T5 - Band Pass Filter using OPAMP

Integrated Master in Physics Engineering

João Lehodey (96538), Jorge Silva (96545), Pedro Monteiro (93156)

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1 Introduction

In this laboratory assignment we seek to build a bandpass filter using an OP-AMP. Particularly we seek to maximize our **merit figure**, M, given by:

$$M = \frac{1}{Cost(VoltageGainDeviation + CentralFreqDeviation + 10^{-6})}$$

where the voltage gain deviation is the absolute value of the difference between the gain at 1000 Hz and 40 dB; and the central frequency deviation is the absolute value of the difference between the central frequency and 1000 Hz. The central frequency, f_c , is given by the geometric mean of the low cut-off frequency and the high cut-off frequency:

$$f_c = \sqrt{f_H f_L}$$

The circuit used was the following:

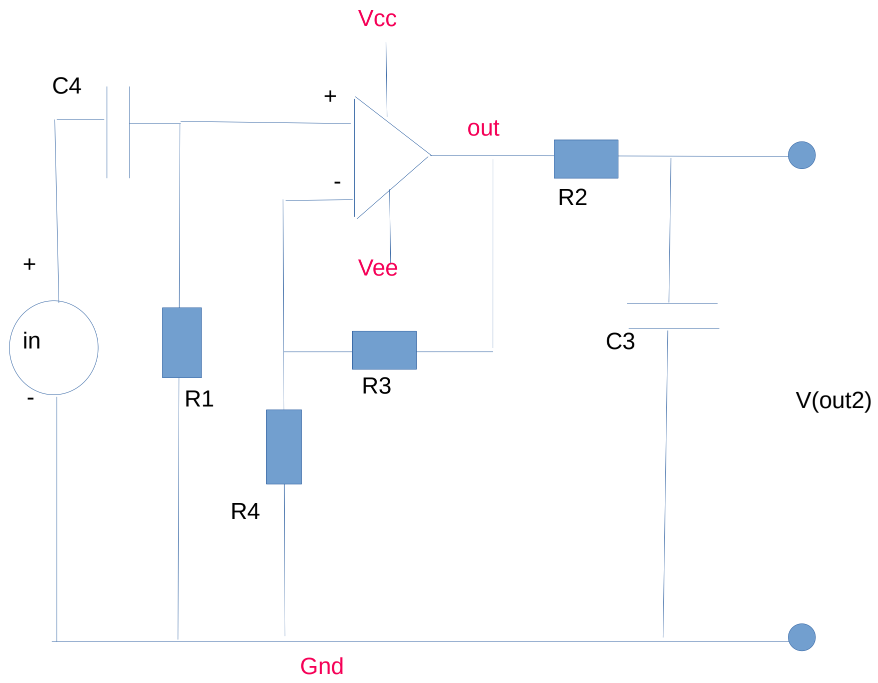

Figure 1: Circuito utilizado

2 Presential Lab

In this lab assigment we were also able to implement this circuit in real life, where we able to measured the gain and the cut-off frequencies. For the circuit configuration, we chose the following components:

R1	R2	R3	R4	C3	C4
1000KΩ	500 Ω	1000KΩ	500 Ω	220nF	220nF

With these components we were able to get a voltage gain of approximately $Gain = 40$ dbs, and cut-off frequencies of $330\,Hz$ and $2.23\,KHz$, corresponding f_L and f_H, respectively. Using ngpsice, we simulated the same circuit, where we obtained the following results:

Cost	13426.472038661
Central frequency, f_0	847.6288757705225
Central frequency deviation $(diff_{F_0})$	152.3711242294775
gain at $1000Hz$, G (db)	42.42502
Gain deviation, $Diff_G$	2.425020000000004
Merit	1.649708859507576e-07
Low Cut off	3.95392e+02
High Cut off	1.81712e+03

3 Theoretical Analysis

3.1 Input and output impedances.

To determine the input and output impedances, we first replace the Op-Amp with its equivalent circuit, as shown in figure (2).

