

# 3-D Vibration Measurement Using a Single Laser Scanning Vibrometer by Moving to Three Different Locations

Dongkyu Kim, Hajun Song, Hossam Khalil, Jongsuh Lee, Semyung Wang, and Kyihwan Park

**Abstract**—3-D vibration measurement is achieved using a single laser scanning vibrometer (LSV) and laser scanner (LS) by moving them to three arbitrarily different locations from the principle that vibration analysis based on the frequency domain is independent of the vibration signal based on time domain. The proposed system has the same effect as using three sets of LSVs, and has an advantage of reducing equipment costs. Analytical approach of obtaining in-plane and out-of-plane vibration of surface is introduced using geometrical relations between three LSV coordinates and vibrations measured at three different locations. The proposed algorithm is verified by comparing the experimental results obtained by a three-axis accelerometer and a developed optical system with an LSV and an LS combined together.

**Index Terms**—3-D vibration measurement, laser Doppler vibrometer (LDV), laser scanner (LS), shape measurement.

## I. INTRODUCTION

**A**CCCELEROMETER is typically used to measure vibration in conventional tests of bulky machinery structures. However, these contact-type sensors have several disadvantages: 1) their loading effect, which can affect the frequency response for a light and flexible structure; 2) the tethering problem, which makes it hard to measure the vibration of a location apart from the sensors; and 3) sensitivity to electromagnetic interference effects, which makes it hard to measure vibration of power electronic components. In addition, the use of contact-type sensors necessitates the use of multiple sensors when the vibration mode shape of a structure must be investigated [1].

These problems can be mitigated by using a noncontact-type laser Doppler vibrometer (LDV) [2]. An LDV provides the velocity signal of an object using the Doppler frequency

change that occurs due to interference between incident light and scattered light that is reflected from the surface of a moving object [3], [4]. Using Doppler effects, LDVs are considered a technology that could rapidly and correctly measure vibration at a desired location. The vibration at multiple points can be easily measured by using a laser scanning vibrometer (LSV) in which laser beam is rotated by using an electric motor.

However, LSVs can only measure speeds parallel to the laser beam direction, i.e., they cannot detect vibration when the surface has vibration that is perpendicular to the laser beam direction. Therefore, three sets of LSVs are required in order to measure components of 3-D vibration. The need for 3-D measurements has increased since various industries have become interested in more accurate and high-speed measurements. For example, Bendel *et al.* [5] measured the vibration of power tools using a 3-D scanning laser Doppler vibrometer (SLDV) to investigate vibration characteristics. In 2006, Miyashita and Fujino [6] proposed a method for measuring 3-D vibration by calibrating the angles and locations of three sets of LDVs. Malley *et al.* [7] conducted research to search for landmines buried underground by acquiring 3-D vibration information obtained.

The use of three sets of LSVs, however, has a notable disadvantage of high cost. For 3-D vibration measurement on an arbitrarily surface, it is possible to measure vibration of the surface at three different locations using a single LSV pointing to the same measurement points. The proposed method has the same effect as using three sets of LSVs since a vibration analysis based on the frequency domain is independent of the vibration signal based on time domain measured. It is usual to assume that vibration condition is steady regardless of the time change in experimental modal analysis when a random signal or a sine sweep is applied as an excitation signal having a sufficient time [8]. Furthermore, the three different locations are easily obtained using a shape measurement device, such as a laser scanner (LS) [9], [10] from the relative geometrical information between the shapes measured at different locations. Hence, the proposed method has an advantage of relieving the use of a mechanical frame or a robot arm to determine the relative geometrical relations employed in the conventional 3-D vibration measurement as well as cost reduction.

This paper describes how to acquire 3-D vibration using a single LSV, which measures vibration at three different

Manuscript received October 23, 2013; revised November 27, 2013; accepted November 28, 2013. Date of publication March 14, 2014; date of current version July 8, 2014. This work was supported by the National Research Foundation of Korea through the Korean Government under Grant 2011-0017876. The Associate Editor coordinating the review process was Dr. Wei Gao.

D. Kim, H. Khalil, J. Lee, S. Wang, and K. Park are with the School of Mechatronics, Gwangju Institute of Science and Technology, Gwangju 500-712, Korea (e-mail: akein@gist.ac.kr; leejongsuh@gist.ac.kr; smwang@gist.ac.kr; khpark@gist.ac.kr).

H. Song is with the School of Information and Communications, Gwangju Institute of Science and Technology, Gwangju 500-712, Korea (e-mail: aroker@gist.ac.kr).

