SOLUTIONS MANUAL

Aerodynamics for Engineering Students

Seventh Edition

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and

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Chapter 3 solutions

1 Solutions to examples and end-of-chapter problems for Chapter 3

This note is a summary of the flat-plate boundary-layer properties prediction methodology required to solve Problems 3.1, 3.2, 3.4 and 3.6. Hence, it summarizes the prediction of skin-friction drag. We start with Prandtl's boundary-layer equations for the case where $dp/dx = 0$. Thus, the Prandtl boundary-layer equations reduce to:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}. \tag{1}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{2}$$

The boundary conditions for this system of equations are

$$u = v = 0, \quad y = 0, \quad 0 \le x \le L.$$

$$u = U, \quad y = \delta, \quad 0 \le x \le L.$$

$$u = U, \quad x = 0, \quad 0 \le y \le \infty.$$

Rearranging Equation (1) using (2), we get

$$\frac{\partial uu}{\partial x} + \frac{\partial vu}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}. \tag{3}$$

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x}. \tag{4}$$

Let us integrate (4) from the wall to the edge of the boundary layer:

$$\int_0^\delta \frac{\partial v}{\partial y} dy = -\frac{\partial}{\partial x}\int_0^\delta u\, dy.$$

Thus,

$$v = -\frac{\partial}{\partial x}\int_0^\delta u\, dy, \tag{5}$$

where $v = 0$ at $y = 0$ was used. Next, let us integrate (3) the same way:

$$\frac{\partial}{\partial x}\int_0^\delta u^2\, dy + \int_0^\delta \frac{\partial vu}{\partial y} dy = \nu\int_0^\delta \frac{\partial^2 u}{\partial y^2} dy.$$

Evaluating the second and third integrals and applying the boundary conditions, we get

$$\frac{\partial}{\partial x}\int_0^\delta u^2\, dy + vu|_{y=\delta} = \nu\frac{\partial u}{\partial y}|_{y=0}.$$

Substituting (5) and applying the definition of the wall shear stress, we get

$$\frac{\partial}{\partial x}\int_0^\delta u^2\, dy - U\frac{\partial}{\partial x}\int_0^\delta u\, dy = -\frac{\tau_w}{\rho}.$$

Since U is a constant for flat-plate thin boundary layers, we can rearrange this equation as follows:

$$\frac{\partial}{\partial x}\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \frac{\tau_w}{\rho U^2}. \tag{6}$$

The momentum thickness is defined by the parameter

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)\, dy. \tag{7}$$

Thus, the momentum integral formula for the flat-plate boundary layer is

$$\frac{d\theta}{dx} = \frac{\tau_w}{\rho U^2}, \tag{8}$$

where

$$\tau_w = \mu \frac{\partial u}{\partial y}|_{y=0}.$$

We can also write the momentum integral formula as follows:

$$\frac{\partial}{\partial x}\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)\, dy = \frac{\nu}{U^2}\frac{\partial u}{\partial y}|_{y=0}, \tag{9}$$

where $\nu = \mu/\rho$ was used. Equation (9) is a useful working relationship. (It was originally proposed by Theodore von Kármán and, hence, its application is sometimes called the Kármán integral method.) Examples of its application are given next.

Example 1: Let us consider the following relatively simple approximation of the velocity profile. Suppose $u = Uy/\delta$. This is a linear profile within the boundary layer. Substituting this into Equation (9) and doing the integrations, we get

$$\frac{1}{6}\frac{d\delta}{dx} = \frac{\nu}{U\delta},$$

or

$$\frac{1}{12}\frac{d\delta^2}{dx} = \frac{\nu}{U},$$

where $\theta = \delta/6$ is the momentum thickness. Integrating this in the x direction and applying the initial condition $\delta = 0$ at $x = 0$, we get

$$\delta = \sqrt{12}\sqrt{\frac{\nu x}{U}},$$

or

$$\frac{\delta}{x} = 3.464\frac{1}{\sqrt{Re_x}}, \tag{10}$$

where $Re_x = Ux/\nu$. Although this is a bit lower than the rough approximation of 5, i.e., the approximation based on the exact result due to Blasius (see the Appendix in this note), considering the crude guess of a linear velocity profile, this result is not all that bad.

