



ieeta instituto de engenharia electrónica e telemática de aveiro



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de aveiro

# **APLICAÇÕES DE INTELIGÊNCIA ARTIFICIAL**

## **APPLICATIONS OF ARTIFICIAL INTELLIGENCE**

### **LECTURE 4: Model Performance Evaluation and Selection**

**Petia Georgieva**  
**(petia@ua.pt)**



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# Outline

**Model performance evaluation:** perf. metrics

- **Model selection: Bias vs. variance**
- **Learning curves**
- **K –fold Cross Validation**

# Performance Evaluation – Confusion Matrix

ACTUAL CLASS	PREDICTED CLASS	
	Class=Yes	Class=No
Class=Yes	a (TP)	b (FN)
	c (FP)	d (TN)

**a: TP (true positive)**

**b: FN (false negative)**

**c: FP (false positive)**

**d: TN (true negative)**

*Python: from sklearn.metrics import confusion\_matrix*

# Performance metric - Accuracy

ACTUAL CLASS	PREDICTED CLASS	
	Class=Yes	Class=No
Class=Yes	(TP)	(FN)
	(FP)	(TN)

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

**Accuracy - fraction of examples correctly classified.**

**1-Accuracy: Error rate (misclassification rate)**

# Limitation of Accuracy

- Consider binary classification (**Unbalanced data set**)
  - Class 0 has 9990 examples
  - Class 1 has 10 examples
- If model classify all examples as class 0, accuracy is  $9990/10000 = 99.9 \%$
- Accuracy is misleading metrics because model does not classify correctly any example of class 1
  - => Use other performance metrics.
  - => Find a way to balance the data set  
(re-sampling methods: oversampling, under-sampling)

# Other Performance Metrics

**True Positive Rate (TPR)**, Sensitivity, Recall  
of all positive examples the fraction of correctly classified

$$TPR = \frac{TP}{TP + FN}$$

**True Negative Rate (TNR)**, Specificity  
of all negative examples the fraction of correctly classified

$$TNR = \frac{TN}{TN + FP}$$

**False Positive Rate (FPR)** - how often an actual negative instance will be classified as positive, i.e. “false alarm”

$$FPR = 1 - TNR = \frac{FP}{FP + TN}$$

**Precision** - the fraction of correctly classified positive samples from all classified as positive

$$\text{Precision} = \frac{TP}{TP + FP}$$

# Combined performance metrics

**F1 Score** - weighted average of Precision and Recall

$$F1 = 2 * (\text{Recall} * \text{Precision}) / (\text{Recall} + \text{Precision})$$

**Balanced Accuracy** =  $(\text{Recall} + \text{Specificity}) / 2$

# Class Imbalance problem

**Solution: Re-sampling methods (under-sampling, oversampling)**

Examples

P1 Normal

P2 Normal

P3 Normal

P4 Mass

P5 Normal

P6 Normal

P7 Mass

P8 Normal

Normal  
P1, P2, P3, P5,  
P6, P8

Mass  
P4, P7

Sample 4

Sample 4

Re-Sampled

P3 Normal

P6 Normal

P1 Normal

P8 Normal

P7 Mass

P4 Mass

P7 Mass

P4 Mass



# Definitions for Epoch / Batch Size / Iterations / Train step

**One Epoch** is when an ENTIRE dataset is passed through the model (e.g. forward and backward in a neural network) only ONCE.  
If data is too big to feed to the computer at once one epoch is divided in several smaller batches.

**Batch Size:** Total number of training examples present in a single batch.

**Iterations** is the number of batches needed to complete one epoch.

**Example:** Let's say we have 2000 training examples.  
We can divide the dataset of 2000 examples into batches of 500 then it will take 4 iterations to complete 1 epoch.

**Training run/step** - is one update of the model parameters.  
We update the parameters after one batch or after one epoch.

# Deciding what to do next ?

Suppose you have trained a ML model on some data. When you test the trained model on a new set of data, it makes unacceptably large errors.

What should you do ?

