EXPECTING THE UNEXPECTED

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Introduction

"A puzzle that has had some currency from 1943 onward" (Quine, 1953, p. 65), but "has, unexpectedly, withstood examination" (Nerlich, 1961, p. 503) is still, three dozen years and over that many papers later, a lively and lovely one, which somehow refuses to be put to rest. This is the puzzle best known as The Surprise Test Paradox, but also as The Class A Blackout, The Hangman Paradox, The Prediction Paradox, The Surprise Egg, etc. Some continuity in the development of the thinking on the paradox has been provided by the fact that almost half the papers on it were published in *Mind*, but too many of the papers remain obscure and uncited. Interestingly, while many authors apparently succeeded in convincing themselves that they had a good grip on the problem (e.g., "I feel my approach has the advantage of being so simple that anyone who reads it will agree with the solution"(Lyon, 1959, p. 510), others were not so convinced. In fact, the rule, rather than the exception, seems to us to be disregard for the inter-relations between one's own views and those of others, in terms of implications, compatibility, effect, etc. The diversity in opinions, approaches and treatments which have been brought to bear on the paradox is truly impressive.

We believe that this large body of literature deserves the benefit of a review. Philosophical journals do not have a tradition of wide-ranging reviews, and more's the pity. In the case of the Surprise Test literature, this has led to the unnecessary repetition of some arguments, to the positing of others which were passé, and to neglect of important critical issues. A handful of writers enjoyed real impact (Scriven, Quine, Shaw, Binkley), and the ingredients for a comprehensive understanding of the paradox have been in print for over fifteen years. Yet papers continue to appear, with no awareness or no acknowledgement of heritage. For a problem which has been labeled anything from "piddling" to "powerful", which has attracted written attention from many illustrious philosophers in around a dozen journals, which is discussed in many an introductory Philosophy course, the time has come for a comprehensive overview. In this paper, we undertook this task.

But our goal in writing this paper goes well beyond the review. We attempt through it to achieve an integrated point of view on the paradox, which will finally resolve it. We attempt to give a perspective wherein each of the various approaches taken can find its place, and the motivations and inter-relationships between them be properly understood. So this paper should be viewed as explication-by-survey.

An Overview

We begin by presenting a popular version of the paradox (by using the word "paradox" throughout this paper we are not prejudging the issue, but merely bowing to the dictates of convenience and tradition): A teacher announces to his pupils that on one of the days of the following school week (Monday through Friday) he will give them an unexpected test. The students argue that the teacher can in no way fulfill the promise. If he waits until Friday, the students will be expecting it from Thursday night. Hence, the last day on which the teacher could possibly give the unexpected test is Thursday. But the students, knowing this, would by Wednesday night (if as yet untested) be expecting the test to occur on Thursday. Using this method, the students proceed to rule out all the days of the coming week. And, as if this isn't paradoxical enough, an extra twist is added by the fact that the teacher did give a test on Tuesday of that week, and it was quite unexpected!

The first paper to introduce the paradox in print (O'Connor, 1948) used a "Class A Blackout" cover story, rather than the later more popular Surprise Test version. For purposes of uniformity, we shall, however, rephrase arguments in the latter terminology whenever convenient.

Neither O'Connnor nor two subsequent writers (Cohen, 1950; Alexander, 1950) were aware of the devastating extra twist mentioned above, i.e., that the students' argument notwithstanding, the teacher *can* give a surprise test. Rather, in their view the paradox was entirely in the announcement. O'Connor, for example, says: "It is easy to see . . . that the [test] cannot take place at all." O'Connor, Cohen, and Alexander devote their efforts to uncovering why a seemingly innocuous announcement, like the teacher's above, cannot be fulfilled. O'Connor considers the announcement "pragmatically self-refuting", rather than "formally self-contradictory", and the entire paradox "rather frivolous" (p. 358). Cohen, in turn, argues that the paradox arises "from a proposition that can be falsified by its own utterance. [The promise] is rendered false by its public announcement" (p. 86). Alexander goes even further, by denying that the announcement raises any "other difficulties than are raised by any conditional sentence whose condition is unrealizable, like, for instance, 'If I can live without air I will not breathe all day tomorrow'" (p. 538). Gamow & Stern (1958), in their presentation of the paradox, also regarded the teacher's promise as unfulfillable.

Only with the publication of Michael Scriven's paper (1951) was it first pointed out that the paradox "is not frivolous, for a reason which has escaped O'Connor and the others" (p. 403), namely that the teacher can and does give the surprise test. In this new light, Scriven saw the paradox as "a new and powerful" one (p. 403). Scriven was also the first to suggest that "cannot know" (the day in advance) might be identified with "cannot produce a proof of". Having made these breakthroughs, Scriven comes disappointingly short of understanding the paradox. His attempts lie in distinguishing between two "functions of an announcement . . . The first are publicly uttered statements. The second are ordainments." His solution: "The suicide of the announcement as an ordainment [i.e., the fact that it is self-refuting] is accompanied by its salvation as a statement [i.e., by the fact that it is possible to verify it]" (p. 407). As to the students' argument, he states forcefully, but without contending with it, that "the logical gadget . . . has somehow short-circuited . . . this flavour of logic refuted by the world makes the paradox rather fascinating" (p. 403).

The next paper to appear was by Paul Weiss (1952), who both invented the term "prediction paradox", and introduced the Surprise Test formulation. Coming as it did after Scriven's revelation, Weiss saw the challenge of the puzzle in explaining how the students' argument short-circuits. However, in dealing with it, Weiss went "to the desperate extremity of entertaining Aristotle's fantasy that 'It is true that *p* or *q*' is an insufficient condition for 'It is true that *p* or it is true that *q*"(Quine, 1953, p. 65). Weiss claimed that the paradox arises "due to a confusion between two meanings of the term 'or' [as it appears in the detailed version of the teacher's announcement: I shall give a surprise test on Monday *or* on Tuesday *or* on Wednesday . . .], a collective and a distributive" (p. 265).

