + [1 - \phi_3] \left[\frac{N_{RCt}}{N_{NRCt}} \right]^{\frac{\rho_3-1}{\rho_3}} \right)^{\frac{\rho_3(\sigma-1)}{(\rho_3-1)\sigma}} + (1 - \varphi_1 - \varphi_2 - \varphi_3) \left(\frac{N_{RMt}}{N_{NRCt}} \right)^{\frac{\sigma-1}{\sigma}}. \end{aligned}$$

¹⁰See the [Holter et al. \(2014\)](#) for a detailed discussion of the properties of this tax function.

consumption of goods, G_t , public debt interest expenses, $r_t B_t$, and lump sum transfers, g_t . Denoting social security revenues by R_t^{ss} and the other tax revenues as T_t , the government budget constraint is defined as:

$$g_t \left(45 + \sum_{j \geq 45} \Omega_j \right) = T_t - G_t - r_t B_t, \quad (22)$$

$$\Psi_t \left(\sum_{j \geq 45} \Omega_j \right) = R_t^{ss}. \quad (23)$$

Asset Structure

Households hold three asset types: non-equipment capital, k_s , equipment, k_e , and government bonds, b .¹¹ There is no investment-specific technological change in the steady state, i.e., $\xi' = \xi$. The return rate on the bond must satisfy:

$$\frac{1}{\xi} [\xi + (r_e - \xi \delta_e)(1 - \tau_k)] = 1 + r(1 - \tau_k), \quad (24)$$

which follows from non-arbitrage: investing in equipment capital must yield the same return as investing the same amount in bonds. By the same token, the return rate on non-equipment capital must satisfy:

$$\frac{1}{\xi} [\xi + (r_e - \xi \delta_e)(1 - \tau_k)] = 1 + (r_s - \delta_s)(1 - \tau_k). \quad (25)$$

The state variable for the consumer is defined as:

$$h \equiv \xi k_e + b + k_s, \quad (26)$$

Household Problem

On any given period a household is defined by age, j , asset position h , time discount factor $\beta \in \{\beta_1, \beta_2, \beta_3, \beta_4\}$, permanent ability a , a persistent idiosyncratic productivity shock u . A working-age household chooses consumption, c , work hours, n , and future asset holdings, h' , to solve his

¹¹In what follows, we suppress time subscripts for simplicity and use prime (') to denote next period values of a variable.

optimization problem. The household budget constraint is given by:

$$c(1 + \tau_c) + \xi k'_e + b' + k'_s = [\xi + (r_e - \xi \delta_e)(1 - \tau_k)] k_e + [1 + r(1 - \tau_k)] b + [1 + (r_s - \delta_s)(1 - \tau_k)] k_s + q\Gamma + g + Y^N, \quad (27)$$

where Y^L is the household's labor income after social security and labor income taxes. Using 24 and 25, in equilibrium we can rewrite the budget constraint as:

$$c(1 + \tau_c) + h' = (h + \Gamma)[1 + r(1 - \tau_k)] + g + Y^N. \quad (28)$$

The household problem can then be formulated recursively as:

$$\begin{aligned} V(j, h, \beta, a, u) &= \max_{c, n, h'} \left[U(c, n) + \beta \mathbb{E}_{u'} [V(j + 1, h', \beta, a, u')] \right] \\ \text{s.t.:} \\ c(1 + \tau_c) + h' &= (h + \Gamma)[1 + r(1 - \tau_k)] + g + Y^N \\ Y^N &= \frac{nw(j, a, u)}{1 + \tilde{\tau}_{ss}} \left(1 - \tau_{ss} - \tau_l \left[\frac{nw(j, a, u)}{1 + \tilde{\tau}_{ss}} \right] \right) \\ n \in [0, 1], \quad h' &\geq -\underline{h}, \quad h_0 = 0, \quad c > 0. \end{aligned}$$

The problem of a retired household differs in three features: age dependent probability of dying $\pi(j)$, the bequest motive $D(h')$, and labor income, which is replaced by constant retirement benefits. Therefore, the retired household's problem is defined as:

$$\begin{aligned} V(j, h, \beta) &= \max_{c, h'} \left[U(c, n) + \beta(1 - \pi(j))V(j + 1, h', \beta) + \pi(j)D(h') \right] \\ \text{s.t.:} \\ c(1 + \tau_c) + h' &= (h + \Gamma)[1 + r(1 - \tau_k)] + g + \Psi \\ h' &\geq -\underline{h}, \quad c > 0. \end{aligned}$$

Stationary Recursive Competitive Equilibrium

$\Phi(j, h, \beta, a, u)$ is the measure of agents with corresponding characteristics (j, h, β, a, u) . The stationary recursive competitive equilibrium is defined by:

1. Taking factor prices and initial conditions as given, the value function $V(j, h, \beta, a, u)$ and the policy functions, $c(j, h, \beta, a, u)$, $h'(j, h, \beta, a, u)$, and $n(j, h, \beta, a, u)$ solve the household's optimization problem.

2. Markets clear:

$$\xi K_e + B + K_s = \int h + \Gamma d\Phi,$$

$$N_{RM} = \varrho_{RM} \int n_{RM} d\Phi, \quad N_{RC} = \varrho_{RC} \int n_{RC} d\Phi,$$

$$N_{NRM} = \varrho_{NRM} \int n_{NRM} d\Phi, \quad N_{NRC} = \varrho_{NRC} \int n_{NRC} d\Phi,$$

$$C + G + \delta_s K_s + \xi \delta_e K_e = F(K_s, K_e, N_{NRC}, N_{NRM}, N_{RC}, N_{RM}).$$

3. Equations 14-18 hold.

4. The government budget balances:

$$g \int d\Phi + G + rB = \int \tau_k r(h + \Gamma) + \tau_c c + n\tau_l \left[\frac{n\omega(a, u, j)}{1 + \tilde{\tau}_{ss}} \right] d\Phi.$$

5. The social security system balances:

$$\int_{j \geq 45} \Psi d\Phi = \frac{\tilde{\tau}_{ss} + \tau_{ss}}{1 + \tilde{\tau}_{ss}} \left(\int_{j < 45} n\omega d\Phi \right).$$

6. The assets of the deceased at the beginning of the period are uniformly distributed among the living:

$$\Gamma \int \omega(j) d\Phi = \int (1 - \omega(j)) h d\Phi.$$

Wage Premia

The wage premia obtained from the model are:

$$\frac{w^{NRC}}{w^{RM}} = \frac{[1 - \phi_1]\phi_1}{1 - \phi_1 - \phi_2 - \phi_3} \left[\phi_1 \left(\frac{K_e}{N_{NRC}} \right)^{\frac{\rho_1-1}{\rho_1}} + (1 - \phi_1) \right]^{\frac{\sigma-\rho_1}{(\rho_1-1)\sigma}} \left[\frac{h_{RM}}{h_{NRC}} \right]^{\frac{1}{\sigma}} \left[\frac{Q_{NRC}}{Q_{RM}} \right]^{1-\frac{1}{\sigma}}, \quad (29)$$

$$\frac{w^{NRM}}{w^{RM}} = \frac{[1 - \phi_2]\phi_2}{1 - \phi_1 - \phi_2 - \phi_3} \left[\phi_2 \left(\frac{K_e}{N_{NRM}} \right)^{\frac{\rho_2-1}{\rho_2}} + (1 - \phi_2) \right]^{\frac{\sigma-\rho_2}{(\rho_2-1)\sigma}} \left[\frac{h_{RM}}{h_{NRM}} \right]^{\frac{1}{\sigma}} \left[\frac{Q_{NRM}}{Q_{RM}} \right]^{1-\frac{1}{\sigma}}, \quad (30)$$

$$\frac{w^{RC}}{w^{RM}} = \frac{[1 - \phi_3]\phi_3}{1 - \phi_1 - \phi_2 - \phi_3} \left[\phi_3 \left(\frac{K_e}{N_{RC}} \right)^{\frac{\rho_3-1}{\rho_3}} + (1 - \phi_3) \right]^{\frac{\sigma-\rho_3}{(\rho_3-1)\sigma}} \left[\frac{h_{RM}}{h_{RC}} \right]^{\frac{1}{\sigma}} \left[\frac{Q_{RC}}{Q_{RM}} \right]^{1-\frac{1}{\sigma}}, \quad (31)$$

Derivative of $\ln \pi$ wrt K_e/N_{NRC} :

$$\frac{\partial \ln \pi}{\partial (K_e/N_{NRC})} = \frac{\sigma - \rho_1}{\rho_1 \sigma} \frac{\phi_1}{\phi_1 \left(\frac{K_e}{N_{NRC}} \right) + (1 - \phi_1) \left(\frac{K_e}{N_{NRC}} \right)^{\frac{1}{\rho_1}}} \quad (32)$$

Labor share equation:

5 Production function estimation

In this section, we go over the stochastic specification of the production function model, the equations to be estimated, and the results. The data is described on Appendix B.