- **Get more training examples ?**
- **Try smaller sets of features (feature selection) ?**
- **Try getting additional features (feature engineering) ?**
- **Try using different/nonlinear kernels ?**
- **Try other values of the hyper parameters (e.g. regul. parameter) ?**

## **Machine learning diagnostics = Model-centric approach**

Run tests to gain insight what isn't working with the learning algorithm and how to improve its performance.

Diagnostics is time consuming , but can be a very good use of your time.

# Simplest division: Train & Test subsets

- Training set (70%-80 %) : used to train the model
- Test set (30%-20%) : used to test the trained model

- **Optimize the model parameters with training data**  
(minimize some cost/loss function  $J$ )

**After the training stage is over (i.e. the cost function  $J$  converged)**

- **Compute the MSE on test data (for regression problems)**

$$E_{test}(\theta) = \frac{1}{m_{test}} \left[ \sum_{i=1}^{m_{test}} \left( h_{\theta}(x_{test}^{(i)}) - y_{test}^{(i)} \right)^2 \right]$$

**or**

- **Compute the model accuracy or some other metric from the confusion matrix, on test data (for classification problems)**

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

**Mean Squared Error (MSE) is NOT the cost function that is minimised during training !!!**

# Different Cost/Loss Functions

Training data MSE

- **Linear Regression Cost Function** with L2 Regularization (Ridge Regression)

$$J(\theta) = \frac{1}{2m} \left[ \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

Ridge Regression

- **Logistic Regression Cost Function** with L2 Regularization

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m [-y^{(i)} \log(h_{\theta}(x^{(i)})) - (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)}))] + \frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2$$

- **Neural Network Cost Function (no regularization)**

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m \sum_{k=1}^K \left[ -y_k^{(i)} \log((h_{\theta}(x^{(i)}))_k) - (1 - y_k^{(i)}) \log(1 - (h_{\theta}(x^{(i)}))_k) \right]$$

**Mean Squared Error (MSE) is NOT the cost function that is minimised during training !!!**

# 3 way split: Train/Dev/Test Sets

**Choose ML model:** Logistic Regression, Neural Network (NN), etc. ?

**Choose model hyper-parameters:**

- # of layers in NN ?
- # of hidden units (neurons) in NN ?
- Which activation functions in NN ?
- What is the best learning rate ?
- What is the best regularization parameter ( $\lambda$ ) ?
- What is the best polynomial degree ?
- .....

**Devide dataset in 3 sub-sets:**

- Training set
- Cross Validation (CV) set = Development set = 'dev' set
- Test set

Traditional division for Small data set (up to 10000 examples) :

60% - 20% - 20%

Big data (1 million. examples):

98% - 1% - 1%

# Model /hyper parameter selection

**Step 1:** Optimize parameters  $\theta$  (to minimize some cost function  $J$ ) using the same training set for all models. Compute some perf. metrics with the training data (i.e. error, accuracy) :

**Training error =>** 
$$E_{train}(\theta) = \frac{1}{2m} \left[ \sum_{i=1}^m \left( h_{\theta}(x^{(i)}) - y^{(i)} \right)^2 \right]$$

