SOLUTIONS MANUAL

Aerodynamics for Engineering Students

Seventh Edition

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E. L. Houghton, P. W. Carpenter, Steven H. Collicott
and
Daniel T. Valentine

Chapter 6 solutions

1 Chapter 6: Two-dimensional Airfoils

This section provides solutions for the homework problems at the end of Chapter 4. Since the problems are on the analysis and design of two-dimensional airfoils, the formulas required to solve the problems are summarized in the next subsection. The solutions to the homework problems are given in the subsequent subsection.

1.1 Review of formulas from Chapter 6

The lift per unit span acting on the airfoil section is given by the *Kutta-Zhukovsky law*, i.e.,

$$L = \rho U \Gamma,$$

where ρ is the mass density of air, U is the free stream velocity and Γ is the total circulation associated with the *circulatory* parts of the airfoil problem (i.e., the parts that are associated with lift, viz., angle of attack and camber).

The aerodynamic problem is to select (or design) an airfoil that produces a required lift. Since the chord, c, of the airfoil is finite in length, the circulation is spread over the chord. This is done as follows:

$$\Gamma = \int_0^c k\,ds,$$

where $k = k(x)$ is the distribution of "vorticity" at $y = 0$, $0 \leq x \leq c$. We place the vortex sheet (i.e., the distribution of vortex singularities) on the x axis because we are applying the thin-airfoil theory approach described in Sections 4.3 and 4.4 on pages 223-235 in the text. Since airfoils are thin, in general, this approach is practical and it simplifies considerably the application of the idea of representing lifting surfaces by distributions of vortex singularities.

The lift per unit span, in terms of the chordwise distribution of circulation, is

$$L = \rho U \int_0^c k\,ds = \int_0^c \rho U k\,ds = \int_0^c p\,ds,$$

where p is the pressure jump across the vortex sheet representing the airfoil. Note that $p = \rho U k$. Hence, the moment of force about the leading edge can be written as follows:

$$M_{L.E.} = -\int_0^c p x dx = -\rho U \int_0^c k x dx.$$

Applying *Bernoulli's theorem* to the upper and lower surfaces, neglecting the higher-order terms (the squares of small velocity components as illustrated in the text), and subtracting the upper and lower pressures to relate the pressure jump to the jump in velocity across the sheet, we get

$$p = p_2 - p_1 = \rho U (u_1 - u_2).$$

Hence, from the definition of the circulation in the clockwise direction around the airfoil, the distribution of k is related to the jump in pressure jump as follows:

$$k \delta x = (u_1 - u_2) \delta x = \frac{p}{\rho U} \delta x,$$

where $\delta s \approx \delta x$ is an element of the vortex sheet that models the surfaces of the airfoil.

An additional constraint on k is the *Kutta condition* at $x = c$ (at the trailing edge). It is $u_1 = u_2$ or $k = 0$ at $x = c$.

1.1.1 Thin-airfoil theory

Let us consider an airfoil at an angle of attack, α, as illustrated in Fig. 6.10 in the text. The airfoil is a sum of a camber distribution and a thickness form. Its upper and lower surfaces are defined in terms of the camber function (or meanline) and the thickness function as follows:

$$y_u = y_c + y_t, \quad \text{and} \quad y_l = y_c - y_t.$$

where $y_t = y_t(x) = (y_u - y_l)/2$ is the thickness function for $0 \le x \le c$, and y_c is the offset of the meanline for $0 \le x \le c$. The boundary condition on the surfaces of the airfoil to within the thin-airfoil approximation is

$$v' = U\frac{dy_c}{dx} - U\alpha \pm U\frac{y_t}{dx}.$$

The first two terms on the right hand side are called the circulatory parts and the last term the non-circulatory part. This boundary condition is evaluated at $y = 0$, $0 \le x \le c$. This shows that the airfoil problem can be solved in two parts and the solutions can be superimposed. We will deal with the circulatory part because it is the part that produces lift. (The non-circulatory part can be handled by a distribution of sources the same way we examined the Rankine oval in Chapter 3.) The circulatory part can be modeled by a distribution of vortices placed on the x axis from 0 to c. The normal component of the velocity associated with this distribution is

$$v' = \frac{1}{2\pi}\int_0^c \frac{k\,(x)}{x - x_1}dx.$$

This can be equated to the circulatory part of the boundary condition to yield

$$U\left[\frac{dy_c}{dx} - \alpha\right] = \frac{1}{2\pi}\int_0^c \frac{k\,(x)}{x - x_1}dx.$$

This equation relates the camber and angle of attack (or geometric quantities that describe the meanline shape and the orientation of the airfoil with respect to the onset flow) with the chordwise distribution of k. This is the working formula for thin-airfoil theory.

Example 1: Let us examine a camber with constant value of p from the leading to trailing edges of the airfoil. Suppose $\alpha = 0$ and $k = k_o$ is a constant at $y = 0$, $0 \le x \le c$. Let us define the free stream speed as U and assume it is in the x direction. Substituting into the boundary condition, we get

$$\frac{U}{c}\frac{dy_c}{d\xi} = \frac{k_o}{2\pi}\int_0^1 \frac{d\xi}{\xi - \xi_1},$$

where $\xi = x/c$ was used. Thus,

$$\frac{dy_c}{d\xi} = \frac{k_o c}{2\pi U}\lim_{\epsilon \to 0}\left[\int_0^{\xi-\epsilon}\frac{d\xi}{\xi - \xi_1} + \int_{\xi+\epsilon}^1 \frac{d\xi}{\xi - \xi_1}\right].$$

Doing the integrations and taking the limit, we get

$$\frac{dy_c}{d\xi}(\xi_1) = \frac{k_o c}{2\pi U}\left[\ln \xi_1 - \ln\left(1 - \xi_1\right)\right].$$

To find the camber that leads to $k = k_o$ (a constant chordwise load distribution), we integrate this equation from $\xi = 0$ to $\xi = \xi$, where $0 \le \xi \le 1$.

$$\int_0^\xi \frac{dy_c}{d\xi}d\xi = \frac{k_o c}{2\pi U}\int_0^\xi \left[\ln \xi - \ln\left(1 - \xi\right)\right]d\xi.$$

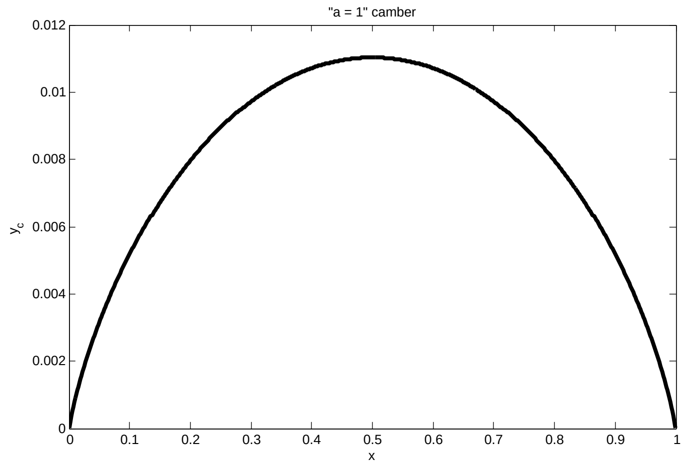

Figure 1: The camber shape.