Stochastic specification

The stochastic elements in our model are the unobserved technology components: (i) the relative price of equipment; (ii) the set of labor-specific efficiency indices; (iii) the factor-neutral technological process.

We assume that the relative price of equipment ($\tilde{\xi}_t = \xi_t/\xi_{t-1}$) is trend stationary, and confirm this with a Dickey-Fuller test. We assume that the labor efficiency index processes have different linear trend for each labor variety. Defining the processes in logs we have:

$$\psi_t \equiv \ln(Q_t), \quad \psi_t = \psi_0 + \psi_1 t + v_t, \quad (33)$$

where ψ_t is a (4×1) vector of the log of the latent efficiency indices, ψ_0 is a (4×1) vector of constants which specify the value of the indices at the beginning of the sample, ψ_1 is a (4×1) vector of growth rates, and ν_t is a (4×1) vector of shock processes that we assume to be multivariate normal, i.i.d. with covariance matrix Ω : $\nu_t \sim N(0, \Omega)$. The i.i.d. assumption simplifies the identification of the factor-neutral technological change, A_t , which is described below.

Equation specification

We will use a system with three sets of equations obtained from the first order conditions of agents in order to estimate the model: (i) the labor share; (ii) the wage bills relative to the routine manual labor variety; and (iii) a no-arbitrage condition between investing in equipment and non-equipment capital. These are defined as follows:

$$\frac{\sum_{s \in S} w_t^s h_{st}}{Y_t} = lsh_t(\psi_t, X_t; \theta), \quad s \in S = \{NRC, NRM, RC, RM\}, \quad (34)$$

$$\frac{w_t^s h_{st}}{w_t^{RM} h_{RMt}} = wbr_{st}(\psi_t, X_t; \theta), \quad s \in S = \{NRC, NRM, RC\}, \quad (35)$$

and

$$1 + [F_{K_s}(\psi_{t+1}, X_{t+1}; \theta) - \delta_{st+1}] = E_t \left(\frac{\xi_{t+1}}{\xi_t} \right) (1 - \delta_{et+1}) + \frac{F_{K_e}(\psi_{t+1}, X_{t+1}; \theta)}{\xi_t} \quad (36)$$

where 36 is obtained from equation 25, assuming that $\xi_t \neq \xi_{t+1}$, and where we substituted the return rates by factor marginal productivities. Note that taxes do not appear in the equation, as they cancel out. Depreciation rates are indexed by time since they change over the time (see Appendix B). Both lsh_t and wbr_{st} are functions of X_t and θ . X_t is the vector of inputs, depreciation rates and the capital income tax rate $\{K_{st}, K_{et}, h_{NRCt}, h_{NRMt}, h_{RCt}, h_{RMt}, \delta_{st}, \delta_{et}, \tau_{kt}\}$. The vector θ is the set of parameters $\{\alpha, \rho_1, \rho_2, \rho_3, \phi_1, \phi_2, \phi_3, \varphi_1, \varphi_2, \varphi_3, \psi_0, \psi_1, \Omega, \eta_\omega\}$. η_ω is the standard deviation of the error term in the equipment prices equation, which we specify below. Like Krusell et al. (2000), we assume that there is no risk premium in equation 36, and that tax treatment is identical between equipment and non-equipment capital returns. Krusell et al. (2000) report that they relax

the latter assumption and find that it produces similar results. Finally, we substitute the first term on the right hand side of equation 36 with $E_t(\xi_{t+1}/\xi_t)(1 - \delta_{et}[1 - \tau_{kt}]) + \omega_t$, where ω_t is the i.i.d. forecast error and $\omega_t \sim N(0, \eta_\omega^2)$. This set of assumptions imply that $A_t = Y_t/G(\cdot)$ from equation 12.

The data for the labor inputs in hours and the hourly (nominal) wages are used to obtain the left side of equations 34 and 35. We use a measure of nominal GDP for Y_t to calculate the labor share and GDP at constant prices to find A_t . The construction of capital stocks and depreciation rates is discussed on Appendix B. Capital income tax rates are constructed using the method of Mendoza et al. (1994).

Given that this is a non-linear system of nine equations with unobserved state variables, standard linear Kalman filter techniques cannot be applied to estimate the parameter vector θ . Ohanian et al. (1997) propose a two-step version of the SPML estimator to find θ for this type of problem, which we detail on Appendix D.

The parameter vector θ has a dimension of 35. Our sample contains 49 observations. We reduce the number of parameters to estimate by calibrating some of the parameters and imposing restrictions. First, we impose that Ω be a diagonal matrix and that the variance of the disturbances be identical for all labor types. Thus, $\Omega = \eta_v^2 I_4$, where η_v^2 is the common innovation variance and I_4 is a (4) identity matrix. We choose to fix ϕ_{NRC0} , the initial level of the labor quality of routine manual workers, which acts as a scaling factor. Finally, we regress the variation rate of the relative price of equipment on a linear trend. We set η_ω to be equal to the estimated standard deviation of the error term in the regression $\tilde{\sigma}_\omega = 0.032$. This reduces the number of parameters to be estimated to 19: the common variance of the latent processes, η_v^2 , the elasticities, $\sigma, \rho_1, \rho_2, \rho_3$, the production function share parameters, $\alpha, \phi_1, \phi_2, \phi_3, \varphi_1, \varphi_2, \varphi_3$, and the parameters governing the state variables, except for ψ_{NRC0} .

Results

6 Calibration

This section describes the calibration of the baseline model to match the U.S. economy in 1980. Parameters are either set directly (i.e., without solving the full model) to match their empirical counterparts, or estimated by simulated method of moments (SMM). Table 1 lists parameter values and sources. The production function parameters are set to equal the estimates described in the previous section.

Preferences

There has been a considerable debate in the literature on the value of the Frisch elasticity of labor supply, η , with estimates ranging from 0.5 to 2 or higher. We set it to 1.0, as in [Brinca et al. \(2016\)](#). Discount factors, disutility from work and the borrowing limit are calibrated by SMM and are discussed below.

Labor productivity

The wage profile through the life cycle (see equation 2) is calibrated directly from the data. We run the following regression, using Panel of Study of Income Dynamics (PSID) data:

$$\ln(w_i) = \ln(w) + \gamma_1 j + \gamma_2 j^2 + \gamma_3 j^3 + \varepsilon_i. \quad (37)$$

where j is the age of individual i . We then use the residuals of the equation to estimate the parameters governing the idiosyncratic shock ρ and σ_ε . The wage differential of each group, a , is calibrated to match the log difference in average wages between groups in 1980. The employment level of each group, which is equal to the probability of being born into a given group, is set to equal its observed weight in total employment in 1980.