Example 2: In Section 3.5.1 the analysis based on the following 4th-order polynomial velocity profile is described. For the case of a flat-plate boundary layer (zero pressure gradient), the profile is

$$\frac{u}{U} = 2\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4.$$

With this profile used to model the laminar boundary layer on a plate, you can show that

$$\delta = 4.64\frac{x}{\sqrt{Re_x}}.$$

Also,

$$\delta^* = 0.375\delta, \quad \theta = 0.139\delta, \quad C_F = \frac{1.293}{\sqrt{Re_L}}.$$

The latter compares quite reasonably with the exact result of Blasius, viz.: $C_F = 1.328/\sqrt{Re_L}$. With this reasonable comparison of the exact with the approximate result, we proceed to the next example, which leads to results that can be applied to predict the boundary-layer parameters for a turbulent boundary layer.

Example 3: Let us apply the momentum integral formulation to examine the turbulent boundary layer profile represented by

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7},$$

where u is the time-averaged velocity profile of a turbulent boundary-layer flow. The results from the momentum-integral approach (due to Theodore von Kármán) are

$$\delta = 0.383\frac{x}{Re_L^{1/5}}, \quad \delta^* = 0.125\delta, \quad \theta = 0.0973\delta, \quad C_F = \frac{0.0744}{Re_L^{1/5}}.$$

This result is for $Re_L > 10^7$. See Figure 3.2 in the text for a complete plot of C_F versus Re_L that compares laminar with turbulent and transitional flows on a flat plate. If the leading part of the plate has a laminar boundary layer and the after part of the plate is turbulent you can estimate the skin-friction coefficient with the following formula:

$$C_F = \frac{0.0744}{Re_L}\left(Re - Re_t + 35.5Re_t^{5/8}\right)^{4/5},$$

where Re_t is the critical Reynolds number at which transition occurs. It is in the range $3\times10^5 \leq Re_t \leq 3\times10^6$. This concludes the review of boundary-layer formulas that we need to solve the homework problems assigned (Problems 3.1, 3.2, 3.4 and 3.6 in Chapter 3 of the text).

2 Solutions to Chapter 3 problems

Problem 3.1: A thin flat plate of length $L = 0.5$ m is held in a uniform water flow such that its horizontal length is parallel to the direction of flow. The flow speed is $V = 1$ m/s. The density of water is $\rho = 1000$ kg/m^3, and its dynamic viscosity is $\mu = 1 \times 10^{-3}$ Pa s. We want to examine the boundary-layer properties at the trailing edge of the plate and to determine the skin-friction force coefficient. We first examine the boundary-layer properties as if the flow was laminar. We then examine the boundary layer as if the flow was turbulent and the velocity profile can be approximated by the 1/7-power-law velocity profile. Finally, we will use the graphical data reported in Figure 8.2 on page 482 do determine C_F for a flow over a plate with transition occurring at $Re_t = 3\times10^5$. In this case the flow over the leading portion of the plate is laminar and the flow past the transition point is turbulent.
Solution: By direct substitution, we find that the Reynolds number based on plate length is $Re_L = 500,000$. If transition is $Re_t > 500,000$, and the free stream is uniform then the boundary-layer flow is laminar. If this is the case, then the Blasius solution applies (see pages 485-496 in the text' also see the Appendix in this note). Thus, at the end of the plate

$$\delta \approx \frac{5L}{\sqrt{Re_L}} = 3.53\,\text{mm}, \quad \delta^* = \frac{1.7208L}{\sqrt{Re_L}} = 1.22\,\text{mm},$$

$$\theta = \frac{1.0444L}{\sqrt{Re_L}} = 0.74\,\text{mm}, \quad C_F = \frac{1.328}{\sqrt{Re_L}} = 0.0019.$$

If the flow is turbulent starting from the leading edge (see pages 521-525 in the text), then

$$\delta = \frac{0.383L}{Re_L^{1/5}} = 13.8\,\text{mm}, \quad \delta^* = \frac{0.0479L}{Re_L^{1/5}} = 1.74\,\text{mm},$$

$$\theta = \frac{0.0372L}{Re_L^{1/5}} = 1.35\,\text{mm}, \quad C_F = \frac{0.0744}{Re_L^{1/5}} = 0.0054.$$

The results for C_F compare favorably with the results graphically reported in Figure 3.2 in the text. If the transition Reynolds number is 3×10^5, then $C_F = 0.0037$. This is between the two values found above as expected.