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Digital Object Identifier 10.1109/TIM.2014.2302244

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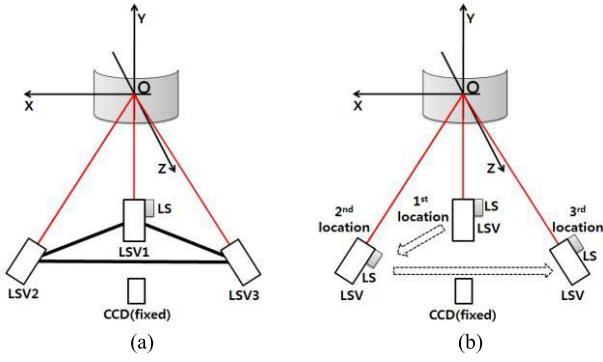


Fig. 1. Comparison of (a) conventional 3-D LSV and (b) proposed optical system.

locations without using a mechanical frame or a robot arm. In Section II, the basic concept of proposed 3-D vibration measurement using a single LSV is introduced. In Section III, a method for obtaining the transformation matrix between each LSV coordinate is explained. In Section IV, the angles between laser beams and in-plane, out-of-plane vectors are calculated. In Section V, the experiments are performed to verify the proposed algorithm.

## II. PRINCIPLE OF PROPOSED 3-D VIBRATION MEASUREMENT

The conventional method for 3-D vibration measurements is composed of three sets of LSVs and one shape measurement device, such as LS, as shown in Fig. 1(a). From the vibration and geometric shapes measured, respectively, from the LSVs and LS, in-plane and out-of-plane vibration components on the measured surface are acquired. Fourier transforms of the time signal into frequency, out-of-plane, and in-plane vibration characteristics of an object are consequently performed. However, this method costs too much to be used for industrial applications. In addition, there are measurement constraints if the measurement locations are constrained by a mechanical frame. In order to mitigate the problem, this paper proposes an algorithm in which only a single LSV with no mechanical frame is used for 3-D vibration measurements.

The proposed optical system consists of a single LSV and LS [Fig. 1(b)]. The LSV moves to three arbitrary locations to measure the vibration, while the LS is used to obtain distance and coordinate at each measurement point of an object. When the system measures vibration at three different measurement locations, a frequency response function (FRF) is obtained from a random signal or a sine sweep applied to the object for a sufficient time. Therefore, it can be assumed that an excitation condition is almost the same at three different measurement locations regardless of the time change. Then, the proposed method has the same effect as using three sets of LSVs. As such, the proposed method can reduce equipment costs.

Fig. 2 shows the geometric relations of the coordinate systems  $(x_1y_1z_1)$ ,  $(x_2y_2z_2)$ , and  $(x_3y_3z_3)$  obtained by moving a single LSV to three different locations. The origins of each coordinate system are attached to the scanning mirrors of

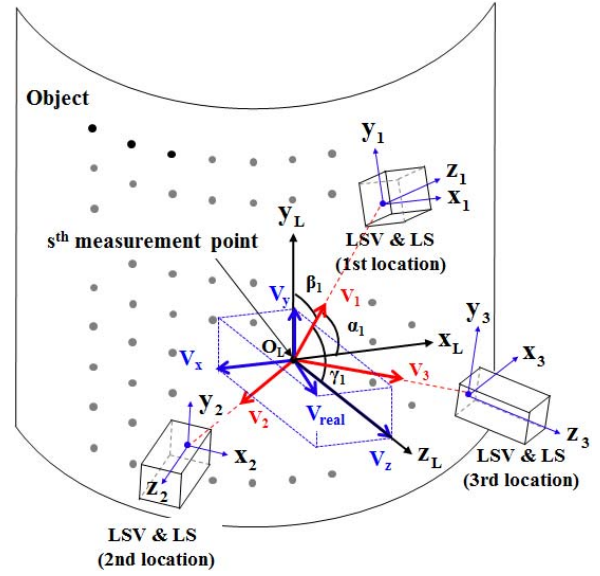


Fig. 2. Relationship between actual speed at arbitrary measurement point on object and speed measured from three different measurement locations.

the LSV. A local coordinate system  $(x_Ly_Lz_L)$  is defined for each measurement point at the surface. The direction of the  $z_L$ -axis of the local coordinate system  $(x_Ly_Lz_L)$  is defined as being perpendicular to the surface of the measurement point, whereas the direction of the  $x_L$ -axis is defined as being parallel to the  $x_1z_1$  plane. Then, the direction of the  $y_L$ -axis is defined as the outer production of the  $z_L$  and  $x_L$  axes. The vibration measured at the three measurement locations is defined as  $V_1$ ,  $V_2$ , and  $V_3$ , respectively, and the angles between each axis of local coordinate system  $(x_Ly_Lz_L)$  and  $V_1$ ,  $V_2$ , and  $V_3$  are defined as  $\alpha_k$ ,  $\beta_k$ , and  $\gamma_k$ ; where  $k = 1, 2, 3$ . For simplicity, in Fig. 2, the angle between each axis of the first coordinate system  $(x_1y_1z_1)$  and  $V_1$  is expressed as  $\alpha_1$ ,  $\beta_1$ , and  $\gamma_1$ . Next, the vibration of the surface  $V_x$ ,  $V_y$ , and  $V_z$  are obtained using, the angles  $(\alpha_k, \beta_k, \gamma_k)$  and vibrations  $(V_1, V_2, V_3)$  as indicated below [11]