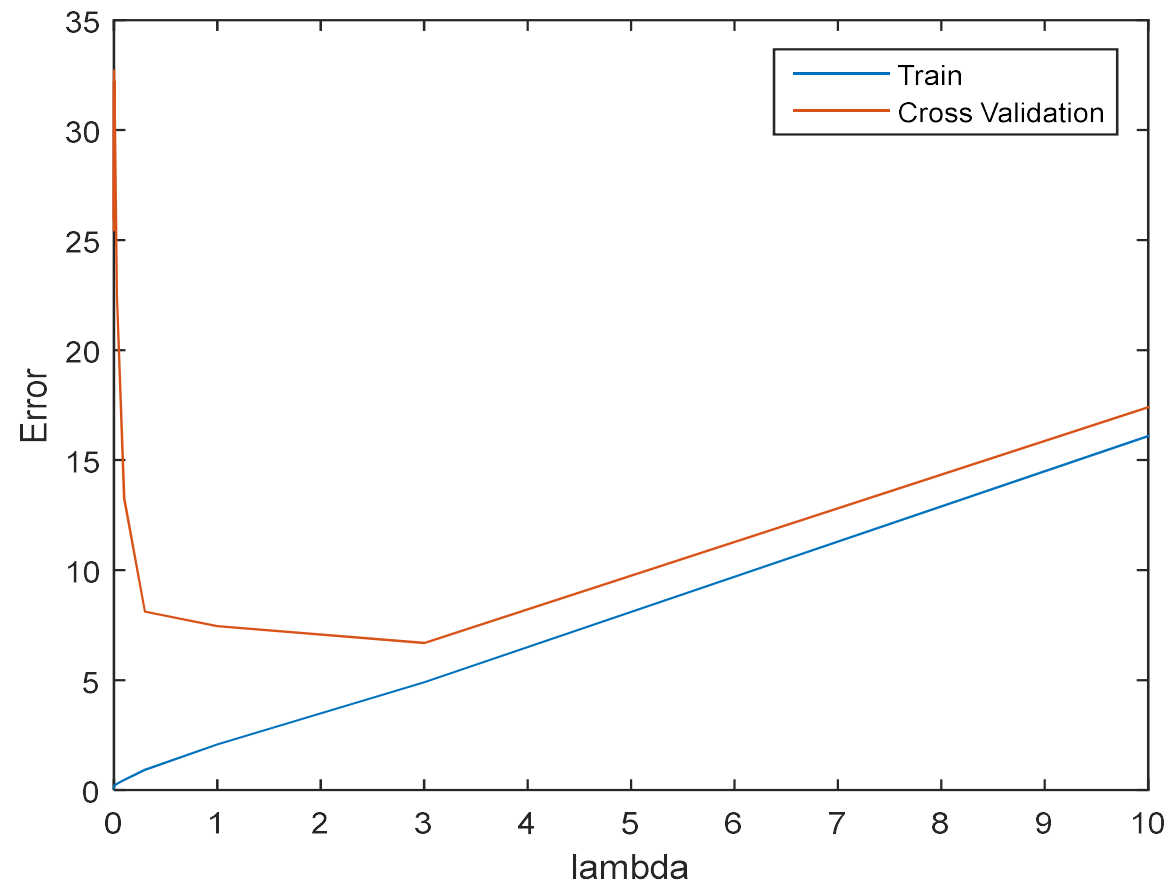
**Step 2:** Test the optimized models from step 1 with the CV set and choose the model with the min CV error (or other performance metric with dev data):

**Cross validation (CV)/dev error =>** 
$$E_{cv}(\theta) = \frac{1}{2m_{cv}} \left[ \sum_{i=1}^{m_{cv}} \left( h_{\theta}(x_{cv}^{(i)}) - y_{cv}^{(i)} \right)^2 \right]$$

**Step 3:** Retrain the best model from step 2 with both train and CV sets starting from the parameters got at step 2. Test the retrained model with test set and compute test data perf. metric (**the real model performance !!!**):