Integrating, we get

$$\frac{y_c}{c} = -\frac{k_o}{2\pi U}\left[\xi \ln \xi + (1-\xi)\ln(1-\xi)\right],$$

where $0 \leq \xi \leq 1$. This is the solution sought. The minus sign is needed because the positive direction of Γ and, hence, k is taken as clockwise in this note. It is this camber that leads to a uniform circulatory load from the leading to the trailing edge of the airfoil.

Let us next examine the theoretical value of the lift due to this camber. From the Kutta-Zhukovsky law

$$L = \rho V \Gamma = \rho U k_o \int_0^c dx = \rho U k_o c,$$

where the fact that k_o is constant in this example was used. Thus, the total circulation for this case is $\Gamma = k_o c$. The moment about the leading edge is

$$M_{L.E.} = -\rho U k_o \int_0^c x\, dx = -\rho U k_o \frac{c^2}{2} = -\rho U k_o c\left(\frac{c}{2}\right).$$

Comparing the lift and moment, we observe that the center of loading is at $x = c/2$ as expected since the distribution of pressure jump is symmetric about midchord.

Recall the definition of the lift coefficient, viz.:

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 c}.$$

With this definition we can rewrite the coefficient of the y_c/x formula as follows:

$$\frac{k_o}{2\pi U} = \frac{L}{\frac{1}{2}\rho U^2 c}\frac{\frac{1}{2}\rho U^2 c}{L}\frac{k_o}{2\pi U} = C_L \frac{\frac{1}{2}\rho U^2 c}{\rho U k_o c}\frac{k_o}{2\pi U} = \frac{C_L}{4\pi}.$$

Thus, the equation for y_c/c can be written in terms of the theoretical lift coefficient, C_L, viz.,

$$\frac{y_c}{c} = -\frac{C_L}{4\pi}\left[\xi \ln \xi + (1-\xi)\ln(1-\xi)\right].$$

To examine this camber numerically the following MATLAB script was executed. It applies a surface distribution of vortex panels to model the camber. The shape of the camber is illustrated in Figure 1. The maximum camber is 0.011 of c.

```
%
% Camber for k=ko=constant
clear;clc
c=1; U=1; ko = .1;
N=600;
C = c*ko/2/pi/U; % C = CL/2;
x1 = .0001; x2 = 1-.0001; DX = x2-x1;
x = x1:DX/N:x2;
for nc = 1:N+1
y(nc) = -C*( (1-x(nc))*log(1-x(nc)) + x(nc)*log(x(nc)) );
end
uinfty = U;
%
% Location of point vortices and
% collocation points:
for j=1:N
    % Point vortices locations
    xvort(j) = x(j) + 0.25*(x(j+1)-x(j));
    yvort(j) = y(j) + 0.25*(y(j+1)-y(j));
    dsP(j) = sqrt((x(j+1)-x(j))^2+(y(j+1)-y(j))^2);
    % Collocation points location
    xc(j) = x(j) + 0.75*(x(j+1)-x(j));
    yc(j) = y(j) + 0.75*(y(j+1)-y(j));
    % Normal to the panel
    normx(j) = (y(j+1)-y(j));
    normy(j) = -(x(j+1)-x(j));
    lengthP = sqrt(normx(j)^2+normy(j)^2);
    normx(j) = normx(j)/lengthP;
    normy(j) = normy(j)/lengthP;
end
% Determination of the velocity at the
% collocation points due to unit vortices:
for j=1:N
    for k=1:N
        dx = xc(j) - xvort(k);
        dy = yc(j) - yvort(k);
        r = sqrt(dx^2+dy^2);
        vx = -1/(2*pi*r)*dy/r;
        vy = 1/(2*pi*r)*dx/r;
        norm_velocity = vx*normx(j) + vy*normy(j);
        A(j,k) = norm_velocity;
    end
    b(j,1) =  uinfty*normx(j) + 0*normy(j);
end
%
vortex_strength_vector = -A\b;
%
for j=1:N
        u(j) = uinfty; v(j) = 0;
        for m=1:N
            dx = xc(j) - xvort(m);
            dy = yc(j) - yvort(m);
            r = sqrt(dx^2+dy^2);
            vx = -1/(2*pi*r)*dy/r;
            vy = 1/(2*pi*r)*dx/r;
            u(j) = u(j) + vortex_strength_vector(m)*vx;
            v(j) = v(j) + vortex_strength_vector(m)*vy;
        end
end
delP = -vortex_strength_vector./dsP';
plot(xvort,delP,'r')
```

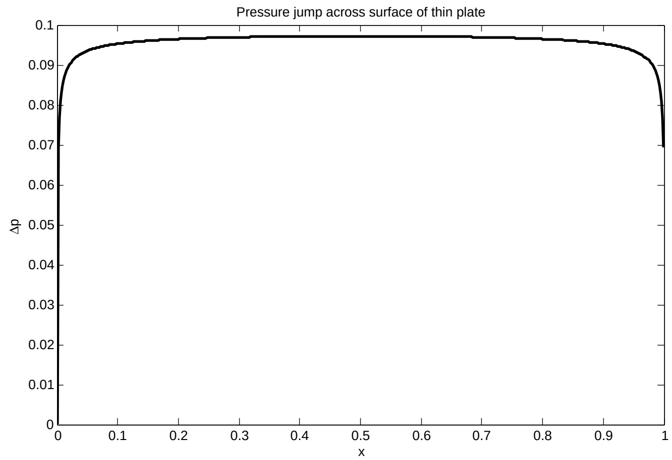

Figure 2: Load distribution numerically estimated.

```
title('Pressure jump across surface of thin plate')
xlabel(' x');ylabel('\Deltap')
%
% Computation of the lift
C = - sum(vortex_strength_vector); xc = xvort';
CMLE = 2*sum(xc.*vortex_strength_vector);
CL = 2*C; disp('The lift and moment coefficients are as follows: ')
disp([' CL = ',num2str(CL)])
CMqt = (CMLE/CL + 0.25)*CL;
disp([' CM about the quarter chord = ',num2str(CMqt)])
disp(' Note that a negative moment is pitch down. ')
%
```

The solution computed for this camber is

```
The lift and moment coefficients are as follows:
        CL = 0.191
        CM about the quarter chord = -0.048
Note that a negative moment is pitch down.
```

The load distribution predicted by applying the MATLAB script is illustrated in Figure 2. The value of the lift coefficient predicted numerically (as given above) is based on a surface distribution of 600 vortex panels. This prediction is 4% lower than the linear theory prediction of 0.2. This comparison illustrates that the linear theory, which provides a formula for the lift coefficient that does not require a computer to evaluate, gives a reasonable approximation as compared with the numerical solution.

