SOLUTIONS MANUAL

Aerodynamics for Engineering Students

Seventh Edition

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for

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E. L. Houghton, P. W. Carpenter, Steven H. Collicott

and

Daniel T. Valentine

Chapter 5 solutions

1 Solutions to Chapter 7 problems

This section provides solutions for typical homework problems at the end of Chapter 5.

Problem 7.1: The solution to this problem can be found by executing the MATLAB file P5_1.m provided with this solutions manual. The script is as follows:

```
%
% Problem 7.1 in Chapter 7 of Houghton, Carpenter, Collicott
% and Valentine (2016).
% Prepared by
% Daniel T. Valentine ........................ 2012/2016.
clear;clc
% This problem is on an elliptic wing (see Section 7.5.3).
L = 73600;% Lift in Newtons.
s = 15.23; % span in meters.
V = 90; % m/s.
rho = 1.2256*.9762; % kg/m/m/m at altitude = 250 m.
Gamma0 = 4*L/(rho*V*pi*s); % Equation (7.29).
Dv = (pi/8)*rho*Gamma0^2/1000; % Equation (7.33).
G0 = round(Gamma0);
disp([' (a) Induced drag = ' num2str(Dv) ' kN'] )
disp([' (b) Mid-span circulation = ' num2str(G0) ' m*m/s'] )
%
```

The output to the Command Window is:

```
 (a) Induced drag = 1.5341 kN
 (b) Mid-span circulation = 57 m*m/s
```

Note that the results are somewhat different than given as answers in the text.

Problem 7.2: This problem is on the glide-path problem. The balance equations for gliding without being under power are

$$C_L \cos\beta_{gp} = C_W, \quad \text{and} \quad C_L \sin\beta_{gp} = C_D,$$

where β_{gp} is equal to the glide angle, C_W is the dimensionless weight of the aircraft and C_D is the drag coefficient of the aircraft. For $AR = 6$ it is given as

$$C_D = 0.02 + 0.06C_L^2.$$

The second term in the drag formula is the induced drag plus a constant. If the wing is an elliptic loaded wing, as is assumed, for $AR = 6$, the induced drag is $C_{D_I} = C_L^2/(\pi\, AR) = 0.0531\, C_L^2$. For $AR = 10$ it is $C_{D_I} = 0.0318\, C_L^2$. Thus, to answer the question we can estimate the change in C_D associated with the change in induced drag (based on the two computations for the elliptic wing). We assume that the difference between the induced drag and the drag proportional to C_L^2 is the same for both aspect ratios. Thus, the glide angles for the two AR values are given by

$$\sin\beta_{gp1} = \frac{0.02}{C_L} + .007C_L + 0.0531C_L,$$

$$\sin\beta_{gp2} = \frac{0.02}{C_L} + .007C_L + 0.0318C_L,$$

respectively. Hence,

$$\sin\beta_{gp1} - \sin\beta_{gp2} = 0.0213C_L.$$

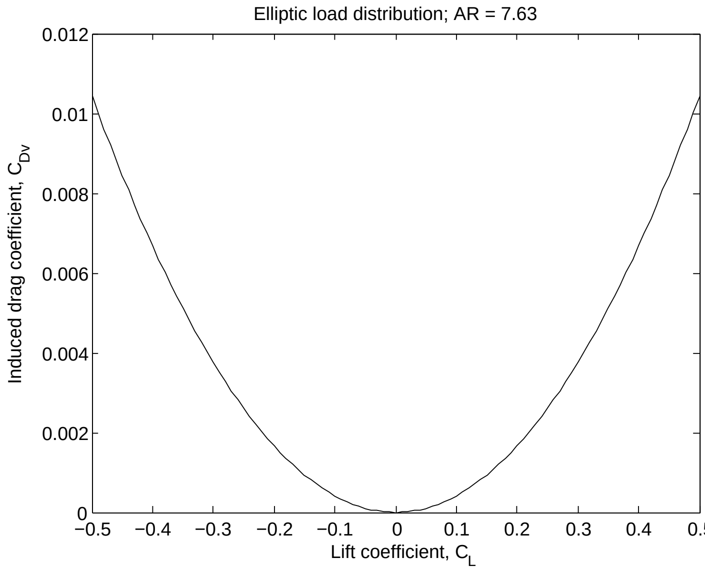

Figure 1: Induced drag versus lift for Problem 5.3.

