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THE PARADOX OF THE UNEXPECTED EXAMINATION

In an article entitled "Pragmatic Paradoxes" (MIND, 1948), Mr. O'Connor gave an example of a paradox resulting from the announcement by the military commander of a certain camp that a 'class A blackout' would take place during the following week. I shall consider the same paradox, but in the form given in Professor Weiss's article "The Prediction Paradox" (MIND 1952).

The Paradox: A headmaster announces to his pupils at the end of term that it is an unbreakable rule of the school that an examination will be given on an unexpected day of the next term. The pupils reason that the examination cannot be given on the last day of term, since if it had not been given until then, they would know on the evening before the last day that it could only take place on the morrow, and so would not be unexpected. Nor could it take place on the next to the last day, for on the previous evening there would only be two days left for the examination and the last day having already been eliminated, the pupils would know that the examination would take place on the morrow, and so again would not be unexpected. Each day of term is eliminated in this way, so that the students, reasoning on the basis that rules of the school are unbreakable, arrive at the conclusion that an unexpected examination cannot take place that term—an eventuality which would itself violate the rules of the school. It would appear, therefore, that the rules of the school are self-contradictory—and yet it would seem that an examination set at any time before (say) the last week of term would *in fact* satisfy the rules of the school.

Several solutions to this paradox have been proposed in previous issues of MIND,[1] but in the following I shall maintain that in essence the paradox is of the familiar *self-referring* type.

To see that this is the case, it is only necessary to state explicitly and unambiguously what is meant by 'knowing' that the examination will take place on the morrow. In their above argument, the students used 'knowing' in the sense 'being able to predict, *provided the rules of the school are not broken*'. It seems to me that this is a perfectly valid meaning to adopt, and if adopted, it is one which should not give rise to contradiction, provided the rules of the school are properly set out. (If instead one adopted a vague common-sense notion of 'knowing', then one could perhaps agree with Professor Quine [2] that an unexpected examination could take place even in a one-day term ; but to my mind, this would be evading the paradox rather than resolving it.) If then we agree to replace 'unexpected' by 'not deducible from certain specified rules of the school', we obtain a purely logical set-up, with the rules acting as axioms, and any

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[1] Peter Alexander, 1951, p. 538 ; Michael Scriven, 1951, pp. 403 ff.; Paul Weiss, 1952, pp. 265ff. ; W. V. Quine, 1953, pp. 65 ff.

[2] *Ibid.*

paradox that results must be a logical one, and not of some other type as suggested in previous discussions.[1]

What do we choose for the rules of the school ?   Suppose we try the following two rules :

Rule 1 : An examination will take place on one day of next term.

Rule 2 : The examination will be unexpected, in the sense that it will take place on such a day that on the previous evening it will not be possible for the pupils to deduce *from Rule 1* that the examination will take place on the morrow.

The last day of term is then ruled out as a possible day for the examination, since it would violate Rule 2.   However, *any other choice for the day of the examination would satisfy both Rule 1 and Rule 2*.   Thus in this case, no paradox results ; the rules are self consistent and are satisfied by any choice of day except the last day of term.

Suppose now we add a third rule :

Rule 3 : The examination will take place on such a day that on the previous evening it will not be possible for the pupils to deduce *from Rules 1 and 2* that the examination will take place on the morrow.

Then, by *Rule 2*, the last day of term is again not a possible day, but now the next to the last day is ruled out also.   For if no examination had taken place when there were only two days of term left, then the pupils could deduce from *Rule 1* that the examination would take place on one of these two days, and from *Rule 2* that it would not take place on the last day, and hence from *Rules 1 and 2* that it would take place on the morrow.   This deduction would violate *Rule 3*, and so the last two days are not possible days for the examination.   However, any other choice of day would satisfy all three rules, and so no paradox results, provided that term lasts more than two days.

If term lasts two days only, then the above three rules are self-contradictory.   In this case, however, no examination can in practice be set satisfying the rules of the school.   For if set on the last day, the pupils will quite rightly claim that, in violation of *Rule 2*, they ' knew ' on the previous evening that it was going to be set that day ; and if set on the next to the last day they will claim that, in violation of *Rule 3*, they ' knew ' it was so going to take place.   The point is that the pupils are able to substantiate their claim that they ' knew ' that the examination was going to take place when it did by appealing to *different* rules for each day of term.   Thus we can say that on the evening before the last day of term they ' knew in the sense of *Rule 1* ' that the examination would take place on the morrow—that is, it had to be set that day if *Rule 1* was not to be broken.   Similarly on the evening before the last two days of term the pupils ' knew in

[1] Peter Alexander, 1951, p. 538; Michael Scriven, 1951, pp. 4C3 ff.; Paul Weiss, 1952, pp. 265 ff.; W. V. Quine, 1953, pp. 65 ff.

the sense of *Rule 2* ' that the examination would take place on the morrow.   Since these two kinds of ' knowing ' violate *Rules 2 and 3* respectively, no examination can *in fact* be set satisfying the rules of the school, and so no paradox results even in a two-day term.

The above clearly generalises to the case of a $n — $ day term with $(r + 1)$ rules (with the obvious definition of *Rules 4, 5,*) . . , $(r + 1)$, If $r < n$, the last r days of term are ruled out as possible days for the examination, but any of the first $(n — r)$ days are allowed.   If $r = n$, then the rules are self-contradictory—but now no examination can in fact be set satisfying the rules of the school.   For if no examination has taken place up to the evening before the last s days of term $(s = 1, 2, . , . , n)$, the pupils will ' know in the sense of *Rule s* ' that the examination will take place on the morrow, and this ' knowing ' violates Rule $(s + 1)$.   Thus in no case does there arise a paradox.

The original paradox arose by taking, in addition to *Rule 1*, only the following rule :

Rule 2 * : The examination will take place on such a day that on the previous evening the pupils will not be able to deduce from *Rules 1 and 2** that the examination will take place on the morrow.

It is clear that the origin of the paradox lies in the self-referring nature of *Rule 2**.

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