Example 2: In this example we examine the flat-plate airfoil. Let us consider the flat-plate airfoil theory, i.e., the solution for the angle-of-attack part of the two circulatory contributions to the lift in thin-airfoil theory discussed on pages 223-235 in the text. The distribution in the jump in pressure across a flat-plate airfoil is given by Equation (4.30). It is

$$p = \rho U k = 2\rho U^2 \alpha \frac{1+\cos\theta}{\sin\theta} = 2\rho U^2 \alpha \frac{\sqrt{1-x}}{\sqrt{x}}.$$

Integrating this jump in pressure from the leading to the trailing edge, we get

$$C_L = 2\pi\alpha,$$

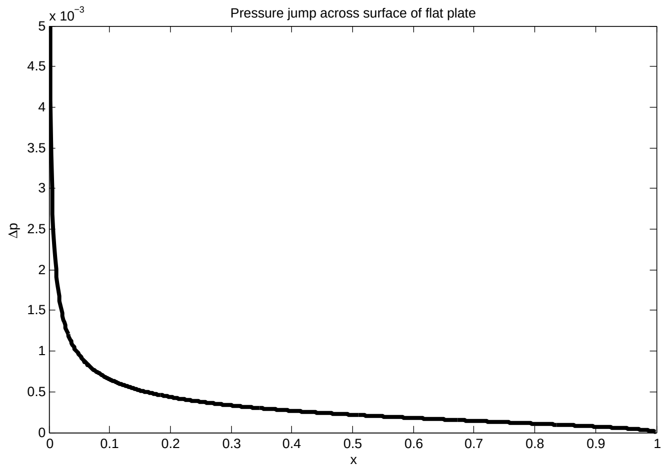

Figure 3: Load distribution on a flat plate at 5 degrees angle of attack.

$$C_{ML.E.} = -\alpha\frac{\pi}{2} = -\frac{C_L}{4}.$$

Thus, the moment about the 1/4-chord point is identically zero. The center pf the lift is at this location. This is also called the "center of pressure", as is done in the text. What does the chordwise distribution of p look like? It is illustrated in Figure 3. Note that it increases singularly at the leading edge; it varies as $1/\sqrt{x}$ as $x \to 0$. Yet, the integral under the curve is finite as indicated by the formula for the lift coefficient.

1.2 Solutions to Chapter 6 problems

Problem 6.1

This problem is similar to the example given in the previous section. We want to examine a chordwise distribution of circulation that is a maximum at the leading edge and decreases linearly to zero at the trailing edge, viz., $k = k_o\,(1-\xi)$. Let us begin by examining the integral for the lift. Substituting for Γ, we get

$$L = \rho U \Gamma = \rho U c \int_0^1 k_o\,(1-\xi)\,d\xi = \frac{1}{2}\rho U k_o c.$$

Thus,

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 c} = \frac{\frac{1}{2}\rho U k_o c}{\frac{1}{2}\rho U^2 c} = \frac{k_o}{U}.$$

Substituting this into the thin-airfoil boundary condition and using the result for C_L, we get

$$\left[\frac{dy_c}{dx} - \alpha\right] = \frac{C_L}{2\pi}\int_0^1 \frac{1-\xi}{\xi-\xi_1}d\xi.$$

Integrating through the singular point by applying the Cauchy approach in the hint and illustrated above, we get

$$\left[\frac{dy_c}{dx} - \alpha\right] = \frac{C_L}{2\pi}\left[(1-\xi)\ln\frac{1-\xi}{\xi} - 1\right].$$

This is the same as the result reported in the problem. To get the function y_c, we integrate

$$\frac{dy_c}{dx} = \frac{C_L}{2\pi}\,(1-\xi)\ln\frac{1-\xi}{\xi}$$

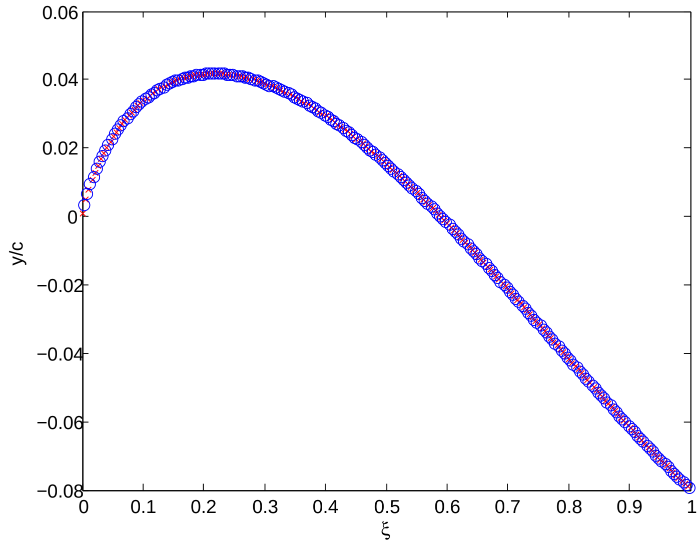

Figure 4: Camber for homework problem 4.1.

from 0 to ξ. The angle of attack is determined from the constant on the right hand side of the boundary condition.

$$\alpha = \frac{C_L}{4\pi}.$$

Integrating the $dy_c/d\xi$ equation from 0 to ξ, we get

$$\frac{y_c}{c} = \frac{C_L}{4\pi}\left[-(1-\xi)^2\ln(1-\xi) + \xi(\xi-2)\ln\xi\right].$$

For numerical verification purposes the coordinates of the meanline including the angle of attack is needed. Combining the two results given above, we get the y offset

$$\frac{y_c}{c} = \frac{C_L}{4\pi}\left[-(1-\xi)^2\ln(1-\xi) + \xi(\xi-2)\ln\xi\right] - \frac{C_L}{4\pi}\xi.$$

The camber plus angle for a $C_L = 1$ is illustrated in Figure 4. Applying the same numerical procedure that was applied in the example in the previous section, the distribution of the difference in pressure across the camber is illustrated in Figure 5. The lift coefficient predicted numerically with the surface distribution of vortices numerical method is $C_L = 0.99813$; it is less than a percent of the linear theory prediction of $C_L = 1$. Also, the angle of attack is the coordinate of the trailing edge below the leading edge. It is a distance of $-C_L/4/\pi = -0.08 = -\tan\alpha = y_{T.E.}/c$ as can be observed in Figure 4.

