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# Structure and Interpretation of Computer Programs

Second Edition



Harold Abelson and  
Gerald Jay Sussman  
with Julie Sussman

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**Structure and  
Interpretation  
of Computer  
Programs**

Second Edition



Harold Abelson and  
Gerald Jay Sussman  
with Julie Sussman

**CATEGORY THEORY  
FOR PROGRAMMERS**

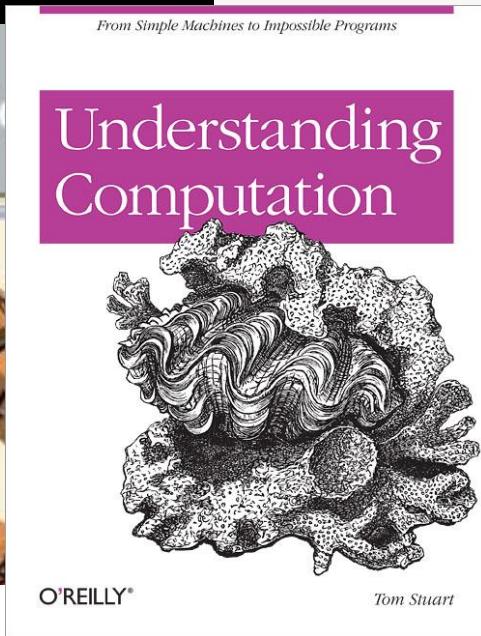
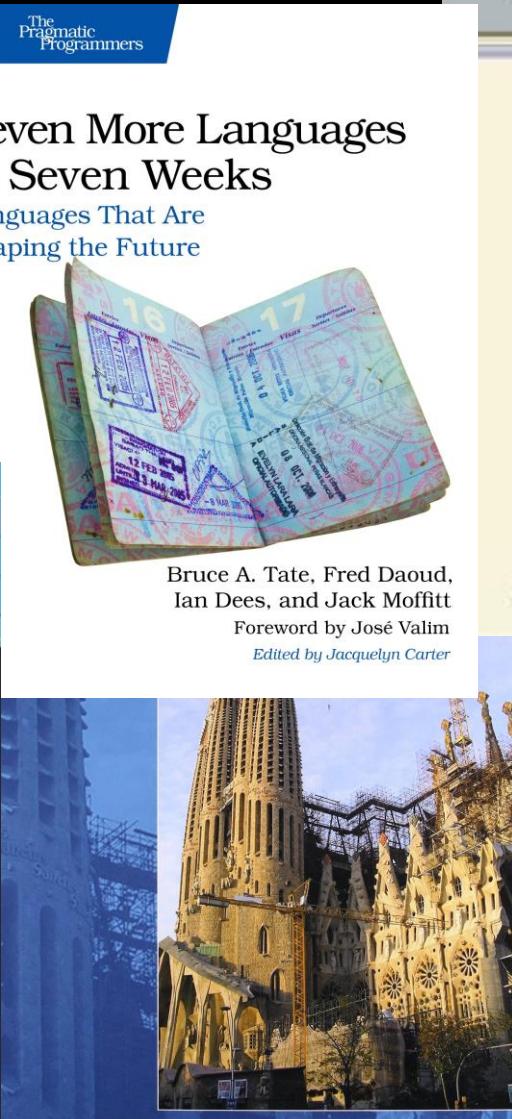
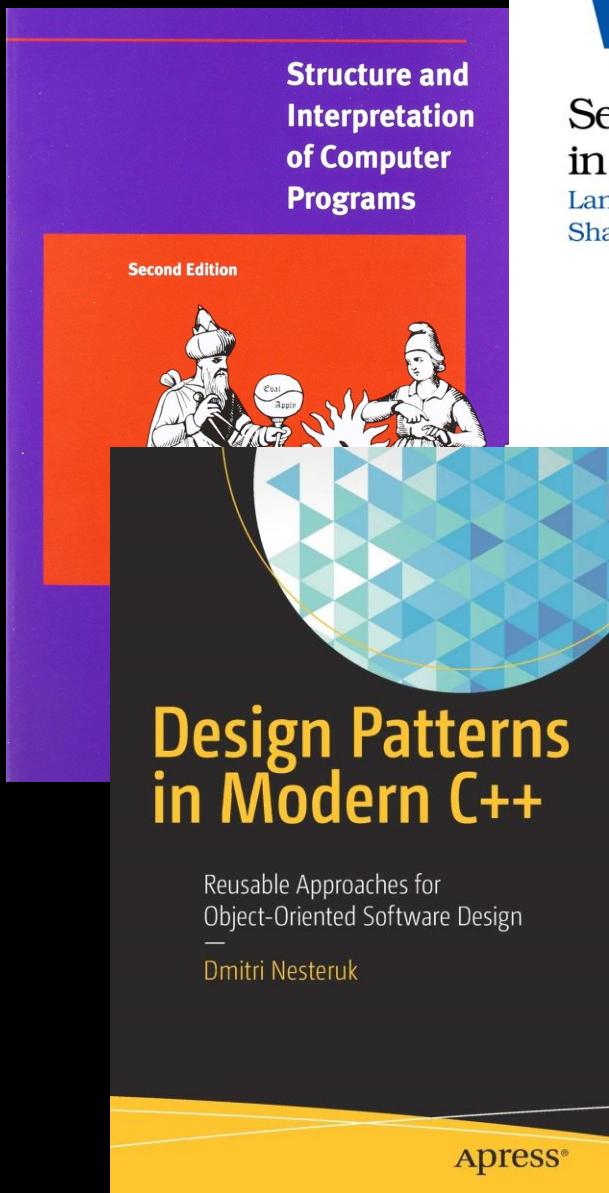