Technology

We use the method by [Eden and Gaggl \(2018\)](#) to estimate the parameters of the production function, described in appendix . Both total factor productivity and the relative price of investment

Table 1: 1980 Calibration Summary

Description	Parameter	Value	Source
Preferences			
Inverse Frisch elasticity	η	1.000	Brinca et al. (2016)
Risk aversion parameter	λ	1.000	Brinca et al. (2016)
Labor productivity			
Parameter 1 age profile of wages	γ_1	0.265	Brinca et al. (2016)
Parameter 2 age profile of wages	γ_2	-0.005	Brinca et al. (2016)
Parameter 3 age profile of wages	γ_3	0.000	Brinca et al. (2016)
Variance of idiosyncratic risk	σ_ϵ	0.307	Brinca et al. (2016)
Persistence idiosyncratic risk	ρ_u	0.335	Brinca et al. (2016)
NRC % wage difference	a_1	0.484	CPS
NRM % wage difference	a_2	-0.251	CPS
RC % wage difference	a_3	0.105	CPS
NRC weight	p_1	0.226	CPS
NRM weight	p_2	0.170	CPS
RC weight	p_3	0.181	CPS
Technology			
Depreciation rate	δ	0.060	Brinca et al. (2016)
Share of the composite	ϕ_1	0.516	Authors' calculations
Share of capital	ϕ_2	0.654	Authors' calculations
EOS routine/composite	ρ	5.628	Authors' calculations
EOS non-routine/capital	σ	0.827	Authors' calculations
Total factor productivity	A	1.000	Normalization
Relative price of investment goods	ξ	1.000	Normalization
Government and SS			
Consumption tax rate	τ_c	0.054	Mendoza et al. (1994)
Capital income tax rate	τ_k	0.469	Mendoza et al. (1994)
Tax scale parameter	θ_0	0.850	Ferriere and Navarro (2018)
Tax progressivity parameter	θ_1	0.160	Ferriere and Navarro (2018)
Government debt to GDP	B/Y	0.320	FRED
Military spending to GDP	G/Y	0.050	World Bank
SS tax employees	τ_{ss}	0.061	Social Security Bulletin, July 1981
SS tax employers	$\tilde{\tau}_{ss}$	0.061	Social Security Bulletin, July 1981

are set to unity in 1980.

Government Budget and Social Security

As described before, to capture the progressivity of both the tax schedule and government transfers, we use the same labor income tax function as [Benabou \(2002\)](#) (equation 21). To estimate θ_0 and θ_1 in 1980 we use the method in [Ferriere and Navarro \(2018\)](#).

For the social security rates we assume no progressivity. Both social security tax rates, on behalf of the employer and on behalf of the employee, are set to 0.06, the average rate in 1980. Finally, we set τ_c and τ_k to match the values obtained in [Mendoza et al. \(1994\)](#) for 1980, i.e, $\tau_c = 0.05$, $\tau_k = 0.47$.

Parameters calibrated using SMM

To calibrate the parameters that do not have any direct empirical counterparts, φ , β_1 , β_2 , β_3 , β_4 , \underline{h} and χ , we use the simulated method of moments so that we minimize the following loss function:

$$L(\varphi, \beta_1, \beta_2, \beta_3, \beta_4, \underline{h}, \chi) = \|M_m - M_d\| \quad (38)$$

with M_m and M_d being the moments in the data and in the model respectively.

Given that we have seven parameters, we need seven data moments to have an exactly identified system. The seven moments we target in the data are the ratio of the average net asset position of households 65 and above relative to the average asset holdings in the economy, four wealth quintiles and the wage premium. Calibration fit is presented on Table 2. Table 3 presents the calibrated parameters. Note that the model fits the target data with an error below 0.001, with the exception of the Q_{80} moment. This inability to match the upper tail of the wealth distribution is the result of a low level of capital-to-output required to achieve the target level of the wage premium. This is due to the fact that we use a measure of productive capital that excludes residential structures, a large portion of household wealth, to estimate the parameters of the production function. It also explains the low level of calibrated discount factors.

Table 2: Calibration fit

Data moment	Description	Source	Data Value	Model value
65-on/all	Average wealth of households 65 and over	US Census Bureau	1.51	1.51
w_{NR}/w_R	Wage Premium	CPS	0.00	0.00
\bar{n}	Fraction of hours worked	PWT	1/3	1/3
$Q_{20}, Q_{40}, Q_{60}, Q_{80}$	Wealth percentiles	WID	-0.01, 0.00, -0.04, 0.17	-0.01, 0.00, -0.04, 0.30

Table 3: Parameters Calibrated Endogenously

Parameter	Value	Description
φ	4.28	Bequest utility
$\beta_1, \beta_2, \beta_3, \beta_4$	0.939, 0.903, 0.902, 0.890	Discount factors
χ	6.1	Disutility of work
\underline{h}	0.02	Borrowing limit

7 Quantitative results

The main experiment conducted in this section is to calculate a new steady state where government and technology parameters are substituted to match more recent values. Concretely, we have chosen 2010 values to calibrate the new steady state. We then calculate the changes in observed inequality statistics and evaluate which parameters are responsible for the most significant changes in those variables. Note that the transition between steady states is not taken into account.

Parameters related to tastes, individual productivity processes and the production function are kept constant between steady states: the age profile of wages ($\gamma_1, \gamma_2, \gamma_3$), the idiosyncratic productivity process (ρ_u and σ_ϵ), preferences ($\lambda, \eta, \beta_1, \beta_2, \beta_3$, and β_4), the borrowing constraint (\underline{h}), depreciation (δ), and production function parameters (ϕ_1, ϕ_2, σ and ρ).

Table 4 displays a comparison between 1980 and the new steady state parameter values. The most relevant changes in the government calibration are in capital income taxes, the labor income tax progressivity parameter, the level of government debt and Social Security taxes. Investment prices are calibrated to match the observed drop from 1980 to 2010. Finally, group employment weights are adjusted to match those observed in 2010.

Results from the experiment are displayed on Table 5. Model pre- and post-tax Gini index

Table 4: Parameter shifts

Parameter	Description	1980	New SS
τ_c	Consumption tax	0.050	0.054
τ_k	Capital income tax	0.469	0.360
θ_1	Tax level parameter	0.850	0.869
θ_2	Tax progressivity parameter	0.160	0.095
B/Y	Government debt	0.320	0.880
τ_{ss}	Employee SS tax	0.061	0.077
$\tilde{\tau}_{ss}$	Employer SS tax	0.061	0.077
ξ	Investment price	1.000	0.586
p_1	NRC weight	0.226	0.392
p_2	NRM weight	0.170	0.134
p_3	RC weight	0.181	0.228

increase by 0.053 and 0.050, respectively, when compared to the 1980 calibration of the U.S. economy, or 42 and 47% of the total increase observed in the data between 1980 and 2010. The gap between the two inequality statistics also increases but only by a fraction of the observed data. The wealth Gini index also increases but only by 0.05, compared to the 0.07 change observed in the data. The non-routine wage premium in the new steady state is nearly one and a half times larger than the one actually observed in the data, indicating that the increase in the Gini index is being driven by an excessive increase in this statistic compared to the one observed in the data.

To counter this drawback we calibrate a new steady state in which the drop in investment prices is such that the non-routine wage premium in 2010 is matched exactly. This implies a 30% drop in the relative price of investment in lieu of the 40% drop observed in the data. This steady state is characterized by a smaller increase in pre- and post-tax income dispersion.

Note that the inequality statistics generated by the model are significantly below their empirical counterparts. This is the result of limited sources of inequality built into the model. Within each of the groups, differences in income between individuals are the result of either age/experience or idiosyncratic risk. Differences in permanent ability as in [Brinca et al. \(2016\)](#), which capture residual inequality within each group, are not modeled. Nevertheless, we are focused on changes rather than levels at this stage.