Problem 6.2

Consider the cubic camber line given by the following formula:

$$y = \frac{y_c}{kc} = \xi(\xi-1)(\xi-2).$$

Expanding the product, we get

$$y = \xi^3 - 3\xi^2 + 2\xi.$$

$$\frac{dy}{d\xi} = 3\xi^2 - 6\xi + 2.$$

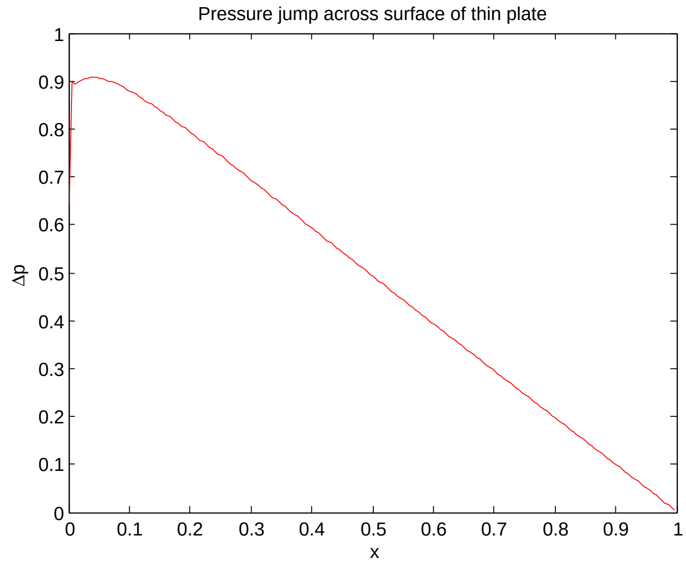

Figure 5: Load distribution for homework problem 4.1.

Setting this equal to zero, we get $\xi = 0.4226467$ as the location of the maximum value of y. At this location $y = 0.3849$; hence, $y_c/c = k(0.3849) = 0.02$ for $k = 0.052$, which is the answer to the first question in this problem.

Next, let $\xi = (1 - \cos\theta)/2$. Thus, we can write $dy/d\xi$ in terms of θ as follows:

$$\frac{dy}{d\xi} = 3\left[\frac{1}{4} - \frac{1}{2}\cos\theta + \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right)\right] - 6\left(\frac{1}{2} - \frac{1}{2}\cos\theta\right) + 2,$$

where the identity $2\cos^2\theta = 1 + \cos 2\theta$ was used. Rearranging terms, we get

$$\frac{dy}{d\xi} = \frac{1}{8} + \frac{3}{2}\cos\theta + \frac{3}{8}\cos 2\theta.$$

Thus,

$$\frac{dy_c}{dx} = \frac{k}{8} + \frac{3k}{2}\cos\theta + \frac{3k}{8}\cos 2\theta.$$

Hence, the Fourier coefficients of the derivative of the camber in this problem are

$$A_o = \frac{k}{8}, \quad A_1 = \frac{3k}{2}, \quad A_2 = \frac{3k}{8}.$$

All other Fourier coefficients are zero. The lift coefficient is, thus,

$$C_L = \pi\left(A_1 - 2A_o\right) + 2\pi\alpha = \pi k\frac{5}{4} + 2\pi\alpha.$$

Since $k = 0.052$ corresponds to a maximum $y_c/c = 0.02$ camber ratio and the specified angle of attack is 3 degrees, by direct substitution, we get $C_L = 0.53$.

The moment about the 1/4 chord is given by Equation (4.47) on page 181 in the text, viz.,

$$C_{M1/4} = -\frac{\pi}{4}\left(A_1 - A_2\right).$$

By direct substitution, we get $C_{M1/4} = -0.046$. Thus, the answers in the text are correct as given for this problem.

Problem 6.3

This problem asks us to determine the lift coefficient for the NACA 8210 wing section by applying thin-airfoil theory. To do this we apply the results in Section 6.8.2. We note that for the NACA 8210 $m = 0.08$, $p = 0.2$ and $t = 0.1$. The formulas for the Fourier coefficients for this series of airfoils are given in the text in terms of m and p. We need to determine θ_P, the location of the maximum camber, first. It is the solution of

$$\frac{x}{c} = p = \frac{1}{2}\left(1 - \cos\theta_p\right).$$

Thus, $\theta_p = 53.13$ degrees or 0.927293 radians. We can now substitute into the equations for A_0 and A_1. We get $A_o = 0.0704$ and $A_1 = 0.3920$. Thus, $C_L = \pi(A_1 - 2 * A_o) = 0.7890$. For the pitching moment you need to determine A_2 as well. It is $A_2 = 0.1723$. Thus,

$$C_{M1/4} = -\frac{\pi}{4}\left(A_1 - A_2\right) = -0.1726.$$

These results are consistent with the answers given in the text. The following MATLAB script was applied to substitute the given information into the equations in the text to obtain the results given above. In addition, at the end of the script the solution to Example 6.2 given in the text is provided as another example of using MATLAB to solve this type of problem.

```
clear;clc
% Problem 3: NACA 8210
m = 0.08; p = 0.2; t = 0.1; alpha = 0;
thetap = acos(1 - 2*p);
Ao = (m/pi/p/p)*((2*p-1)*thetap+sin(thetap)) ...
    + (m/(pi*(1-p)^2))*( (2*p-1)*(pi-thetap)-sin(thetap) );
A1 = (2*m/pi/p/p)*( (2*p-1)*sin(thetap) + sin(2*thetap)/4 ...
    + thetap/2) - (2*m/(pi*(1-p)^2))*( (2*p-1)*sin(thetap) ...
    + sin(2*thetap)/4  - (pi - thetap)/2 );
A2 = (2*m/pi/p/p)*( (2*p-1)*(sin(2*thetap)/4+thetap/2) ...
    + sin(thetap) - sin(thetap)^3/3 ) - (2*m/(pi*(1-p)^2))* ...
    ( (2*p-1)*(sin(2*thetap)/4-(pi-thetap)/2) + sin(thetap) ...
    - sin(thetap)^3/3 );
CL = pi*(A1-2*Ao) + 2*pi*alpha;
CMqc = - pi*(A1-A2)/4;
disp('       Ao       A1        A2        CL       CMqc ')
disp([Ao A1 A2 CL CMqc])
% RESULTS: The CL and the CMqc are the same as in the text.
%         Ao       A1        A2        CL       CMqc
%     0.0704    0.3920    0.1723    0.7890   -0.1726
%
% % Example 6.2 in the text:
% m = 0.04; p = 0.4; t = 0.12; alpha = 0;
% thetap = acos(1 - 2*p);
% Ao = (m/pi/p/p)*((2*p-1)*thetap+sin(thetap)) ...
%     + (m/(pi*(1-p)^2))*( (2*p-1)*(pi-thetap)-sin(thetap) );
% A1 = (2*m/pi/p/p)*( (2*p-1)*sin(thetap) + sin(2*thetap)/4 ...
%     + thetap/2) - (2*m/(pi*(1-p)^2))*( (2*p-1)*sin(thetap) ...
%     + sin(2*thetap)/4  - (pi - thetap)/2 );
% A2 = (2*m/pi/p/p)*( (2*p-1)*(sin(2*thetap)/4+thetap/2) ...
%     + sin(thetap) - sin(thetap)^3/3 ) - (2*m/(pi*(1-p)^2))* ...
%     ( (2*p-1)*(sin(2*thetap)/4-(pi-thetap)/2) + sin(thetap) ...
%     - sin(thetap)^3/3 );
% CL = pi*(A1-2*Ao) + 2*pi*alpha;
```

```
% CMqc = - pi*(A1-A2)/4;
% disp('       Ao       A1        A2        CL       CMqc ')
% disp([Ao A1 A2 CL CMqc])
% % RESULTS: The following agree with what is in the text.
% %        Ao       A1        A2        CL       CMqc
% %     0.0090    0.1630    0.0228    0.4556   -0.1101
```

Problems 6.4 and 6.5

You could examine the solution by using the MATLAB script in Problem 6.3 to examine NACA 9410. In this case use $m = 0.09$, $p = 0.4$ and $t = 0.1$. Follow a similar procedure ("trial and error") to examine the solutions provided to help solve Problem 6.5.

