DISCUSSIONS

Two Forms of the Prediction Paradox

In an article by one of the authors in *Mind* [1] it was argued that the prediction paradox, in the form there given, could not be resolved within the framework of classical two-valued logic because, it was claimed, one was able from true premisses to draw a false conclusion. It is shown below that this is not so and only appeared to be so because of an insufficiently sophisticated use of logic.

This form of the paradox can be stated as follows: On Sunday a captain has decided to give his men a route-march on the following Wednesday, but merely tells them that on one of the six following days he will give them a *surprise* route-march. He defines this as a march for which the day of occurrence cannot validly be deduced beforehand. The argument now goes that the march could not occur on the last day, Saturday, because on Saturday morning it would be deducible that it would occur on that, the then only possible, day and therefore would not be a surprise. Nor could it occur on Friday, since on Friday morning it would be known that Saturday was ruled out, that therefore it would occur on Friday and so would not be a surprise. Similarly it can be shown, in succession, that it could not occur on Thursday, Wednesday, Tuesday and Monday. Nevertheless, it *does* occur on Wednesday and, apparently, this was not deducible. So, apparently, the captain's statement was true but a false conclusion could be drawn from it—namely, that the march would not take place at all.

But in fact the captain's statement was *not* true. It was a conjunction of two parts: the first asserted that there would be a march—which was indeed true; the second asserted that the day on which it occurred was not deducible beforehand. This is not true, for the argument given above has simply not been pushed far enough. It was there shown that the march could not take place on Monday, Tuesday, Thursday, Friday or Saturday. On the other hand the premisses stated that the march would occur on one day of the week. It follows immediately that the march would occur on Wednesday, so that the day of occurrence *was* deducible.

This reasoning has an appearance of legerdemain about it, but this is usually so in applications of the valid logical principle that from a contradiction one can deduce any proposition whatsoever. A well-known example is Bertrand Russell's deduction from an arithmetical falsehood that he is the Pope.

The need for the use of this principle disappears if we consider the one-day form of the paradox, as Quine [2] does. In this case the captain states there will be a march

[1] Meltzer, B., 'The Third Possibility', *Mind*, **73**, 1964, pp. 430–433

[2] Quine, W V., 'On a So-called Paradox', *Mind*, 1953, **62**, pp. 65–67. Quine is concerned with a problem different from ours, since it is for him a matter of what the soldiers *know*—a matter that goes outside pure logic. As he shows, the form of the problem that he considers is not paradoxical.

today and it is not deducible that this is so. This is immediately seen to be a false statement because from a proposition A one can certainly deduce A.

It is interesting to consider another form of the paradox, in which one changes the definition of a surprise march. The ordinary meaning of 'surprise' involves the notion of probability, and if this ordinary meaning is used, the resolution of the paradox becomes trivial. The captain used a non-probabilistic interpretation of surprise, as described above, but there is another non-probabilistic interpretation that he could have used, perhaps somewhat more artificially. We shall now say that a march will be termed a surprise if it occurs on a day on which it is deducible beforehand that it will not occur! We may now again argue that the march could not occur on Saturday; because if it had not occurred by Friday night it would *not* be deducible that it would not occur the next day, so that the Saturday march would not be a surprise. And, as before, one can argue back day by day and show that all six days are ruled out. So in this case the captain's statement is apparently true, since the march *does* take place on Wednesday and it has been deduced beforehand that it will not take place on Wednesday (among other days). Nevertheless from this apparently true statement an apparently false conclusion has been drawn, namely, that the march does not take place on Wednesday.

The authors do not agree on the resolution of this paradox. One would suggest that a three-valued logic is required, on the lines that he discussed in *Mind*.[1] But the other takes a view regarding this logical antinomy, and most others, which can be best presented by considering the one-day form of the problem:

The captain makes a statement S which is of the form: 'A, and not-A is deducible from the whole of this statement.' Here A is meaningful and true by hypothesis, and, if S were meaningful, then an argument similar to that of the last paragraph but one shows that it would be the conjunction of two true statements and would therefore be true, and could not lead to contradictions. Since, however, it does lead to contradictions, it follows that it is meaningless. But the conjunction of two meaningful statements is meaningful (this may be taken as an axiom of semantics), and A is meaningful; therefore both S and the second clause of S are meaningless.

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THE LOGIC AND PSYCHOLOGY OF SCIENCE:

Reply to Professor Kapp

IN his review of my book, *The Origins of Science* (this *Journal*, 1965, 15, 333), R. O. Kapp ascribes to me a view which is exactly the opposite of what I have said. On page 135, I have written of Euclid that, 'He was the first to overcome the horror of the infinite and to show that it could be brought under some sort of control: we speak today of the "countable" or "denumerable" kind of infinity.' Since the Greeks had been occupied with the problem of infinity ever since they began to speculate about nature, it is absurd to suggest that 'Euclid was the first to be overcome by the horror of the infinite', as is done in Kapp's review (p. 340).

[1] Meltzer, op. cit.