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} \cos \alpha_1 & \cos \beta_1 & \cos \gamma_1 \\ \cos \alpha_2 & \cos \beta_2 & \cos \gamma_2 \\ \cos \alpha_3 & \cos \beta_3 & \cos \gamma_3 \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}. \quad (1)$$

## III. TRANSFORMATION MATRIX BETWEEN THE COORDINATE SYSTEMS

We propose a method for obtaining the transformation matrix between the coordinate systems defined in Section II by comparing the curved surface information measured using an LS at each measurement location. Fig. 3 shows the shapes of the object measured at three measurement locations by using an LS. The arrows are normal vectors that are perpendicular to the surface.

The normal vectors of measurement points are obtained using the method of mean weight by areas of adjacent triangles (MWAATs) [12]. For the shape measured at the coordinate system  $(x_1y_1z_1)$ , as shown in Fig. 3(a), suppose that  $(\mathbf{n}_s)_{x_1y_1z_1}$  is a normal vector of the measurement point  $P_s$  whose vertex

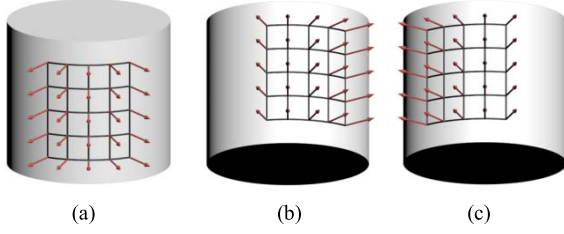


Fig. 3. Shapes and normal vectors measured at (a) 1st location, (b) 2nd location, and (c) 3rd location.

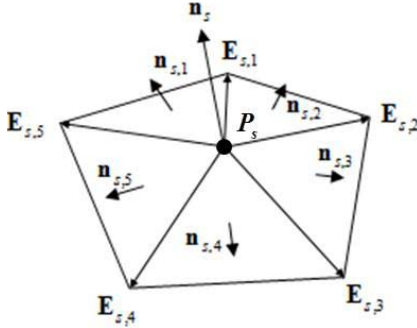


Fig. 4. Normal vector for measurement point and its surrounding vectors.

is shared with the surrounding surfaces, as shown in Fig. 4.  $\{\mathbf{n}_{s,1}, \mathbf{n}_{s,2}, \mathbf{n}_{s,3}, \mathbf{n}_{s,4}, \mathbf{n}_{s,5}\}$  is a set of vertex normal vectors of the surrounding surfaces,  $\{\mathbf{E}_{s,1}, \mathbf{E}_{s,2}, \mathbf{E}_{s,3}, \mathbf{E}_{s,4}, \mathbf{E}_{s,5}\}$  is the edge vectors of these surfaces, and the normal vector  $(\mathbf{n}_s)_{x_1y_1z_1}$  is obtained using the MWAAT algorithm as

$$\mathbf{n}_s = \frac{\sum_{i=1}^n (\mathbf{n}_{s,i} |\mathbf{E}_{s,i} \times \mathbf{E}_{s,i+1}|)}{\left| \sum_{i=1}^n (\mathbf{n}_{s,i} |\mathbf{E}_{s,i} \times \mathbf{E}_{s,i+1}|) \right|}. \quad (2)$$

For simplicity the expression of coordinate system  $(x_1y_1z_1)$  is omitted.

For the surfaces measured at the coordinate systems  $(x_2y_2z_2)$  and  $(x_3y_3z_3)$ , as shown in Fig. 3(b) and (c), normal vectors  $(\mathbf{n}_s)_{x_2y_2z_2}$ ,  $(\mathbf{n}_s)_{x_3y_3z_3}$  can be also obtained using a similar method. The normal vector set at every measurement point is then expressed as

$$(\mathbf{N})_{x_ky_kz_k} = [\mathbf{n}_1, \mathbf{n}_2, \dots, \mathbf{n}_s, \dots, \mathbf{n}_n]_{x_ky_kz_k} \quad (3)$$

where  $k = 1, 2, 3$ .

The normal vector sets obtained, respectively, at the coordinate systems  $(x_2y_2z_2)$  and  $(x_3y_3z_3)$  can be coincided with the normal vector set obtained at the coordinate systems  $(x_1y_1z_1)$  if they are rotated at angles that are represented in the transformation matrix.