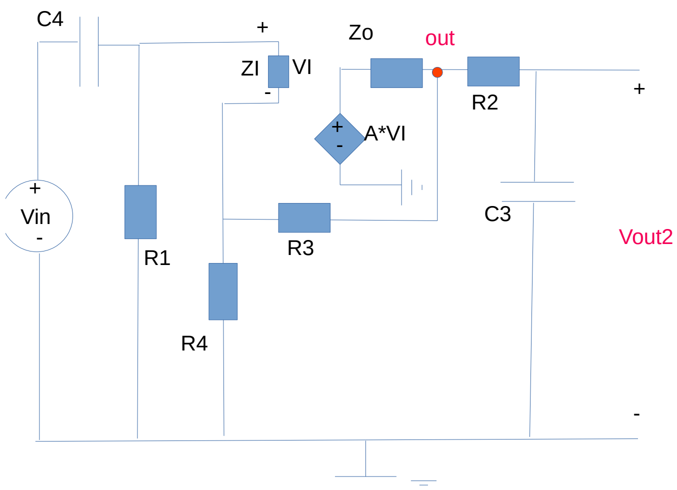

Figure 2: Pass-band circuit, with the amp-pop replaced with its equivalent circuit.

Considering the amp-op configuration is a non-inverting amplifier(and the ideal amp-op model), we get that the output and input impedances, Z_O and Z_I, are 0 and ∞, respectively, and that the gain A is equal to : $(1 + \frac{R_3}{R_4})$. Therefore we get the following circuit, in figure (3)

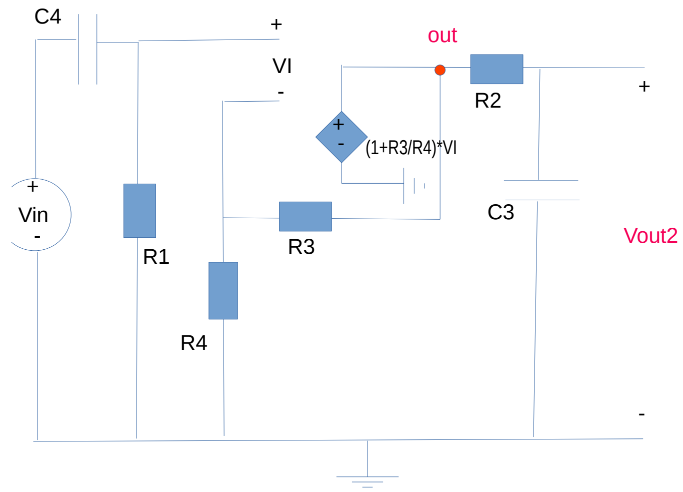

Figure 3: Pass-band circuit, with the amp-pop replaced with its equivalent circuit, using the ideal model aproximation.

Finally, we can deduce the expressions for the input and output impedances for the circuit, Z_I and Z_{0ut}, (as seen by V_{in} and V_{out}, respectively). From the circuit in figure (3), we get that (there is no effect on the first part of the circuit, by V_{out2}, therefore it is not required to short-circuit the output):

$$Z_I(\omega) = Z_{C_4} + R_4 = \frac{1}{j\omega C_4} + R4 \tag{1}$$

As for the output impedance, we need to short-circuit the input, hence we get the circuit in figure (4), from the V_{out_2} terminals:

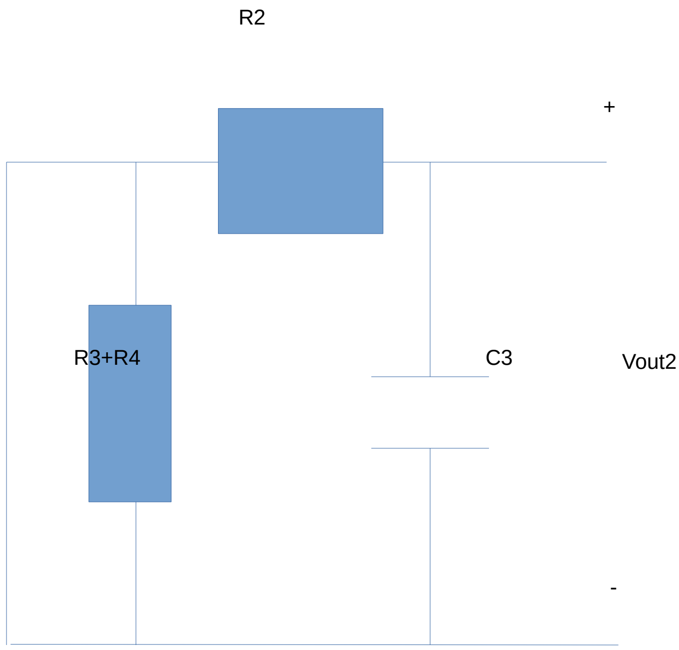

Figure 4: Equivalent circuit seen by the terminals of V_{out_2}, when $V_I = 0$.

