



## T5 - Band Pass Filter using OPAMP

Integrated Master in Physics Engineering

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### Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Presential Lab</b>	<b>2</b>
<b>3</b>	<b>Theoretical Analysis</b>	<b>3</b>
3.1	Input and output impedances. . . . .	3
3.2	Transfer function . . . . .	5
3.3	Cut-off frequencies . . . . .	6
<b>4</b>	<b>Simulation Analysis</b>	<b>7</b>
<b>5</b>	<b>Conclusion</b>	<b>11</b>

### 1 Introduction

In this laboratory assignment we seek to build a bandpass filter using an OP-AMP. Particularly we seek to maximize our **merit figure**,  $M$ , given by:

$$M = \frac{1}{\text{Cost}(\text{VoltageGainDeviation} + \text{CentralFreqDeviation} + 10^{-6})}$$

where the voltage gain deviation is the absolute value of the difference between the gain at 1000 Hz and 40 dB; and the central frequency deviation is the absolute value of the difference between the central frequency and 1000 Hz. The central frequency,  $f_c$ , is given by the geometric mean of the low cut-off frequency and the high cut-off frequency:

$$f_c = \sqrt{f_H f_L}$$

The circuit used was the following:

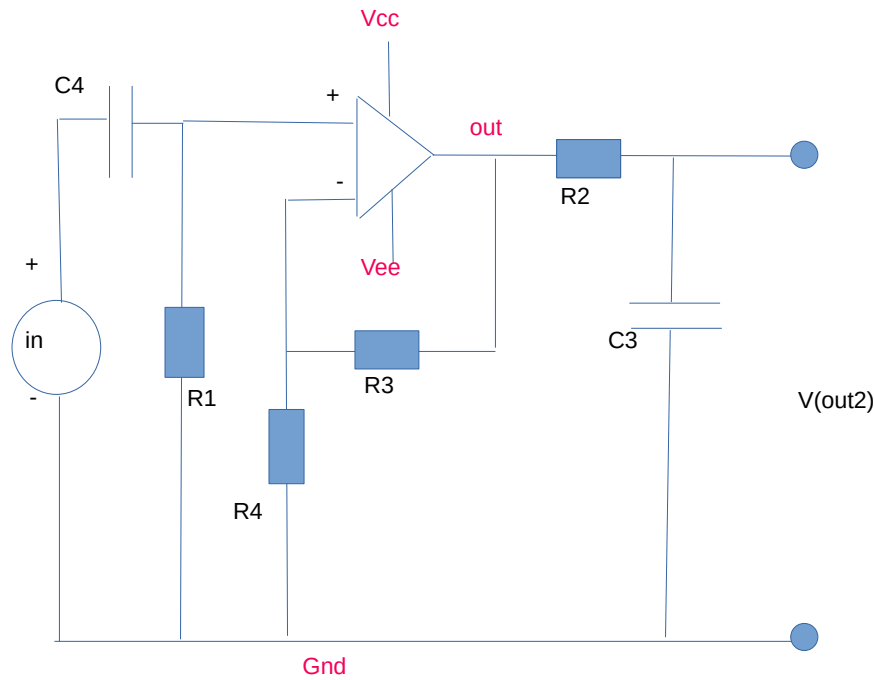


Figure 1: Circuito utilizado

## 2 Presential Lab

In this lab assigment we were also able to implement this circuit in real life, where we able to measured the gain and the cut-off frequencies. For the circuit configuration, we chose the following components:

R1	R2	R3	R4	C3	C4
1000K $\Omega$	500 $\Omega$	1000K $\Omega$	500 $\Omega$	220nF	220nF

With these components we were able to get a voltage gain of approximately  $Gain = 40$  dbs, and cut-off frequencies of  $330\text{ Hz}$  and  $2.23\text{ KHz}$ , corresponding  $f_L$  and  $f_H$ , respectively. Using ngpsice, we simulated the same circuit, where we obtained the following results:

Cost	13426.472038661
Central frequency, $f_0$	847.6288757705225
Central frequency deviation ( $diff_{F_0}$ )	152.3711242294775
gain at $1000\text{ Hz}$ , G (db)	42.42502
Gain deviation, $Diff_G$	2.4250200000000004
Merit	1.649708859507576e-07
Low Cut off	3.95392e+02
High Cut off	1.81712e+03

### 3 Theoretical Analysis

#### 3.1 Input and output impedances.

To determine the input and output impedances, we first replace the Op-Amp with its equivalent circuit, as shown in figure (2).