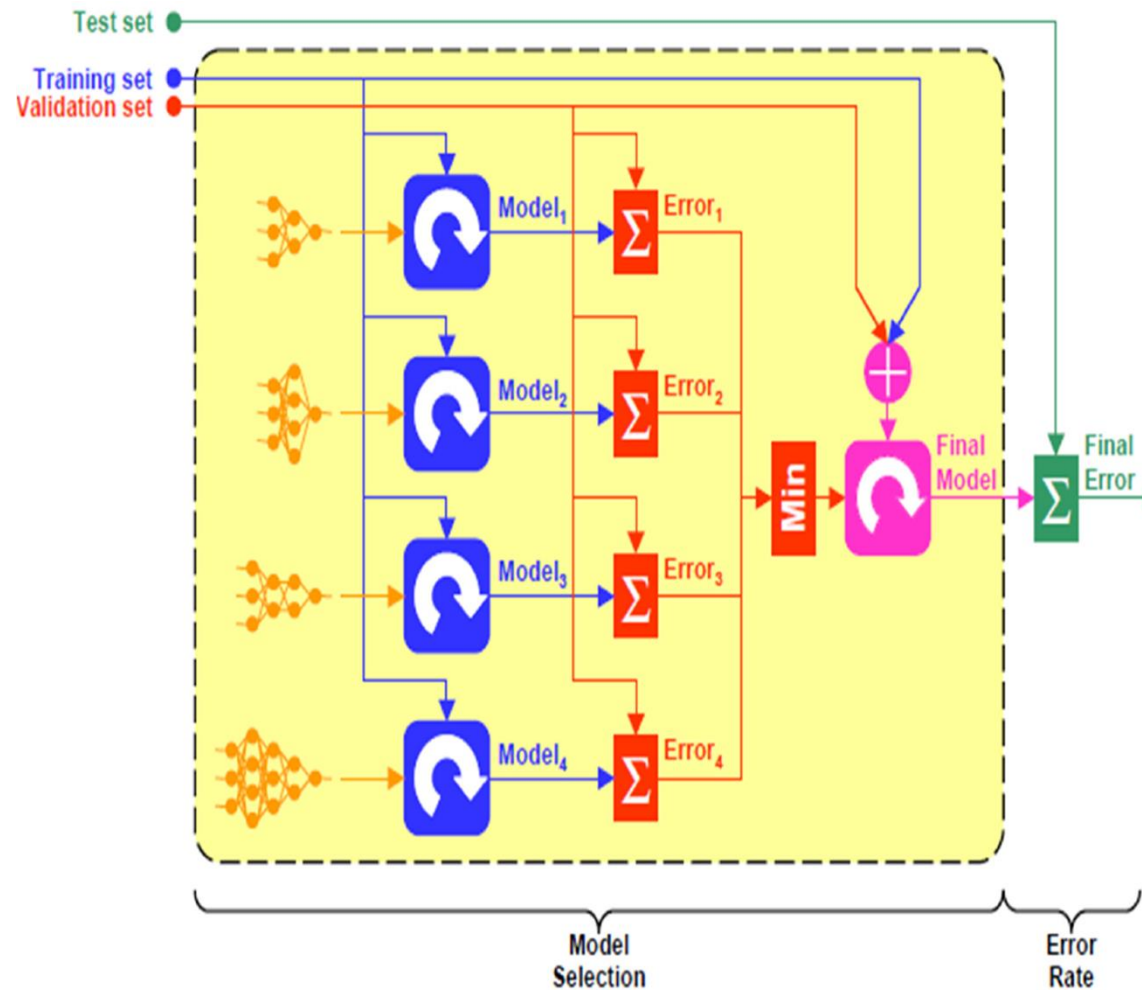
**Test error =>** 
$$E_{test}(\theta) = \frac{1}{2m_{test}} \left[ \sum_{i=1}^{m_{test}} \left( h_{\theta}(x_{test}^{(i)}) - y_{test}^{(i)} \right)^2 \right]$$

