Aprendizagem 2023

Lab 3: Bayesian learning

Practical exercises

I. Probability theory

1. Consider the following registry where an experiment is repeated six times and four events (A, B, C and D) are detected.

	A	B	C	D
$\mathbf{x}_1$	1	1	0	0
$\mathbf{x}_2$	1	1	1	0
$\mathbf{x}_3$	0	0	0	1
$\mathbf{x}_4$	0	0	0	1
$\mathbf{x}_5$	0	0	0	0
$\mathbf{x}_6$	0	0	0	0

Considering frequentist estimates, compute:

$p(A)$

$p(A, B)$

$p(B|A)$

$p(A, B, C)$

$p(A|B, C)$

$p(A, B, C, D)$

$p(D|A, B, C)$

2. Considering the following two-dimensional measurements {(-2,2),(-1,3),(0,1),(-2,1)}.

a) What are the maximum likelihood parameters of a multivariate Gaussian distribution for this set of points?

b) What is the shape of the Gaussian? Draw it approximately using a contour map.

II. Bayesian learning

3. Consider the following dataset where:

- 0: False and 1: True
- y1: Fast processing
- y2: Decent Battery
- y3: Good Camera
- y4: Good Look and Feel
- y5: Easiness of Use
- class: iPhone

	y_1	y_2	y_3	y_4	y_5	class
$\mathbf{x}_1$	1	1	0	1	0	1
$\mathbf{x}_2$	1	1	1	0	0	0
$\mathbf{x}_3$	0	1	1	1	0	0
$\mathbf{x}_4$	0	0	0	1	1	0
$\mathbf{x}_5$	1	0	1	1	1	1
$\mathbf{x}_6$	0	0	1	0	0	1
$\mathbf{x}_7$	0	0	0	0	1	1

And the query vector $\mathbf{x}_{\text{new}} = [1\ 1\ 1\ 1\ 1]^T$

a) Using Bayes' rule, without making any assumptions, compute the posterior probabilities for the query vector. How is it classified?

b) What is the problem of working without assumptions?

c) Compute the class for the same query vector under the naive Bayes assumption.

d) Consider the presence of missings. Under the same naive Bayes assumption, how do you classify $\mathbf{x}_{\text{new}} = [1\ ?\ 1\ ?\ 1]^T$

4. Consider the following dataset

	weight (kg)	height (cm)	NBA player
$\mathbf{x}_1$	170	160	0
$\mathbf{x}_2$	80	220	1
$\mathbf{x}_3$	90	200	1
$\mathbf{x}_4$	60	160	0
$\mathbf{x}_5$	50	150	0
$\mathbf{x}_6$	70	190	1

And the query vector $\mathbf{x}_{\text{new}} = [100\ 225]^T$

a) Compute the most probable class for the query vector assuming that the likelihoods are 2-dimensional Gaussians

b) Compute the most probable class for the query vector, under the Naive Bayes assumption, using 1-dimensional Gaussians to model the likelihoods

5. Assuming training examples with *m* Boolean features.

a) How many parameters do you have to estimate considering features are Boolean and:

i. no assumptions about how the data is distributed

ii. naive Bayes assumption

b) How many parameters do you have to estimate considering features are numeric and:

iii. multivariate Gaussian assumption

iv. naive Bayes with Gaussian assumption

Programming quests

Resources: *Classification* and *Evaluation* notebooks available at the course's webpage

6. Reuse the **sklearn** code from last lab where we learnt a decision tree in the *breast.w* data:

a) apply the naïve Bayes classifier with default parameters

b) compare the accuracy of both classifiers using a 10-fold cross-validation