We run additional experiments to isolate the contribution of each set of parameters. Figure 3

Table 5: Experiment Results

	Data		Model									
	1980	2010	1980	New SS	New SS*	$\tilde{\tau}_{SS}, \tau_{SS}$	τ_k	θ_2	B/Y	Employment	ξ	ξ^*
Labor share	0.636	0.564	0.643	0.574	0.578	0.644	0.637	0.641	0.647	0.629	0.623	0.630
Gini index (pre-tax)	0.458	0.586	0.315	0.369	0.346	0.294	0.322	0.322	0.308	0.308	0.371	0.354
Gini index (post-tax)	0.374	0.480	0.229	0.279	0.265	0.206	0.233	0.246	0.230	0.224	0.252	0.245
Gini gap	0.085	0.107	0.086	0.090	0.082	0.088	0.089	0.076	0.079	0.085	0.119	0.109
NR wage premium (%)	0.000	0.143	0.000	0.334	0.143	-0.025	0.084	0.012	-0.060	-0.161	0.375	0.217
Wealth Gini index	0.81	0.88	0.70	0.73	0.72	0.70	0.69	0.69	0.69	0.69	0.74	0.72
Q_{20}	-0.01	-0.03	-0.01	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
Q_{40}	0.00	-0.03	0.01	0.01	0.01	-0.01	0.00	0.00	0.00	-0.01	0.01	0.00
Q_{60}	0.04	0.00	0.05	0.07	0.08	0.05	0.06	0.07	0.06	0.06	0.07	0.07
Q_{80}	0.17	0.12	0.31	0.22	0.23	0.31	0.31	0.32	0.32	0.32	0.21	0.23

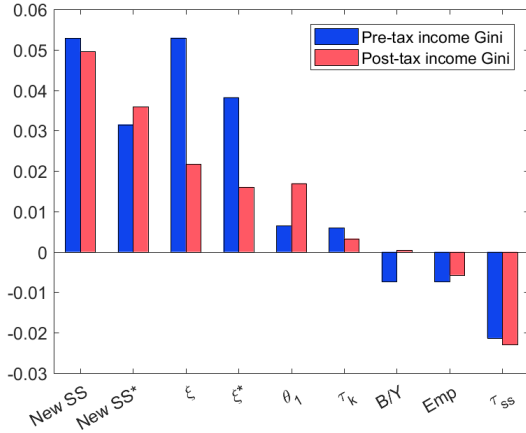
Notes: table indicates the values of each statistic from the data and the different model calibrations. A star indicates the model where the drop in the relative price of investment is set to match the rise in the non-routine wage premium. The isolated impacts of τ_c , θ_0 and government spending are not shown due to their residual contribution to the changes of the statistics of interest.

provides a visualization of the results, which are detailed on Table 5. The parameter which contributes the most to the pre-tax income Gini is the change in the relative price of investment. This is due to its significant impact on the non-routine wage premium, which creates a large wedge between non-routine and routine wages. However, the impact of this parameter is dampened by the progressivity of the labor tax schedule (which is kept at 1980 levels), with the increase of the post-tax income Gini being limited to almost half of the pre-tax increase.¹² By itself, the change in investment prices is responsible for 21% of the increase in the pos-tax Gini index. In the experiment where the non-routine wage premium is matched, this figure drops to 15%.

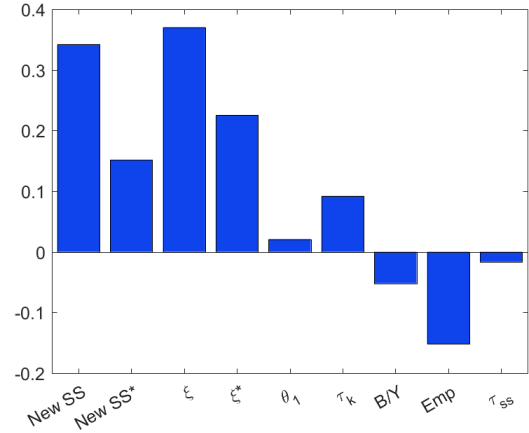
The drop in progressivity also increases income dispersion by reducing the distortion on the labor supply at the top of the earnings distribution. In particular, it has a significant impact on the pos-tax inequality. Coupled with the drop in investment prices, it is responsible for generating the large increase in the post-tax income Gini in the full model. Alone, this channel accounts for 16% of the change in the post-tax Gini observed in the data.

The reduction in capital income taxes fosters capital accumulation, increasing the marginal productivity of non-routine labor relative to routine labor and raising the wage premium. Fur-

¹²Note that changes in the non-routine wage premium have a non-trivial effect on the earnings distribution: while they increase the wage of the non-routine skilled group, it also reduces the negative differential of non-routine unskilled workers, bringing their earnings closer to the average.



(a) Income dispersion



(b) Change in wage premium

Figure 3: Impact of changes in parameters. A star indicates the model where the drop in the relative price of investment is set to match the rise in the non-routine wage premium. The y-axis indicates the difference observed in each calibration with respect to the 1980 calibration.

thermore, it also produces greater post-tax capital income, contributing to the increase in income dispersion, albeit only modestly.

Some of the observed structural shifts during our period of analysis have also contributed to a dampening in the effects produced by investment-specific technological change and the reduction in progressivity. The large increase in government debt-to-GDP produces a crowding out of private capital, reducing the relative productivity of non-routine labor and the wage premium. The employment shifts in this period, in particular the surge in the relative employment of non-routine workers, has also counteracted wage premium growth in a significant manner, generating a modest reduction in overall income inequality by itself.

Finally, the increase in Social Security contributions has a large negative effect income dispersion. This is the result of the highly progressive nature of the Social Security bloc in the model: contributions are collected as a flat tax on labor income and redistributed lump sum to retirees.¹³ This apparent modeling limitation cushions the increase in income inequality from other sources, implying that the increase in the income Gini in the full model would be larger if the impact of social security distributions would be more muted.

¹³Both the data and the model pre-tax income Gini include pensions, but exclude Social Security contributions.

Our future work will involve the nesting of a total of four labor varieties in the production function, which will allow us to generate an endogenous skill premium and increase the quantitative significance of the changes in the income Gini index.

8 Conclusion

We propose a framework which can quantitatively explain the rise in inequality in the U.S. since 1980. Our calculations show that structural changes in technology, government policy and employment are able to produce an increase of the income Gini index which is one third of the change in post-tax income Gini observed in the data.

The main mechanisms at play are the rise in the non-routine wage premium, which increases the dispersion in the earnings distribution and the ability of a fraction of the population to accumulate greater amounts of wealth relative to the lower quintiles. Additionally, the reduction in the progressivity of the labor income tax schedule reduces labor supply distortions and increases the post-tax income Gini index. We show that ISTC alone accounts for 15% of the change in observed post-tax income Gini, while the reduction in progressivity accounts for 16%.

Our next steps involve expanding the model to account for the role of capital-skill complementarity, in the same manner that capital-non-routine complementarity was introduced. This will allow us to study the effects of technological change on both the non-routine and the skill wage premium in a single framework, and attempt to observe the impact of these two theories of the impact of technological change on earnings inequality. We will also refine our experiment by recasting it as unexpected change in the trend of ISTC, enabling us to analyze the path of income dispersion measures in a more realistic way relative to the comparison of steady states.

A Data sets

A.1 CPS

Imputation. From survey year 1968 to 1975, hours worked in the previous year are not available. We follow [Autor and Acemoglu \(2011\)](#) and impute these by running a regression of hours worked on the previous year on hours worked in the current year, on an indicator variable for whether the individual worked 35+ hours last year or not, on the current labor force status, on an interaction variable between the two previous variables, and on the sector the individual worked in the previous year for the survey years 1976-1978. We then use the estimated equation to assign hours worked in the previous year to the 1968-1975 observations.