Therefore we get that:

$$Z_O(\omega) = R_2 || C_3 = \frac{R_2}{j\omega C_3 R_2 + 1} \tag{2}$$

,

3.2 Transfer function

The transfer function is defined as the ration between the output and the input. In our case, the output is $v0$ and the input vs:

$$T(s) = \frac{v0}{vs}$$

after a little algebra, we get to the following expression:

$$T(s) = \frac{R_1 C_1 s}{1 + R_1 C_1 s}(1 + \frac{R_3}{R_4})(\frac{1}{1 + R_2 C_2 s}) \tag{3}$$

where, as usual

$$s = j\omega$$

3.3 Cut-off frequencies

The theoretical cut-off frequencies, f_L and f_H, can be calculated by the Short Circuit Time Constants Method. They are given by[1]:

$$f_L = \frac{1}{R_1 C_1} \tag{4}$$

$$f_H = \frac{1}{R_2 C_2} \tag{5}$$

where f_H is the hight cut-off frequency and f_L is the low cut-off frequency. Experimentally, the cut off frequencies will be calculated through the following expression:

$$f = \frac{V_{max}}{\sqrt{2}}$$

where f can be either f_H or f_L.

The Results obtained using octave were the following:

Total Cost	13458.69203866100
Central Freq	714.25510087083
Central Frequency diference	285.74489912917
Gain	39.72291000000
Gain Theoric	98.42407740711
Cut off low	222.94100000000
Cut off high	2288.32000000000
Gain Diference	0.27709000000
ZO	474.82104761502
ZI	1000.00000000000
Merit	0.00000097900

Table 1: Values used as parameters for the circuit studied.

[1] If you want to see the deduction in detail, you may visit the following link: https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-012-microelectronic-devices-and-circuits-fall-2009/lecture-notes/MIT6_012F09_lec23.pdf?fbclid=IwAR3ezEOiIWVJOLyNLNp49EwgcpWSC-_IQFO6wASvf9cKXiGx2_0zBplPnb8

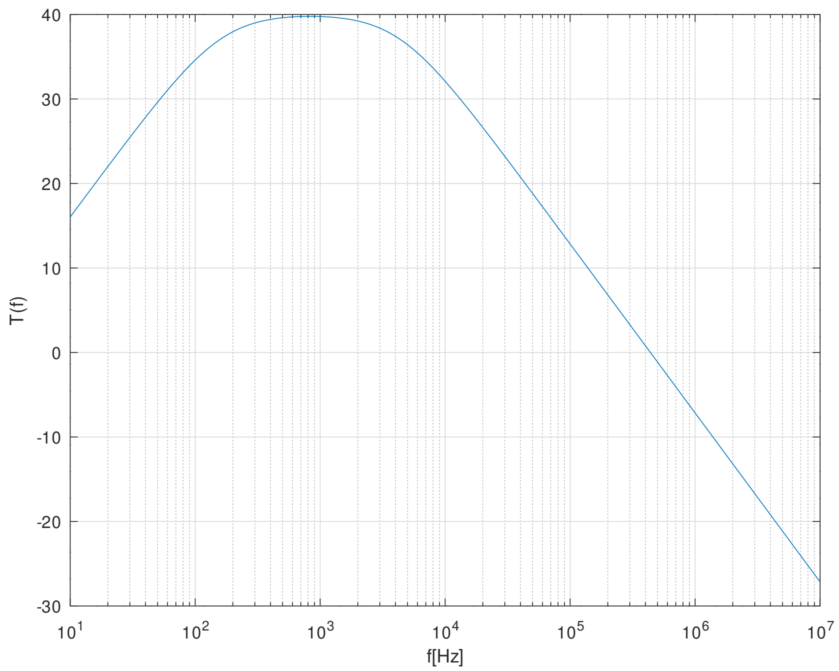

Figure 5: Forced sinusoidal response.