One of the best-known papers on this "so-called paradox" is by Quine (1953), who opens it with the words: "There is a false notion abroad that actual paradox is involved" (p. 65). In fact, however, the student, K (actually Quine's K denoted a sentenced man, not a student, for Quine used the Hangman version) has merely used "a faulty argument" to persuade himself that the surprise test could not be given. Quine proceeds to reconstruct K's argument to detect where it went wrong. According to Quine's analysis, come Thursday night four exhaustive and exclusive possibilities exist: "(a) the event will have occurred at or before that time; (b) the event will (in keeping with the decree) occur [on Friday], and K will (in violation of the decree) be aware [of this]; . . . (c) the event will (in violation of the decree) fail to occur [on Friday]; (d) the event will (in keeping with the decree) occur [on Friday], and K will (in keeping with the decree) remain ignorant meanwhile of that eventuality" (p. 66). K is trying to prove that Friday is not a viable day for the test to occur on in keeping with the decree. If he succeeds, then in domino fashion the rest of the days of the week will also be eliminated, to make his proof complete. But Quine denies that K has proven Friday away. K "erred in not recognizing that either (a) or (d) could be true even compatibly with the decree" (p. 66). His diagnosis: the puzzle may arise from "a wrong association of K's argument with *reductio ad absurdum*" (p. 66). It is imperative to distinguish between the assumption that the teacher's promise will be kept and the assumption that K knows the teacher's promise will be kept. The latter is clearly false, while the former may be true. Quine's reconstruction even justifies a reduction of the story to the one-day case. I.e., even if "the [teacher] tells K on Sunday afternoon that he, K, will be [tested] on the following noon and will remain ignorant of the fact till the intervening morning," no paradox arises.

While no subsequent writers have found fault with Quine's logical treatment of the paradox *given his interpretation*, many accused him of "evading the paradox rather than resolving it" (Shaw, 1958, p. 382), "offering formulations which can be shown not to be paradoxical, rather than discovering an exact formulation which is genuinely paradoxical" (Kaplan & Montague, 1960, p. 79), or "picking out a trivial paradox for solution, leaving the more difficult one unsolved" (Medlin, 1964, p. 66). We shall show in the next section that while Quine may have oversimplified in his interpretation, his approach contains the potential for a broader solution.

Shaw's paper (1958) marks a milestone, too. Since Scriven's discovery of the fact that the teacher *can*, after all, give an unexpected test, efforts had been directed at finding the flaw in the students' argument. Shaw, on the other hand, shows that the fault may well rest with the teacher's announcement, leaving the students' reasoning impeccable. He first shows that the teacher's announcement in the popular version can be given more than one interpretation. He suggests that "we agree to replace 'unexpected' by 'not deducible from certain specified rules of the school'", thereby obtaining "a purely logical set up, with the rules acting as axioms" (p. 382). If a paradox still results, it "must be a logical one." Different interpretations hinge on the different possible choices of such a set of rules. For example, Quine's solution, according to this view, would take the set of the school's rules to be void.

Shaw considers two kinds of sets of rules. In one we have:

> "Rule 1: An examination will take place on one day of next term.
> Rule [i]: The examination will be unexpected in the sense that it will take place on such a day that on the previous evening it will not be possible for the pupils to deduce [from rules 1 to i-1] that the examination will take place on the morrow" (p. 383).

Rule i is defined by regression, for $i \geqslant 2$. A school may have any number of such rules, so that the above is really a family of school rules, one for each r, where r is the number of rules. We can now distinguish two cases. Those where $r \geqslant n$, and those where $r < n$, where n is the number of days in the term. If $r \geqslant n$, the rules are self-contradictory. That is, regardless of if and when an examination will take place, it will violate one of the rules, and so the students' argument is correct. If, on the other hand, $r < n$, then a genuine surprise examination can be set for any of the first n-r days of the term, and the students cannot prove otherwise.

A second possible set of rules for the school is the following:

> Rule 1: An examination will take place on one day of next term.
> "Rule 2*: The examination will take place on such a day that on the previous evening the pupils will not be able to deduce from *Rules 1 and 2** that the examination will take place on the morrow. (p. 384).

This set of rules is satisfiable under no circumstances. Shaw ends his "most illuminating" (Nerlich, 1967, p. 503) little paper with the words: "It is clear that the origin of the paradox lies in the self-referring nature of Rule 2*" (p. 384). Actually, though, Shaw himself proposed a set of non-self-referring, yet contradictory, rules (the first set, with $r \geqslant n$).

The impact of Shaw's paper rivals that of Quine's. In short succession, his paper was followed by others (Lyon, 1959; Kaplan & Montague, 1960; Nerlich, 1961; Medlin, 1964; Fitch, 1964), all of which formalize the problem, and share the view that it can be formulated in a manner that leads to genuine paradox, or at least contradiction. Nevertheless, each is critical of at least some of the claims made by at least some of its predecessors. We shall not review these papers in full, since a good review can be found in Bennett (1965) and in Cargile (1965). We do, however, want to mention some points made by these authors, especially novel ones. (i) Lyon uses our paradox for "an attempted application ... to the problem of free will" (p. 510), which is, essentially, "that neither determinism nor indeterminism is incompatible with free will, and that the apparent conflict arises . . . through not examining particular-case uses of the word 'predict' " (p. 517). (ii) Nerlich, like Quine, believes that a surprise test can be given even if the teacher waits until as late as Friday, though he calls Friday "a queer case", and his own position on it "very uncomfortable" (p. 512). However, unlike Quine, it seems that his only justification for endorsing Friday is that anything else would send the students' argument into infinite regress, where they would go over the same grounds again and again – hardly a satisfying reason. (iii) Medlin, aside from dismissing all previous papers but Shaw's with great disdain, generalizes the paradox to any finite number of days, n, or indeed to "any set which is ordinally similar to the set of negative integers under the relation *less than*" (p. 70). (iv) Fitch formalizes the problem using "merely concepts of logic, syntax, and elementary number theory", along with "techniques devised by Gödel" (p. 161), and as a solution offers the suggestion that the surprise test was never, in practice, intended by the teacher to "be a surprise *whenever* it occurs, but only when it occurs on some day *other than the last*" (p. 163), in which case no problems arise. All in all, it doesn't seem to us that this post-Shaw series made any significant progress in our understanding of the Surprise Test Paradox.

A slightly different, though still formal, approach was taken by Kaplan and Montague (1960), which makes heavy use of epistemological concepts. These authors set out to deliberately "discover an exact formulation ... which is genuinely paradoxical" (p. 79), as contrasted with Shaw's formulation, which is "merely incapable of fulfillment" (p. 84). Their "genuinely paradoxical decree", D, is a self-referring conjunction that, roughly speaking, says of itself that unless K (the student) knows that D is false on the day that it is given, K will be tested on such a day that on the basis of D, K cannot know in advance that it is the day. A simpler version would be "K knows that this statement is false". Kaplan and Montague labeled their paradox 'The Knower', "in view of certain obvious analogies with . . . the Liar" (p. 88). The lesson they derive from their paradox is that "some intuitively plausible epistemological principles . . . are incompatible with the principles of elementary syntax", unless "a number of restrictions [are] imposed on a formalized theory of knowledge", such as that "the predicate 'knows' would occur only in the meta-language, and would significantly apply only to sentences of the object language" (p. 87, 88).