Weeks worked last year are not available for 1968-1975 also. We compute mean weeks worked last year by race and gender for the years 1976-78 for each bracket and impute those means for the 1968-1975 period.

Top-coding. To obtain accurate estimates of earnings inequality and wage premia, we have to account for the top-coding in the CPS earnings data. We use the variables *INCWAGE*, *INCLONGj* and *OINCWAGE*, in the taxonomy of [Flood et al. \(2018\)](#). We proceed in two steps: (i) identify top-coded observations; (ii) assuming the underlying distribution is Pareto, we forecast the mean value of top-coded observations by extrapolating a Pareto density fitted to the non-top-coded upper end of the observation distribution. For details on the procedure to approximate the tail of a Pareto distribution see [Heathcote et al. \(2010\)](#).

Top-coding thresholds in the ASEC change across variables and time. Information on top-coding thresholds can be found on the [IPUMS](#) website. Prior to the 1996 survey year, there is little documentation available regarding the thresholds, but the effective top-coding thresholds are provided by IPUMS based on [Larrimore et al. \(2008\)](#). From 1996 onward, the Census Bureau began reporting top-coding thresholds for a set of income variables.

In addition, the Census Bureau has changed its top-coding procedure through time: from 1996 until 2011, the values for top-coded observations were replaced with values based on the

individual's characteristics (so-called cell/group means). From 2011 onward, the Census Bureau shifted from an average-replacement value system to a rank proximity swapping procedure.

Ideally, we would like to use a consistent procedure for handling top-coding across time. However, since the Census Bureau started publishing top-coding procedures in 1996, they drastically reduced public use censoring thresholds. [Heathcote et al. \(2010\)](#) found that the Pareto-extrapolation procedure does not perform well in this case. Therefore, we only apply this procedure until survey year 1995. [Heathcote et al. \(2010\)](#) use the extrapolation until survey year 1999, but we find that this produces a large jump in earnings inequality in the late 90's which does not seem plausible.

Bottom-trimming. According to [Flood et al. \(2018\)](#), there is no publicly available information on bottom-coding thresholds of income variables in the ASEC. To deal with this shortcoming, a common practice in the literature is to select a bottom threshold on earnings for inclusion in the sample. We use the procedure of [Heathcote et al. \(2010\)](#): the final sample only includes observations where the hourly wage is above the minimum threshold of one half of the federal minimum wage in each year (end-year federal minimum wage data for farm and non-farm workers is retrieved from FRED).

Variable definitions. All variables are computed as explained in [Autor and Acemoglu \(2011\)](#).

Sample selection. We build two samples, labeled A and B. Table 6 shows the number of records at each stage of the selection process.

The initial sample is a cleaned version of the raw data, which excludes individual records which are either: below the age of 16 in the previous year, not part of the universe, not wage workers, did not work in the previous year, have zero or missing weights, missing age, or have positive earnings but no weeks worked in the previous year, or vice-versa. In 2014, two distinct samples were drawn because of sample redesign. We keep the sample which is consistent with previous surveys.

Sample A excludes all records where the hourly wage is lower than one half of the federal

Table 6: CPS sample selection (survey years 1968-2017)

	Dropped	Remaining
Initial sample		4,089,617
Wage > 0.5 × min. wage	116,608	3,973,009
Sample A		3,973,009
Age 25-64	861,598	3,111,411
Hours worked per week last year > 6	19,308	3,092,103
Sample B		3,092,103

hourly minimum wage. We assume that this sample is representative of the (non-institutionalized) U.S. population. In order to validate the data, we compare a set of sample statistics on wages and hours worked to their aggregate (NIPA) counterpart. This is shown on Figure [A.1](#).

There is an average absolute deviation of 5% between the NIPA (Table 2.1, line 3) and the CPS wage bill. Regarding hours of part and full-time employees, the NIPA series (Tables 6.9B-D, line 2) is lower by 3.3%, on average, and 6.5% after 1986. The BEA uses BLS data to calculate its hours worked series, but the variables are based on the Quarterly Census of Employment and Wages (QCEW) data, rather than on the ASEC variable “usual hours worked per week last year” used in this paper. The total number of full- and part-time employees is much closer to the NIPA series (Table 6.4B-D, line 2), albeit the gap is still 2.7% on average.

Sample B excludes individuals between 25 and 64 years old in the previous year. We consider that 25 years old is a reasonable cutoff age, where individuals’ occupation choice has stabilized. According to the BLS, for 2018 the labor force participation rate drops from 65% to 27%, on average, between the 55-64 and the 65 and older age brackets, which justifies our upper bound for inclusion in the sample. We also exclude records where individuals usually worked less than 6 hours per week in the previous year. This is the sample we use to calculate inequality and wage premia statistics. For comparison, [Heathcote et al. \(2010\)](#) have 2,578,035 individual records in their individual-level database, covering the 1967-2005 survey years. This implies that we have around 63,000 records per year, on average, while [Heathcote et al. \(2010\)](#) have 68,000.

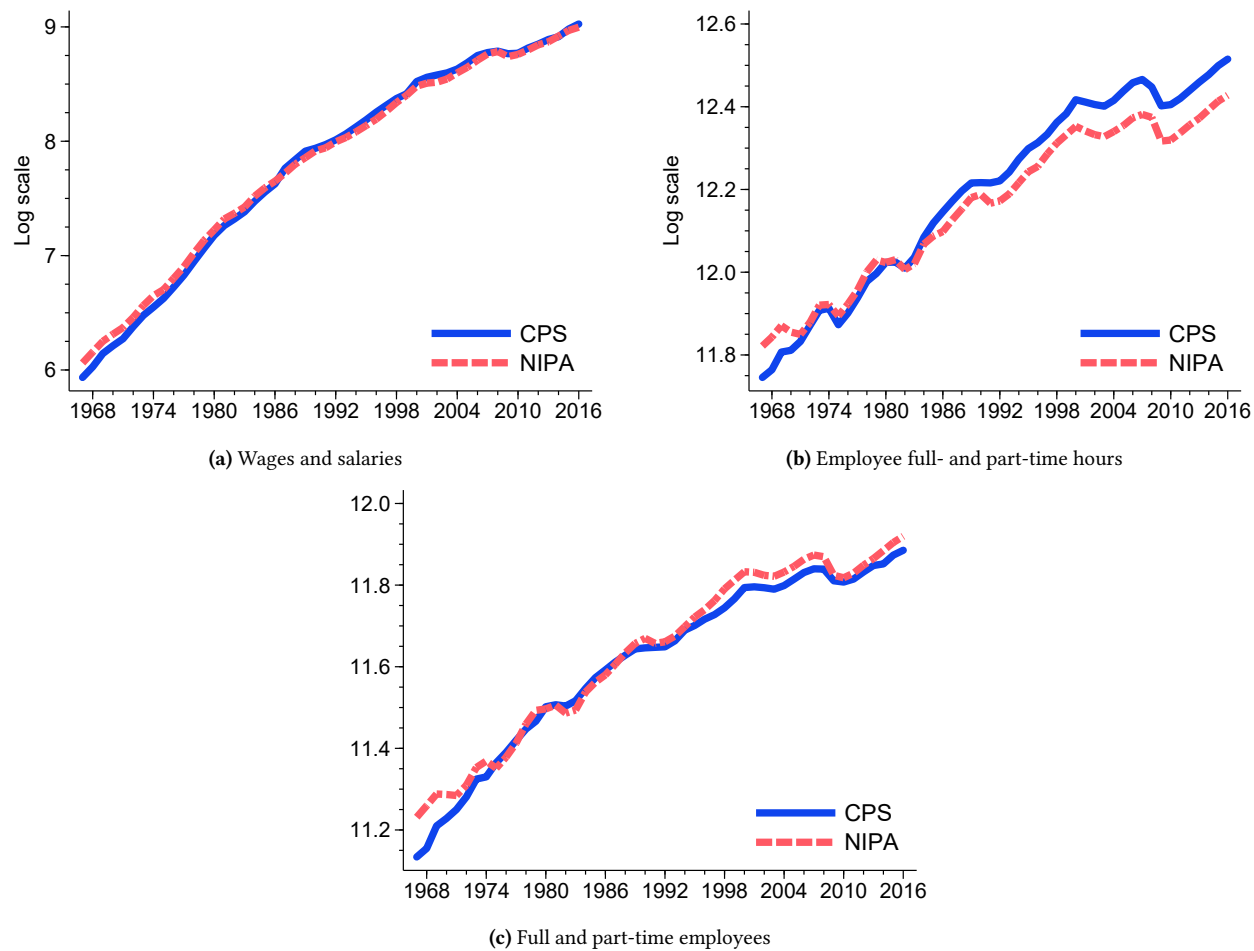


Figure A.1: Comparison between aggregate labor variables in the CPS and in the NIPA.