Bartosz Milewski

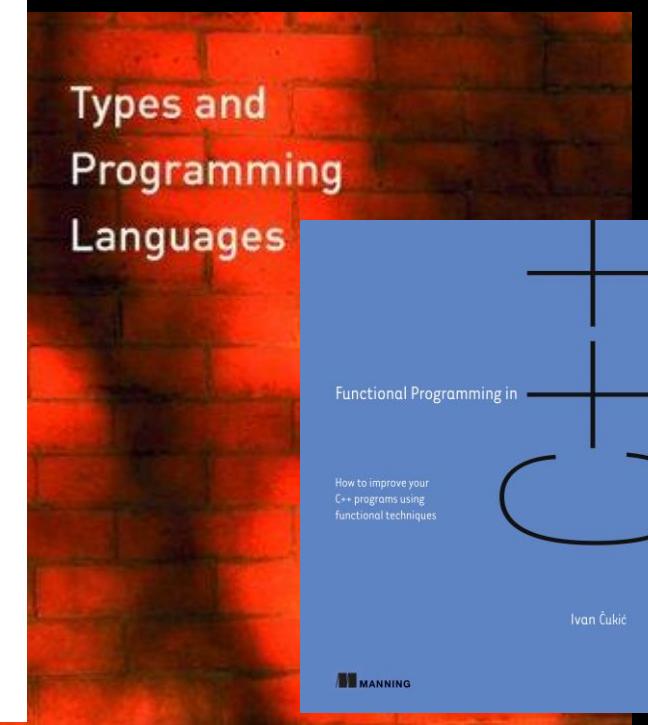
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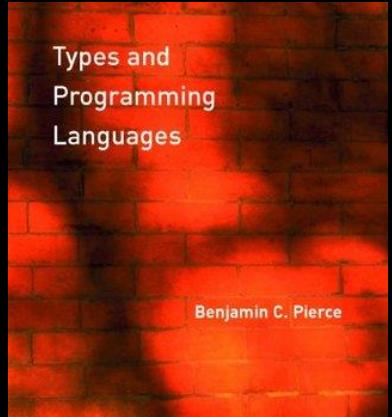
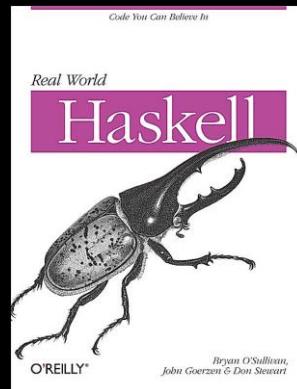
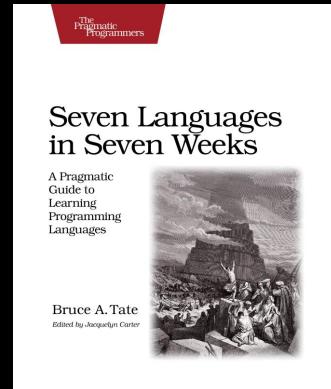
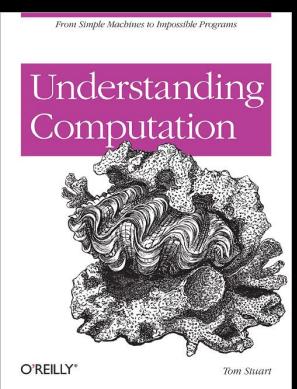
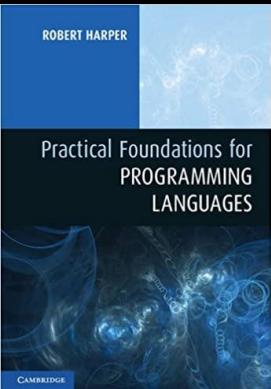
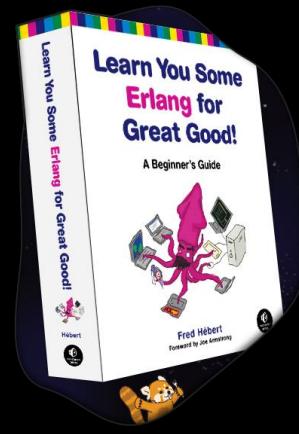
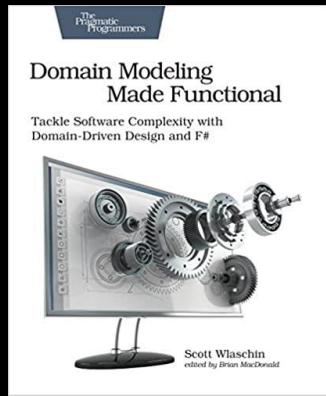