Like Kaplan & Montague, Kanger (1976), in a "mini essay", also seems to think that the paradox can be reduced to an inconsistency inherent in some of our epistemological notions, but does little to resolve or even illuminate it.

If Kaplan and Montague saw the Surprise Test Paradox as *analogous* to the Liar, Peter Windt (1973) claims that it *is* actually "another of the large family of paradoxes collected under the heading of 'The liar' " (p. 65). To show this, he "rephrase[s] the example, . . . in terms of the possibility of a sound deduction [i.e., a valid deduction of a true statement]" (p. 66). The announcement then reduces to the following set of admissible premises:

"(a) . . . [a test] will occur during the week in question . . .
(b) On any day during the week, . . . either . . . the [test] has occurred . . . or . . . it has not, whichever is actually the case.
(c) . . . no sound deduction of the date . . . can be derived from . . . (a)–(c) *before* that date actually arrives" (p. 66).

Using the definition of "sound", Windt shows that if (a) and (b) are true, then "(c) is true if and only if false" (p. 67). Since his paper earlier shows that premises such as (c) hide a Liar component, this establishes Windt's initial claim. He offers no particular resolution to this paradox, since he is convinced that it will be resolved along with "an acceptable treatment of the family of Liar paradoxes" (p. 67).

Kaplan & Montague and Windt arrived at formulations of the teacher's decree that make it genuinely paradoxical – i.e., one which "oscillates like the statement 'This sentence is false' " (Keifer & Ellison, 1965, p. 427) – rather than merely self-contradictory or unfulfilable. Cargile (1965) remarks that " 'knower' type paradoxes are just Liar-family paradoxes in which knowing is involved only in that it entails truth" (p. 103).

While this stream of formal papers were appearing, O'Beirne (1961) was discussing the puzzle informally with the lay audience of New Scientist. According to his analysis, the students' reconstructed argument assumes three statements: (1) "There will be an examination ... next week"; (2) "before the examination is announced, pupils will not be able to deduce that it is about to take place"; and (3) "the pupils may regard both the previous statements as unconditionally true" (p. 464). Contradiction is derived from *all three* in conjunction, but is resolved by the realization that though "our first two statements *are* true, it becomes *logically absurd* to assume that the pupils ... could properly be *convinced* that both were true [in advance]" (p. 464). Their truth could only be established "in retrospect" (p. 465).

Another writer to discuss the paradox with a lay audience was Martin Gardner (1963). In his inimitable fashion, he laid out the paradox in several of its versions, fleshing it out with many a tantalizing detail to drive home the power of the paradox. His own position is sympathetic to that of O'Beirne ("the key to resolving the paradox lies in recognizing that a statement about a sure event can be known to be a true prediction by one person but not known to be true by another until after the event" (p. 154), and he anticipates an argument offered by a much later author, Cargile, about the students' options in a two-day case: "although you would be wise to bet that [the test is set for Thursday], because it probably is, ... to win the bet you have to do more than that: you have to prove [it] ... with iron logic. This you cannot do" (p. 150). Gardner also shows how the students' argument gets them into a "vicious circle of contradictions", which seems to him analogous to yet another paradox, the Jourdain or Langford Visiting Card Paradox.[1] He plays some more with this association in Gardner (1962) (see also Popper, 1962).

Lastly in this genre, Woodall (1967) in a very brief but very lucid presentation, argues in the same vein as his two predecessors that the implicit (and untenable) hidden assumption behind the students' argument is "that *they know* ... [for] absolutely certain ... that [the teacher] is telling the truth in every minutest detail". All their argument proves, however, is that "they cannot possibly know with absolute certainty that he is telling the truth" (p. 32), for that would lead them to argue from inconsistent hypotheses.

The string of papers appearing in *Mind*, broken since Nerlich's 1961 paper, was resumed in 1964 with a flurry of Discussion Notes, to which we now turn. Even though this series begins at about the same time that Bennett and Cargile published their respective reviews, many of the papers seem to deal with more "primitive" points than one would expect in the aftermath of previous achievements. Thus, for example, Sharpe (1965) chooses to regress to an ambiguous usage of the term "surprise", and to allow the teacher his test if the students fail to come up with a proof to the contrary, even for extra-logical reasons. "The key to the puzzle," he says, "is the knowledge or ignorance of either party [the students or the teacher] of the rule . . . [which provides] the argument for excluding unsuitable days". Sharpe seems to be ambiguous as to his own solution as well, for he says: "If both boys and master know of the rule and apply it, no day can be chosen which satisfies the conditions for a surprise," but also: "the occurrence of the examination on any of the other days [but the last] would be a surprise for the boys whether or not they thought the master knew of and intended to apply the rule." (p. 255).

Meltzer (1965) considers the situation posed by the teacher's announcement to be one which, though "otherwise most paradoxical" can be "clear[ed] up simply" by "the recognition of [the] third possibility" (p. 432), the one excluded by the Law of the Excluded Middle... Showing that both with respect to ordinary language concepts (e.g., baldness) and mathematical ones (e.g., truth) this Law is "myopic and wrong" (p. 430), he suggests that here, too, just prior to the onset of the last day, "there is no finite procedure by which [the test's] occurrence or non-occurrence on that day could be definitely deduced" (p. 432).

A year later, in a joint paper with I.J. Good (1965), Meltzer admitted to "an insufficiently sophisticated use of logic" in his previous paper, but still stuck to his basic view "that a three-valued logic is required" (p. 51). His co-author, however, did not agree. Good, rather, thinks that the teacher's announcement is of the self-referring form: " 'A, and not-A is deducible from the whole of this statement' " (p. 51), which is meaningless because its second conjunct is.