Suppose that the coordinate system  $(x_2y_2z_2)$  is obtained by rotating the coordinate system  $(x_1y_1z_1)$  by  $\theta_{12}$  based on the  $y_1$ -axis and  $\phi_{12}$  based on the  $x'_1$ -axis rotated from the  $x_1$ -axis by  $\theta_{12}$ , as shown in Fig. 5. The coordinate systems  $(x_3y_3z_3)$  are similarly obtained by rotating the coordinate system  $(x_1y_1z_1)$  by  $\theta_{13}$  based on the  $y_1$ -axis and  $\phi_{13}$  based on the  $x'_1$ -axis.

The normal vectors  $(\mathbf{n}_s)_{x_2y_2z_2}$  and  $(\mathbf{n}_s)_{x_3y_3z_3}$  represented in coordinate systems  $(x_2y_2z_2)$ ,  $(x_3y_3z_3)$  can be transformed

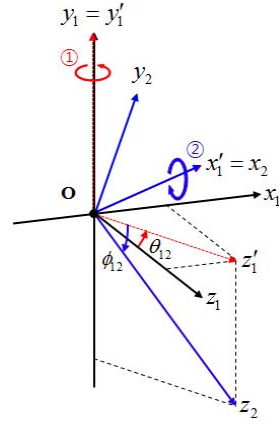


Fig. 5. Change of axis angle by the transformation matrix.

into the normal vector  $(\mathbf{n}_s)_{x_1y_1z_1}$  represented in the coordinate system  $(x_1y_1z_1)$  using transformation matrices  $\mathbf{R}_{12}$ ,  $\mathbf{R}_{13}$ , and such

$$(\mathbf{n}_s)_{x_1y_1z_1} = \mathbf{R}_{1k} \times (\mathbf{n}_s)_{x_ky_kz_k} + \mathbf{e}_{s,1k} \quad (4)$$

where  $k = 2, 3$ .

Similarly, suppose that the coordinate system  $(x_Ly_Lz_L)$  is obtained by rotating the coordinate system  $(x_1y_1z_1)$  by  $\theta_{1L}$  based on the  $y_1$ -axis and  $\phi_{1L}$  based on  $x'_1$ -axis. Then, the normal vector  $(\mathbf{n}_s)_{x_Ly_Lz_L}$  represented in coordinate systems  $(x_Ly_Lz_L)$  can be transformed into the normal vector  $(\mathbf{n}_s)_{x_1y_1z_1}$  represented in the coordinate system  $(x_1y_1z_1)$  using transformation matrices  $\mathbf{R}_{1L}$ .

From the rotation angle between the coordinate systems, the transformation matrices  $\mathbf{R}_{12}$ ,  $\mathbf{R}_{13}$ , and  $\mathbf{R}_{1L}$  are calculated as

$$\mathbf{R}_{1k} = \begin{bmatrix} \cos \theta_{1k} & \sin \theta_{1k} \cdot \sin \phi_{1k} & \sin \theta_{1k} \cdot \cos \phi_{1k} \\ 0 & \cos \phi_{1k} & -\sin \phi_{1k} \\ -\sin \theta_{1k} & \cos \theta_{1k} \cdot \sin \phi_{1k} & \cos \theta_{1k} \cdot \cos \phi_{1k} \end{bmatrix} \quad (5)$$

where  $k = 2, 3, L$ .

Note that  $\mathbf{e}_{s,12}$ ,  $\mathbf{e}_{s,13}$ , and  $\mathbf{e}_{s,1L}$  are errors associated with normal vectors experimentally obtained from the surface information using the LS such as

$$\mathbf{e}_{s,1k} = \begin{bmatrix} e_{s,x_{1k}} \\ e_{s,y_{1k}} \\ e_{s,z_{1k}} \end{bmatrix} \quad (6)$$

where  $k = 2, 3, L$ .

Then,  $\mathbf{R}_{12}$ ,  $\mathbf{R}_{13}$ , and  $\mathbf{R}_{1L}$  are obtained from angles that minimize the sum of errors,  $\varepsilon_{\text{LSM},1k}$  in each measurement point using the least squares method, such that

$$\varepsilon_{\text{LSM},1k} = \sum_{s=1}^n (|\mathbf{e}_{s,1k}|^2) = \sum_{s=1}^n \left( (\mathbf{n}_s)_{x_1y_1z_1} - \mathbf{R}_{1k} \times (\mathbf{n}_s)_{x_ky_kz_k} \right)^2 \quad (7)$$

where  $k = 2, 3, L$ .