Problem 6.6

A thin two-dimensional flat-plate airfoil is fitted with a trailing edge flap of chord $100e\%$ of the airfoil chord. Show that the flap effectiveness,

$$\frac{a_2}{a_1} = \frac{\partial C_L/\partial \eta}{\partial C_L/\partial \alpha},$$

where α is the angle of attack and η is the flap angle, is approximately $4\sqrt{e}/\pi$ for flaps of small chord.

From Equation (6.52) we get

$$\frac{\partial C_L}{\partial \alpha} = 2\pi, \quad \text{and} \quad \frac{\partial C_L}{\partial \eta} = 2\left(\pi - \phi + \sin\phi\right).$$

Thus,

$$\frac{a_2}{a_1} = \frac{2\left(\pi - \phi + \sin\phi\right)}{2\pi} \approx \frac{4\sqrt{e}}{\pi},$$

where $\phi \to \pi$, $\sin\phi = \sqrt{1-\cos^2\phi}$, and $\cos\phi = 2e - 1$ were used. In addition, $\sin\phi = \sqrt{1-(2e-1)^2} = \sqrt{1-(1-4e+4e^2)}$; thus, $\sin\phi \to \sqrt{4e} = 2\sqrt{e}$. This is half the result. The other half comes from expanding ϕ as follows. Note that $\cos\phi = \cos\left(\phi - \epsilon\right)$, where ϵ is a small number. Applying the addition theorem $\cos\left(\phi - \epsilon\right) = \cos\pi\cos\epsilon - \sin\pi\sin\epsilon = -\cos\epsilon$. For small ϵ

$$\cos\epsilon \approx 1 - \frac{\epsilon^2}{2} + \ldots.$$

Thus,

$$\cos\phi = 2e - 1 \approx -1 + \frac{\epsilon^2}{2}.$$

Thus, $\epsilon = \sqrt{4e} = 2\sqrt{e}$. This is the second half of the final approximate result, viz., $4\sqrt{e}/\pi$.

This can be evaluated graphically by applying the MATLAB script:

```
%
% Problem 6.6 check on flap effectiveness
%
F = 0:.001:.5; c=1;
cosPhi = 2.*F-1;
Phi = acos(cosPhi);
a1 = 2*pi;
a2 = 2.*(pi - Phi + sin(Phi));
eff = a2./a1;
effCk = 4.*sqrt(F)./pi;
plot(F,eff,F,effCk,'o')
```

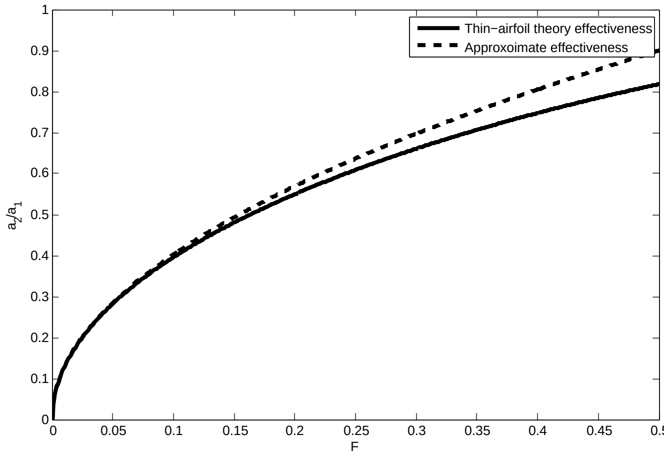

Figure 6: Comparison of approximate and exact fin effectiveness formulas.

Fig. refeffect illustrates that the approximate formula is quite good for $F = e < 0.1$ or flap chords less than 10 percent of the airfoil chord.

Problem 6.7

The camber line of interest in this problem is

$$\frac{y_c}{c} = k\left[\frac{1}{4} - \left(\frac{x'}{c}\right)^2\right],$$

where $x' = x - c/2$. Substituting this relationship into the formula above, we get

$$\frac{y_c}{c} = k\left[\frac{1}{4} - \left(\frac{x}{c} - \frac{1}{2}\right)^2\right],$$

where $0 \leq x \leq c$. Select k so that the maximum camber is 0.025 times the chord. Determine the thin-airfoil theory lift and pitching moment coefficients for this camber. Describe (in a sentence or two) one method by which you could reduce the pitching moment without changing the maximum camber or the lift.

Solution: You were asked to examine the circular-arc camber distribution with a maximum camber divided by chord ratio equal to 2.5% of the chord. To determine the k that corresponds to $y_{cmax} = 0.025c$, we set the following derivative to zero to determine the location of the maximum camber:

$$\frac{dy_c/c}{dx/c} = -2k\left(\frac{x}{c} - \frac{1}{2}\right).$$

Thus, the maximum camber is located at $x/c = 1/2$. Substituting this location into the camber distribution, we find that the maximum camber is $y_{cmax}/c = k/4$. Thus,

$$k = 0.1$$

for a 2.5% camber ratio.

To solve for C_L and $C_{M_{1/4}}$ we apply the method in Section 4.4.2 (pages 231-235). It is applied to a cubic camber distribution in Section 4.8 (page 245). The first step is to expand the derivative with respect to x of the camber distribution in a cosine series. To apply the procedure described in the text, we need to convert x to θ by substituting

$$\frac{x}{c} = \frac{1}{2}(1 - \cos\theta)$$

into the formula for dy_c/dx given above. Thus,

$$\frac{dy_c}{dx} = -2k\left(-\frac{1}{2}\cos\theta\right) = k\cos\theta.$$

This is the Fourier series expansion of the derivative of the camber distribution sought. By inspection, $A_o = A_2 = 0$, and $A_1 = k$. Thus,

$$C_L = \pi(A_1 - 2A_o) = \pi k, \quad C_{M_{1/4}} = -\frac{\pi}{4}(A_1 - A_2) = -\frac{\pi k}{4}.$$

Substituting for k, we get

$$C_L = 0.1\pi, \quad C_{M_{1/4}} = -0.025\pi.$$

How can we reduce the moment for fixed values of maximum camber and lift? *One approach is to shift the maximum camber towards the quarter-chord to reduce the moment arm. Since angle of attack does not contribute to the moment about the quarter-chord, any loss in lift by the shift in the location of the maximum camber could be made up by increasing the angle of attack. Hence, the maximum camber and the lift would not change.*

Problem 6.8

The camber line of a circular-arc airfoil is given by

$$\frac{y_c}{c} = 4h\frac{x}{c}\left(1 - \frac{x}{c}\right),$$

where $0 \le x \le c$. Derive an expression for the load distribution (pressure difference across the airfoil) at incidence α. Show that the zero-lift angle $\alpha_o = -2h$, and sketch the load distribution at this incidence. Compare the lift curve of this airfoil with that of a flat plate.

