THE PREDICTION PARADOX

IN MIND, 1948, Mr. O'Connor published an article entitled "Pragmatic Paradoxes". He there gave the following case: "The military commander of a certain camp announces on a Saturday evening that during the following week there will be a 'Class A blackout' . . . an exercise which the participants cannot know is going to take place . . . it follows from the announcement of this definition that the exercise cannot take place at all. It cannot take place on Saturday because if it has not occurred on the first six days of the week it must occur on the last. And the fact that the participants can know this violates the condition which defines it. Similarly . . . it cannot take place on Friday either, because when Saturday is eliminated, Friday is the last available day". If Friday is eliminated, Thursday will be eliminated for the same reason, and so on back through the week, with the result that the blackout cannot occur at all.

I hope Mr. O'Connor will not mind my giving his paradox the new and somewhat more appropriate name of "the prediction paradox". I hope, too, he will not mind my using a different illustration of a more practical, poignant and permanent interest than his, and that he will excuse my refining his formulation a little.* (Mr. O'Connor so states the paradox that it is possible for the announcement to be rescinded, with the consequence that the non-occurrence of the blackout rather than its occurrence could be predicted.)

The Paradox: A headmaster says, "it is an unbreakable rule in this school that there be an examination on an unexpected day." The students argue that the examination cannot be given on the last day of the school year, for if it had not been given until then, it could be given only on that day and would then no longer be unexpected. Nor, say they, can it be given on the next to the last day, for with the last day eliminated, the next to the last day will be the last, so that the previous argument holds, and so on and so on. Either the headmaster gives the examination on an expected day or he does not give it at all. In either case he will break an unbreakable rule; in either case he must fail to give an examination on an unexpected day.

The paradox arises I think due to a confusion between two meanings of the term "or", a collective and a distributive. When we predict we refer to a range of possibilities which are as yet undistinguished one from the other. They are connected by means of a collective "or", prohibiting the separation of any one of them from the others, without the introduction of some power or factor not included in the concept of the range. Since predictions always refer to a range and never to the specific determinations of it produced in fact, the predictions must be supplemented by history or the imagination if we are to

* I owe the illustration to George B. Burch.

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select and eliminate first one and then another alternative. What is selected and eliminated in history or in the imagination will be something distinct, focussed on, actualized, connected with others by means of a distributive " or ". If we avoid confusing these two meanings of " or ", our paradox, I think, will disappear. Perhaps this will be somewhat clearer from an examination of the following five cases.

1. On the last day of the term, the headmaster says, " Oh, by the way, I forgot to tell you about an unbreakable rule in this school. We must have an examination on a previously unexpected day. Please take out your pencils and write down and answer the following questions." There is no doubt but that the day and the examination will be totally unexpected. Instead of prohibiting the giving of the examination, as Mr. O'Connor seems to infer it must, the announcement will require the examination, to the consternation of totally unprepared boys. If we wish to produce a paradox we must suppose that the announcement be made not on the last day but on some other. This points up the fact that the paradox is a function, not of the announcement, but of the relation of the announcement to the announced occurrence.

2. The headmaster, on some day of the term other than the last, announces : " There is an unbreakable rule that we must have an examination on a previously unexpected day, and this is it ". Once again the day will be unexpected; once again there will be no paradox. If we wish to produce a paradox we must insist that the announcement be made before the day on which the examination is given. The first case discussed above is evidently a special case of this ; the general rule is not that the announcement must be made on a day before the last, but that it must be made on a day before the examination is given.

It might be argued that if we were to treat the announcement as definitory of a range of occurrences, determining what events we are thereafter to consider, the unexpectedness will disappear. On this view the element of unexpectedness relates solely to the act of announcing a rule, and has nothing to do with what in fact takes place under the aegis of that rule. If we could eliminate the element of unexpectedness involved in the act of announcing we should then have nothing unexpectable left. The position cannot, I think, be maintained, for firstly, we cannot eliminate the element of unexpectedness in the act of announcing the rule, and if we could eliminate it, there will still be left an unexpectedness connected with the occurrence of the examination.

Instead of meeting the question at issue, the above gives it a new location. It leaves us with the question : can we expect the announcing of an announcement which says that it will be announced unexpectedly ? To this no answer is given. Yet it seems evident that we have no grounds in an announcement for an expectation that the announcement will be announced in the particular way or time

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in which it is announced. There is something unexpectable in the act of announcing, just as surely as there is something unexpectable in the occurrence of the events covered by the announcement. Between a possible and an actual announcement there is a distance. He who starts with the first will find that the second comes unexpectedly, and for the same reason that the fulfilment of an announced rule comes unexpectedly—because it is a particular not knowable from the vantage point of what, with respect to it, is only a possibility.

If we have only one day left on which an examination can be given we know the day of the examination, but we cannot know or predict its length, its tediousness, etc. These are subordinate specifications of the examination and depend for their existence on the giving of the examination. I can say that I will positively do 'something or other' tomorrow and yet not know positively what it will be. I can be positive that "this or that will be", *i.e.* of a range of possible occurrences connected by a collective "or", which allows no one of those occurrences to stand apart or be distinct from the others. This does not make it possible for me to know that just this will be done, or that something other than this will be done. There is a great difference between "it is true that either x or non-x is the case", and "it is true that x is the case or it is true that non-x is the case",[1] between $f(x \vee \bar{x})$ and $f(x) \vee f(\bar{x})$. The former is a necessary truth, the latter is true only when one has isolated the x and the non-x, an act which requires one to leave the realm of possibility for the realm of time, history, becoming. To become is to convert the collective into the distributive "or".