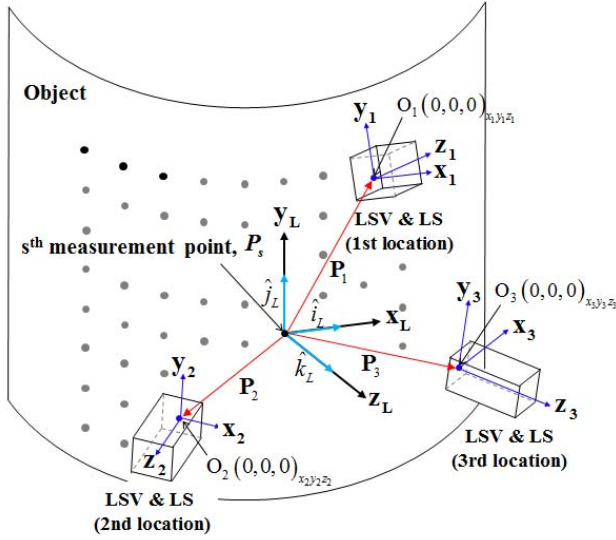


Fig. 6. Direction vectors of laser beam.

#### IV. IN-PLANE AND OUT-OF-PLANE VIBRATION

Suppose that the coordinate of a measurement point  $P_s$  in three coordinate systems  $(x_1y_1z_1)$ ,  $(x_2y_2z_2)$ , and  $(x_3y_3z_3)$  are obtained using a LS as  $P_1(p_{x1}, p_{y1}, p_{z1})_{x_1y_1z_1}$ ,  $P_2(p_{x2}, p_{y2}, p_{z2})_{x_2y_2z_2}$ , and  $P_3(p_{x3}, p_{y3}, p_{z3})_{x_3y_3z_3}$ , as shown in Fig. 6. Suppose that the vectors  $\mathbf{P}_1$ ,  $\mathbf{P}_2$ , and  $\mathbf{P}_3$  are laser direction vectors pointing toward the origin of each coordinate system from the point  $P_s$ , respectively. They have magnitudes of distances from  $P_s$  to the origin of each coordinate system. Then,  $\mathbf{P}_1$ ,  $\mathbf{P}_2$ , and  $\mathbf{P}_3$  can be represented in vector forms as

$$\begin{aligned} (\mathbf{P}_1)_{x_1y_1z_1} &= \begin{bmatrix} -p_{x1} \\ -p_{y1} \\ -p_{z1} \end{bmatrix}, & (\mathbf{P}_2)_{x_2y_2z_2} &= \begin{bmatrix} -p_{x2} \\ -p_{y2} \\ -p_{z2} \end{bmatrix} \\ (\mathbf{P}_3)_{x_3y_3z_3} &= \begin{bmatrix} -p_{x3} \\ -p_{y3} \\ -p_{z3} \end{bmatrix}. \end{aligned} \quad (8)$$

$(\mathbf{P}_2)_{x_2y_2z_2}$  and  $(\mathbf{P}_3)_{x_3y_3z_3}$  can be transformed into vectors represented in the coordinate system  $(x_1y_1z_1)$  using transformation matrixes  $\mathbf{R}_{12}$  and  $\mathbf{R}_{13}$  such that

$$(\mathbf{P}_2)_{x_1y_1z_1} = \mathbf{R}_{12} \times (\mathbf{P}_2)_{x_2y_2z_2} \quad (9)$$

$$(\mathbf{P}_3)_{x_1y_1z_1} = \mathbf{R}_{13} \times (\mathbf{P}_3)_{x_3y_3z_3}. \quad (10)$$

Knowing that  $\hat{i}_L$ ,  $\hat{j}_L$ ,  $\hat{k}_L$  are unit vectors on the axes  $x_L$ ,  $y_L$ ,  $z_L$  of local coordinate system  $(x_Ly_Lz_L)$ , the vectors  $(\hat{i}_L)_{x_Ly_Lz_L}$ ,  $(\hat{j}_L)_{x_Ly_Lz_L}$ ,  $(\hat{k}_L)_{x_Ly_Lz_L}$  represented in the coordinate system  $(x_Ly_Lz_L)$  are expressed as

$$\begin{aligned} (\hat{i}_L)_{x_Ly_Lz_L} &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, & (\hat{j}_L)_{x_Ly_Lz_L} &= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ (\hat{k}_L)_{x_Ly_Lz_L} &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \end{aligned} \quad (11)$$

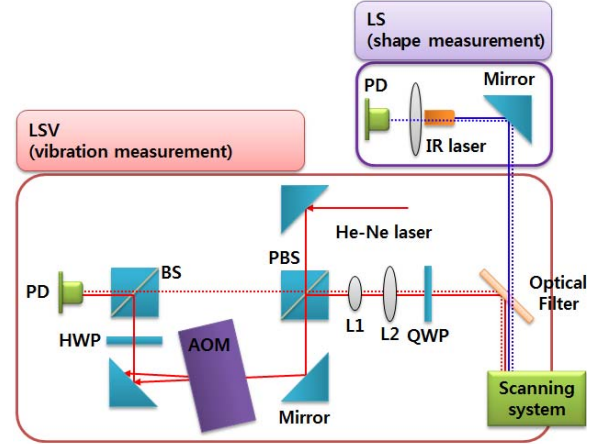


Fig. 7. Developed optical system with LSV and LS combined together.