Chapman and Butler (1965) argue that the boys never have a "valid logical argument" which isn't "open to contradiction" that the test will be held on the last day (if not by then), since "the conclusion that the examination must be held on the last day (based on the promise that there *will* be an examination), is just as warranted as the conclusion that it cannot be held (based on the promise that it would be unexpected)" (p. 424). They seem unperturbed to be saying that, if the examination is held on the last day, then the announcement both leads to a contradiction ("on the morning of the last day, it is necessarily the case that the examination both is and is not held on that day ") (p. 424) and is confirmed ("the examination, even if it is held on the last day, will be unexpected in *the required sense*" (italics ours)) (p. 424). They also make the erroneous claim that "nobody has yet proved that (a) and (b) are incompatible," where (a) and (b) are: "(a) that there would be an examination on one of the afternoons of the following week; and (b) that the examination would be unexpected" (p. 424).

Keifer and Ellison (1965) define two meanings of "surprise" by allowing two different premises, respectively, to be used by the students in their deduction. These premises are much like Shaw's Rules 2 versus 2*. They find the self-referring definition to be self-contradictory, and the simpler one to be uncontroversially satisfiable. The authors clearly acknowledge the resemblance to Shaw, and we fail to see where they made an additional contribution.

Schoenberg's (1966) somewhat muddled diagnosis as to what is wrong with the students' argument, which she describes as "oddly resistant to criticism" in spite of its vulnerability (p. 125) is that "it is fundamentally one of contradiction concealed by incomplete inference" (p. 127). By this she refers, presumably, to "the fact that the premise used to establish the [students'] argument . . . [is] that the examination has not been given under the prescribed conditions"; and that this, being "in plain contradiction with the headmaster's order" . . . "cannot be considered to be an inference from the defined conditions" (pp. 125, 126).

Wright (1967) contributes a novel interpretation of the headmaster's declaration, the main fault of which is that it is far enough removed from any natural interpretation to be hardly a solution to the standard puzzle. By choosing a day for the test to be set using a lottery procedure which gives Friday a chance to be chosen equal to that of any other day, the headmaster nullifies the students' argument, albeit at the cost of incurring a non-zero probability of having to forego the test altogether (and thus breaking his promise), should Friday indeed be chosen. If Quine was accused of evading the paradox, this accusation can be applied with a vengeance against Wright. As to the standard interpretation, Wright readily accepts that *it* does involve a paradoxical announcement.

In a clever little note, Austin (1969), noting that even after the students offer their proof "there still remains the problem . . . of the master setting the examination say, on Wednesday, and it being unexpected," outlines the practical steps the students must take to prevent such an examination being unexpected, "these being to construct on each evening their proof up to and including (but no further than) the line 'The examination must take place tomorrow' ". The result of this procedure is "that if the examination is held it will be entirely expected." While Austin refrains from commenting on his own facetious advice, it highlights a point made, implicitly or explicitly, by others: that there is no natural stop rule to determine when the students' proof, as they unfold it, is complete; and by taking as proven the last step in any chain of inferences, the students seem to be able to prove any conclusion they wish – or continue in circles indefinitely.

Ten years later, Austin developed this theme somewhat in Austin (1979). There he distinguishes two cases. In one, a teacher, D, promises a test such that "the students will not be able to prove on the day before the exam, *that* it will be the next day" (p. 63). In the second, a teacher, W, promises to give a test such that "the students will not be able to prove on a day before the exam on *which* of the remaining days it will be" (p. 64). Both D and W make inconsistent promises, says Austin, but whereas "D is (merely) unable to keep his word," W, were he to give a test on Thursday, say, would keep it!

The most recent paper which we found in *Mind* is by Ayer (1973). Ayer criticizes Quine's solution as "valid ad hominem, but . . . [failing to] get to the root of the puzzle" (p. 125). He transforms the problem to one of predicting, while running in sequence through a finite stack of well-shuffled cards, where a given card is. It is impossible "to predict *before the sequence is run through* when the event in question will occur," but might be possible to do so "in the course of the run" (p. 126) – specifically, if the target card is last. "It is *only* [italics ours] because the two cases are not distinguished in the formulation of the puzzle that the appearance of an antinomy arises" (p. 126).

Ayer's formulation highlights what other authors have also perceived, namely that the time factor in itself is nonessential to the paradox, because any ordered sequence can be substituted for the progression of days. Nevertheless, Fraser (1966) conceptualizes our problem as "a powerful illustration of the antinomies of time . . . the paradox is produced when arguments involving temporal order are mixed with arguments disregarding time" (p. 575).

The idea put forth by Wright that the teacher might employ a randomizing strategy for selecting the test day, and that this strategy might assign Friday a non-zero chance of being selected, is explored in depth in Cargile (1967). Cargile (who also authored the review of Kaplan and Montague's paper mentioned above) holds the view that the standard interpretations of the paradox "are of some logical interest, but they represent no new problem" (p. 550). He tells the story in a way that leads to a new kind of paradox. Not a logical one, but a decision- or game-theoretic one. He uses the two-day case, and the following formulation: the teacher must choose either Thursday or Friday for the exam; the students are trying to outguess his choice; if they succeed, he loses, and otherwise he wins; both of them are "ideally rational agents". What strategy of choice should the teacher employ in order to optimize his prospects? In his discussion, Cargile takes for granted that which to some has seemed the crux of the matter, i.e., that "the probability of giving a test that is a surprise on Friday is 0" (p. 556), and that the probability of giving a test that is a surprise on Thursday may be non-zero (as an empirical rather than a logical possibility). He stipulates what "would appear to be an essential truth about ideal rationality: if two ideally rational agents are asking independently whether a given proposition is true and if both have exactly the same relevant data and exactly the same knowledge about what is relevant, then they will both reach the same conclusion" (p. 557). Then he arrives at his own version of the paradox: "Thursday is the ideally rational choice if and only if it is not" (p. 558). After many convolutions, Cargile solves this new paradox by persuading (himself) that if the teacher set the examination for Thursday, a "perfectly rational judge" would rule that he had won, i.e., that the students don't have a tight enough justification that this is what the teacher had to do, though it is (what the teacher had to do). Cargile's paradox is an intriguing one, but its problematics are quite different from those of the standard interpretation, and we shall not include it in our later discussion.