Solution: Take the derivative of y_c/c with respect to x/c. We get

$$\frac{dy_c}{dx} = 4h\left(1 - 2\frac{x}{c}\right),$$

Next, substitute $x/c = (1 - \cos\theta)/2$. We get

$$\frac{dy_c}{dx} = 4h[1 - (1 - \cos\theta)] = -4h\cos\theta.$$

This is a Fourier cosine series for dy_c/dx; hence, $A_o = 0$, $A_1 = -4h$ and $A_2 = A_3 = \ldots = 0$. From the general solution given in the text, we get the pressure distribution requested in this problem. It is

$$p = \rho U k = 2\rho U^2\alpha\frac{1 + \cos\theta}{\sin\theta} + \rho U^2 8h\sin\theta.$$

The first term is the angle of attack contributions and the second term is the camber line contribution. The lift coefficient, including angle of attack, is

$$C_L = 2\pi\alpha + 4\pi h.$$

Thus, the angle of zero lift is $\alpha = -2h$. Thus we can write

$$C_L = 2\pi(\alpha - \alpha_o)$$

The slope of this line is the same as the slope of C_L versus α for the flat plate. The difference is that this curve is shifted to the left of the origin by α_o.

Problem 6.9

The computational solution for this problem was found by applying the following MATLAB script that models the plate as a distribution of lumped-vortex panels. Note that in this case we used $U = 1$ and $\alpha = 0$. This corresponds to the approximate theoretical result reported in the problem statement. This fact was verified graphically by executing this script.

```
% PROBLEM 9 in Chapter 6 of the text.
% Note that the increment of CL is approximately equal
% 2*(4*Gam)/(3*pi*h*h); it is twice the dimensional ratio
% given in the text because in this code it is CL and not L
% that is computed!
%
% Potential flow around a flat plate by
% using the 1/4-3/4-chord 'Weissinger' panels.
% N = number of panels.
%
% Daniel T. Valentine ............ March 2012/August 2016.
clear;clc
N = 100;
% Geometry of a cylinder
%theta = 0:2*pi/N:2*pi;
%R = 1;
% for j=1:N+1
% x(j) = R*cos(theta(j));
% y(j) = R*sin(theta(j));
% end
% VORTEX NEAR PLATE INTERACTION
% Geometry of a flat plate at angle of attack
alpha = 0; % degrees
% x = 0:1/N:1;
for j=1:N+1;
x(j) = cos(alpha*pi/180)*(j-1)/N;
y(j) = -sin(alpha*pi/180)*(j-1)/N;
end
% Vortex
Gam = 1;
xnu = 0.5; % Above mid-chord location
hv = 5:1:20;
for ih = 1:length(hv)
    h = hv(ih);
ynu = h; % >> xnu as per Problem 9 in the text.
% Free stream
uinfty = 1;
%
% Location of point vortices and
% collocation points
%
for j=1:N
% Point vortices locations
xvort(j) = x(j) + 0.25*(x(j+1)-x(j));
yvort(j) = y(j) + 0.25*(y(j+1)-y(j));
% Collocation points location
xc(j) = x(j) + 0.75*(x(j+1)-x(j));
yc(j) = y(j) + 0.75*(y(j+1)-y(j));
% Normal to the panel
normx(j) = (y(j+1)-y(j));
```

```
normy(j) = -(x(j+1)-x(j));
lengthP = sqrt(normx(j)^2+normy(j)^2);
ds(j) = lengthP;
normx(j) = normx(j)/lengthP;
normy(j) = normy(j)/lengthP;
end
% Determination of the velocity at the
% collocation points due to unit vortices
% distance from the vortices to the collocation
% POINT
for j=1:N
for k=1:N
dx = xc(j) - xvort(k);
dy = yc(j) - yvort(k);
r = sqrt(dx^2+dy^2);
vx = -1/(2*pi*r)*dy/r;
vy = 1/(2*pi*r)*dx/r;
norm_velocity = vx*normx(j) + vy*normy(j);
A(j,k) = norm_velocity;
end
vcx(j) = Gam*(ynu-yc(j))/( 2*pi*(xnu-xc(j))^2 + (ynu-yc(j))^2 );
vcy(j) = -Gam*(xnu-xc(j))/( 2*pi*(xnu-xc(j))^2 + (ynu-yc(j))^2 );
b(j,1) = (uinfty+vcx(j))*normx(j) + (0+vcy(j))*normy(j);
end
% Replace last equation by ones and set b = 0; this
% ensures that the net circulation is zero (works without this!!!!!)
% A(N,:) = ones(1,N);
% b(N,1) = 0.0;
vortex_strength_vector = -A\b;
% The -A\b vector is the solution sought
%
% Computation of the lift
vcc = uinfty + vcx';
dx = -normy';
dC = dx.*vcc.*vortex_strength_vector;
CL = -sum(dC);
disp(['alpha in degrees = ',num2str(alpha),', h = ',num2str(h),', CL = ',num2str(CL)])
CLhv(ih) = CL;
CLap(ih) = 2*(4/3/pi/h/h);
end
plot(hv,CLhv,'k',hv,CLap,'+k')
```

The result printed to the Command Window were

```
alpha in degrees = 0, h = 5, CL = 0.035921
alpha in degrees = 0, h = 6, CL = 0.024612
alpha in degrees = 0, h = 7, CL = 0.017872
alpha in degrees = 0, h = 8, CL = 0.013548
alpha in degrees = 0, h = 9, CL = 0.010614
alpha in degrees = 0, h = 10, CL = 0.0085359
alpha in degrees = 0, h = 11, CL = 0.0070109
alpha in degrees = 0, h = 12, CL = 0.0058595
alpha in degrees = 0, h = 13, CL = 0.0049694
alpha in degrees = 0, h = 14, CL = 0.0042672
alpha in degrees = 0, h = 15, CL = 0.0037036
alpha in degrees = 0, h = 16, CL = 0.0032445
alpha in degrees = 0, h = 17, CL = 0.0028656
```

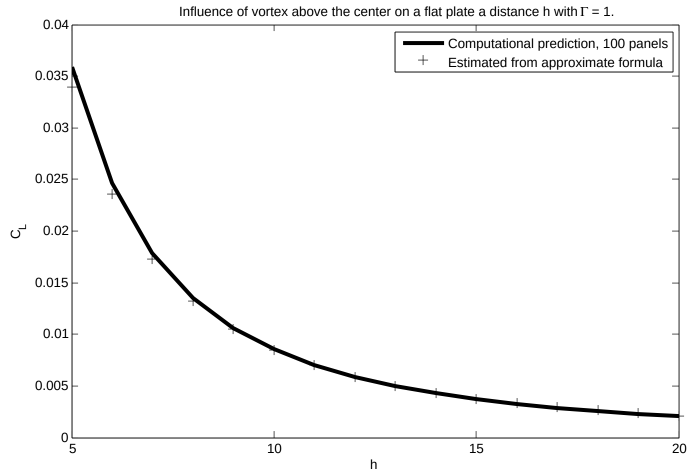

Figure 7: Comparison of approximate lift with numerical prediction of lift.

```
alpha in degrees = 0, h = 18, CL = 0.0025493
alpha in degrees = 0, h = 19, CL = 0.0022826
alpha in degrees = 0, h = 20, CL = 0.0020555
```

These results are essentially the answers given by the formula in the problem statement; note that the results from the formula in the text must be multiplied by 2 to compare with the above result because the above results are lift coefficients as opposed to lift. In Fig. 7 we compare the results from the formula with the numerical predictions over the range $5 \leq h \leq 20$. The graph indicates that the approximation is very good for $h > 8$; it isn't bad for $h = 5$ as well.

