Fine Structure Constant Measurement via Compton Scattering Cross Section

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Abstract

The purpose of this experiment was to make measurements of the Compton cross-section and to determine the value of the Fine Structure Constant. The experiment consisted of a gamma ray source which would pass through an absorber to reach a photon detector at the end of a lead collimator. Measuring the photon count for different absorbers of varying materials and thicknesses leads to finding the total cross-section of the photon with each material, and this data was used to find a value for the Compton cross-section. Using the Compton cross-section, I determined the value of the fine structure constant (α) to be $\alpha = 0.007209 \pm 5\%$ where its actual value is 0.007297, which agrees within uncertainty.

Introduction

Compton scattering was one of the first experimental results that corroborated Einstein's theory that light was made from particles. In his original experiment Compton used a beam of X-rays aimed at an absorber material, the resulting light wave was then filtered out through a slit to reach a detector that simply measured the total energy deposited over time. He explained his results by detailing a process in which the individual photons collide with an electron from the absorber material and transfer it enough energy to eject said electron. However, unlike the photoelectric effect, where the photon is absorbed and the difference between the initial energy of the photon and the energy required to eject the electron becomes the kinetic energy of the electron, with the Compton effect this energy difference is much larger, so the photon gets re-emitted with much of its original energy soon after the electron gets ejected. This process can be modelled as an inelastic collision between the photon and the electron. Compton then developed an equation that showed the relationship between the incident and scattered photon wavelengths based on the scattering angle of the photon[1]:

$$\lambda' = \lambda + \frac{h}{m_e c}(l - cos\theta) \tag{1}$$

Compton scattering is one of the three different ways that photons can interact with electrons. At lower energies, an incoming photon gets fully absorbed and transfers all its energy to the ejected electron, in a process known as the photoelectric effect. In slightly higher energy levels Compton Scattering occurs as well as the photoelectric effect. And at even higher energies (>1.022MeV) a process called pair production becomes possible and more prevalent, where the energy of the photon gets converted to create an electron-positron pair. Lastly, it's possible that photons collide with the electrons elastically. For this experiment we assume effect due to elastic collision and pair production to be non-existent or negligible. With this assumption, we get the following equation for the total cross-section, where σ_{pe} is photoelectric cross-section and σ is the Compton cross-section:

$$\sigma_{tot} = \sigma_{pe} + \sigma \quad (2)$$

The actual experiment comprised of measuring the total photon intensity for varying different absorber materials and thicknesses. There is a simple equation that models the scenario[2]:

$$I(x) = I(0)\exp(-n\sigma_{tot}x) \quad (3)$$

Where n is the atomic density of the absorbers (atoms/m3), σ_{tot} is the total cross section per atom for all processes of gamma ray interaction with the absorbers and x is the absorber thickness.

The Compton Cross Section is a measure with dimensions of area, and it represents the likelihood that a photon of a specific energy will interact with an electron. This value can be calculated using the Klein-Nishina formula[3], a quantum electrodynamics calculation:

$$\sigma = 2\pi r_o^2\{\frac{1+\gamma}{\gamma^2}[\frac{2(1+\gamma)}{1+2\gamma} - \frac{1}{\gamma}ln(1+2\gamma)] + \frac{1}{2\gamma}ln(1+2\gamma) - \frac{1+3\gamma}{(1+2\gamma)^2}\} \quad (4)$$

Where $\gamma = \frac{E}{m_e c^2}$ and E is the energy of the photon, which is 662KeV for this experiment[2]. Lastly, r_o^2 is the classical electron radius, which is defined below in terms of the fine structure constant[2]:

$$r_e = \frac{\alpha h_{bar} \times c}{m_e \times c^2} \quad (5)$$

Experimental Techniques (Methods and Materials/ Apparatus and Procedure)

Apparatus

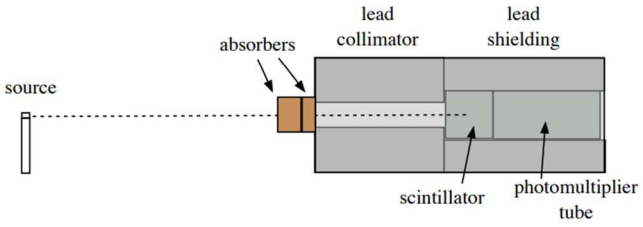

Figure 1: Sketch of Compton experimental set-up. [2]

In this experiment the setup consisted of a radioactive source (Cesium-137), an absorber, a lead collimator, a power source, and a photon detector (comprised of a scintillator and photomultiplier tube). A scintillator is a material that absorbs high energy radiation (such as the gamma rays used in this experiment), and converts it to near visible light, which gets absorbed by the photomultiplier tube and converted into an electrical signal[4].The setup is also directly connected to a lab computer so that this electrical signal generated by the photomultiplier tube gets automatically sent to a software for analysis called UCS30. Additionally, 5 different materials were tested as absorbers: carbon, aluminum, lead, copper, and bismuth. Each material was tested for 5 different thicknesses. Lastly, a caliper was used to measure the thickness of each absorber.