A.2 PSID

Data set structure. The PSID is a panel data set of U.S. individuals and family units. The original 1968 sample was drawn from two independent sub-samples: an over-sample of roughly 2000 poor families selected from the Survey of Economic Opportunities (SEO), and a nationally representative sample of roughly 3000 families designed by the Survey Research Center (SRC) at the University of Michigan. PSID surveys were annual from 1968 to 1997, and biennial since then.

Since 1968, the PSID has interviewed the individuals from the originally sampled families, which have either remained in the 1968 family unit, or have split off, forming their own. Although some information is collected for each individual in the family unit, the greatest detail is for the so-called husband/reference person and the wife/spouse, when present. In particular, information about wages, occupation, and hours worked are often limited to these two family members, which is the reason why we will focus on these two when analyzing PSID data. See the PSID website for the rules on how the reference person is selected for each family unit.

Because the SRC sample was representative of the U.S. population in 1968, we will restrict our analysis to those families and their split-offs (with a 1968 interview number below 5000). No weights are used for this reason. The main issue with this choice is the inflow of immigrants since 1968. In 1990, the PSID added 2000 Latino households, which covered a major immigration group but missed out on a range of post-1998 immigrants, such as Asians. Because of this short-coming, this sample was dropped in 1995. A new sample of 441 immigrant families, including Asians, was added in 1997 (the so-called “Immigrant” sample).

Variable definitions. To maintain consistency, we use the variable definitions of [Autor and Acemoglu \(2011\)](#), which we used for the CPS data set and which are close to those of [Heathcote et al. \(2010\)](#).

Top-coding.

Bottom-trimming. As with the case of the CPS, we eliminate records where the hourly wage is below one half of the end-year federal minimum wage.

Sample selection. As with the CPS, our data cleaning procedure and sample definition pro-

cedure is described in this subsection. We build two samples, labeled A and B. Table 7 shows the number of records at each stage of the selection process.

Table 7: PSID sample selection (survey years 1968-2017)

	Dropped	Remaining
Initial sample		260,449
Wage > $0.5 \times \text{min. wage}$	69,638	190,811
Sample A		190,811
Age 20-64	9,959	180,852
Hours worked last year > 260	5,704	175,148
Sample B		175,148

The initial sample is a cleaned version of the raw data on heads and spouses only, and excludes individual records which are: below the age of 16 in the previous year, not wage workers, did not work in the previous year, missing age, or have positive earnings but no weeks worked in the previous year, or vice-versa.

B Measures

B.1 Labor supply and wages

We follow the procedure of [Krusell et al. \(2000\)](#) to build measures of wages and the labor supply for each of the labor categories (NRC, NRM, RC, RM). The sample used for this purpose is the same as the one used for the regression analysis described on section 3, apart from the fact that we include workers which did not work full-year or full-time. The reason for this is that in the regression analysis we were aiming to identify the wage premia by observing workers in a similar labor market situation. Here, the aim is to construct measures of labor inputs and wages which will be used in the estimation of the production function. We use these bins in order to exclude phenomena such as the increased labor force participation of women from the estimation. Since the labor supply of part-time workers contributes to real GDP, it is necessary to account for

those. We do not, however, include self-employed individuals in the analysis. In what follows, the subscript t denotes the year and i denotes an individual observation.

For each worker we record the following variables: hours usually worked per week last year, weeks worked last year, earnings last year, potential experience, race, gender, years of education, occupation category and ASEC weight. Potential experience is divided into 5 five-year groups. Race into white, black and other. There are two sexes. Education is divided into 5 categories: no high school, high school graduate, some college, college graduate, and post college education. Occupation groups are defined as before.

Each worker is assigned to one group defined by the variables described. There are 600 groups, each one denoted by $g \in G$. For each group, we construct a measure of the labor input and labor earnings. The individual labor input is defined as $l_{it} = h_{it}wk_{it}$, where h_{it} is hours usually worked last year and wk_{it} is weeks worked last year. The individual wage is defined as $w_{it} = y_{it}/l_{it}$. Therefore for each group g we define:

$$l_{gt} = \frac{\sum_{i \in g} l_{it} \mu_{it}}{\mu_{gt}},$$

$$w_{gt} = \frac{\sum_{i \in g} w_{it} \mu_{it}}{\mu_{gt}},$$

where μ_{it} is the individual ASEC weight and $\mu_{gt} = \sum_{i \in g} \mu_{it}$. We aggregate the set G of 600 sets into the occupation categories previously defined $s \in \{NRC, NRM, RC, RM\}$. From this aggregation we obtain total annual labor input per group, N_{st} , and its hourly wage, w_t^s . We assume that the groups within a category are perfect substitutes, and for aggregation we use as weights the group wages of 1980. Thus, for each category s , we have:

$$N_{st} = \sum_{g \in s} l_{gt} w_{g80} \mu_{gt},$$

$$w_t^s = \frac{\sum_{g \in s} w_{gt} l_{gt} \mu_{gt}}{N_{st}},$$

where μ_{it} is the individual ASEC weight and $\mu_{st} = \sum_{i \in s} \mu_{it}$. This yields a measure of the total labor input in hours by category $(h_{NRCt}, h_{NRMt}, h_{RCt}, h_{RMt})$, as well as average hourly wages $(w_t^{NRC}, w_t^{NRM}, w_t^{RC}, w_t^{RM})$.

B.2 Capital, prices and output

Table 8 shows the definitions of main variables compared with those of [Krusell et al. \(2000\)](#).

Table 8: Comparison with [Krusell et al. \(2000\)](#)

Variable	Definition	Definition (KORV)
Output	Business non-farm gross value added	Private domestic product (excluding housing and farm)
Structures	Non-residential structures (private)	Non-residential structures (private)
Equipment	Equipment (private)	Non-military equipment (private)
Equipment price	Equipment price deflator (BEA)	Authors' calculations based on Gordon (1990)

Capital. Our main source for capital data are the BEA's fixed asset accounts and the NIPA. We use the method of [Berlemann and Wesselhöft \(2014\)](#) to construct measures of the capital stock at constant 2012 prices for equipment and non-residential structures. We only include private capital in our measure. Nominal investment for each asset category is deflated using the investment price index from the BEA. The resulting measures for non-residential structures, K_{et} , and equipment capital, K_{st} . Like [Krusell et al. \(2000\)](#), we interpret these values as being measured in efficiency units.

Equipment prices. To obtain the price of equipment in each year, we aggregate investment price indices from the BEA fixed asset accounts (Table 5.3.4) across equipment types using a Törqvist index. We then divide the resulting average equipment price by the BLS consumer price index for all urban consumers to obtain the relative price of investment.