In the papers we have reviewed up to this point, we can see a consensus emerging to the effect that a "surprise" test is best thought of as a test whose date of occurrence (which possibly includes the null date, i.e., no test) cannot, previous to the actual occurrence, be deduced from some set of premises. Binkley (1968), and later Kvart (1976), takes a different approach. In his view, the paradox (to which he devotes considerable attention in spite of calling it "piddling") "cannot be handled in a formulation that makes no use of epistemological or pragmatical concepts, as the deducibility formulation seeks to do" (p. 128), since "such an approach misses the nub of the difficulty, which is that the flawless reasoning of the students is somehow rudely brought to nothing by the actual occurrence of the promised, but apparently impossible, exam" (p. 128). Alternatively, he conceptualizes the student as an "ideal seeker after knowledge ... an ideal of rationality, [whom] circumstances may conspire to ... lead ... into false belief" (p. 128). He proceeds by giving an axiomatic model of his ideal knower, composed of what he defends as "familiar", "plausible" and uncontroversial demands.

Next he proves that the teacher's decree is such that an ideal knower could not possibly believe it (because that would lead him into contradictory beliefs), even though it might be true. This renders the paradox a member of "the same family as Moore's paradox"[2] (p. 135). "So . . . the surprise examination paradox reduces to the phenomenon of incredible though possibly true propositions" (p. 136). Unfortunately, we do not believe Binkley comes to grips with the question of just what it is about the announcement that renders it incredible. Although modal logic doesn't assume for belief, as it does for knowledge, that from the fact that x believes that p it follows that p, we venture to say that, as in the Kaplan & Montague case, the resort to epistemological notions would be unnecessary if a clarification of the logical status of the announcement and the deduction were achieved. We shall elaborate in the next section.

Binkley, in spite of considerable differences, aligned himself with Quine in what he calls "a tradition of skepticism with respect to authorities" (p. 131). This tradition came under interesting criticism in a paper by Jorge Bosch (1972). Motivated by a need to set up a criterion for judging what formulations of our (or any) paradox "respect the 'spirit' of the original" (p. 505) and what constitutes a solution. Bosch formulated his "coherence requirement" (p. 505): and analysis of the paradox should preserve the intuitive plausibility of the announcement, and should show where, if at all, the students' argument is erroneous. "'To explain' or 'to solve' the paradox signifies to give the announcement an interpretation such that the informal proof be still relevant, and to decide – within the framework of this interpretation – whether the informal proof is correct or not and where does the 'cause' of the paradoxical effect lie" (p. 507). Now, if one "take[s] into account from the beginning the possibility that the teacher's announcement will not be fulfilled" (p. 508), (which, Bosch says, is problematic in itself, unless one first clarifies what constitutes a fulfillment of the announcement, i.e., what constitutes 'to know in advance'), then it is the announcement which is doubtful, even paradoxical, whereas the students' argument simply becomes irrelevant. We do not believe that Bosch's criticism is correctly applied to Quine, but it can be applied not just to Binkley-Kvart, but to Wright, Fitch, and others.

Bosch suggests, following the implicit view of Bennett (1965), that the paradoxical effect of our puzzle, from a psychological point of view, arises from the failure to distinguish between A_2 (or something like it) and A_3 (or something like it) below:

A_2: On no day by which a test has not yet been given will it be possible to deduce from that fact and from the promise of a test that the test must be on the following day.

A_3: On no day by which a test has not yet been given will it be possible to deduce from that fact and from the promise of a test *and from* A_3 that the test must be given on the following day.

This failure may be due to the fact that "as a pragmatical teacher's announcement, the most plausible is $A.A_2$ [where A is the promise that there will be a test], and with respect to the [students'] informal proof, the most adequate is $A.A_3$" (p. 512). Not only a naive reader might confuse the two, but even some writers (e.g., Lyon; Nerlich; see Bennett, 1965). Under either interpretation of the teacher's announcement the students' argument is relevant, but it is wrong in the first case and right in the second.

A novel contribution of Bosch was to formalize the puzzle in set-theoretical terms (rather than predicate-calculus ones as hitherto), which does away with self-reference. Lastly, Bosch outlines a theory of formal prediction, within which he shows that while self-reference may not be the *origin* of the paradox, "self contradiction seems to arise *because unpredictability is accepted as a premise to deduce . . . unpredictability*" (p. 524), a practice which "in ordinary scientific work" (p. 525) is highly "irregular" (p. 523).

Wright and Sudbury (1977) broaden the list of criteria which they feel an "account given of the content of the announcement" should satisfy in order to be "intuitively satisfying" (p. 42). For example, it should allow the teacher to keep his promise, even if made to the pupils, "since, palpably, he can" (p. 42). Clearly, many previous accounts failed to comply with at least some of Wright & Sudbury's criteria.

Their own analysis proceeds by interpreting "surprise test" as one in the occurrence of which on a given day the students cannot have "reasonable belief, in advance of its actually being given, detailing a theory of reasonable belief, and proving from it that the teacher's announcement is perfectly alright, but the students' argument isn't. In other words, the teacher can fulfil his promise even if he waits till the last day to do so, and the students are not entitled to reasonably believe the test will be given on a certain day even if it hasn't been given before the last one, and even if they are privy to the teacher's announcement.

Wright & Sudbury are meticulous in showing how their solution answers the requirements they set, thus meeting Bosch's challenge as well. But while Bosch insisted that "it is out of the question to introduce any epistemological ... consideration" (p. 506) into the solution, Wright & Sudbury's is wholly in epistemological terms. It is their theory of reasonable belief which shoulders the burden of the puzzle.

The last few years have shown "signs of a revival of interest in the Paradox" (Champlin, 1976, p. 349), and it is to this recent work which we turn at the close of our review. It may be difficult to find something refreshingly new to say at this stage (another point which speaks for a definitive review), and indeed Slater's (1974) paper brings old arguments. Slater, in a manner reminiscent of the very earliest writers, points out that what "the teacher *says* is quite consistent as evidenced by the fact that he both gives a test, *and* nobody quite expects it, and "the pupils' reasoning is therefore invalid" (p. 49), but that it is "reprehensible" of the teacher to make such an announcement, since "while not making contradictory remarks, by *making* the first remark [that there will be a test] ... [he] influences the truth of the second that the pupils won't know it in advance" (p. 50).

Edman (1976) is also, unfortunately, old hat. He employs the by-now familiar scheme, initiated by Shaw, of comparing what emerges when the announcement does refer to itself as an admissible premise in deducing the date of the test, versus what happens when it doesn't. He adds two other rather uninteresting interpretations, which are, moreover, "irrelevant" in Bosch's sense.