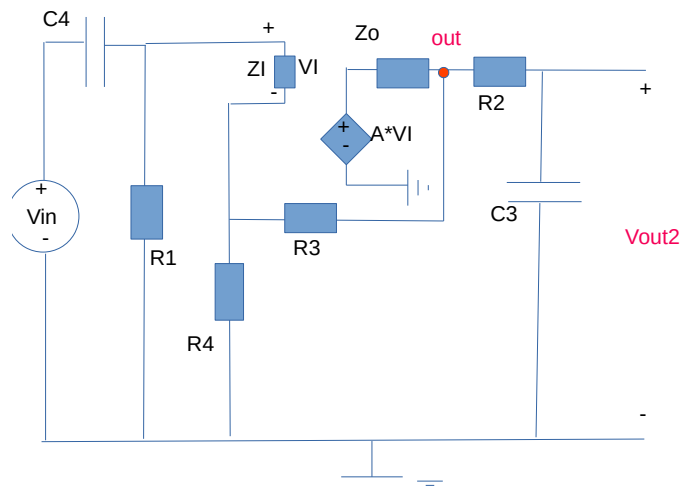


Figure 2: Pass-band circuit, with the amp-pop replaced with its equivalent circuit.

Considering the amp-op configuration is a non-inverting amplifier (and the ideal amp-op model), we get that the output and input impedances,  $Z_O$  and  $Z_I$ , are 0 and  $\infty$ , respectively, and that the gain  $A$  is equal to:  $(1 + \frac{R_3}{R_4})$ . Therefore we get the following circuit, in figure (3)

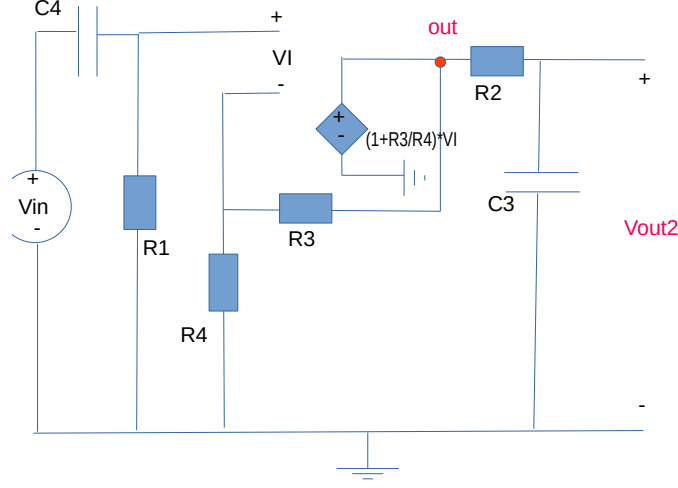


Figure 3: Pass-band circuit, with the amp-pop replaced with its equivalent circuit, using the ideal model approximation.

Finally, we can deduce the expressions for the input and output impedances for the circuit,  $Z_I$  and  $Z_{out}$ , (as seen by  $V_{in}$  and  $V_{out}$ , respectively). From the circuit in figure (3), we get that (there is no effect on the first part of the circuit, by  $V_{out2}$ , therefore it is not required to short-circuit the output):

$$Z_I(\omega) = Z_{C_4} + R_4 = \frac{1}{j\omega C_4} + R_4 \quad (1)$$

As for the output impedance, we need to short-circuit the input, hence we get the circuit in figure (4), from the  $V_{out2}$  terminals:

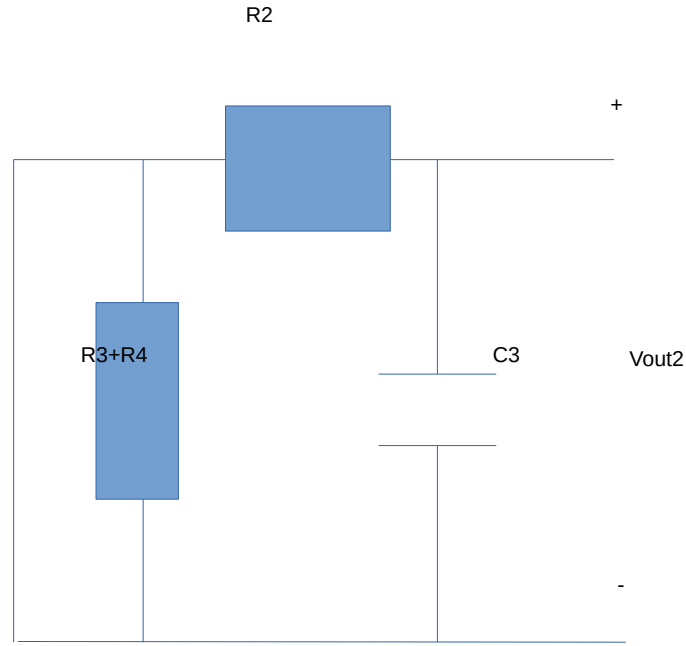


Figure 4: Equivalent circuit seen by the terminals of  $V_{out2}$ , when  $V_I = 0$ .