Therefore, for constant C_L, $\beta_{gp1} > \beta_{gp2}$ for $AR_1 < AR_2$. Since $\beta_{gp2} < \beta_{gp1}$, the higher the AR the smaller is the glide-path angle indicating that the plane with higher AR will glide a longer distance.

Problem 7.3: The minimum induced drag of a wing is associated with elliptic loading because it is for this loading that the downwash is uniform. Section 5.5.5 provides further elaboration of this issue. Fig.1 illustrates the relationship between induced drag and lift coefficient for $AR = 7.63$ as requested in the problem statement. The script executed to plot the figure is as follows:

```
%
% Chapter 7 Problem 3.
CL = -.5:.01:.5;
delta = 0;
deltad = 0;
AR = 7.63;
CDV = (1+delta)*CL.^2./(pi*AR);
CDVd = (1+deltad)*CL.^2./(pi*AR);
plot(CL,CDV,'k')
title('Elliptic load distribution; AR = 7.63')
ylabel('Induced drag coefficient, C_{Dv}')
xlabel('Lift coefficient, C_L')
```

Problem 7.4: Obtain an expression for the downward induced velocity behind a wing of span $2s$ at a point at distance y from the center of the span, the circulation around the wing at any point y being Γ. If the circulation is parabolic, i.e.,

$$\Gamma = \Gamma_o \left(1 - \frac{y^2}{s^2}\right),$$

calculate the value of the induced velocity w at mid-span, and compare this value with that obtained when the same lift is distributed elliptically.

Solution: Since y is a point along the span, the integral equation for the downwash is

$$w(y_1) = -\frac{1}{4\pi}\int_{-s}^{s} \frac{d\Gamma}{dy}\frac{1}{y-y_1}\,dy$$

The derivative of the span circulation is

$$\frac{d\Gamma}{dy} = -2\Gamma_o\frac{y}{s^2}$$

Thus, for this circulation distribution

$$w(y_1) = \frac{\Gamma_o}{2\pi s^2}\int_{-s}^{s}\frac{y}{y-y_1}\,dy$$

At mid-span this reduces to

$$w(0) = \frac{\Gamma_o}{2\pi s^2}\int_{-s}^{s} dy = \frac{\Gamma_o}{\pi s}$$

How does this compare with the downwash of an elliptically loaded wing with the same lift? The lift for the given circulation distribution is

$$L = \rho V \Gamma_o \int_{-s}^{s}\left(1-\frac{y^2}{s^2}\right)dy = \frac{4}{3}\rho V \Gamma_o s$$

The formula for the lift for an elliptically loaded wing is

$$L_e = \frac{\pi}{2}\rho V \Gamma_{oe} s$$

Thus, for the same lift

$$\Gamma_{oe} = \frac{2}{\pi}\frac{4}{3}\Gamma_o$$

The downwash at mid-span for the elliptic wing is

$$w_e(0) = \frac{2}{\pi}\frac{4}{3}\frac{\Gamma_o}{4s} = \frac{2}{3}\frac{\Gamma_o}{\pi s}$$

Hence, the downwash associated with an elliptic wing is 2/3 of the downwash of the wing loading specified in this problem.