# Example: Select best $\lambda$



**Best  $\lambda = 3$**

# Training/Valid (Dev)/Test subsets

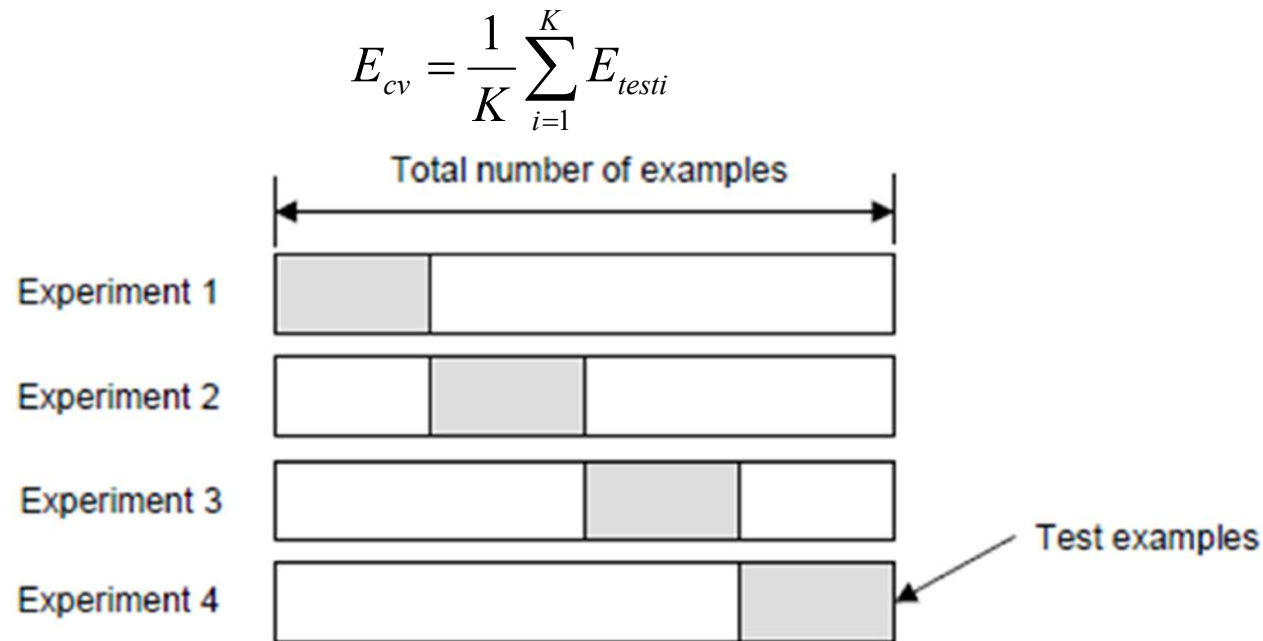


**The most credible is the performance metric with test data, not used for training or validation of the model.**



# K –fold Cross Validation

- Divide data into Training and Test subsets.
- Divide Training data into K subsets (K-fold).
- Use K-1 subsets for training and the remaining subset for CV.
- The final validation error is the average CV error of K experiments.
- Choose the best model /hyper-parameter the one that minimise the average CV error.

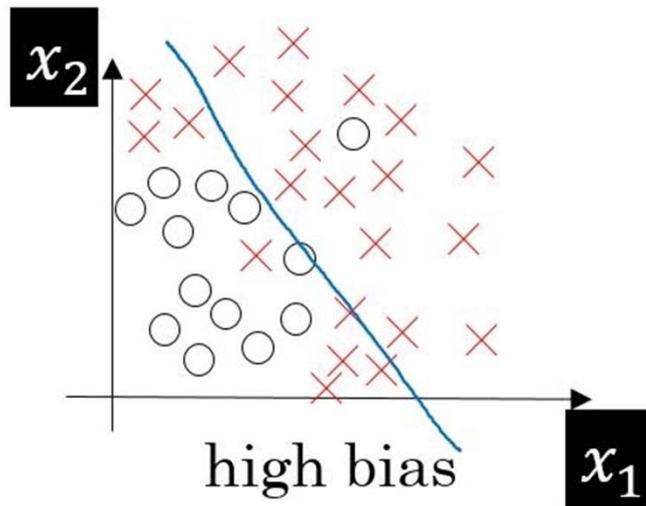


# Bias vs. Variance

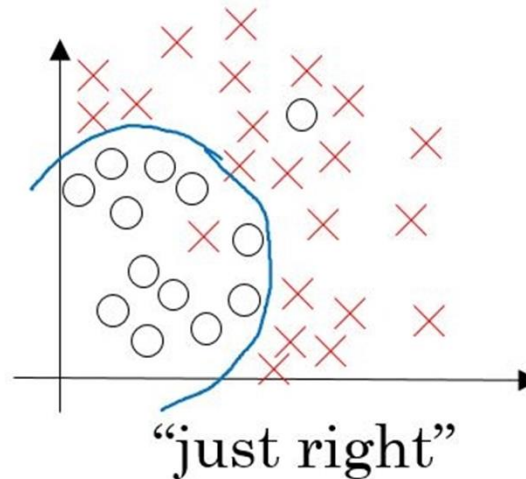
An important concept in ML is the bias-variance tradeoff.