Depreciation rates. These are obtained using the method by [Eden and Gaggli \(2018\)](#). We use BEA data on the net current cost of the stock of capital, $P_{it}NetStock_{it}$, and depreciation at current cost, $P_{it}Dep_{it}$, to compute depreciation rates, which are given by the following formula:

$$\delta_{jt} = \frac{P_{it}Dep_{it}}{P_{it}NetStock_{it} + P_{it}Dep_{it}}.$$

We compute average depreciation rates for equipment and non-residential structures, with weights given by the capital stocks at constant prices.

Output and the labor share. We use the pre-1999 revision measure of labor share created by [Koh et al. \(2019\)](#), which excludes the accounting impact of intellectual property capital, since we do not explicitly model this type of capital. As the measure of output, we use the BEA's gross national product (Table 1.7.5, line 4), which is the definition consistent with the labor share measure of [Koh et al. \(2019\)](#).

C Wage premium regression

The following cross-sectional regression model is estimated separately for each gender and year t , across individuals, i , weighted with ASEC weights:

$$\ln w_{it} = \alpha_0 t + \alpha_{1t} X_{it} + \epsilon_{it} \quad \text{for } t \in \{1967, \dots, 2016\}$$

The dependent variable is the log of hourly wages, obtained by dividing the yearly labor earnings by the number of hours worked in the corresponding period. The categories contained in X_{it} are:

1. **Five education categories:** high school dropouts, high school graduates, some college, college graduate, and greater than college;
2. **Four occupation categories:** non-routine cognitive, non-routine manual, routine cognitive, routine manual;
3. **Three race categories:** white, black, and non-white other;
4. **Potential experience:** 5, 15, 25, 35, 45 years. Potential experience is as defined in [Autor and Acemoglu \(2011\)](#).

X_{it} also contains interaction terms between occupation type and education, a quartic in experience and interactions of the education dummies and the experience quartic. We use white males with college education and 45 years of potential experience in routine manual occupations as the base set of categories. We then build a set of 600 demographic groups with a constant composition equal to the one observed in 1980. We take the average predicted log wage by group weighted by the composition adjusted hours worked. The wage premia are obtained as the log-difference between the average wage of each of the labor categories and that of the routine manual category. This composition adjusted method must be used to identify the premia because the model is non-linear.

D Production function estimation method

To estimate the production function, we use the two-step SPML estimator proposed by [Ohanian et al. \(1997\)](#). First, we write the non-linear state space model formally. Next, we briefly describe the methods used to estimate it.

Our non-linear state-space system of equations is of the form:

$$\begin{aligned} \text{Measurement equations :} \quad & Z_t = f(X_t, \psi_t, \omega_t; \theta), \\ \text{State equations :} \quad & \psi_t = \psi_0 + \psi_1 t + v_t. \end{aligned}$$

$f(\cdot)$ contains the labor share equation, the three wage bill equations and the no-arbitrage condition. Z_t is thus a (5×1) vector, which is a function of the variables X_t , the log of the unobservable labor quality indices ψ_t , which is a (4×1) vector, and v_t and ω_t which are (5×1) and (4×1) vectors, respectively, of i.i.d. normally distributed disturbances. Like [Krusell et al. \(2000\)](#), we assume that A_{t+1} and ψ_{t+1} are known when investment decisions are made.

The model is estimated in two steps: (i) instrument the variables which are potentially endogenous; and (ii) apply the SPML estimator. We assume that the capital stocks, K_{st} and K_{et} , are exogenous at date t . However, we allow for the possibility that date t values of the labor inputs may respond to realization of the technology and labor quality shocks. To instrument these variables, we run a first stage regression of the labor inputs on a constant, current and lagged equipment and non-equipment capital stocks, the lagged relative price of equipment, a trend and the lagged value of the OECD composite leading indicator of business cycles. \tilde{X}_t is the vector of K_{st} , K_{et} , the instrumented values of the labor inputs, the depreciation rates and the capital income tax.

The SPML procedure is as follows. Given the distributional assumptions on the error terms, for each t we generate S realizations of the dependent variables, each indexed by i , starting at $t = 1$ in two steps:

$$\begin{aligned} \text{Step 1 :} \quad & \psi_t = \psi_0 + \psi_1 t + v_t. \\ \text{Step 2 :} \quad & Z_t^i = f(\tilde{X}_t, \psi_t^i, \omega_t^i, \theta). \end{aligned}$$

In Step 1, we draw a realization of v_t from its distribution (conditional on our guess of Ω) and use it to

construct a date t value for ψ_t . In Step 2, we use our realization of ψ_t , ψ_t^i , together with a draw of ω_t (conditional on our guess of η_ω), to generate a realization of Z_t , Z_t^i . By using this procedure to generate S realization, we can obtain first and second simulated moments, respectively, of Z_t :

$$m_S(\tilde{X}_t; \theta) = \frac{1}{S} \sum_{i=1}^S Z_t^i,$$

$$V_S(\tilde{X}_t; \theta) = \frac{1}{S-1} \sum_{i=1}^S (Z_t^i - m_S(\tilde{X}_t; \theta)) (Z_t^i - m_S(\tilde{X}_t; \theta))'.$$

From this procedure, we will obtain $2T$ moments, which we will use to construct an objective function. Denoting the vector of all actual observations of the dependent variables by Z^T :

$$L_S(Z^T; \theta) = -\frac{1}{2T} \sum_{t=1}^T \left[[Z_t - m_S(\tilde{X}_t; \theta)]' V_S(\tilde{X}_t; \theta)^{-1} [Z_t - m_S(\tilde{X}_t; \theta)] \ln \det(V_S(\tilde{X}_t; \theta)) \right].$$

The SPML estimator, $\tilde{\theta}_{ST}$, is the maximizer of this expression. It is very important that throughout the maximization procedure of the objective function the same set of $(T \times S)$ random realizations of the dependent variables. Otherwise, the likelihood becomes a random function.

E Solution algorithm

To characterize the stationary competitive equilibrium of the model we must find the ratios $\frac{K_s}{N_{NRC}}, \frac{K_e}{N_{NRC}}, \frac{N_{NRM}}{N_{NRC}}, \frac{N_{RC}}{N_{NRC}},$ and $\frac{N_{RM}}{N_{NRC}}$ which clear the capital and labor markets. In addition, we have to fit the tax function, clear the government and social security budget and find the value of Γ which, given a distribution for the state variable h , uniformly distributes the assets of the dead among the living. The algorithm is as follows:

1. Make a guess on $\frac{K_e}{N_{NRC}}, \frac{N_{NRM}}{N_{NRC}}, \frac{N_{RC}}{N_{NRC}},$ and $\frac{N_{RM}}{N_{NRC}}.$
2. Obtain the value of $\frac{K_s}{N_{NRC}}$ which is consistent with the remaining ratios given the no-arbitrage condition 25 using a bisection method. Compute marginal productivities 14-18 with these guesses.
3. Guess g, Ψ, Γ and average earnings.
4. Compute value and policy functions for the retired and active agents by backward induction, given processes for the transitory and permanent shocks. Both shocks are discretized using the Tauchen procedure (Tauchen, 1986), with 4 and 20 states, respectively. We use 20 states for the permanent shock so that we have 5 states for each group supplying a different labor variety. This allows us to calibrate both within-group and between group earnings inequality. The grids for h and n have 24 and 100 points, respectively. In between the grid points, the values of the functions are interpolated using cubic splines.
5. Simulate the model for 120,000 agents, where assets holdings are zero for every agent entering the labor market. Obtain total savings (asset demand), $\int h + \Gamma d\Phi$, and quantities of each labor variety supplied, $N_{NRC}, N_{NRM}, N_{RC}, N_{RM}.$
6. Compute output given the labor supply of households. The quantity of government bonds is obtained by multiplying output by the government debt-to-GDP ratio. The remainder of asset demand must be allocated between non-equipment and equipment capital. The quantity of structures is obtained by multiplying the initial guess of $\frac{K_s}{N_{NRC}}$ by the quantity of labor supplied by households $N_{NRC}.$ The quantity of equipment, measured in consumption units, is the residual of asset demand. If this residual is negative, we set the quantity of equipment to be 10% of the guess for the non-equipment stock, which allows the algorithm to proceed.