Champlin (1976) wrote his paper with the purpose of "exposing a defect in [Quine's] widely acclaimed solution, not ... devising a fresh solution" (p. 349). Quine pointed out himself that "K's fallacy way be brought into sharper relief by taking *n* as 1 . . . [Teacher] tells K on Sunday afternoon that he, K, will be [tested] the following noon and will remain ignorant of the fact" (Quine, p. 66). Champlin disagrees that the seven day case is just like the one day case (or, in other words, that the last is to be dealt with in identical fashion to any other day). For "where the [teacher] says 'You will be [tested] on one of the next seven noons and you will not know the day in advance' ... it is conceivable that ... [he] take[s] himself to have said nothing more problematic than 'You will be [tested] one day next week, but I don't know which'. In sharp contrast, it is not conceivable that a [teacher] could speak sincerely [when saying, 'You will be [tested] tomorrow noon and will not know the date in advance'] in the perfectly normal course of events" (p. 351). Whether the next-day decree is self-contradictory or not, says

Champlin, can only be decided after the story is fleshed out some more, as circumstances can swing it one way or the other. "The key question," which Quine 'side-steps" is " 'Could a sincere and sober [teacher] deliver his [decree, in the seven-day case] to a reasonable K without self-contradiction?" (p. 351). Champlin closes his paper with the hint of a "suspicion that the much maligned regress argument may be valid after all" (p. 351).

Understanding the Surprise Test Paradox

In the course of our careful reading of the literature for the purpose of writing the survey contained in the previous section, we encountered some of our beliefs in the writings of others, as well as having been influenced by others in the shaping of our own beliefs. At the outset of this section we wish, therefore, to humbly acknowledge our intellectual debt, and to warn the reader that much of what we shall presently say will inevitably, at this juncture, sound familiar. We believe, nevertheless, that by reconciling, juxtapositioning, generalizing, putting in perspective, and altogether clarifying the various points of view that have been put forth over the years, we shall be able to draw an integrated, maybe even innovative picture of this paradox. This we shall do in the course of tracing its intellectual evolvement.

Let us begin with the teacher's decree as it might have been given in a simple, informal, unsophisticated manner: "Next week I shall give you a surprise test." This is certainly a commonplace enough announcement, which could (though not necessarily must) be taken to mean merely: "Next week I shall give you a test whose precise day of occurrence I shall not presently announce." This is probably just what flesh-and-blood teachers in real-life classrooms mean when they make such announcements, for the obvious reason that it encourages students to be prepared for a test on each day at least prior to the actual administration of the test). If the teacher in his announcement makes no claim as to the inferability of the day of the test in advance, no problems arise. The teacher can clearly fulfill the promise, and the students clearly have no argument to the contrary.

Consider now the announcement: "Next week I shall give you a test whose week of occurrence I shall not presently announce." This announcement is clearly refuted by its own rendering, but still no problems arise, since clearly the teacher *cannot* give the test as promised (though he can give a surprise test). Both these formulations are more or less compatible with some sense of "surprise test", and both are not just paradox-free, but even obviously so. However, they also fail to respect the spirit of the paradox.

Suppose, therefore, that we interpret 'surprise test' as a test given on such a day that the stucents cannot know of it in advance. This is where the students' argument becomes relevant and trouble begins. Even here, if the teacher will hedge his promise by adding "unless I give the test on Friday", he can avoid the students' argument. So to preserve the puzzle, the hedge must be omitted.

We are not committed to seeking out deliberately a genuinely paradoxical interpretation of the teacher's announcement (as did, e.g., Kaplan & Montague, and as Quine has been accused of shirking), but we agree with Bosch and with Wright & Sudbury that it is only fair to require an interpretation which is challenging. In other words, one to the validity of which the students may, as in the original story they do, object.

The formulation: "Next week I shall give a surprise test, i.e., a test given on such a day that the students won't know it in advance", is indeed the one held – with elaboration, modification, rigorization, etc. – by practically all of the writers, including, we shall argue, those who avoided reference to epistemological terms.

Initially, there would seem to be nothing the matter with this promise. It isn't self-contradictory in the purely propositional-logic sense, a fact which is driven forcefully home by the realization that if the announcement is never made, but only resolved internally by the teacher, then he can calmly go about fulfilling ot. Obviously, or so it seems, it is the fact that the students hear the announcement, or rather *understand* it (for to hear it in Sanskrit would not be very helpful), or anyway have it at their disposal, which gives the situation its bite. The very earliest writers, and some later anachronistic ones, addressed themselves to just this angle of our puzzle, attempting to analyze the means whereby the bringing of a decree to the attention of the students creates conflict with the prediction contained in the decree. "Self-refuting" is more apt a description of the decree thus understood than "self-contradictory", and in this approach it came to be labelled a "pragmatic paradox".

Quine for one, however, would deny that the decree is even that. Even if the teacher means by 'surprise test' a test whose day of occurrence is not known in advance, and even if he brings his intention of giving such a test to his class's attention, there is little his students can make of this. Since they can't even "know" that the teacher will keep his promise, they certainly can't know, even if untested by Thursday night, that the test will be given on Friday. The announcement in itself gives them no grounds for knowing anything about when or if a test will be given.

This bind can be resolved, of course, if we assume that the students *do* know that the promise will be kept. But to engineer circumstances under which this assumption is even half-way plausible is not merely difficult, but may be begging the question: it is clearly problematic to assume 'knowledge' that the promise will be kept for the express purpose of proving that it can't. (Of course, the students may assume the truth of the promise as a step in constructing a reductio ad absurdum argument, but Quine took pains to show the difference between such an assumption, used internally in some argument, and imposing the assumption on the reader externally by manipulating the story). The difficulty arises from the principle, accepted in epistemic logic, that from "K knows that p" it follows that p is true. Thus the statement "K knows that the promise will be kept" contradicts the final conclusion "the promise can't be kept," casting doubt on K's 'knowledge' in the first place.

It is, all the same, quite easy to grant the students some more limited knowledge. In particular, there is nothing problematic in allowing the students to 'know' that a test will take place, though not that a *surprise* test will take place. The administration of a test can be independently guaranteed. This was in fact achieved by some of the versions presented in the literature (such as, e.g., Ayer's Pack of Cards story, where an ace of spades is placed among other cards, and the announcement claims that its ordinal place in the pack cannot be known in advance). In a classroom version, we could have the teacher commit himself, before the announcement is made, to some date next week by notifying the principal of the school. Indeed, regardless of whether he ultimately actually gives the test or not, we could have the students deal with the date *planned* for the surprise test. This story has the additional virtue of avoiding the question of 'knowing' with respect to future events, for it is a past event, though presently unknown to them, that the students are reasoning about.