Models with **high bias** are not complex enough and **underfit** the training data.

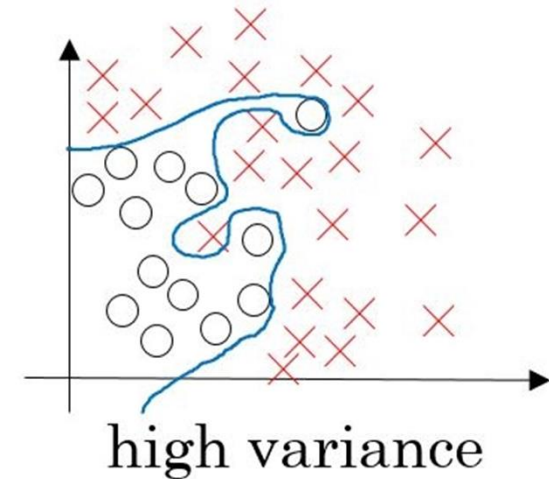
Models with **high variance** are too complex and **overfit** the training data.



**underfitting data**  
(very simple model)



(good model)



**overfitting data**  
(very complex model)

# Diagnosing Bias vs. Variance

How to diagnose if we have a high bias problem or high variance problem ?

## **High Bias (underfitting) problem:**

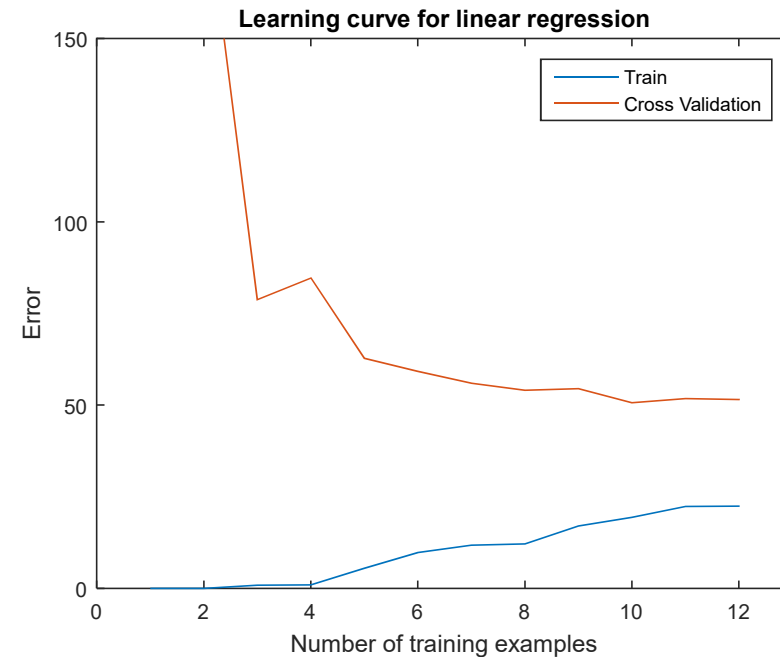
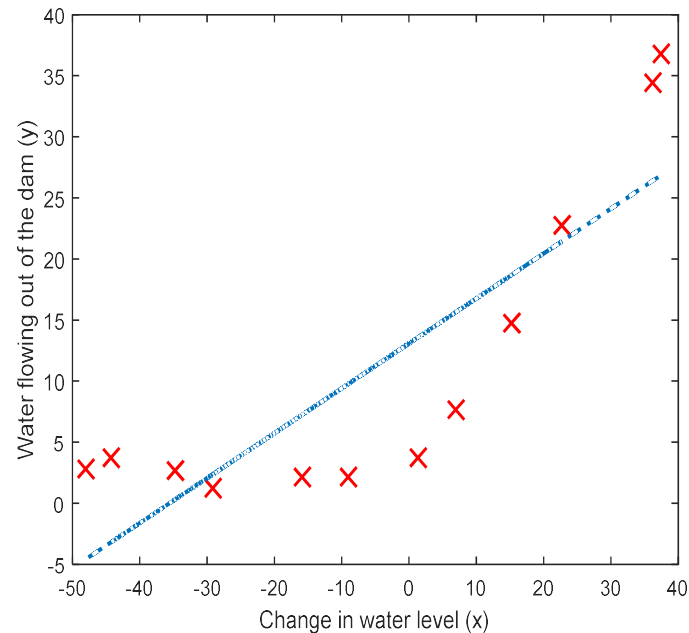
Training error ( $E_{train}$ ) and Validation/dev error ( $E_{cv}$ ) are both high