Problem 3.2: In this problem we are asked to examine the boundary layer on a flat plate with length $L = 1$ m in a uniform stream $V = 25$ m/s in air with properties: $\rho = 1.2$ kg/m^3 and $\mu = 1.8 \times 10^{-5}$ Pa s. The transition is assumed to occur at $Re_t = 500,000$. What is the distance from the leading edge of the plate that corresponds to the transition location? It is, by definition of the Reynolds number,

$$x_t = Re_t \frac{\mu}{\rho V} = 300\,\text{mm}.$$

This is the solution to part (i) of the problem. Thus, the flow is laminar from the leading edge to this point on the plate. At this location the boundary-layer properties are

$$\delta \approx \frac{5L}{\sqrt{Re_L}} = 2.13\,\text{mm}, \quad \delta^* = \frac{1.7208L}{\sqrt{Re_L}} = 0.73\,\text{mm},$$

$$\theta = \frac{1.0444L}{\sqrt{Re_L}} = 0.44\,\text{mm}, \quad C_F = \frac{1.328}{\sqrt{Re_L}} = 0.0019.$$

Part (ii) asks for the equivalent length of the plate if the plate was fully turbulent yet has the same momentum thickness at the trailing edge as the actual boundary layer. This requires us to apply the formulas for mixed boundary layers in Section 3.6.8. The fictitious starting point for the turbulent boundary layer is

$$x_{tT} = 35.5 \frac{\mu}{\rho V} Re_t^{5/8} = 77.7\,\text{mm}.$$

Thus, the effective length of the plate (the solution for this part of the problem) for a flow that is fully turbulent is $L_T = L - x_t + x_{tT} = 778$ mm.

The third part, (iii), asks us to examine the drag associated with the laminar part of the plate and the drag associated with the turbulent part of the plate to compare the separate contributions to the skin-friction drag. For the first 300 mm of the plate we apply the Blasius solution given above. Thus, for a plate 300 mm in length with laminar flow leads to

$$C_{DF} = 2C_F = 0.0038.$$

The total skin-friction coefficient for this problem is

$$C_F = \frac{0.0744}{Re} \left(Re - Re_t + 35.5 Re_t^{5/8} \right)^{4/5} = 0.0035,$$

where $Re = VL/\nu = 1.7 \times 10^6$, $\nu = \mu/\rho = 1.5 \times 10^{-5}$ m^2/s were used. The total drag coefficient is $C_{DF} = 2C_F = 0.0070$; hence, the contribution to the total drag due to the turbulent part of the boundary layer is 0.0032.

Part (iv) asks us to compute the total drag on the plate. Since

$$C_{DF} = \frac{D_F}{\frac{1}{2}\rho V^2 LB} = 0.0070,$$

$$D_F = C_{DF}\frac{1}{2}\rho V^2 LB = 2.6\,\mathrm{N/m}.$$

or 1.3 N per side of the plate, where B is the breadth of the plate.

Part (v) asks for the percentage of the drag due to the turbulent part of the boundary layer. It is $0.0032/0.0070 \times 100$, or 46%.

The MATLAB script applied to compute the results given in the solution to this problem is:

```
% Problem 3.2
clear;clc
 Ret = 500000;
 L= 1; V = 25; B = 1;
 rho = 1.2; mu = .000018; nu = mu/rho;
 xt = 1000*Ret*mu/rho/V;
 xtT = (35.5*nu/V)*Ret^(5/8)*1000;
 LT = L*1000-xt+xtT;
 Re = V*L/nu;
 CF = (0.0744/Re)*( Re - Ret + 35.5 * Ret^(5/8) )^(4/5);
 DF = (2*CF)*rho* V^2 *L *B/2
%
```