Problem 7.5: For a wing with modified elliptic loading, such that at a distance y form the center of the span, the circulation os given by

$$\Gamma = \Gamma_o\left(1+\frac{1}{6}\frac{y^2}{s^2}\right)\sqrt{1-\frac{y^2}{s^2}},$$

where s is the semi-span, show that the downward induced velocity at y is

$$\frac{\Gamma_o}{4\,s}\left(\frac{11}{12}+\frac{y^2}{2\,s^2}\right).$$

Also prove that for such a wing of aspect ratio A_R the induced drag coefficient at the lift coefficient, C_L, is

$$C_{D_o} = \frac{628}{625}\frac{C_L^2}{\pi\,A_R}.$$

Solution: Let us apply $y/s = -\cos\theta$ to the formula for the circulation. Thus,

$$G = \frac{\Gamma}{\Gamma_o} = \left(1 + \frac{1}{6}\cos^2\theta\right)\sqrt{1-\cos^2\theta}$$

Let us next apply the identities $\sin^2\theta + \cos^2\theta = 1$ and $2\,\cos^2\theta = 1 + \cos 2\theta$. Thus,

$$G = \left[1 + \frac{1}{12}\left(1+\cos 2\theta\right)\right]\sin\theta$$

$$G = \frac{13}{12}\sin\theta + \frac{1}{12}\cos 2\theta\,\sin\theta$$

Next, we apply the following trigonometric addition theorems

$$\sin\left(\alpha+\beta\right) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$-\sin\left(\alpha-\beta\right) = -\sin\alpha\cos\beta + \cos\alpha\sin\beta$$

Summing these equations, we get

$$\sin\left(\alpha+\beta\right) - \sin\left(\alpha-\beta\right) = 2\,\cos\alpha\sin\beta.$$

Substituting $\alpha = 2\theta$ and $\beta = \theta$, we get

$$\cos 2\theta\,\sin\theta = \frac{1}{2}\left(\sin 3\theta - \sin\theta\right).$$

With this identity we can write G as follows:

$$G = \frac{13}{12}\sin\theta + \frac{1}{24}\left(\sin 3\theta - \sin\theta\right)$$

Rearranging, we get

$$G = \frac{25}{24}\sin\theta + \frac{1}{24}\sin 3\theta$$

So solve for the downwash, we need to find $dG/d\theta$. It is as follows:

$$\frac{dG}{d\theta} = \frac{25}{24}\cos\theta + \frac{3}{24}\cos 3\theta$$

Substituting this into the downwash integral equation given in the text, we get

$$w = \frac{\Gamma_o}{4\pi s}\int_0^\pi \frac{\frac{25}{24}\cos\theta + \frac{3}{24}\cos 3\theta}{\cos\theta - \cos theta_1}\,d\theta$$

Applying the results given on the integral table in Appendix C, we get

$$w = \frac{\Gamma_o}{4\pi s}\left(\frac{25}{24}\pi + \frac{3}{24}\pi\frac{\sin 3\theta}{\sin\theta}\right)$$

Next, we apply the identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. Thus,

$$w = \frac{\Gamma_o}{4\pi s}\left(\frac{25}{24}\pi + \frac{3}{24}\pi\frac{3\sin\theta - 4\sin^3\theta}{\sin\theta}\right)$$

$$w = \frac{\Gamma_o}{4\pi s}\left[\frac{25}{24}\pi + \frac{3}{24}\pi\left(3 - 4\sin^2\theta\right)\right]$$

$$w = \frac{\Gamma_o}{4\pi s}\left[\frac{25}{24}\pi + \frac{3}{24}\pi\left(-1 + 4\cos^2\theta\right)\right]$$
$$w = \frac{\Gamma_o}{4\pi s}\left(\frac{22}{24}\pi + \frac{12}{24}\pi\cos^2\theta\right)$$

Thus,

$$w = \frac{\Gamma_o}{4s}\left(\frac{11}{12} + \frac{1}{2}\cos^2\theta\right) = \frac{\Gamma_o}{4s}\left(\frac{11}{12} + \frac{1}{2}\frac{y^2}{s^2}\right)$$

This is what we wanted to demonstrated.