$(\hat{i}_L)_{x_Ly_Lz_L}$ ,  $(\hat{j}_L)_{x_Ly_Lz_L}$ ,  $(\hat{k}_L)_{x_Ly_Lz_L}$  also can be transformed into vectors represented in the coordinate system  $(x_1y_1z_1)$  using transformation matrix  $\mathbf{R}_{1L}$  such that

$$(\hat{i}_L)_{x_1y_1z_1} = \mathbf{R}_{1L} \times (\hat{i}_L)_{x_Ly_Lz_L} \quad (12)$$

$$(\hat{j}_L)_{x_1y_1z_1} = \mathbf{R}_{1L} \times (\hat{j}_L)_{x_Ly_Lz_L} \quad (13)$$

$$(\hat{k}_L)_{x_1y_1z_1} = \mathbf{R}_{1L} \times (\hat{k}_L)_{x_Ly_Lz_L}. \quad (14)$$

Then, we have the following:

$$(\hat{i}_L)_{x_1y_1z_1} \cdot (\mathbf{P}_k)_{x_1y_1z_1} = |(\hat{i}_L)_{x_1y_1z_1}| |(\mathbf{P}_k)_{x_1y_1z_1}| \cos \alpha_k \quad (15)$$

$$(\hat{j}_L)_{x_1y_1z_1} \cdot (\mathbf{P}_k)_{x_1y_1z_1} = |(\hat{j}_L)_{x_1y_1z_1}| |(\mathbf{P}_k)_{x_1y_1z_1}| \cos \beta_k \quad (16)$$

$$(\hat{k}_L)_{x_1y_1z_1} \cdot (\mathbf{P}_k)_{x_1y_1z_1} = |(\hat{k}_L)_{x_1y_1z_1}| |(\mathbf{P}_k)_{x_1y_1z_1}| \cos \gamma_k. \quad (17)$$

where  $k = 1, 2, 3$ .

And finally

$$\cos \alpha_k = \frac{(\hat{i}_L)_{x_1y_1z_1} \cdot (\mathbf{P}_k)_{x_1y_1z_1}}{|(\hat{i}_L)_{x_1y_1z_1}| |(\mathbf{P}_k)_{x_1y_1z_1}|} \quad (18)$$

$$\cos \beta_k = \frac{(\hat{j}_L)_{x_1y_1z_1} \cdot (\mathbf{P}_k)_{x_1y_1z_1}}{|(\hat{j}_L)_{x_1y_1z_1}| |(\mathbf{P}_k)_{x_1y_1z_1}|} \quad (19)$$

$$\cos \gamma_k = \frac{(\hat{k}_L)_{x_1y_1z_1} \cdot (\mathbf{P}_k)_{x_1y_1z_1}}{|(\hat{k}_L)_{x_1y_1z_1}| |(\mathbf{P}_k)_{x_1y_1z_1}|} \quad (20)$$

where  $k = 1, 2, 3$ .

Then,  $V_x, V_y, V_z$  are obtained using (1).

#### V. EXPERIMENTS

For the experiment, an optical system with LSV and LS combined together is developed using an optical filter, which reflects He-Ne laser for the vibration measurement and passes IR laser for the shape measurement as shown in Fig. 7 so that the LSV and LS can measure the same point. Fig. 8 shows an experimental setup, which can simply verify the proposed algorithm by measuring 3-D vibration of a single point in upper left quadrant of an object using a three-axis accelerometer and the developed system. The object is a curved plate with dimension measuring 284.0 mm × 200.0 mm × 0.5 mm (curvature:  $2.8 \times 10^{-3} \text{ mm}^{-1}$ ) with mass density of 8055 kg/m<sup>3</sup>. A shaker is used to excite central