Problem 3.4: This is a practical question on the estimate of the drag on a submarine (or any other object like a submarine, e.g., the fuselage of a low-speed aircraft) that is primarily due to skin-friction drag. It is also practical since it points out that the viscosity of the fluid will influence the drag. Let us assume that the drag can be estimated using the flat-plate drag formulas for turbulent boundary layers with 1/7-power-law velocity profiles. Thus,

$$C_F = \frac{F}{\frac{1}{2}\rho V^2 LC} = 0.0744\frac{1}{Re_L^{1/5}},$$

where $C = 2\pi R = 50$ m as given; this is the "breadth of the equivalent flat plate". The equivalent length of the vehicle is given as $L = 130$ m. We are asked to calculate the power required to overcome the skin-friction drag in 0^oC water at $V = 16$ m/s. The Reynolds number for this vehicle at this speed is $Re_L = VL/\nu = (16\,\mathrm{m/s})(130\,\mathrm{m})/(1.79 \times 10^{-6}\,\mathrm{m^2s} = 1.16 \times 10^9$. Thus, the power is $P = FV = 15.2$ MW. The speed is increased due to the change in viscosity going from 0^oC water to 20^oC to $V = 16.65$ m/s at the same power. (The method applied to determine this speed was to compute the power required at 16 m/s in water with viscosity at 20^oC. Then the speed was increased incrementally until the speed at which the power required equaled 15.2 MW.)

The MATLAB script applied to compute the results given in the solution to this problem is:

```
% Chapter 8 HW
% Problem 8.4
 L= 130; V = 16.65; B = 50;
```

```
 rho = 1000; nu = .00000101; nmu = nu*rho;
 Re = V*L/nu
 CF = 0.0744/(Re^(1/5));
 DF = (CF)*rho* V^2 *L *B/2;
 Power = DF*V
%
```

Problem 3.6: A light aircraft cruises at $V = 55$ m/s in air with $\nu = 15 \times 10^{-6}$ m^2/s. We want to estimate the coefficient of skin-friction drag of the wing. The equivalent length (or chord) of a flat plate to be used to model the skin-friction drag of the wing is $L = 2$ m. Laminar to turbulent boundary-layer transition is known to occur at $x = 0.75$ m from the leading edge. Thus, the critical Reynolds number at transition is $Re_t = 2.75 \times 10^6$, which is in the correct range of transition Reynolds numbers for flat plate boundary layers. The wing Reynolds number is $Re_L = 7.33 \times 10^6$. Hence,

$$C_F = \frac{0.0744}{Re}\left(Re - Re_t + 35.5 Re_t^{5/8}\right)^{4/5} = 0.0023,$$

or

$$C_{DF} = 0.0046.$$

The skin-friction coefficient is consistent with the answer in the text.

The MATLAB script applied to compute the results given in the solution to this problem is:

```
% Problem 3.6
format compact
 L= 2; V = 55;
 rho = 1; nu = .000015; mu = nu*rho;
 xt = 0.75;
 Ret= V*xt/nu
% xtT = (35.5*nu/V)*Ret^(5/8);
% LT = L*1000-xt+xtT;
 Re = V*L/nu
 CF = (0.0744/Re)*( Re - Ret + 35.5 * Ret^(5/8) )^(4/5)
% DF = (2*CF)*rho* V^2 *L *B/2
format
```

Appendix: Laminar, flat-plate boundary-layer theory

In this appendix the computational results originally due to Blasius (1908) (under the direction of Prandtl) is discussed. It is known as a similarity solution, the meaning of which will be discussed as well. A computational method is applied to solve the nonlinear ordinary differential equation derived and solved by Blasius. The results predicted by the relatively crude numerical scheme applied herein are found to be in excellent agreement with results reported elsewhere.

In Section 3.2.1 in the text the similarity solution of the Prandtl boundary-layer equations and its application are described. This solution is possible because the coordinate transformation

$$\eta = y\sqrt{\frac{U_\infty}{2\nu x}}$$

reduces the boundary-layer equations to the following ordinary differential equation:

$$\frac{d^3 f}{d\eta^3} + f\,\frac{d^2 f}{d\eta^2} = 0,$$

where f is proportional to the stream function, $df/d\eta = f' = u/U_\infty$ is the dimensionless velocity, and $d^2 f/d\eta^2 = f''$ is proportional to the shear stress at $y = 0$.

The boundary conditions for this problem reduce to

$$f = \frac{df}{d\eta} = 0 \quad \text{at} \quad \eta = 0; \quad f \to 1 \quad \text{as} \quad \eta \to \infty.$$

The solution to this ordinary-differential-equation boundary-value problem can be found numerically by applying the MATLAB code given at the end of this appendix. The computed results are summarized in Table 1. The velocity profile is illustrated in Fig. 1.