7. Obtain implied values for g , Ψ , Γ and average earnings. Compare with guesses made in step 4. If the difference between guesses and implied values is within a preset tolerance interval, proceed to step 8. If not, update the guesses of each variable and go back to step 4.¹⁴
8. Compute the difference between the ratios implied by the labor supply and asset demand of households with the initial guesses. If these differences are within a preset tolerance level, the solution has been reached with sufficient accuracy. If not, update the guesses and go back to step 2.

¹⁴Our algorithm uses the homotopy procedure to update all the guesses. That is, if v is the initial guess and v' is the value implied by the simulation, then the updated guess is $v'' = v + a(v' - v)$, where a is a constant chosen by the researcher which controls the size of the update and the rate of convergence of the algorithm.

References

- Acemoglu, D. and Restrepo, P. (2017a). Low-Skill and High-Skill Automation. NBER Working Papers 24119, National Bureau of Economic Research, Inc.
- Acemoglu, D. and Restrepo, P. (2017b). Robots and jobs: Evidence from us labor markets. Working Paper 23285, National Bureau of Economic Research.
- Acemoglu, D. and Restrepo, P. (2018). Artificial Intelligence, Automation and Work. Boston University - Department of Economics - Working Papers Series dp-298, Boston University - Department of Economics.
- Aiyagari, S. R. (1994). Uninsured Idiosyncratic Risk and Aggregate Saving. *The Quarterly Journal of Economics*, 109(3):659–684.
- Aiyagari, S. R. (1995). Optimal capital income taxation with incomplete markets, borrowing constraints, and constant discounting. *Journal of Political Economy*, 103(6):1158–1175.
- Autor, D. and Acemoglu, D. (2011). Skills, tasks and technologies: Implications for employment and earnings. In *Handbook of Labor Economics*. Elsevier.
- Autor, D. H., Levy, F., and Murnane, R. J. (2003). The Skill Content of Recent Technological Change: An Empirical Exploration. *The Quarterly Journal of Economics*, 118(4):1279–1333.
- Benabou, R. (2002). Tax and education policy in a heterogeneous agent economy: What levels of redistribution maximize growth and efficiency? *Econometrica*, 70:481–517.
- Berleemann, M. and Wesselhöft, J.-E. (2014). Estimating Aggregate Capital Stocks Using the Perpetual Inventory Method: A Survey of Previous Implementations and New Empirical Evidence for 103 Countries. *Review of Economics*, 65(1):1–34.
- Bewley, T. F. (2000). Has the Decline in the Price of Investment Increased Wealth Inequality? Unpublished.
- Brinca, P., Faria-e Castro, M., Ferreira, M. H., and Holter, H. A. (2019). The nonlinear effects of fiscal policy. Working Paper.
- Brinca, P., Holter, H. A., Krusell, P., and Malafry, L. (2016). Fiscal multipliers in the 21st century. *Journal of Monetary Economics*, 77:53–69.
- Brinca, P., Homem Ferreira, M., Franco, F. A., Holter, H. A., and Malafry, L. (2018). Fiscal consolidation programs and income inequality. Available at SSRN 3071357.
- Chamley, C. (1986). Optimal taxation of capital income in general equilibrium with infinite lives. *Econometrica*, 54(3):607–622.

- Civale, S. (2016). Has the Decline in the Price of Investment Increased Wealth Inequality? Unpublished.
- Cortes, G. M., Jaimovich, N., Nekarda, C. J., and Siu, H. E. (2016). The Micro and Macro of Disappearing Routine Jobs: A Flows Approach. NBER Working Papers 20307, National Bureau of Economic Research, Inc.
- Delaney, K. J. (2017). Droid duties: The robot that takes your job should pay taxes, says bill gates. *Quartz*.
- DiCecio, R. (2009). Sticky wages and sectoral labor comovement. *Journal of Economic Dynamics and Control*, 33(3):538–553.
- Eden, M. and Gaggl, P. (2018). On the welfare implications of automation. *Review of Economic Dynamics*, 29:15 – 43.
- Ferriere, A. and Navarro, G. (2018). The Heterogeneous Effects of Government Spending: It’s All About Taxes. Working paper.
- Flood, S., King, M., Rodgers, R., Ruggles, S., and Warren, J. R. (2018). Integrated public use microdata series, current population survey: Version 6.0 [dataset].
- Gordon, R. (1990). *The Measurement of Durable Goods Prices*. National Bureau of Economic Research, Inc.
- Greenwood, J., Hercowitz, Z., and Krusell, P. (1997). Long-Run Implications of Investment-Specific Technological Change. *American Economic Review*, 87(3):342–362.
- Guerreiro, J., Rebelo, S., and Teles, P. (2017). Should robots be taxed? NBER Working Paper.
- Heathcote, J., Perri, F., and Violante, G. L. (2010). Unequal we stand: An empirical analysis of economic inequality in the united states, 1967–2006. *Review of Economics Dynamics*, 13:15–51.
- Heathcote, J., Storesletten, K., and Violante, G. L. (2019). Optimal Progressivity with Age-Dependent Taxation. CEPR Discussion Papers 13550, C.E.P.R. Discussion Papers.
- Holter, H. A., Krueger, D., and Stepanchuk, S. (2014). How Does Tax Progressivity and Household Heterogeneity Affect Laffer Curves? PIER Working Paper Archive 14-015, Penn Institute for Economic Research, Department of Economics, University of Pennsylvania.
- Hubmer, J., Krusell, P., and Smith Jr, A. A. (2017). The Historical Evolution of the Wealth Distribution: A Quantitative-Theoretic Investigation. CEPR Discussion Papers 11743, C.E.P.R. Discussion Papers.
- Hugget, M. (1993). The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies. *Journal of Economic Dynamics and Control*, 17:953–969.
- Karabarbounis, L. and Neiman, B. (2014). The global decline of the labor share. *The Quarterly Journal of Economics*, 129(1):61–103.

- Koh, D., Santaeuàlia-Llopis, R., and Zheng, Y. (2019). Labor Share Decline and Intellectual Property Products Capital. Unpublished.
- Krusell, P., Mukoyama, T., and Şahin, A. (2010). Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations. *Review of Economic Studies*, 77(4):1477–1507.
- Krusell, P., Ohanian, L. E., Ríos-Rull, J. V., and Violante, G. L. (2000). Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica*, 68(5):1029–1053.
- Larrimore, J., Burkhauser, R. V., Feng, S., and Zayatz, L. (2008). Consistent cell means for topcoded incomes in the public use march cps (1976-2007). *Journal of Economic and Social Measurement*, 33(2/3).
- Lucas, R. E. (1990). Supply-side economics: An analytical review. *Oxford Economic Papers, New Series*.
- Mendoza, E., Razin, A., and Tesar, L. (1994). Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption. *Journal of Monetary Economics*, 34(3):297–323.
- Michaels, G., Natraj, A., and Reenen, J. V. (2010). Has ICT Polarized Skill Demand? Evidence from Eleven Countries over 25 years. NBER Working Papers 16138, National Bureau of Economic Research, Inc.
- Mirrlees, J. A. (1971). An Exploration in the Theory of Optimum Income Taxation. *Review of Economic Studies*, 38(2):175–208.
- Ohanian, L. E., Violante, G. L., Krusell, P., and Ríos-Rull, J. V. (1997). Simulation-based estimation of a nonlinear latent factor aggregate production function. In Mariano, R. S., Schuermann, T., and Weeks, M., editors, *Simulation-Based Inference in Econometrics: Methods and Applications*. Cambridge University Press, Cambridge.
- Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economic Letters*, 20:177–181.