Section 6.10 in the text on computational methods

The FORTRAN code in Section 5.10 of the text was converted to the following MATLAB function:

```
function [AN,AT,XC,YC,NHAT,THAT] = InfluSour(XP,YP,N)
% Influence coefficients for source distribution over a
% symmetric body.
% AN is the Nij matrix and AT is the Tij matix in Eqn (3.99).
%
for J = 1:N
if J==1
XPL = XP(N);
YPL = YP(N);
else
XPL = XP(J-1);
YPL = YP(J-1);
end
XC(J) = 0.5*(XP(J) + XPL);
YC(J) = 0.5*(YP(J) + YPL);
S(J) = sqrt( (XP(J) - XPL)^2 + (YP(J) - YPL)^2 );
THAT(J,1) = (XP(J) - XPL)/S(J);
THAT(J,2) = (YP(J) - YPL)/S(J);
NHAT(J,1) = - THAT(J,2);
NHAT(J,2) = THAT(J,1);
end
%Calculation of the influence coefficients.
for I = 1:N
for J = 1:N
if I==J
AN(I,J) = pi;
AT(I,J) = 0;
else
DX = XC(I) - XC(J);
DY = YC(I) - YC(J);
XQ = DX*THAT(J,1) + DY*THAT(J,2);
YQ = DX*NHAT(J,1) + DY*NHAT(J,2);
VX = -0.5*( log( (XQ + S(J)/2 )^2 + YQ*YQ )...
-log( (XQ - S(J)/2 )^2 + YQ*YQ ) );
VY = -( atan2( (XQ + S(J)/2),YQ) - atan2((XQ - S(J)/2),YQ));
NTIJ = 0;
NNIJ = 0;
TTIJ = 0;
TNIJ = 0;
for K = 1:2
NTIJ = NHAT(I,K)*THAT(J,K) + NTIJ;
NNIJ = NHAT(I,K)*NHAT(J,K) + NNIJ;
TTIJ = THAT(I,K)*THAT(J,K) + TTIJ;
TNIJ = THAT(I,K)*NHAT(J,K) + TNIJ;
end
```

```
AN(I,J) = VX*NTIJ + VY*NNIJ;
AT(I,J) = VX*TTIJ + VY*TNIJ;
end
end
end
```

In Section 6.10 the modification described were implemented and the resulting function given in this section of the text. For completeness, the modified function is:

```
function [AN,AT,XC,YC,NHAT,THAT] = InfluSourV(XP,YP,N,NT,NTP1)
% Influence coefficients for source distribution over a
% symmetric body.
%
NP1 = N + 1;
for J = 1:N
AN(J,NP1) = 0;
AT(J,NP1) = pi;
if J==1
XPL = XP(N);
YPL = YP(N);
else
XPL = XP(J-1);
YPL = YP(J-1);
end
XC(J) = 0.5*(XP(J) + XPL);
YC(J) = 0.5*(YP(J) + YPL);
S(J) = sqrt( (XP(J) - XPL)^2 + (YP(J) - YPL)^2 );
THAT(J,1) = (XP(J) - XPL)/S(J);
THAT(J,2) = (YP(J) - YPL)/S(J);
NHAT(J,1) = - THAT(J,2);
NHAT(J,2) = THAT(J,1);
end
%Calculation of the influence coefficients.
for I = 1:N
for J = 1:N
if I==J
AN(I,J) = pi;
AT(I,J) = 0;
else
DX = XC(I) - XC(J);
DY = YC(I) - YC(J);
XQ = DX*THAT(J,1) + DY*THAT(J,2);
YQ = DX*NHAT(J,1) + DY*NHAT(J,2);
VX = -0.5*( log( (XQ + S(J)/2 )^2 + YQ*YQ )...
-log( (XQ - S(J)/2 )^2 + YQ*YQ ) );
VY = -( atan2((XQ + S(J)/2 ),YQ) - atan2((XQ - S(J)/2),YQ ));
NTIJ = 0;
NNIJ = 0;
TTIJ = 0;
TNIJ = 0;
for K = 1:2
```

```
NTIJ = NHAT(I,K)*THAT(J,K) + NTIJ;
NNIJ = NHAT(I,K)*NHAT(J,K) + NNIJ;
TTIJ = THAT(I,K)*THAT(J,K) + TTIJ;
TNIJ = THAT(I,K)*NHAT(J,K) + TNIJ;
end
AN(I,J) = VX*NTIJ + VY*NNIJ;
AT(I,J) = VX*TTIJ + VY*TNIJ;
AN(I,NP1) = AN(I,NP1) + VY*NTIJ - VX*NNIJ;
AT(I,NP1) = AT(I,NP1) + VY*TTIJ - VX*TNIJ;
end
end
end
for n = 1:NP1
AN(NP1,n) = -(AT(NT,n) + AT(NTP1,n));
AT(NP1,n) = 0;
end
AT(NP1,NP1) = pi;
```