Now that we have the induced velocity we can compute the induced drag. The induced drag is given by the formula

$$D_o = \int_{-s}^{s} \rho w \Gamma \, dy.$$

Starting with this equation, the equation for lift, the definition of C_L, etc., the following set of symbolic computations was executed in MATLAB.

```
%
% Problem 7.5: Solution of induced drag
% x = y/s, GG = Go/4/s, G = G/Go
clear;clc
syms x G Go s rho U A
w = (Go/4/s)*(11/12+x^2/2)
G = Go*(1 + x^2/6)*sqrt(1-x^2)
intgr = rho*w*G
Do = s*int(intgr,'x',-1,1)
intgL = rho*U*G
L = s*int(intgL,'x',-1,1)
CDo = Do/(rho*U^2* A/2)
CL = L/(rho*U^2* A/2)
f = CDo/CL^2
subs(f,A,'4*s^2/AR')
```

Executing this script, we find that

$$C_{D_o} = \frac{628\, C_L^2}{625\,\pi A_R}$$

This is the answer we sought to demonstrate.

Problem 7.6: In this problem we are asked to consider a rectangular, untwisted wing of aspect ratio 3 with airfoils with $a_\infty = 6$. Assume the following distribution of circulation:

$$\Gamma = 4sV \sum A_n \sin n\theta$$

where $y = -s\cos\theta$ is the assumed coordinate transformation in the direction of the span and s is the semi-span. (If s is assumed to be the span, then the series is as given in the problem.) Determine the approximate circulation distribution with a two-term expansion, i.e.,

$$\Gamma = 4sV\left(A_1 \sin\theta + A_3 \sin 3\theta\right)$$

Apply this expression at $\theta = \pi/4$ and $\pi/2$ to solve for A_1 and A_3. The procedure to be applied is illustrated in detail, by example, in Example 5.6 on pages 319-321. The solution to this problem was found by applying the following MATLAB script file:

```
% Solution to Problem 7.6 -------------------------------
% in Chapter 7 of "Aerodynamics for Engineering Students",
% 7th Edition (2016) by E. L. Houghton, et al.
% This code was written by Daniel T. Valentine, 2012/2016
   clear;clc;
%
  rho = 1.2256; % Density of fluid, kg/m^3.
  V = 1;        % Free-stream speed, m/s.
% Wing geometric characteristics:
  s = 3/2;      % Wing semi-span, m.
  b = 2*s;      % Wing span.
  AR = 3;       % Aspect ratio.
  S = b^2/AR;   % Plan area.
  cbar = S/b;   % Standard mean chord (SMC).
% Fourier analysis of circulation distribution:
% Two-term expansion as requested in the problem statement:
NZ = 2;
theta = 0:pi/2/NZ:pi/2;
cc = 1;
ainff = 6;
alphaGive = 1*180/pi;  % alpha in degrees
sinthe = sin(theta);
muu = cc.*ainff/(8*s); % s, in this code, is semi span.
mualphsin = muu.*alphaGive.*sinthe*pi/180;
for mm = 1:NZ+1
     mt = mm;
    for nz = 1:NZ+1
        nn = 2*nz - 1;
CC(mm,nz) = sin(nn*theta(mt))*(sinthe(mt) + nn*muu);
    end
end
for mm = 1:NZ
    for nn=1:NZ
        C(mm,nn) = CC(mm+1,nn);
    end
end
C;
b = mualphsin(2:end);
Cinv = inv(C);
A = C\b'
```

The numerical results for the two coefficients asked for in this problem are $A_1 = 0.372\alpha$ and $A_3 = 0.0231\alpha$. These results compare exactly with the answers given.

Problem 7.7: We are asked to examine an elliptical plan area wing with symmetric airfoil cross sections when the angle of attack is 2^o at the center of the span. The wing is designed with a circulation distribution given as

$$\Gamma = \Gamma_o \left[1 - \left(\frac{y}{s}\right)^2\right]^{3/2}$$

where s is assumed to be the semi span of the wing (note that in the formula given in the problem statement in the text s was assumed to be the span). Let $y = -s\cos\theta$; the $-s \le y \le s$ is transformed to $0 \le \theta \le \pi$. Thus,

$$G = \frac{\Gamma}{\Gamma_o} = \left(1 - \cos^2\theta\right)^{3/2} = \left(\sin^2\theta\right)^{3/2} = \sin^3\theta$$