In the above two cases we suppose that there is an actual announcement of a rule, and that there is only one case to which the rule applies. The actual announcement comes unexpectedly and so does the application of the announcement. Let us ignore the first unexpectedness and concentrate on the second. The transition from a rule announced to an instance of it requires us to go beyond the conditions expressed in the rule. It makes no difference if the rule have many cases or only one case to which it can apply. To put it still another way, $f(x)$ is not deducible from $f(x \text{ or } y)$ even when it is the case that y is 0.

In the above two cases what is unexpectable—or unpredictable or undeducible, if one likes—is both the occurrence of the only available instance of a rule and the announcing of the rule. In the following cases we take the rule for granted, thereby removing what was a source of confusion in the previous cases. The question that then remains is whether or not one can deduce that there cannot be an unexpected examination inside the time-span which the announcement defines and limits in advance. It is my contention that there can be such an unexpected examination. This is a consequence of the fact that the announcement makes use of the collective "or", and that the examination occurs only when detached from other events and times by means of the distributive "or". The occurrence

[1] *Cf.* Aristotle's, *De Interpretatione*, 18b ff.

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of the distributive "or" and the items it disjoins are always unexpectable from the standpoint of what is constituted by the collective "or".

3. The headmaster makes his statement in the evening, two days before the last. It is then true that "the examination will be given the next day or the day after"; it will not then be true that "it will be given the next day" or true that "it will be given on the last day". There is as yet no distinct next day or day after on which the examination could be given. If the examination is not given the next day, when that day comes around, there will, of course, be no alternative left but that of the last day. It is no surprise then that if we think of ourselves as at the last day, as the paradox requires, and then proceed to eliminate that day, we will find there is no day left on which the examination could be given, the other days having already been eliminated in the act of coming to the last day. When the announcement is made it is not predictable which of two days will be the day on which the examination will be given, but once we have eliminated the first of these days in fact or in imagination, the examination must occur on the second day. But until the days have been distinguished they are not alternatives, excluding one another. He who supposes that they are, and therefore argues that the examination cannot be given, will find to his surprise that the examination is given and at a totally unexpected time.

4. Suppose the examination had not been given by the time the last day came around. Suppose, too, that a student had paid no attention to the announcement when it was made some time previous to the last day. Suppose, finally, that the student suddenly remembered the announcement that day. The examination will certainly come on an unexpected day for him, though there is, in fact, only one day on which it could occur. This conclusion does not go counter to our observation above. A student to whom the announcement was not communicated (because of his inattention or otherwise) is confronted with the possibility not of "examination today or not today" but with the possibility "examination or no examination today". He does not know, before he remembers the announcement, whether an examination will be given or whether an examination will not be given. In remembering the announcement he eliminates, as no longer pertinent to him, the alternative "no examination today". The elimination is unpredictable; it cannot be determined by a consideration of the range of possibility open to him. When, though, the alternative of "no examination today" has been eliminated by him, there is, of course, no alternative left but that of "examination today". What will then be unpredictable will be the exact time or nature of the examination. This he can know when he is in the actual presence of the examination, not before.

When and as the examination occurs it is not unexpected; it is unexpected only before it occurs. Or, if by "examination" we mean to represent a possibility of long or short, fair or unfair, etc.,

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examination, an examination of some sort will be rightly expected but one could not predict its length or fairness—prediction being understood to be grounded in present data and to infer, with warrant, what this entails.

5. If the headmaster were to make his statement before school opened, the students would have to be prepared for an examination on any day at all. That examination will occur some time in school term. But they cannot know, they cannot predict, they cannot rightly expect the day on which it will occur. If the examination is not given by the time the last day comes around, the examination will have to be given on that last day. That day will have been unexpected all along. When it comes around it will, of course, no longer be an unexpected day, for what is now present is no longer unexpected. But the last day cannot be dislocated from the other days at the time the announcement is made, expect by a kind of theoretical anticipation of actual history, and thus by moving away from the announcement of a possibility to that state of affairs where possibilities are specified and distinguished one after the other. The announcement tells us that some one day will be selected as the day of the examination. Whatever one it be, it will be unexpected, so far as we view it from the standpoint of the announcement; it will be expectable so far as we have eliminated the others and thus made it distinct from them—a status it does not have in the announcement.

In all the above cases the students can be certain that some one of a number of alternatives in a given range must occur. They can know with surety the range as a tissue of possibilities connected by collective "or"; they will not then know the items in that range so far as these are connected by a distributive "or". The range might be a range of dates for a possible examination, in which case the students can be certain that an examination will occur but cannot be certain as to just when it will occur. Or it might be a range of events which can be given the common name "examination or no examination". In such a case it will be certain that something will occur, but it will not be certain as to just what this will be. Should we succeed in eliminating all alternatives but one within a given range, there will be no uncertainty regarding the necessity for the remaining alternative. The elimination of that alternative will leave us with no alternative at all; it will involve the cancellation of the entire range.

The paradox of prediction arises because it is forgotten that there is no distributive "or" until we do something to the range of items connected by a collective "or", or more concretely, because we forget that the dates on which an examination can occur are, on the date of an announcement made before the last day, not yet distinguished, and that we have already eliminated all other days when we treat the last day as a day distinct from all the rest.

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