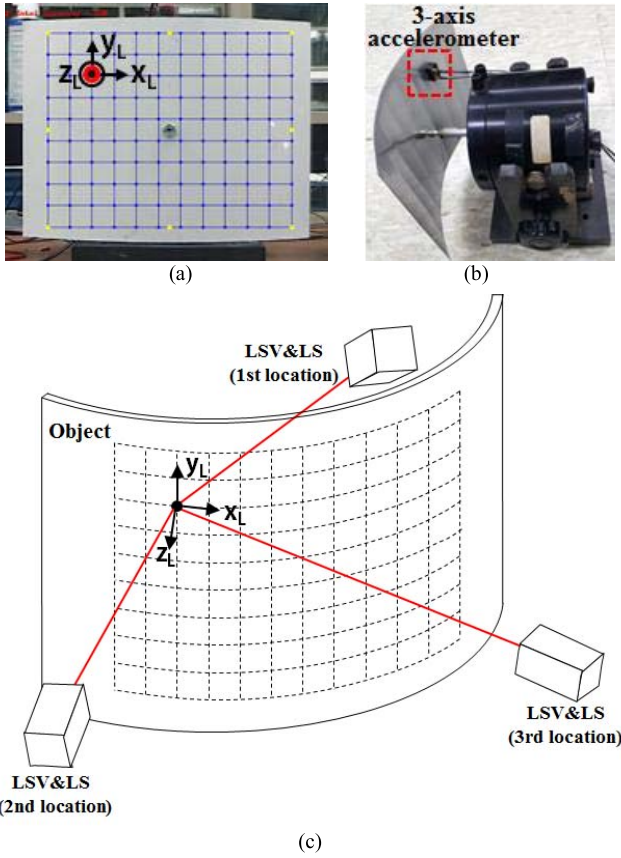


Fig. 8. Experimental setup for 3D vibration measurement using a curved object (a) meshes to be measured for shape and one point to be measured for vibration, (b) experimental setup of the 3-axis accelerometer, and (c) experimental setup of the developed optical system.

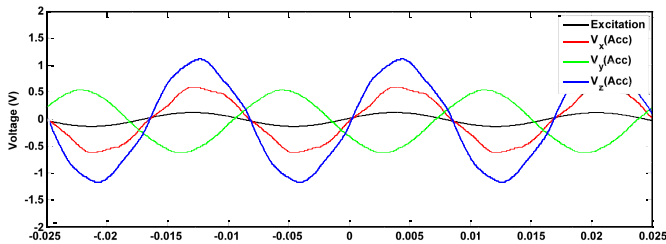


Fig. 9. 3-D vibration components results obtained by using the three-axis accelerometer.

point of the object in the  $z$ -direction at its fundamental natural frequency. The accelerometer is attached to the back of the plate at the same point as the single point we will measure, as shown in Fig. 8(b).

The 3-D vibration components,  $(V_x)_a$ ,  $(V_y)_a$ , and  $(V_z)_a$  of the object are directly obtained from a three-axis accelerometer amplifier, as shown in Fig. 9.

In order to obtain the 3-D vibration components  $(V_x)$ ,  $(V_y)$ , and  $(V_z)$  of the object from the LSV, the experiment begins with measuring the vibration,  $(V_1)$ ,  $(V_2)$ , and  $(V_3)$  of the object using the developed optical system at three different measurement locations. Then, the shapes measured at three different measurement locations are obtained using the developed optical system, as shown in Fig. 10. After that (18)–(20) are used to calculate the angles  $(\alpha_k, \beta_k, \gamma_k; k = 1, 2, 3)$  between the laser direction vectors  $(\mathbf{P}_1, \mathbf{P}_2, \text{ and } \mathbf{P}_3)$  at

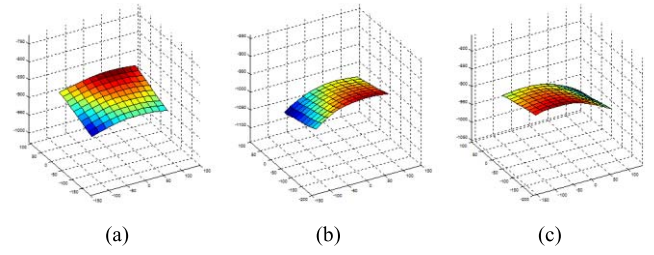


Fig. 10. Shapes of object measured at (a) 1st location, (b) 2nd location, and (c) 3rd location by using the developed optical system.

TABLE I  
TRUE AND ESTIMATED ANGLES BETWEEN THE  
LASER BEAM AND LOCAL AXIS

Angles	True angles	Estimated angles	Angle error
$\alpha_1$	$65^\circ$	$64.23^\circ$	0.77
$\beta_1$	$77^\circ$	$75.34^\circ$	1.66
$\gamma_1$	$29^\circ$	$29.72^\circ$	0.72
$\alpha_2$	$84^\circ$	$82.69^\circ$	1.31
$\beta_2$	$105^\circ$	$103.06^\circ$	1.94
$\gamma_2$	$14^\circ$	$13.01^\circ$	0.99
$\alpha_3$	$63^\circ$	$61.29^\circ$	1.71
$\beta_3$	$104^\circ$	$102.82^\circ$	1.18
$\gamma_3$	$34^\circ$	$33.00^\circ$	1.00