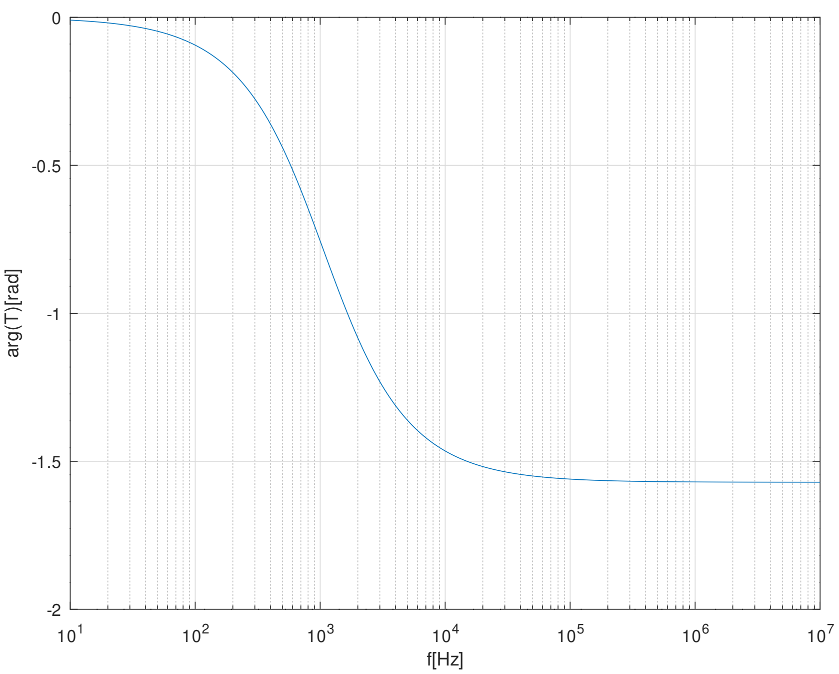

Figure 6: Forced sinusoidal response.

4 Simulation Analysis

The Operating point analysis is the following:

The graphs are the following:

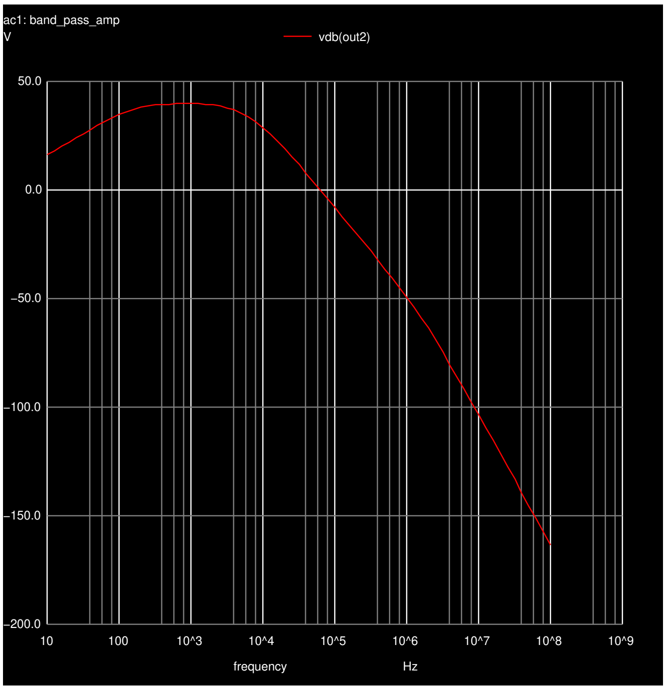

Figure 7: Time analysis

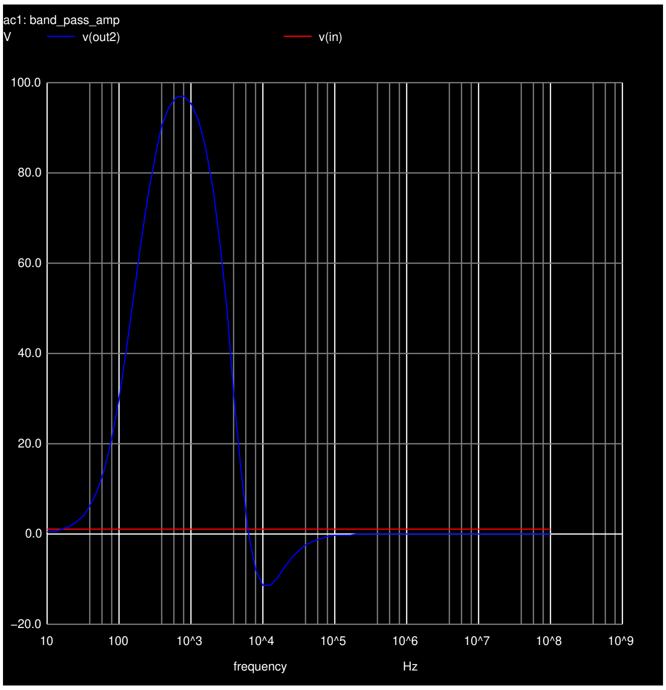

Figure 8: Frequency analysis

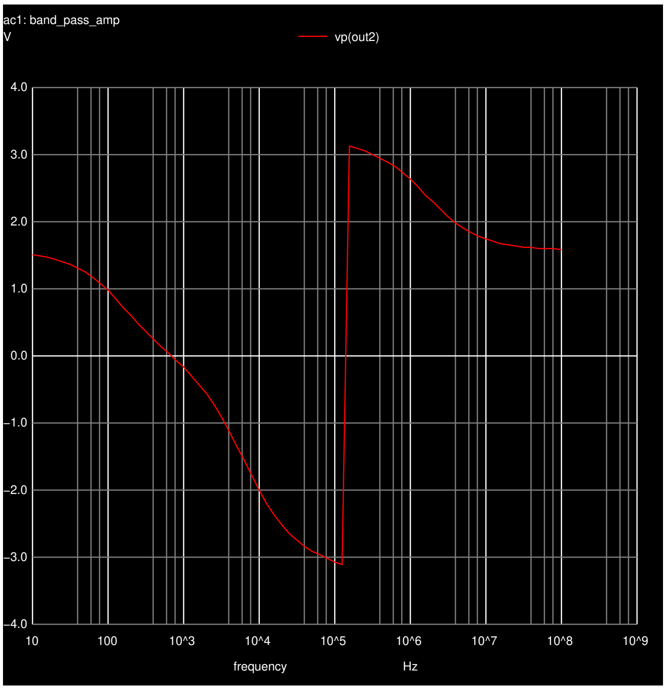
ac1: band_pass_amp
V
vp(out2)
4.0
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
-4.0
10
100
10^3
10^4
10^5
10^6
10^7
10^8
10^9
frequency
Hz

Figure 9: —-

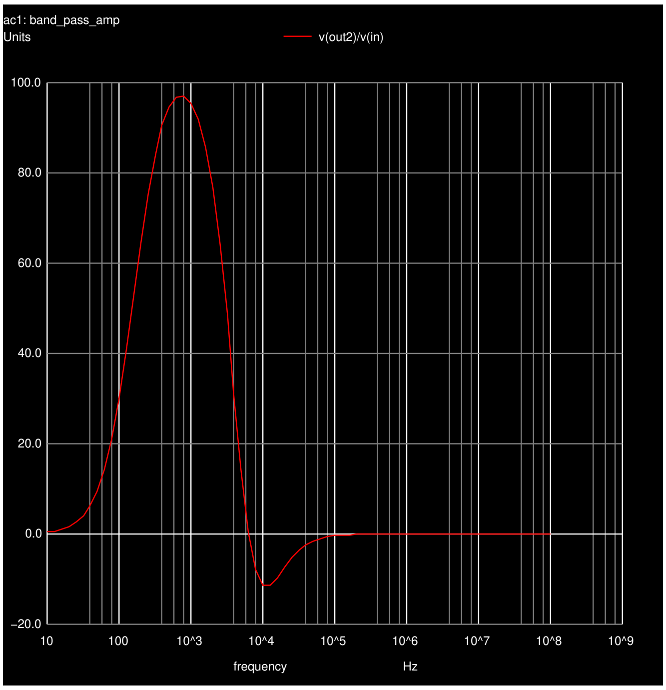

Figure 10: v(out)/v(in)

5 Conclusion