On the other hand, the reference in the decree to what the students will or will not know may not be such that its truth can also be independently guaranteed. Indeed, its very structure should light a red signal, for in it "the formal unpredictability of a proposition is referred to a system which includes the formal unpredictability of the same proposition" (Bosch, p. 524) – i.e., smacks strongly of self-reference.

The distinction between the status of the two constituents which make up the teacher's decree – the test, and its unpredictability – is one which Quine chose to overlook, thus making his solution unnecessarily restricted, and incurring more criticism than he deserved. Quine's brand of skepticism is more sweeping than the case calls for, but the spirit of his solution can be readily generalized. For suppose the students know that the test will be given (a supposition which is clearly attainable), but don't know, at least not in advance, that it will be a surprise test (a supposition which is inevitable). Even so they don't have an argument to rule out the possibility of the teacher keeping his promise, *provided only that the teacher has at least two days on which he can schedule a test*. The role played with respect to Friday by what we might call Noday (i.e., not giving a test altogether) can, if Noday is ruled out by prior assumption, be played with respect to Thursday by Friday. All that is needed, in other words, to apply a Quinean solution, is that on at least one of the days in question the teacher will have at least two recourses of action open to him. Specifically, let us examine the students' argument at the close of the week under circumstances where they are sure that a test will be given. They are right then, if untested by Thursday night, to argue that they *know* it will take place on the morrow. They are, moreover, right in inferring from this that the latest the teacher can give his test *and* comply with his own decree is, therefore, Thursday. But this is downright insufficient for them to *know* that therefore he can't give it on Thursday. From the knowledge that the test has not yet been given by Thursday night the students know only that *if the teacher is to keep his word he must give the test on Thursday*, but not *that the teacher will keep his word, hence that he will give the test on Thursday!*

How would matters stand if we assume only that the students *believe*, rather than *know*, that the promise will be kept? This will solve two problems. First, beliefs may be mistaken, even if held by rational people, and so from "K believes that p", p does not follow. This might help if not to avoid trouble altogether, then at least to defer it. Second, it is quite easy to engineer circumstances in which is it plausible to attribute to the students *belief* in the promise, if not knowledge. Suffice it, for example, to say that the teacher is known from previous experience to be truthful and reliable. But even though such experience provides some positive grounds for believing the teacher, there might be interfering circumstances. One would be justified in doubting even a remarkably trustworthy person if he were to make some outrageous claim (say, that he had just been offered the throne of France). What about a claim such as: "Next week I shall give you a surprise test." Is *it* outrageous? As we saw, the answer may not be independent of what we take 'surprise' to mean. Nevertheless, the promise seems, on the face of it, innocent enough. Compare this with the promise: "*Tomorrow* I shall give you a surprise test." While the credibility of this promise also hinges on the exact meaning of 'surprise', it is bizarre enough, on the face of it, to seem *in*credible. Many writers have been quite facile in making the leap from many to one days, and claiming that nothing is lost in the reduction. At least intuitively, however (and formally, too, as we just saw), the many days and the one-day cases are not always equivalent. Anyhow, we are back to our task of examining whether the innocuous looking statement "Next week I shall give you a surprise test," spoken by a generally truthful and reliable teacher, is one which can be believed.

Note that the students needn't answer this on the basis of appearances alone. They can carry out a "Gedankenexperiment," imagining the teacher entering the classroom on, say, Tuesday, and dictating a test. Would they be surprised? And if by 'surprise test' we mean one whose day of occurrence cannot be known in advance, would they not even be surprised in the appropriate sense? Thus we see that the students' judgment of the believability of the teacher's announcement must depend on the nature of the announcement and cannot be decided on the basis of extraneous considerations solely.

Belief can be introduced via the teacher's announcement as well. The announcement may promise a test on a day such that the students will not believe, rather than know, that it will be the day in advance. This was the interpretation favoured by Binkley (1968) and Kvart (1976). Both of them diagnosed the paradox as being closely related to Moore's Paradox. That is, the teacher's announcement is tantamount to saying: "I will give you a test tomorrow, and you do not now believe that I will give you a test tomorrow" (in fact, in the one-day case it reduces to just that). This might very well be true – since both conjuncts may well be true – but is nevertheless from the student's point of view incredible. Since it is an incredible announcement, an ideal rational person could not utilize it to inform his beliefs. According to Binkley and Kvart, therefore, the student's argument is flawed, while the teacher's announcement, though incredible to the students, is still easily fulfillable.

The use of epistemological notions can be sidestepped altogether by reformulating the teacher's announcement yet again, in purely logical terms. Under this formulation, the teacher's decree is presented as a set of *rules* or *premises*, and the question transforms to that of what can and what cannot be deduced from these premises, and whether or not they are self-contradictory.

The logical formulation has often been presented as contrasting with the epistemological one. In fact, however, the two frameworks offer quite similar solutions. It has always been clear that the only legitimate, or at least relevant, means by which the students could have come to know, or to expect, the test to be given on any day were those that were deduced from the decree and from the knowledge that the test had not yet been given, with some adding the certainty that some test will be given. Other means of coming by the knowledge of the test – such as bribing the school secretary, peeking in the teacher's diary, etc. – were excluded. Similarly, the ignorance of a student who happened not to have heard the announcement is also dismissable.

At the same time, the typical epistemological treatment attributed to the students the property of deductive closure – that is, they were credited with knowing logic and with knowing anything which was deducible from what they knew. Thus, it seems to us that clarifying what is soundly deducible on any given day, determines what is known on that day, or even what is reasonably believed. It is, of course, important to remember that something can logically follow from the announcement and yet not be known to be true – since the announcement itself may not be known to be true.

Within the logical framework, Shaw and others demonstrated convincingly several things: (i) that there are more than one set of rules which can be substituted for the teacher's decree; (ii) that the differences between them are often subtle and easy to overlook, even with careful analysis, though they are crucial to the issue at hand; (iii) that some, though not all, of the rules are genuinely self-contradictory; (iv) that some, though not all, allow the possibility of a surprise test actually being given.

It has been customary within this approach to set apart the premise: "There will be a test next week" from the premise that says something about the unexpectedness or the deducibility of the test's date of occurrence. This is not merely a matter of convenience. It goes to the heart of the matter by effectively separating what is harmless in the announcement from what is potentially a source of trouble. It separates the knowable from the unknowable, the believable from the unbelievable, the ascertainable from the unascertainable.