Therefore we get that:

$$Z_O(\omega) = R_2 || C_3 = \frac{R_2}{j\omega C_3 R_2 + 1} \quad (2)$$

### 3.2 Transfer function

The transfer function is defined as the ration between the output and the input. In our case, the output is  $v_0$  and the input  $v_s$ :

$$T(s) = \frac{v_0}{v_s}$$

after a little algebra, we get to the following expression:

$$T(s) = \frac{R_1 C_1 s}{1 + R_1 C_1 s} \left(1 + \frac{R_3}{R_4}\right) \left(\frac{1}{1 + R_2 C_2 s}\right) \quad (3)$$

where, as usual

$$s = j\omega$$

### 3.3 Cut-off frequencies

The theoretical cut-off frequencies,  $f_L$  and  $f_H$ , can be calculated by the Short Circuit Time Constants Method. They are given by<sup>1</sup>:

$$f_L = \frac{1}{R_1 C_1} \quad (4)$$

$$f_H = \frac{1}{R_2 C_2} \quad (5)$$

where  $f_H$  is the high cut-off frequency and  $f_L$  is the low cut-off frequency. Experimentally, the cut off frequencies will be calculated through the following expression:

$$f = \frac{V_{max}}{\sqrt{2}}$$

where  $f$  can be either  $f_H$  or  $f_L$ .

The Results obtained using octave were the following:

Total Cost	13427.69537166100
Central Freq	712.82860548662
Central Frequency difference	287.17139451338
Gain	39.72250000000
Cut off low	218.11200000000
Cut off high	2329.65000000000
Gain Difference	0.27750000000
Merit	0.00000097467

Table 1: Values used as parameters for the circuit studied.

<sup>1</sup>If you want to see the deduction in detail, you may visit the following link:  
[https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-012-microelectronic-devices-and-circuits-fall-2009/lecture-notes/MIT6\\_012F09\\_lec23.pdf?fbclid=IwAR3ezEOiIWVJOLyNLNp49EwgcpWSC-\\_IQF06wASvf9cKXiGx2\\_OzBp1Pnb8](https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-012-microelectronic-devices-and-circuits-fall-2009/lecture-notes/MIT6_012F09_lec23.pdf?fbclid=IwAR3ezEOiIWVJOLyNLNp49EwgcpWSC-_IQF06wASvf9cKXiGx2_OzBp1Pnb8)

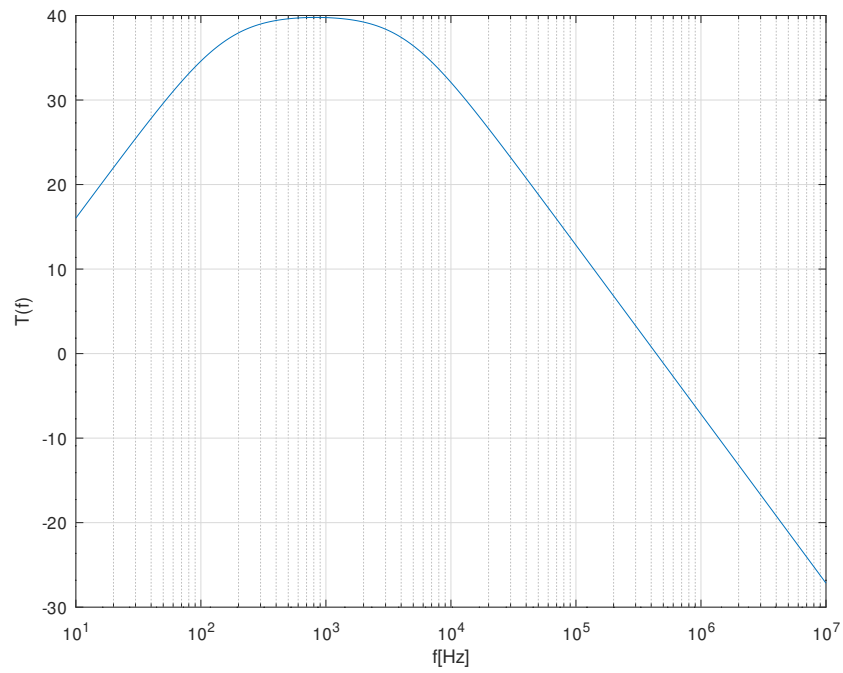


Figure 5: Forced sinusoidal response.