This MATLAB function was used to plot Fig. 6.25 is as follows:

```
% N is number of panels used to model elliptical surface.
clear;clc
%
% NACA 4-digit airfoil: 4412
%
% clear;clc
c = 1;
Nthe = 201;
the = 0:-2*pi/Nthe:-pi;
x = (1 + cos(the) )/2;
m=4/100; p=4/10; t = 0.12;
yt = 5*t*c .* (.2969*x.^0.5 - .126*x ...
    - .3516*x.^2 + .2843*x.^3 - .1015*x.^4); %yt(length(x))=0;
for n=1:length(x)
  if x <= p
    yc(n) = (m*c/p)*(2*p*x(n)-x(n)^2);
  else
    yc(n) = (m*c/(1-p)^2)*((1-2*p) + 2*p*x(n) - x(n)^2);
  end
end
XPU = x; YPU = yt + yc;
for n = 1:length(x)-1
    XPL(n) = x(length(x)-n);
    YPL(n) = yc(length(x)-n) - yt(length(x)-n);
end
XP = [XPU(1:end) XPL(1:end-1)];
YP = [YPU(1:end) YPL(1:end-1)];
% Inherent camber angle of attack = atan(0.022)*180/pi = 1.3
alpha = 4.2; alp = pi*alpha/180; N = length(XP);
NT = 1; NTp1 = N;
% for m = 1:N
%     YP(m) = YP(m) - sin(alpha*pi/180)*XP(m);
% end
figure(1);plot(XP,YP,'-o')
%
```

```
[AN,AT,XC,YC,NHAT,THAT] = InfluSourV(XP,YP,N,NTp1,NT);
% uinfty = 1;
 for j=1:N
 b(j,1) =  cos(alp)*NHAT(j,1) + sin(alp)*NHAT(j,2);
 end
  b(N+1,1) = - ( THAT(NT,1) + THAT(NT+1,1));
 Sources = AN\b;
 THAT(N+1,1) = 0; THAT(N+1,2) = 0;
 ut = AT*Sources - THAT(:,1);
 cp = 1 - ut.^2;
figure(2);
plot(XC,1-cp(1:end-1),'k')
xlabel(' x '),ylabel('  1-Cp '),axis([0 1 0 3])
title(' NACA 4412 airfoil at C_L \approx 1.09')
%
hold on
A = [1.0000   0.134
0.9792   0.167
0.9486   0.180
0.8990   0.203
0.8491   0.211
0.7492   0.231
0.6494   0.244
0.5448   0.250
0.4998   0.262
0.4490   0.268
0.3998   0.265
0.3490   0.290
0.2996   0.293
0.2490   0.313
0.1998   0.321
0.1494   0.345
0.0996   0.402
0.0738   0.462
0.0494   0.568
0.0292   0.748
0.0166   0.916
0.0092   1.013
0.0036   0.905
0.0000   0.157
0.0000  -1.000
0.0044  -1.740
0.0094  -1.793
0.0170  -1.743
0.0294  -1.647
0.0490  -1.547
0.0750  -1.432
0.0996  -1.391
0.1258  -1.350
0.1492  -1.308
0.1744  -1.272
0.1996  -1.239
0.2244  -1.224
0.2492  -1.163
0.2744  -1.122
0.2988  -1.071
```

```
0.3498  -0.982
0.3990  -0.880
0.4480  -0.809
0.4992  -0.690
0.5492  -0.601
0.5994  -0.541
0.6490  -0.456
0.6986  -0.371
0.7490  -0.285
0.7992  -0.199
0.8488  -0.106
0.8988  -0.009
0.9490   0.079
0.9800   0.120 ];
plot(A(:,1),1-A(:,2),'ok')
Drela=  [    1.00000  0.18788
   0.99392  0.17318
   0.98332  0.14468
   0.97097  0.10993
   0.95716  0.07020
   0.94236  0.02829
   0.92695 -0.01410
   0.91123 -0.05511
   0.89533 -0.09452
   0.87933 -0.13197
   0.86328 -0.16771
   0.84720 -0.20166
   0.83108 -0.23430
   0.81494 -0.26571
   0.79879 -0.29600
   0.78261 -0.32532
   0.76643 -0.35418
   0.75022 -0.38206
   0.73401 -0.40973
   0.71778 -0.43672
   0.70155 -0.46342
   0.68530 -0.49006
   0.66905 -0.51620
   0.65280 -0.54230
   0.63654 -0.56849
   0.62028 -0.59456
   0.60402 -0.62065
   0.58776 -0.64680
   0.57151 -0.67335
   0.55525 -0.69992
   0.53901 -0.72692
   0.52278 -0.75424
   0.50655 -0.78225
   0.49035 -0.81066
   0.47416 -0.83994
   0.45799 -0.87032
   0.44186 -0.90204
   0.42579 -0.93564
   0.40980 -0.97231
   0.39397 -1.01717
   0.37833 -1.05592
```

0.36279	-1.08967
0.34732	-1.12116
0.33191	-1.15066
0.31655	-1.17857
0.30126	-1.20503
0.28603	-1.23025
0.27087	-1.25409
0.25578	-1.27652
0.24079	-1.29823
0.22589	-1.31839
0.21110	-1.33793
0.19644	-1.35591
0.18193	-1.37340
0.16758	-1.38971
0.15344	-1.40546
0.13953	-1.42067
0.12593	-1.43555
0.11269	-1.44998
0.09993	-1.46503
0.08776	-1.47915
0.07634	-1.49247
0.06582	-1.50382
0.05633	-1.52458
0.04792	-1.56921
0.04059	-1.58460
0.03426	-1.60531
0.02881	-1.62630
0.02412	-1.64583
0.02008	-1.66365
0.01659	-1.67754
0.01357	-1.68606
0.01094	-1.68724
0.00867	-1.67756
0.00670	-1.65374
0.00501	-1.61075
0.00359	-1.54117
0.00241	-1.43814
0.00147	-1.29278
0.00077	-1.09919
0.00029	-0.85909
0.00004	-0.58168
0.00001	-0.28605
0.00021	0.02077
0.00065	0.32660
0.00138	0.59604
0.00239	0.79958
0.00369	0.92722
0.00528	0.98828
0.00713	0.99891
0.00926	0.97656
0.01168	0.93422
0.01440	0.88214
0.01746	0.82493
0.02091	0.76709
0.02480	0.71033
0.02922	0.65555

0.03427	0.60408
0.04008	0.55536
0.04680	0.50994
0.05460	0.46962
0.06364	0.43483
0.07401	0.40097
0.08570	0.37226
0.09862	0.34895
0.11256	0.33033
0.12730	0.31570
0.14264	0.30445
0.15843	0.29574
0.17455	0.28924
0.19092	0.28450
0.20751	0.28066
0.22425	0.27805
0.24114	0.27594
0.25813	0.27433
0.27519	0.27280
0.29228	0.27161
0.30934	0.27018
0.32635	0.26870
0.34332	0.26659
0.36025	0.26424
0.37715	0.26084
0.39405	0.25549
0.41101	0.24725
0.42803	0.24268
0.44511	0.23917
0.46224	0.23649
0.47939	0.23425
0.49656	0.23235
0.51374	0.23081
0.53090	0.22945
0.54805	0.22806
0.56518	0.22690
0.58229	0.22572
0.59939	0.22469
0.61647	0.22353
0.63354	0.22217
0.65060	0.22122
0.66765	0.21987
0.68469	0.21863
0.70172	0.21718
0.71875	0.21582
0.73577	0.21406
0.75280	0.21246
0.76981	0.21090
0.78683	0.20898
0.80385	0.20727
0.82087	0.20524
0.83789	0.20323
0.85491	0.20137
0.87191	0.19918
0.88889	0.19739
0.90581	0.19580

```
    0.92257  0.19400
    0.93901  0.19324
    0.95482  0.19284
    0.96952  0.19391
    0.98258  0.19568
    0.99368  0.20266
    1.00000  0.18788];
plot(Drela(:,1),1-Drela(:,2),'--k')
```

The code XFOIL, made available to the community by Drela at MIT, was used to take account of viscous effects.