## **High Variance (overfitting) problem:**

Training error ( $E_{train}$ ) is low  
and Validation/dev error ( $E_{cv}$ ) is much higher than  $E_{train}$

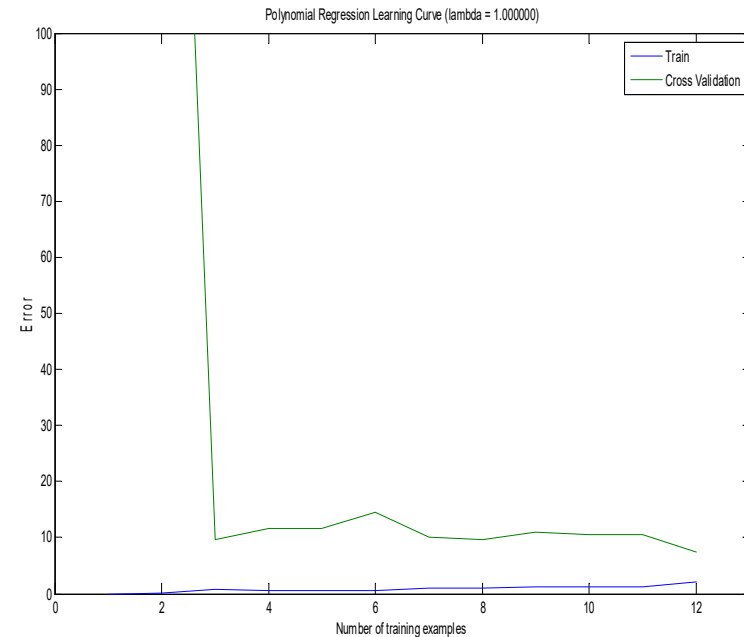
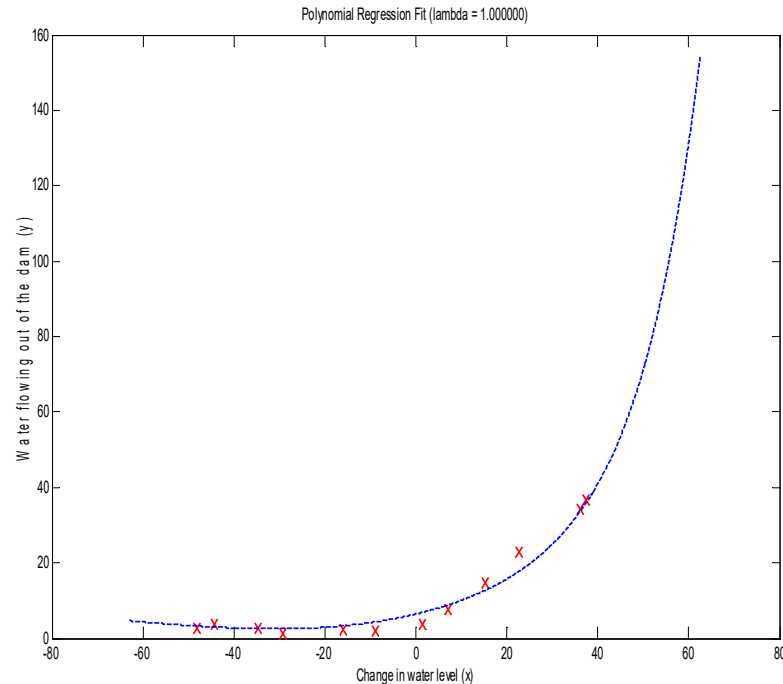
# Learning Curves

