THE UNEXPECTED EXAMINATION

THIS well known paradox has also appeared under the name of the 'unexpected execution'. Suppose a class of boys are warned that they can expect a surprise examination the following week. They can reason that it cannot occur on Friday (it is a five-day week) because they would then know on Thursday night that it must occur the following day; having ruled out Friday, it cannot take place on Thursday because they would then know on the evening of Wednesday that it can only take place the following day. By this method, all the days can be excluded in turn so that no day is left on which the surprise examination can be held.

The argument offers a rule for excluding unsuitable days for the examination; as it happens, all the days are ruled out. Thus, and this is the crucial assumption, if both boys and master know of the rule and apply it, no day can be chosen which satisfies the conditions for a surprise. Had the rule excluded all but one day, it would still have made the choice of a surprise day impossible but had it excluded all but two or more days we would have been able to pick a day which satisfies all the conditions for a surprise even if both parties knew of and applied the rule.

The key to the puzzle is the knowledge or ignorance of either party of the rule; this is an essential ingredient in the surprise. We may reasonably assume that the master knew of the boys' method of predicting an examination on Friday from Thursday evening since it is the most obvious application of the general rule but was either ignorant of or chose to ignore their rule for working back. Then the occurrence of the examination on any of the other days would be a surprise for the boys whether or not they thought the master knew of and intended to apply the rule. For if they thought he did, he surprised them by breaking it, and if not, they had no reason to expect it on any one day. (Friday, because it is an obvious application of the rule, would probably be excluded.)

If both parties know of and apply the rule then no day can be chosen. But a surprise may be due to ignorance of the others' choice or to ignorance of the rule by which the other chooses. Since the rule here excludes all days in the week as possible days for the examination, to choose a day at all will be a surprise in the sense of displaying ignorance of or a deliberate breaking of the rule. An element of self-reference arises from the fact that on the terms by which the paradox can occur, the master must take into account the boys' own prediction before choosing a day. Since he cannot choose days which they have predicted, they negatively affect the choice and if they have played a part in making the choice it is difficult to see how it can surprise them. But the puzzle arises because the rule of exclusion eliminates all the days. If it only eliminated all but two or more, in the absence of another elimination rule he could choose a surprise day despite the fact that his choice is still negatively dependent on the boys' prediction. The paradox disappears but the self-reference remains. Its presence is therefore not sufficient for the occurrence of the paradox.

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