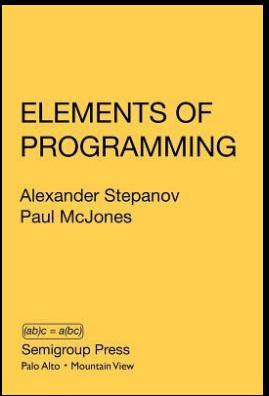
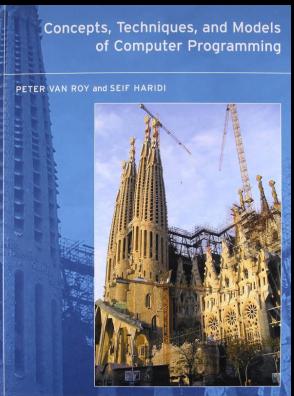
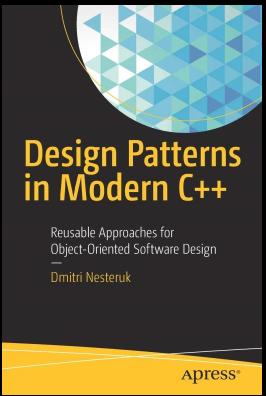
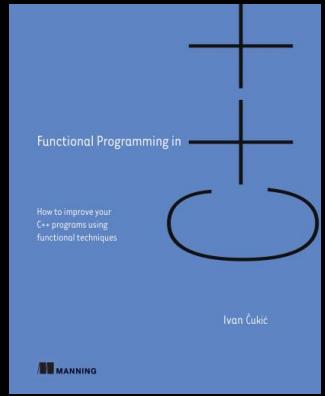
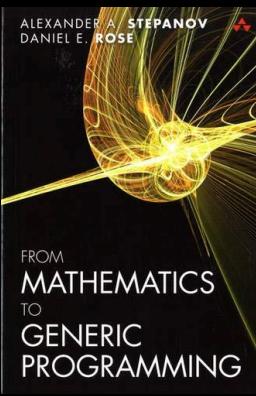
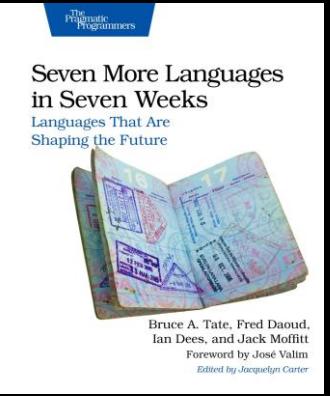
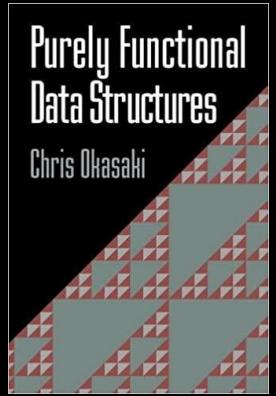
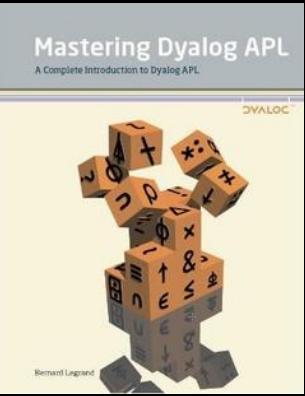
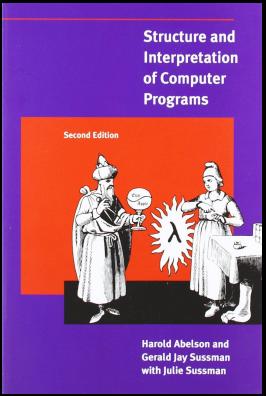
- We voted