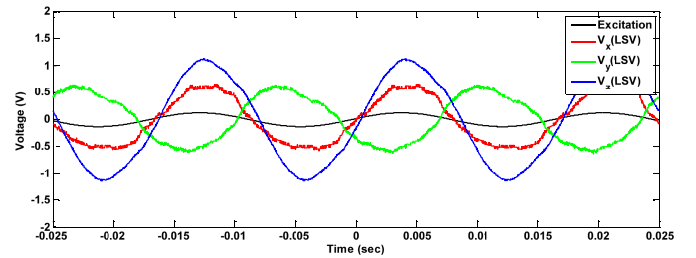


Fig. 11. 3-D vibration components results obtained by using the developed optical system.

three different measurement locations and unit vectors  $(\hat{i}_L, \hat{j}_L, \hat{k}_L)$  of the local coordinate system  $(x_L y_L z_L)$ . They are listed in Table I. Finally, the 3-D vibration components  $(V_x)$ ,  $(V_y)$ , and  $(V_z)$  are obtained from  $(V_1)$ ,  $(V_2)$ , and  $(V_3)$  by using (1).

Fig. 11 shows  $(V_x)$ ,  $(V_y)$ , and  $(V_z)$  obtained by using the developed optical system. They are noisier than  $(V_x)_a$ ,  $(V_y)_a$  and  $(V_z)_a$  measured by using the three-axis accelerometer since the 3-D vibration components measured by using the three-axis accelerometer are integrated from acceleration signal. However, the amplitude and phase of the 3-D vibration components measured by using the developed optical system are almost the same as them of the 3-D vibration components measured by using the three-axis accelerometer. From these results, we can conclude the proposed algorithm is valid. If  $(V_x)$ ,  $(V_y)$ , and  $(V_z)$  are transformed into frequency domain, we can obtain FRFs in

three-axis ( $x_L$ ,  $y_L$ ,  $z_L$ ), which can be used for in-plane and out-of-plane mode shapes.

## VI. CONCLUSION

In this paper, we proposed an algorithm for 3-D vibration measurement using a single LSV and verified the algorithm by comparing with the time signal of three axis accelerometer. 3-D vibration measurement is achieved using the developed optical system by moving it to three arbitrarily different locations from the principle that vibration analysis based on the frequency domain is independent of the vibration signal based on time domain. The proposed system has the same effect as using three sets of LSVs. The proposed system has advantage of reducing the equipment cost caused by using three sets of LSV for 3-D vibration measurement. Using the geometrical relation obtained by the LS and three vibrations obtained at three different measurement locations, in-plane and out-of-plane vibration of all measurement points can be measured. Then, the obtained in-plane and out-of-plane helps us to understand the real vibrational property of the objects.

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- Dongkyu Kim** received the B.S. degree in mechanical engineering from Hanyang University, Seoul, Korea, in 2011, and the M.S. degree in mechatronics from the Gwangju Institute of Science and Technology, Gwangju, Korea, in 2013, where he is currently pursuing the Ph.D. degree.
- His current research interests include vibration measurement in 3-D and vibration measurement of rotating object using laser Doppler vibrometer.
- Hajun Song** received the B.S. degree in electronics computer engineering from Chonnam National University, Gwangju, Korea, in 2009, and the M.S. degree in mechatronics from the Gwangju Institute of Science and Technology (GIST), Gwangju, Korea, in 2011.
- He is currently studying mechatronics at the Gwangju Institute of Science and Technology, Gwangju, Korea. His current research interests include THz generation using photo-mixing technique and applications.
- Hossam Khalil** received the B.S. degree in industrial electronics and control engineering and the M.S. degree in automatic control engineering from Menoufiya University, Menoufia, Egypt, in 2004 and 2010, respectively.
- He is currently a student at the School of mechatronics with the Gwangju Institute of Science and Technology, Gwangju, Korea. His current research interests include control engineering, vibration measurement, and analysis.
- Jongsuh Lee** received the B.S. degree in mechanics from Kyounghee University, Gyeonggi-do, Korea, in 2007, and the M.S. degree in mechatronics from the Gwangju Institute of Science and Technology, Gwangju, Korea, in 2009, where he is currently pursuing the Ph.D. degree.
- His current research interests include experimental modal analysis and vibration reduction with passive design.
- Semyung Wang** received the Ph.D. degree in mechanical engineering from the University of Iowa, Ames, IA, USA.
- He is currently a Professor and Dean at the School of Mechatronics, Gwangju Institute of Science and Technology, Gwangju, Korea. His current research interests include vibration control and multidisciplinary design optimization.
- Kyihwan Park** received the B.S. degree in mechanical design and production engineering from Seoul National University, Seoul, Korea, in 1985, and the Ph.D. degree in mechanical engineering from the University of Texas at Austin, Austin, TX, USA, in 1993.
- He has been a Professor with the School of Mechatronics, Gwangju Institute of Science and Technology, Gwangju, Korea, since 1995.