We can expand this formula for G by applying a trigonmetric identity, i.e.,

$$G = \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta$$

In the text, following Prandtl, the circulation was expanded in a sine series as follows:

$$G = \frac{4sV}{\Gamma_o}\sum A_n \sin n\theta$$

where s in this solution is assumed to be half the span. Comparing the two formulas, we get

$$A_1 = \frac{3}{4}\frac{\Gamma_o}{4sV}, \quad A_3 = -\frac{1}{4}\frac{\Gamma_o}{4sV}$$

Thus, the formula for the downwash is

$$\frac{w}{V} = A_1 + 3A_3\frac{\sin 3\theta}{\sin\theta} = \frac{3}{4}\frac{\Gamma_o}{4sV}\left(1 - \frac{\sin 3\theta}{\sin\theta}\right)$$

This was one of the items requested in the problem statement. The second was the angle of the section approaching the tip. The shape of the wing plan area is an ellipse. Integrating the area of one side, we get $S = \pi c_m s/2$, where s is half the span. Note that $c = c_m \sin\theta$. Thus, the μ in Prandtl's lifting line formula is, by definition,

$$\mu = \frac{c_m a_\infty}{8s},$$

where s is te semi-span. The aspect ratio is defined as

$$A_R = \frac{b^2}{S}$$

where $b = 2s$ is the span and S is the plan area. We were given $A_R = 7$ and $a_\infty = 5.8$. We were also given the angle of the wing section at midchord. This is enough information to compute or estimate the angle of the tip for the given load distribution. To solve this problem the following MATLAB script was developed and applied:

```
% P7_7: G = gam/gam0
clear;clc
% y = -1:.1:1;
th = 0:.01:pi;
% GG = (1 - cos(th).^2).^(3/2)
yy = cos(th);
ainF = 5.8; AR = 7;
mu0 = ainF/pi/AR; % This is correct because s in this solution
                  % assumes the span is 2s. (Note that in many
                  % of the formulas Chapter 7 in 7e of this book
                  % the symbol s is assumed to be the span.
% C = 3*gam0/16/s/V;
C = (2*pi/180)*mu0/( sin(pi/2) - sin(3*pi/2)/3 + mu0*(1 - sin(3*pi/2)/sin(pi/2)));
mu = mu0.*sin(th);
alpha = (C./mu).*( sin(th) - sin(3.*th)/3 + mu.*(1 - sin(3.*th)./sin(th)));
plot(yy,180*alpha/pi); alpha(2)*180/pi,alpha(end-1)*180/pi
```

The angle at the tip was found to be equal to -0.566^o. Hence, a minus sign must be added to the answer given in the text. The script was checked by truncating all terms in the `C` and the `alpha` formulas in the script except the first `sin` terms. In this case the load is elliptic and we should expect a constant angle of 2^o from tip to tip. This modified script was executed and it produced the expected result quite accurately.

Problem 7.8: The solution for this problem is given in Example 7.7 in the text. In this problem $A_1 = 0$ and A_2 is essentially what is found in the example.

Problem 7.9: The series expansion was re-examined by applying MATLAB as follows:

```
% Problem 9: Consider a symmetric rectangular wing with AR=5.
% AR = 2s/c for rectangular wing. Let us examine N Fourier
% terms.
a_infinity = 2*pi; AR = 5;
alpha = 10*pi/180;
mu = a_infinity/4/AR;
% Matrix of coefficients to compute An's
 N = 10;
 theta = pi/N:pi/N:pi;
for n=1:length(theta)
    for m=1:length(theta)
        AA(n,m) = sin(m*theta(n))*(1+m*mu/sin(theta(n)));
        if abs(AA(n,m)) < 10^-6
            AA(n,m) = 0;
        end
    end
end
for m=1:length(theta)
    b(m) = mu*alpha;
end
AI = inv(AA);
A = AI*b';
CL = A(1)*pi*AR
% Note: if change CL with CL =.691; then CDi is as given.
delta = 3*A(3)^2/A(1)^2 + 5*A(5)^2/A(1)^2 + 7*A(7)^2/A(1)^2 ...
    + 9*A(9)^2/A(1)^2;
CDI = CL^2/(pi*AR) * (1 + delta)
%
```