From Table 1 we find that at $\eta \approx 3.6$, $u/U_\infty \approx 0.99$; this is the definition of the edge of the boundary layer. Hence,

$$\delta_{0.99} \approx 3.6\sqrt{\frac{2\nu x}{U_\infty}} = 5\sqrt{\frac{\nu x}{U_\infty}},$$

which is the same result obtained by Blasius and applied in Example 8.2 on page 496.

The wall shear stress is

$$\tau_w = 0.4697\,\mu\sqrt{U_\infty 2\nu x} = 0.332\,\mu\sqrt{U_\infty \nu x}$$

which is the same result given in Eq. (8.33) on page 495.

This appendix is provided as a verification of the results reported in the text. We could have used the ODE solvers available in MATLAB to solve this problem more elegantly. This is left as an exercise for the reader.

```
% NUMERICAL SOLUTION OF
% Blasius's ODE:
%       fppp + f fpp = 0,
% where fppp = d3fdn3 and fpp = d2fdn2.
% Boundary conditions:
%       f = fp = 0 at n=0,
% where fp = dfdn, and
%       f -> 1 as n -> infinity.
%
% Let v = fp = dfdn; thus, we can write an ODE for v, viz.:
```

Table 1: Numerical solution of the Blasius flat-plate ODE

η	$f(\eta)$	$f'(\eta)$	$f''(\eta)$
0	0	0	0.4697
0.2000	0.0094	0.0939	0.4691
0.4000	0.0376	0.1876	0.4664
0.6000	0.0844	0.2807	0.4598
0.8000	0.1497	0.3721	0.4478
1.0000	0.2330	0.4608	0.4292
1.2000	0.3337	0.5454	0.4036
1.4000	0.4508	0.6246	0.3711
1.6000	0.5830	0.6970	0.3325
1.8000	0.7290	0.7614	0.2895
2.0000	0.8869	0.8170	0.2444
2.2000	1.0551	0.8636	0.1995
2.4000	1.2317	0.9014	0.1573
2.6000	1.4151	0.9309	0.1196
2.8000	1.6035	0.9531	0.0876
3.0000	1.7959	0.9693	0.0617
3.2000	1.9909	0.9806	0.0418
3.4000	2.1878	0.9881	0.0272
3.6000	2.3859	0.9930	0.0170
3.8000	2.5849	0.9960	0.0102
4.0000	2.7843	0.9978	0.0059
4.2000	2.9839	0.9989	0.0033
4.4000	3.1838	0.9994	0.0017
4.6000	3.3837	0.9997	0.0009
4.8000	3.5836	0.9999	0.0004
5.0000	3.7836	0.9999	0.0002

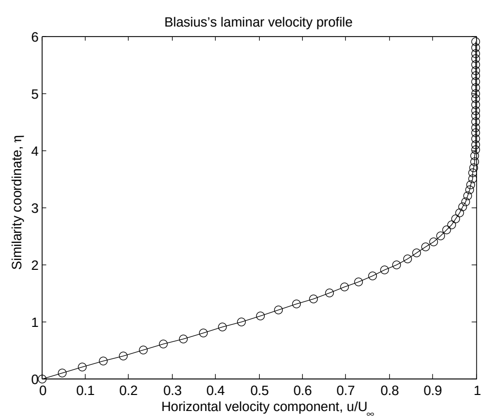

Figure 1: The velocity profile for laminar flow over a flat plate.

```
%       d2vdn2 + f * dvdn = 0.
% This is done to help develop a numerical method to solve
% this two-point boundary-value problem.
clear;clc
dn = .1;
v(61) = 1;
f(61) = 1;
v(1) = 0;
f(1) = 0;
y(1) = 0;
for it = 1:150000
for m = 2:60
    y(m) = y(m-1) + dn;
    f(m) = (v(m)+v(m-1))*dn/2 + f(m-1);
    v(m) = (v(m+1) + v(m-1))/2  + f(m) * dn* (v(m+1) - v(m-1))/4;
    fpp(m-1) = (v(m)-v(m-1))/dn;
end
end
plot(v(1:end-1),y,'-ok')
title('Blasius''s laminar velocity profile')
xlabel('Horizontal velocity component, u/U_\infty')
ylabel('Similarity coordinate, \eta')
% The method applied is not sophisticated but it works. The
% results compare excellently with the results reported in
% White, F. M., VISCOUS FLUID FLOW, McGraw-Hill (1974).
```