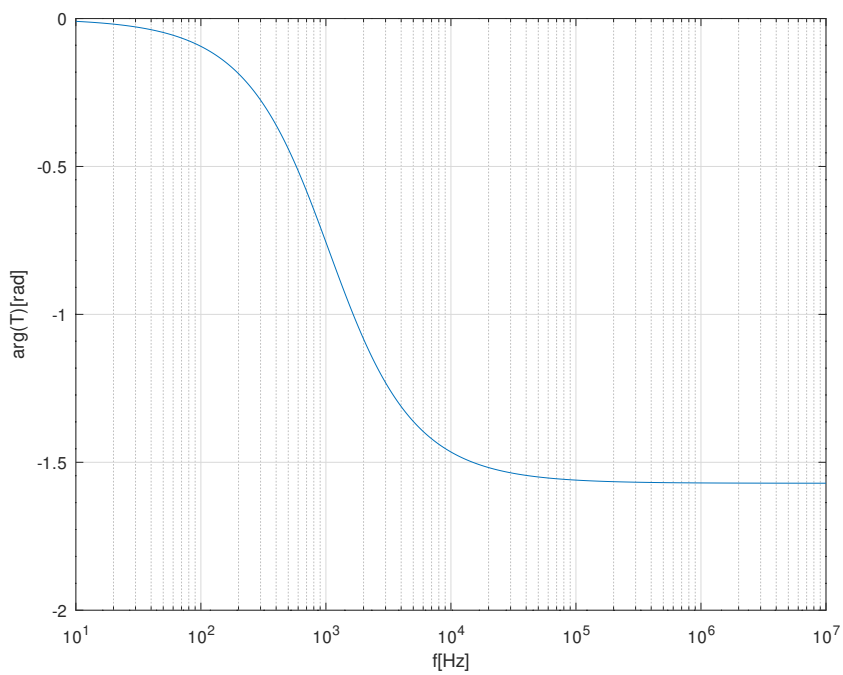


Figure 6: Forced sinusoidal response.

## 4 Simulation Analysis

The Operating point analysis is the following:

The graphs are the following:

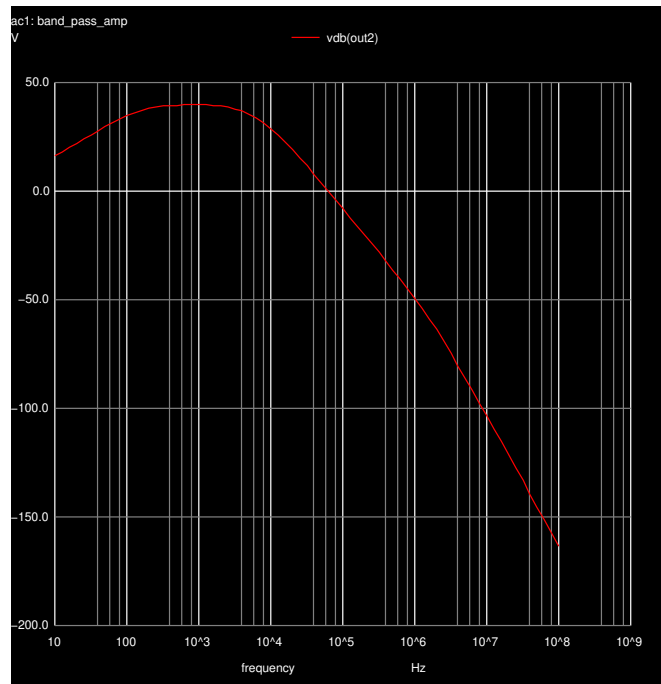


Figure 7: Time analysis



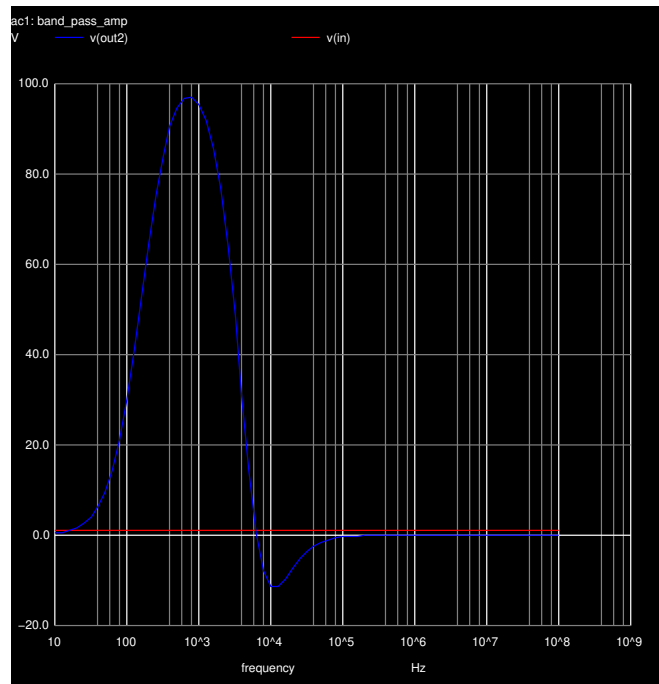


Figure 8: Frequency analysis

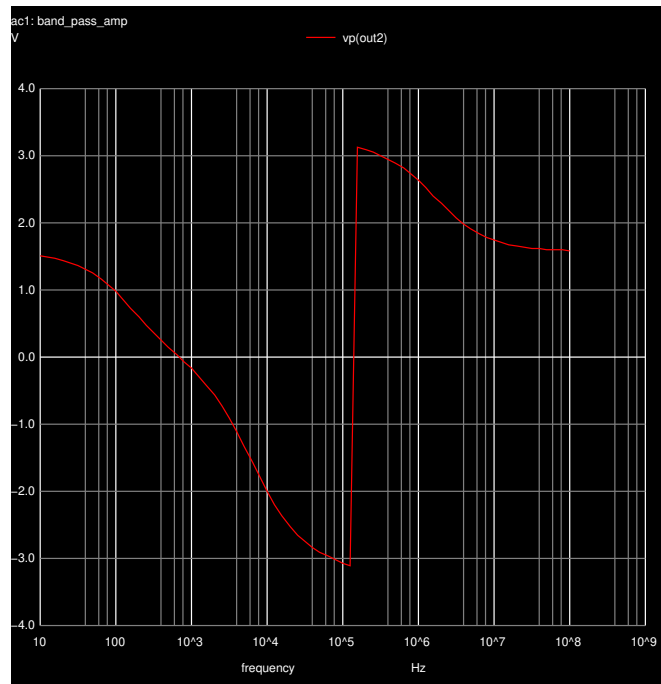


Figure 9: —

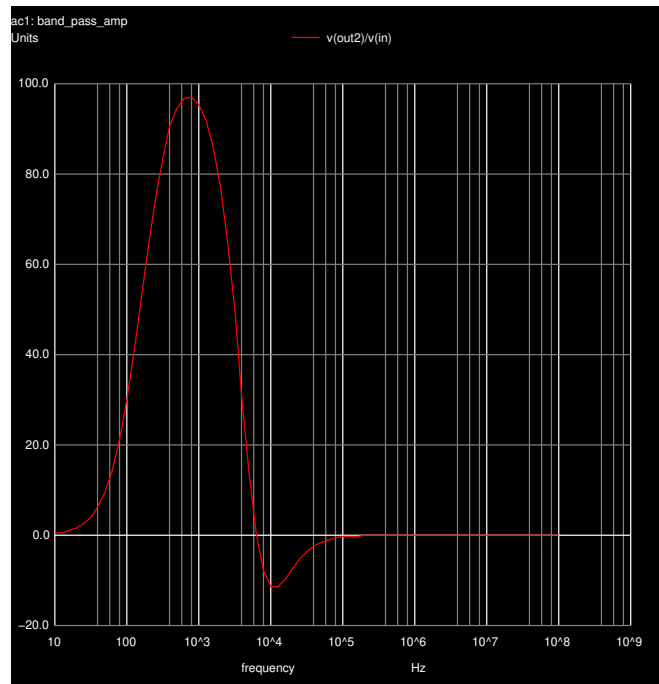


Figure 10:  $v(out)/v(in)$

## 5 Conclusion