Executing the script, we get the Fourier coefficients

$$A_1 = 0.047924, \quad A_3 = 0.005211, \quad A_5 = 0.001003, \quad A_7 = 0.0002491$$

for the data provided in the problem statement. This is close to the given distribution. Using the data provided in the problem statement, we obtain

$$C_L = \pi(AR)\, A_1 = \pi(5)\, 2(0.0234) = 0.735.$$

For this case

$$C_{Di} = \frac{C_L^2}{\pi\,(AR)}\,[1+\delta]\,,$$

where

$$\delta = 3\left(\frac{A_3}{A_1}\right)^2 + 5\left(\frac{A_5}{A_1}\right)^2 + 7\left(\frac{A_7}{A_1}\right)^2 = 0.044212,$$

and, hence

$$C_{Di} = 0.0359.$$

(If the C_L given as an answer is used to compute C_{Di}, then the answer given for C_{Di} is obtained. Hence, the δ computed above is correct. Alternatively, if the angle of attack from the zero lift angle in the analysis is changed from 10^o to 9.2^o, the answers given in the text are obtained by applying the MATLAB script given above.)

Problem 7.10: The following script is a direct application of the Biot-Savart formula, viz., Eq. (7.3) with the parameters in the equation defined in Fig. 7.7. Ground effects are described in Section 7.3.2. The image of the horseshoe modeling the wing is the same distance below the ground as the airplane is above the ground. However, the circulation is in the opposite direction. Fig. 7.11 illustrates the configuration. To determine the downwask at the tailplane located at $(18.3, 0, 6.1)$ m in the plane of the horsehoe representing the wing, the following script was applied:

```
% P 7.10: The script below applies to points along the
% centerline perpendicular to the span.
clear;clc
%
L = 250*1000; % N of lift
b = 34;       % m span
s = b/2;
V = 40;       % m/s forward speed
rho = 1.23;   % kg/m/m/m
X = 18.3;     % m behind center of wing
H = 6.1;      % m above ground
% Assume elliptic loading
sprime = s * pi/4; % Half distance between trailing vortices.
% tanbeta = sprime/L;
beta = atan2(sprime,X);
Go = L/(rho*V*2*sprime);
wp = (Go/(2*pi*sprime))*(1+sec(beta))/V;
wp = wp*V;
% ang = atan2(wp,V)*180/pi
% The change in downwash due to image
% IMAGE
% Biot-Savart Law of the image vortex system:
Go = -Go;
% Induction by lifting line
x1 = 0; y1 = -sprime; z1 = -H; % End 1 of vortex element
x2 = 0; y2 =  sprime; z2 = -H; % End 2 of vortex element
x3 = X; y3 =  0; z3 = H;       % Field point on center line
c2 = (x2-x1)^2 + (y2-y1)^2 + (z2-z1)^2; c = sqrt(c2);
a2 = (x3-x2)^2 + (y3-y2)^2 + (z3-z2)^2; a = sqrt(a2);
b2 = (x3-x1)^2 + (y3-y1)^2 + (z3-z1)^2; b = sqrt(b2);
cosA = (b2 + c2 - a2)/(2*b*c);
cosB = (a2 + c2 - b2)/(2*a*c);
h = sqrt(x3^2 + (z3 - z2)^2);
ui = (Go/4/pi/h)*(cosA+cosB);
costh = x3/h;
w = ui*costh;
% Induction of the trailers
```

```
ht1 = sqrt(sprime^2+(z3-z1)^2);
cosAt1 = x3/sqrt(ht1^2+x3^2);
uit1 = (Go/4/pi/ht1)*(cosAt1+1);
costht1 = sprime/ht1;
wt1 = 2*uit1*costht1;
%
 wtotali = wt1+w;
 ang = atan2(wp + wtotali,V)*180/pi
```

The computed angle is 3.38^o.