

FOR ALL $N \geq 1$,
THEOREM: IF $\begin{array}{c} \sim \\ \boxed{U} \\ \sim \end{array}^N$ IS UNITAL UPPER TRIANGULAR, THEN
 THERE EXISTS $\begin{array}{c} \sim \\ \boxed{M} \\ \sim \end{array}$ UNITAL UPPER TRIANGULAR SUCH THAT

$$\begin{array}{c} \sim \\ \boxed{U} \\ \sim \end{array} = \begin{array}{c} \sim \\ \boxed{M} \\ \sim \end{array}$$

PROOF: (PROOF BY INDUCTION ON N)

BASE CASE: $N=1$

$$\begin{array}{c} \sim \\ \boxed{U} \\ \sim \end{array}^1 = \text{---} \quad (\text{UNITAL UPPER TRIANGULAR BASE CASE})$$

$$= \begin{array}{c} \sim \\ \boxed{M} \\ \sim \end{array} \quad (\text{IDENTITY IS UNITAL UPPER TRIANGULAR})$$

INDUCTIVE STEP

$$\begin{array}{c} \sim \\ \boxed{U} \\ \sim \end{array} = \begin{array}{c} \text{---} \quad \boxed{U_{11}} \quad \text{---} \\ \quad \quad \quad \diagup \quad \diagdown \\ \quad \quad \quad \boxed{U_{12}} \quad \quad \quad \text{---} \\ \quad \quad \quad \diagdown \quad \diagup \\ \text{---} \quad \boxed{U_{22}} \quad \text{---} \end{array} \quad \begin{array}{l} \text{WITH } U_{11} \text{ AND } U_{22} \text{ U.U.T.} \\ (\text{RECURSIVE DEFINITION OF U.U.T.}) \end{array}$$

$$= \begin{array}{c} \text{---} \quad \boxed{M_{11}} \quad \text{---} \\ \quad \quad \quad \diagup \quad \diagdown \\ \quad \quad \quad \boxed{M_{12}} \quad \quad \quad \text{---} \\ \quad \quad \quad \diagdown \quad \diagup \\ \text{---} \quad \boxed{M_{22}} \quad \text{---} \end{array} \quad \begin{array}{l} \text{WITH } M_{11} \text{ AND } M_{22} \text{ U.U.T.} \\ (\text{INDUCTION}) \end{array}$$

$$= \begin{array}{c} \text{---} \quad \boxed{M_{11}} \quad \text{---} \\ \quad \quad \quad \diagdown \quad \diagup \\ \quad \quad \quad \boxed{M_{12}} \quad \quad \quad \text{---} \\ \quad \quad \quad \diagup \quad \diagdown \\ \text{---} \quad \boxed{M_{22}} \quad \text{---} \end{array} \quad \begin{array}{l} (\text{TRIANGULAR ANTIPODE}) \\ \text{L3 MHA} \end{array}$$

$$= \begin{array}{c} \text{---} \quad \boxed{M_{11}} \quad \text{---} \\ \quad \quad \quad \diagdown \quad \diagup \\ \quad \quad \quad \boxed{M_{11}} \quad \boxed{M_{12}} \quad \boxed{M_{22}} \\ \quad \quad \quad \diagup \quad \diagdown \\ \text{---} \quad \boxed{M_{22}} \quad \text{---} \end{array} \quad \begin{array}{l} (\text{DICTIONARY FOR FUNCTIONS}) \end{array}$$

$$= \begin{array}{c} \sim \\ \boxed{M} \\ \sim \end{array} \quad \begin{array}{l} M \text{ IS U.U.T.} \\ (\text{RECURSIVE DEFINITION OF U.U.T.}) \end{array}$$