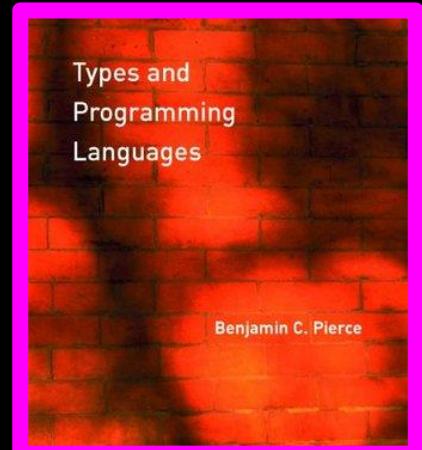
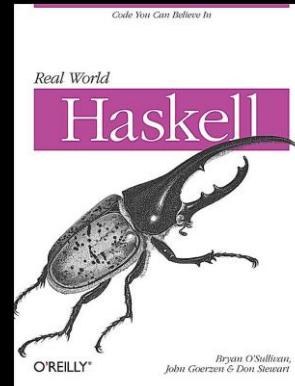
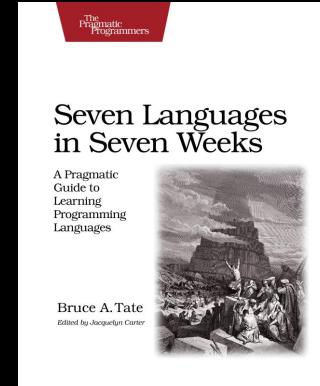
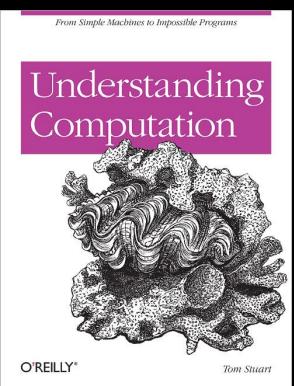
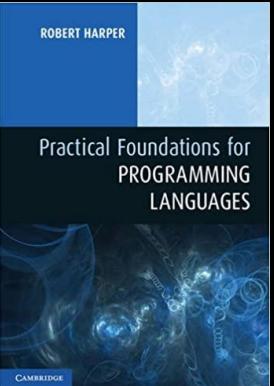
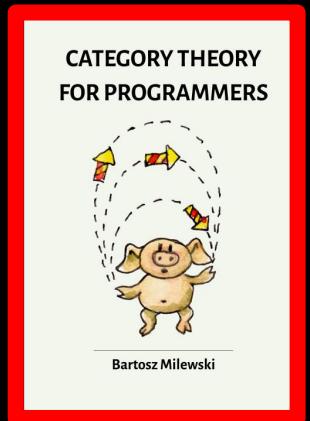
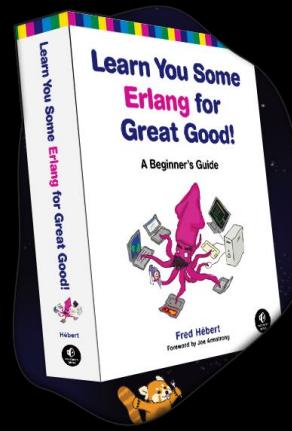
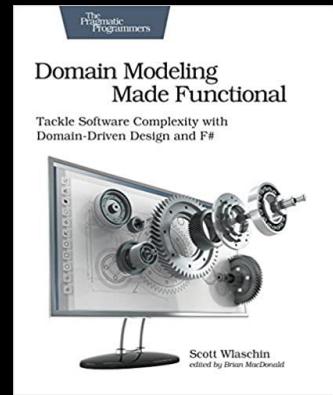
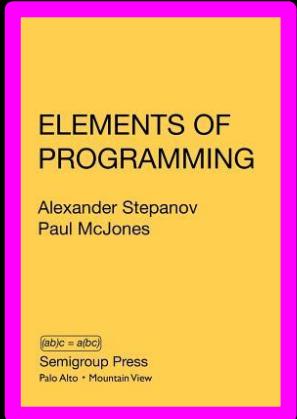
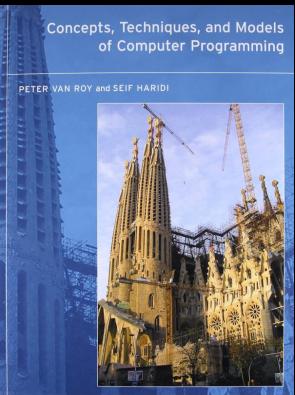
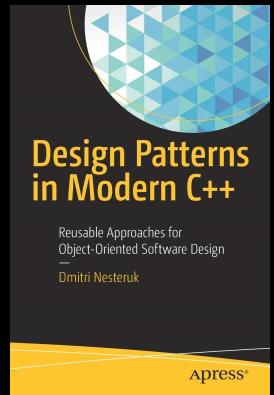
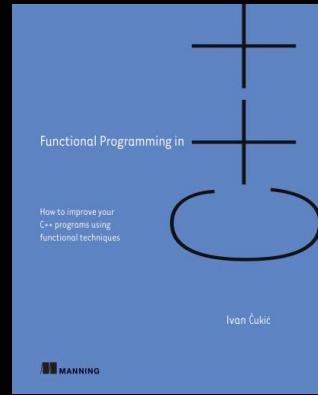
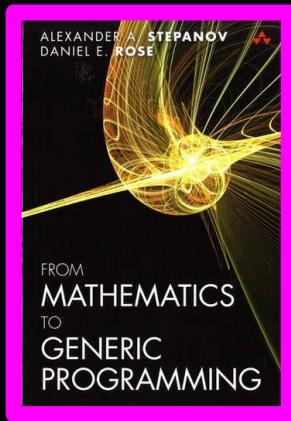
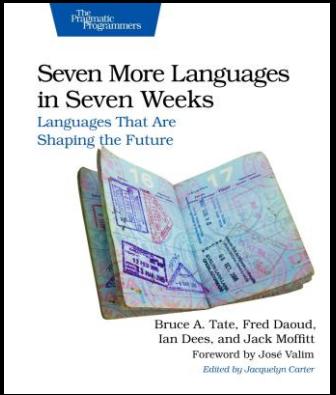
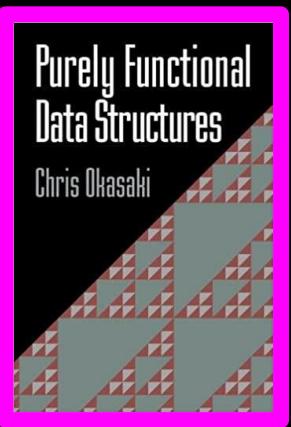
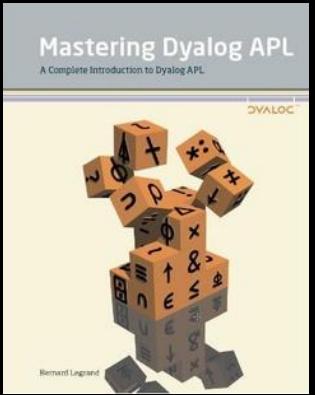
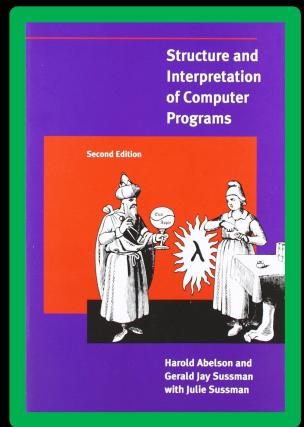
# Books to read:



ACKET  
ON TO LANGUAGE-ORIENTED  
USING RACKET  
TTERICK · VERSION 1.6







# Goal of PLVM

- Work through books on programming languages together
- Grow knowledge on PLs and PL:
  - principles
  - design
  - implementation
- This ultimately will lead to ability to write **code** that is more:
  - readable & expressive
  - maintainable & scalable
  - beautiful & idiomatic

# Format of PLVM

- 1.5 hour meeting once a week
- ~15 min presentation at beginning
- ~45 of discussion afterwards
- I will pre-record presentation and upload to YouTube for those unable to attend

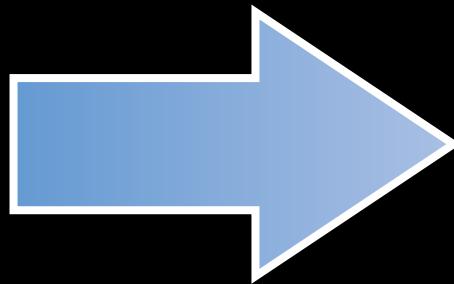
CATEGORY THEORY  
FOR PROGRAMMERS



---

Bartosz Milewski

Category  
Theory  
for  
Programmers  
Chapter 1:  
Category

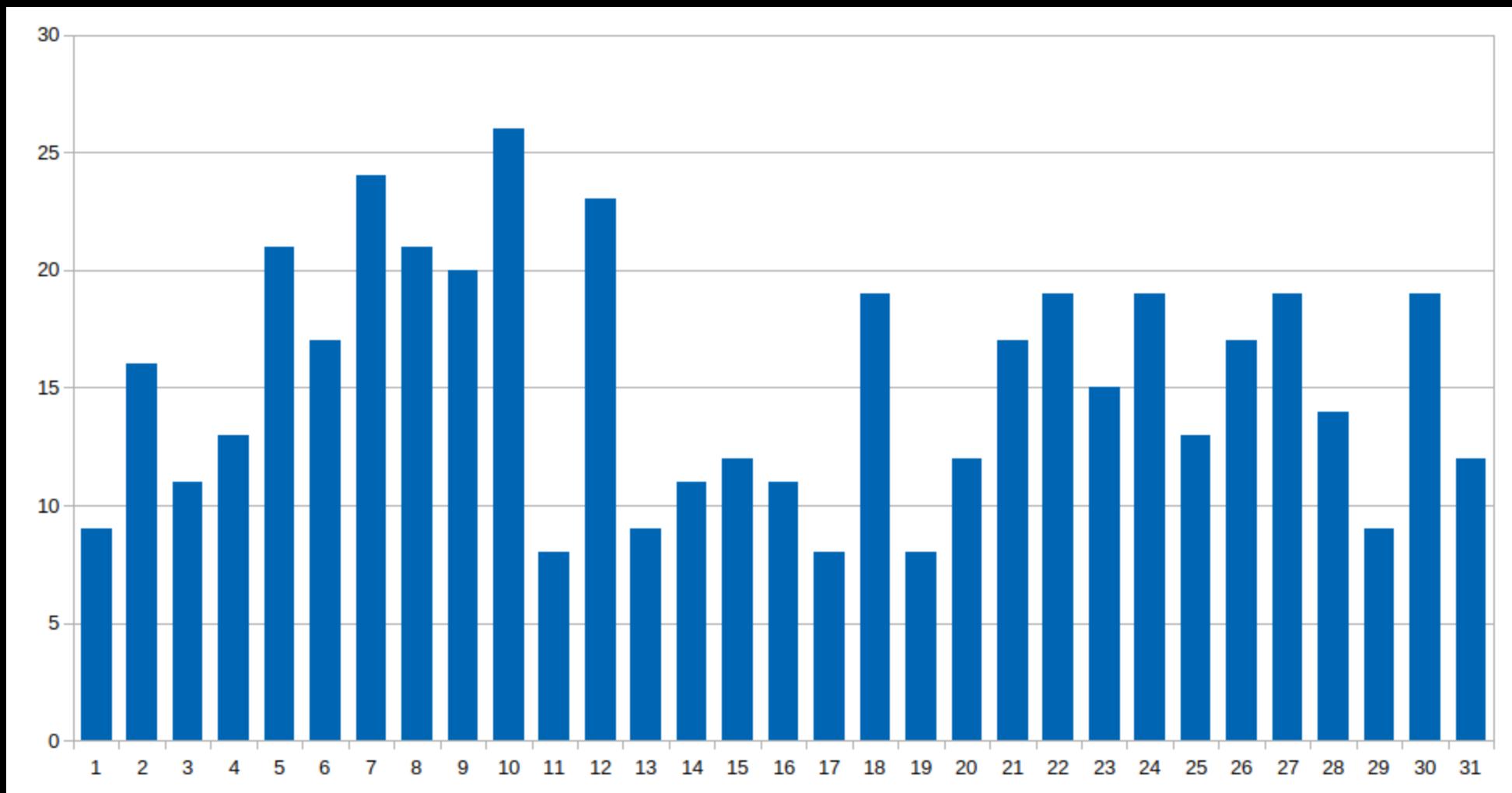


## Talks

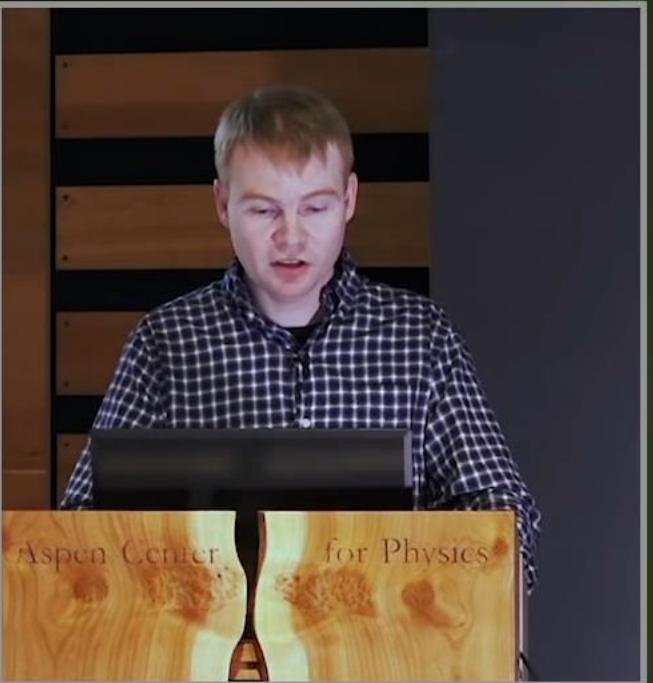
- Bartosz Milewski talks
  - [Functional Patterns in C++, 1. Functors](#), 2012
  - [Functional Patterns in C++, 2. Currying, Applicative](#), 2012
  - [Functional Patterns in C++, 3. Async API, Monoid, Monad](#), 2012
  - [Compile-Time/Run-Time Functional Programming in C++](#), Bartosz Milewski, Eric Niebler, BoostCon, 2012
  - [Haskell -- The Pseudocode Language for C++ Template Metaprogramming \(Part 1\)](#), BoostCon, 2013
  - [Haskell -- The Pseudocode Language for C++ Template Metaprogramming \(Part 2\)](#), BoostCon, 2013
  - [Re-discovering monads in C++](#), C++ User Group Novosibirsk, 2014
  - [Functional techniques in C++](#), CDays14, 2014
  - [Categories for the Working C++ Programmer](#), C++ Russia, 2015
  - [Monads for C++](#), itCppCon17, 2017