Had it been apparent to all that the formulations which allow for a surprise test to actually be given in compliance with the rules are a subset of those which are not self-contradictory, the paradox from the purely logical viewpoint would have ended then and there. Amazingly, however, some of the writers who took the nonepistemological, symbolic-logical, approach to the puzzle seem to have countenanced a state of affairs where the teacher's announcement is self-contradictory, yet is simultaneously capable of being instantiated! Surely this is no less mind boggling than a four-sided triangle! This is what Scriven called the puzzle's "flavor of logic refuted by the world. The logician goes pathetically through the motions that have always worked the spell before, but somehow the monster, Reality, has missed the point and advances still" (p. 403). If the Surprise Test Paradox is not an old one in disguise, that must be the heart of its new angle.

The resolution of this new paradox is wonderfully simple. The test, even in the presence of a valid argument showing its impossibility of complying with the rules "may nevertheless occur on some one of the specified set of days, and when it does occur it does constitute *a sort* of surprise [italics ours]" (Fitch, p. 161). But not any sort of surprise test complies with the rules. In every case where a writer considered the possibility that an unfulfillable promise was fulfilled anyway (see Scriven, Weiss, Nerlich, Bennet, Cargile, O'Beirne, Gardner, Fitch, Sharpe, Chapman & Butler, Schoenberg, Wright, Austin, Ayer, Binkley, Kvart, Champlin) we have, by careful scrutiny, found either that the students' proof is erroneous (typically along Quinean lines), or that the teacher, even if he gives a surprising test, actually fails to comply with the precise terms laid out by his own decree. We shall spare the reader the need to follow us in the exercise of doing this paper by paper, but assure him or her that we have done it ourselves, to our satisfaction. Here we shall deal with only one example, modeled after Gardner (1963), who "reduce(d) the paradox to its essence."

Suppose the teacher announces: "Tomorrow there will be a test, and it is not possible to deduce from this announcement that tomorrow there will be a test." The first conjunct of this announcement appears to contradict the second conjunct. For surely from "to morrow there will be a test," it is possible to deduce "tomorrow there will be a test" contrary to "it is not possible to deduce from this announcement that tomorrow there will be a test," as the second conjunct says. On the other hand, says Gardner, since the teacher has two ways of violating his announcement – either by giving a test whose occurrence is deducible from the announcement or by not giving a test at all – the students have "no rational basis for choosing between these alternatives" therefore *cannot* deduce that tomorrow there will be a test, and so the teacher can surprise them by actually giving a test. Thus, says Gardner of this announcement: "A statement that yesterday appeared to be nonsense, that plunged [the students] into an endless whirlpool of logical contradictions, has today [upon the giving of a test] suddenly been made perfectly true and non-contradictory" (p. 154).

Not so, we claim. Statements are either true or false. Promises, on the other hand, are either kept or broken. If we regard the announcement as a promise (the epistemological point of view) then from the promise: "Tomorrow there will be a test" it does *not* follow that tomorrow there will be a test, as anyone who has ever been given a promise only to have it later broken will attest.[3] Thus the promise made by the teacher is not self-contradictory, and can be fulfilled. This, of course, is Quine's point. Suppose, however, that we regard the announcement as a statement (the non-epistemological point of view). As such it is self-contradictory. Since anything is deducible from a contradiction, in particular the two statements: "Tomorrow there will be a test" and "Tomorrow there will not be a test" are, but neither of them can be known, since the deduction, though valid, is not sound. Furthermore, if a test is now given, one can by no means agree with Gardner's claim that the statement has thus "been made . . . true", even if the test is unexpected. The announcement-qua-statement was, is, and forever will be, self-contradictory. (After the fact, its first conjunct is true, but its second is still false). This, of course, is Shaw's point.

The beauty of this example is in showing simultaneously how either the students' argument is erroneous (if they argue about the implications of the announcement as a promise), or the teacher cannot render his announcement true (if the students argue about the implications of the announcement as a premise).

Though this example places the source of the paradoxical effect on the failure to distinguish between the announcement-as-promise and the announcement-as-premise, there are other potential sources of confusion which may contribute to the paradoxical effect. Bennett and Bosch show how subtle, though significant, the differences are between the possible premises perceived to underly the teacher's announcement, and how easy it is to fail to distinguish between them adequately. Ayer shows how two senses of 'to know in advance' can be confused. Shaw, Edman and Kieffer & Ellison show how several meanings of 'surprise test' can be confused, etc.

Summary

We conclude with a brief summary. Our paradox has the following form: I. Teacher makes a simple promise. II. Students argue that teacher can't possibly keep his promise. III. Teacher keeps it anyway. Some writers have focussed on the first two parts only. Why is a perfectly innocuous promise rendered impossible to keep, just by the procedure of actually making it? Most writers, however, have focussed on the last two parts, addressing themselves to the question of how a proof can be refuted by an actuality. We have attempted to integrate all the existing views into a comprehensive framework. And we believe that not much that is real paradox remains in this framework. When the teacher's announcement is construed as a promise we are faced at most with what may be called a pragmatic paradox, namely, a promise whose fulfillment is hindered by its rendering, though the uttered statement is not a logical falsehood (i.e., contradictory). When, on the other hand, the teacher's announcement is interpreted as a set of premises, we are faced at most with a Liar type paradox, i.e., with a premise that manages, by referring to itself, to be demonstrably true if and only if false.

The informal presentation of the paradox, which is intuitively more powerful than the various formalizations, is open to almost as many interpretations as a Rorschach Card. "Surprise Test" has been understood in a multitude of manners. It has reminded authors of Moore's Paradox, Gödel's Theorem, The Visiting Card Paradox, The Liar Paradox, etc. It has been used as a jumping board for the discussion of many philosophical notions, such as free will, knowledge, etc. Happily, however, it still allows us to go about doing logic, since what it never really demonstrated was how the world refutes logic.

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NOTES

1 The Visiting Card Paradox consists of a card, on one face of which appears the sentence, "The sentence on the other side of this card is true," and on the other face of which appears the sentence, "The sentence on the other side of this card is false."

2 Moore's Paradox consists of the announcement by someone "p, but I don't believe that p," where p is any statement.

3 This brings to mind the status of hearsay evidence in a court of law, as opposed to direct evidence. If X testifies: "Y said S", this cannot, normally, be used to establish that S, but only that Y said that S. If, however, Y testifies: "S", this can be used to establish that S.

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