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$



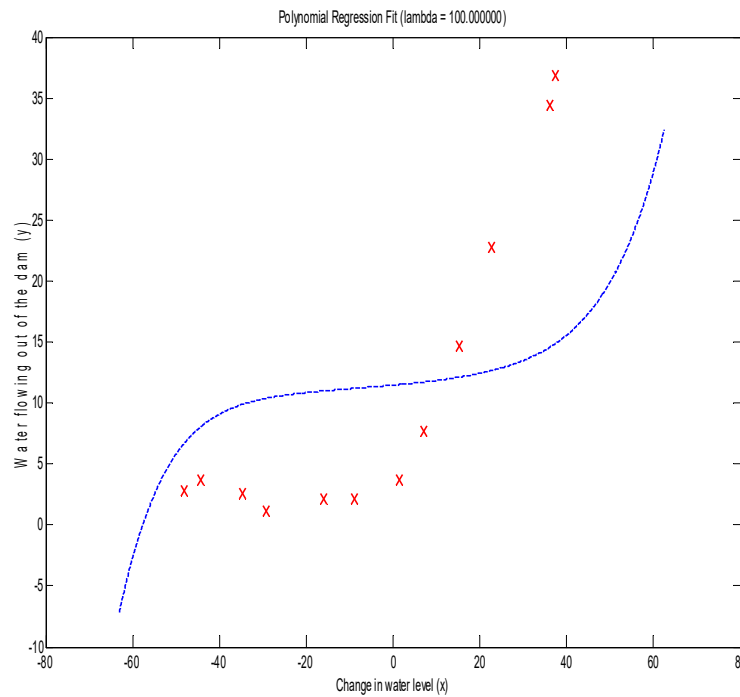
**If a learning algorithm is suffering from high bias, getting more training data will not help much**

# Learning Curves

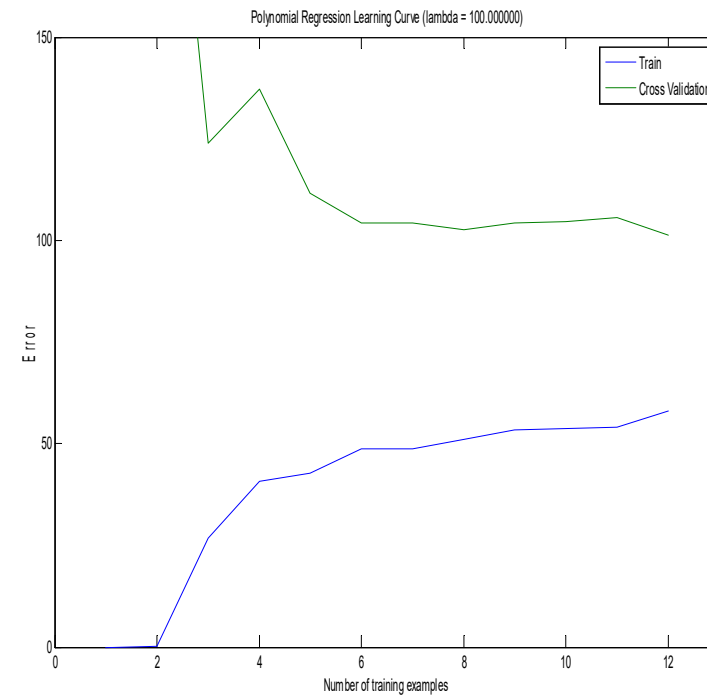


**If a learning algorithm is suffering from high variance, getting more training data is likely to help**

# Regularization and Learning Curves



**Polynomial regression,  $\lambda = 100$**



**Learning curve,  $\lambda = 100$**

# Hints to improve ML model

Suppose you have learned a data model (hypothesis). However, when you test your hypothesis on a new set of data, you find that it makes unacceptably large errors in its prediction (regression or classification). What should you try next?

- **Get more training examples – fixes high variance**
- **Try smaller sets of features – fixes high variance**
- **Try getting additional features – fixes high bias**
- **Try adding polynomial features - fixes high bias**
- **Try decreasing  $\lambda$  – fixes high bias**
- **Try increasing  $\lambda$  – fixes high variance**