[https://github.com/graninas/cpp\\_functional\\_programming](https://github.com/graninas/cpp_functional_programming)

<b>Part One</b>	<b>2</b>
<b>1 Category: The Essence of Composition</b>	<b>2</b>
1.1 Arrows as Functions . . . . .	2
1.2 Properties of Composition . . . . .	5
1.3 Composition is the Essence of Programming . . . . .	8
1.4 Challenges . . . . .	10



If you're an experienced programmer, you might be asking yourself: I've been coding for so long without worrying about category theory or functional methods, so what's changed? Surely you can't help but notice that there's been a steady stream of new functional features invading imperative languages. Even Java, the bastion of object-oriented programming, let the lambdas in. C++ has recently been evolving at a frantic pace — a new standard every few years — trying to catch up with the changing world. All this activity is in preparation for a disruptive change or, as we physicists call it, a phase transition. If you keep heating water, it will eventually start boiling. We are now in the position of a frog that must decide if it should continue swimming in increasingly hot water, or start looking for some alternatives.

**Ben Deane**

---

Identifying Monoids:  
Exploiting Compositional  
Structure in Code

---

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## PAUSE: WHY ARE MONOIDS IMPORTANT?

Why is "abstract mathematics" important for programming?

*"Being abstract is something profoundly different from being vague... the purpose of abstraction is not to be vague, but to create a new semantic level in which one can be absolutely precise."*

-- EWD

**Category Theory is to Programming what  
Chemistry is to Baking ...**

**“whether or not you know about monoids  
doesn’t change the fact that they are in  
your code.”**

**Ben Deane  
Identifying Monoids, C++Now 2019**

<https://www.youtube.com/watch?v=INnattuluiM>

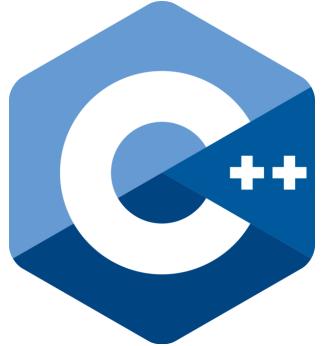
## **Part One** 2

<b>1 Category: The Essence of Composition</b>	<b>2</b>
 1.1 Arrows as Functions . . . . .	2
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A CATEGORY is an embarrassingly simple concept. A category consists of *objects* and *arrows* that go between them.

**A**CATEGORY is an embarrassingly simple concept. A category consists of *objects* and *arrows* that go between them.

Think of arrows, which are also called *morphisms*, as functions. You have a function  $f$  that takes an argument of type  $A$  and returns a  $B$ . You have another function  $g$  that takes a  $B$  and returns a  $C$ . You can compose them by passing the result of  $f$  to  $g$ . You have just defined a new function that takes an  $A$  and returns a  $C$ .



```
B f(A a);  
C g(B b);  
  
C g_after_f(A a) {  
    return g(f(a));  
}
```



|  $f :: A \rightarrow B$

Similarly:

|  $g :: B \rightarrow C$

Their composition is:

|  $g . f$



|  $f :: A \rightarrow B$

Similarly:

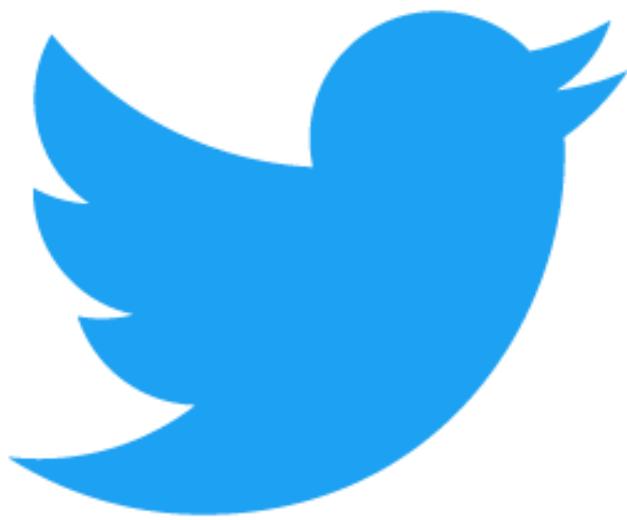
|  $g :: B \rightarrow C$

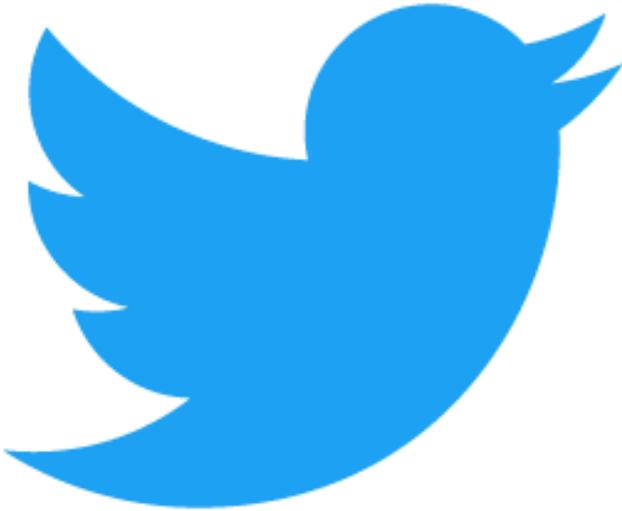
$g \circ f$

Their composition is:

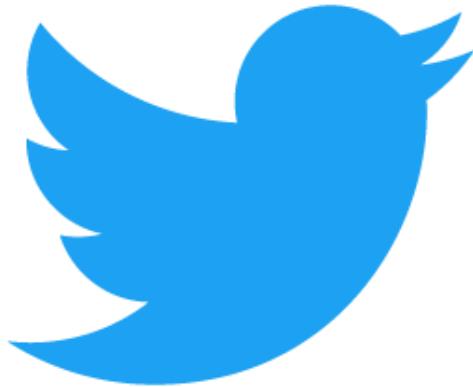
|  $g . f$

$A \rightarrow C$





```
Prelude> :t (.)  
(.) :: (b -> c) -> (a -> b) -> a -> c
```



Prelude> :t (.)

(.) :: (b -> c) -> (a -> b) -> a -> c

**bluebird :: (b -> c) -> (a -> b) -> a -> c**

B combinator or function composition



2. Implement the composition function in your favorite language. It takes two functions as arguments and returns a function that is their composition.





o

g ° f

## **Part One** 2

<b>1</b>	<b>Category: The Essence of Composition</b>	<b>2</b>
 1.1	Arrows as Functions . . . . .	2
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1. Composition is associative. If you have three morphisms,  $f$ ,  $g$ , and  $h$ , that can be composed (that is, their objects match end-to-end), you don't need parentheses to compose them. In math notation this is expressed as:

$$h \circ (g \circ f) = (h \circ g) \circ f = h \circ g \circ f$$

In (pseudo) Haskell:

```
f :: A -> B
g :: B -> C
h :: C -> D
h . (g . f) == (h . g) . f == h . g . f
```

2. For every object  $A$  there is an arrow which is a unit of composition. This arrow loops from the object to itself. Being a unit of composition means that, when composed with any arrow that either starts at  $A$  or ends at  $A$ , respectively, it gives back the same arrow. The unit arrow for object  $A$  is called  $\mathbf{id}_A$  (*identity* on  $A$ ). In math notation, if  $f$  goes from  $A$  to  $B$  then

$$f \circ \mathbf{id}_A = f$$

and

$$\mathbf{id}_B \circ f = f$$



```
template<class T> T id(T x) { return x; }
```



| **id** :: a → a  
**id** x = x



1. Implement, as best as you can, the identity function in your favorite language (or the second favorite, if your favorite language happens to be Haskell).



ト

## **Part One** 2

### **1 Category: The Essence of Composition** 2

	1.1 Arrows as Functions . . . . .	2
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This process of hierarchical decomposition and recombination is not imposed on us by computers. It reflects the limitations of the human mind. Our brains can only deal with a small number of concepts at a time. One of the most cited papers in psychology, **The Magical Number Seven, Plus or Minus Two<sup>1</sup>**, postulated that we can only keep  $7 \pm 2$  “chunks” of information in our minds. The details of our understanding of the human short-term memory might be changing, but we know for sure that it’s limited. The bottom line is that we are unable to deal with the soup of objects or the spaghetti of code. We need structure not because well-structured programs are pleasant to look at, but because otherwise our brains can’t process them efficiently. We often describe some piece of code as elegant or beautiful, but what we really mean is that it’s easy to process by our limited human minds. Elegant code creates chunks that are just the right size and come in just the right number for our mental digestive system to assimilate them.

# Making Things Easy



- Bring to hand by installing
  - getting approved for use
- Become familiar by learning, trying
- But mental capability?
  - not going to move very far
  - make things near by simplifying them



```
std::reverse(v.begin(), v.end());  
std::transform(v.begin(), v.end(), v.begin(),  
    [](auto e) { return e + 1; });
```



map (1+) . reverse



```
std::reverse(v.begin(), v.end());  
std::transform(v.begin(), v.end(), v.begin(),  
    [](auto e) { return e + 1; });
```



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std::reverse(v.begin(), v.end());  
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[](auto e) { return e + 1; });
```



map (1+) . reverse



1 + φ



3. Write a program that tries to test that your composition function respects identity.

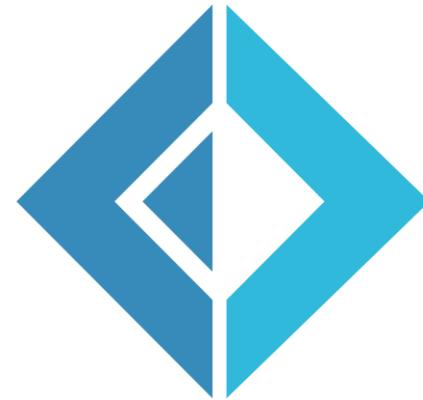


test  $\leftarrow \{ (\alpha\alpha \circ \textcolor{red}{\top} \equiv \alpha\alpha) \omega \}$   
 $\phi$  test  $\iota 5$

1

$1 \circ \phi$  test  $\iota 5$

1





# Bartosz Milewski

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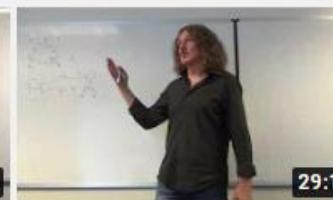
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Monad algebras part 3

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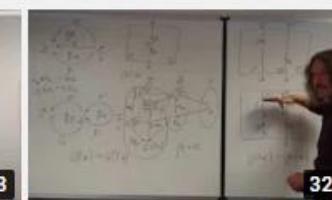
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Adjunctions and monads

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Diagrams part 2

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Category Theory III 1.1:  
Overview part 1

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Category Theory II 9.2:  
Lenses categorically

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Category Theory II 9.1:  
Lenses

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Category Theory II 8.2:  
Catamorphisms and...

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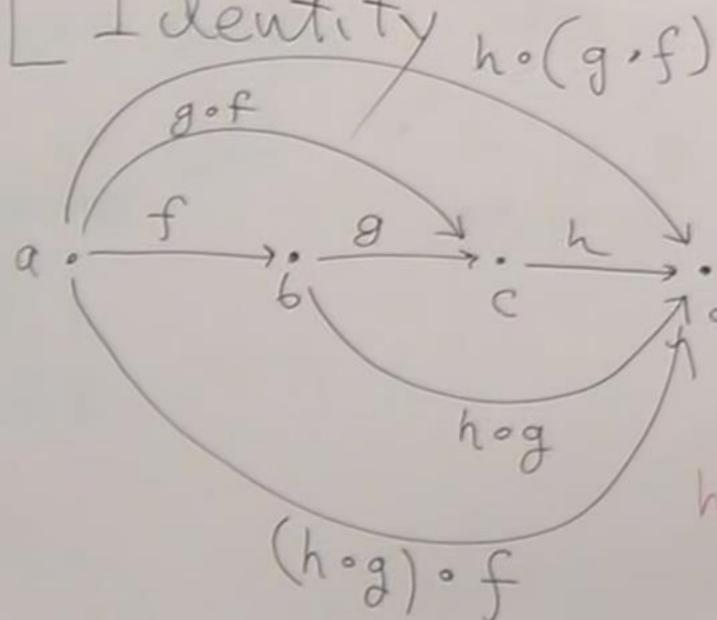
Category Theory II 8.1: F-  
Algebras, Lambek's lemma

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Abstraction

✓ Composition

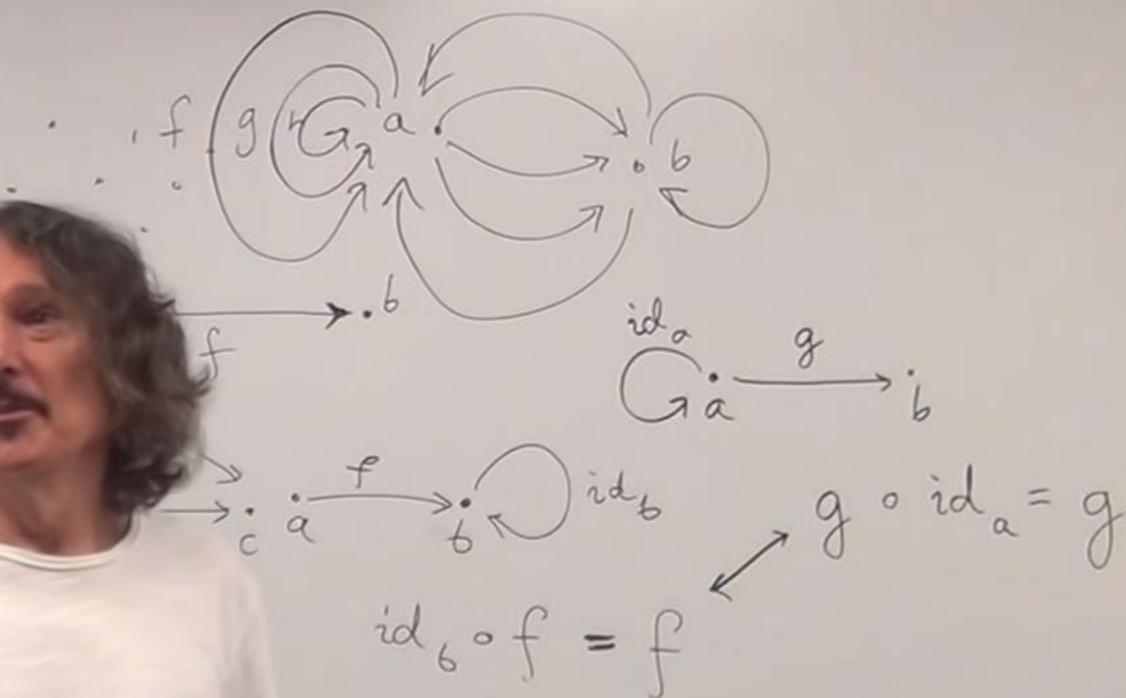
Identity



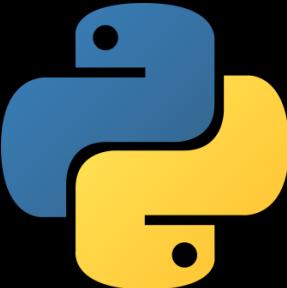
Category

Objects

Morphisms



7



*meetup*