Procedure

The procedure for the data collection for each absorber sample was the same. Started by acquiring the Cs-137 and placing it in its designated position inside the lead collimator tunnel, then placing the absorber in its correct position and using the lead pieces to fully close the tunnel such that there were no gaps through which light could enter. Then the power source and the computer were turned on, and the software was opened. In the software I then preset a recording time of 900 seconds (15min) and hit run to start collecting the data. The time the experiment is run for can be arbitrary if the photon count is normalized afterwards. However, I choose to use the same time for each experiment to avoid the additional normalization step, thus, 900 seconds was chosen as it was a long enough time such that enough photons could accumulate, and a clear peak would be distinguishable even for the thickest absorbers used.

At the beginning of the experiment, I let the computer record data over night without the Cs-137 source, this data was then used by the software to do a background removal for each subsequent experiment. For each experiment then, once the data collection period of 15min was through, I removed the background (thus reducing systematic uncertainty), saved the resulting file, and sent it to my email. This procedure was repeated 25 times (5 different thicknesses for each of the 5 different absorber materials). The materials used and their respective thicknesses are detailed in the table below:

Aluminum (mm)	Copper (mm)	Carbon (mm)	Bismuth (mm)	Lead (mm)
6.35±0.01	1.61±0.01	25.50±0.01	3.19±0.01	3.28±0.01
12.71±0.01	3.19±0.01	50.84±0.01	6.31±0.01	6.77±0.01
13.11±0.01	6.35±0.01	76.28±0.01	6.39±0.01	13.36±0.01
13.20±0.01	12.75±0.01	101.71±0.01	12.84±0.01	25.52±0.01
25.43±0.01	24.42±0.01	127.04±0.01	25.41±0.01	51.55±0.01

Table 1: Table of absorber materials and thicknesses

Data Analysis

The first part of data analysis comprised of fitting the data from the software to a gaussian curve and integrating it, the value found for the integral was the value used for I(x), where x was the thickness of the absorber in question.

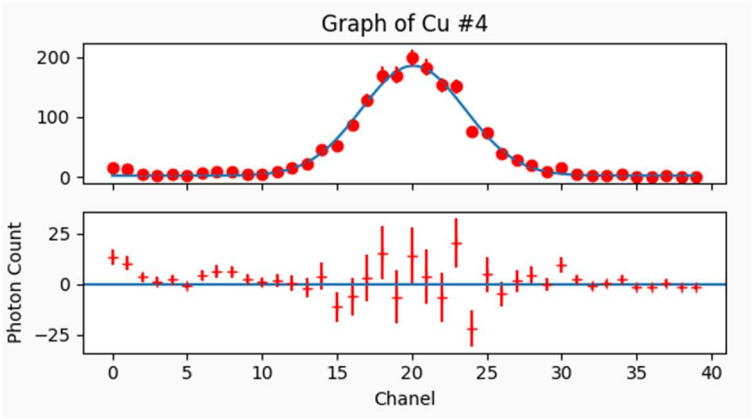

Figure 2: Graph of Photon Count vs Energy Channel for Copper absorber sample

For example, this is the graph of the data and residuals for one of the Copper absorbers. The red dots represent the photon count for each channel (which represents a respective energy band the photomultiplier detected the photon to have) with their respective uncertainty (vertical error bars), which was calculated using Poisson[6] for counting uncertainty $\mu = \frac{1}{\sqrt{N}}$, where N is the total photon count for each respective energy channel. A curve fitting program[5] was used to analyze this data and fit it to a Gaussian curve, the blue line is the graph of the best fit Gaussian curve overlaid over the actual data points. The bottom graph shows the plot of the residuals with the respective error bar for each data point.

Once a Gaussian function of best fit was found for each absorber, the function was integrated, and the result was used as the photon intensity I(x) value from equation 3. Furthermore, the fitting program also returned uncertainty values for each of the parameters in the equation fitted for. The Gaussian equation fitted for was the following:

$$y = B + Ae^{--\frac{(x-\mu)^2}{2\sigma^2}} \quad (6)$$

Where B represents a constant background, A is the peak height above the background, μ is the central value, and σ represents the standard deviation. To find the total uncertainty in the intensity value I made two different gaussian functions using parameters with their added uncertainties, one such that the integral value would be maximized and the other such that it would be minimized. I took the absolute value of the maximum difference between the max and the original and the min and the original to be the uncertainty in the photon intensity.

Then, the I(x) and x data for each material was fitted to the following equation:

$$I(x) = I(0)\exp(— kx). \quad (7)$$

This is the same as equation 3 except that the product of the atomic density and cross section was substituted by a parameter k to be found by the python fitting program[5].

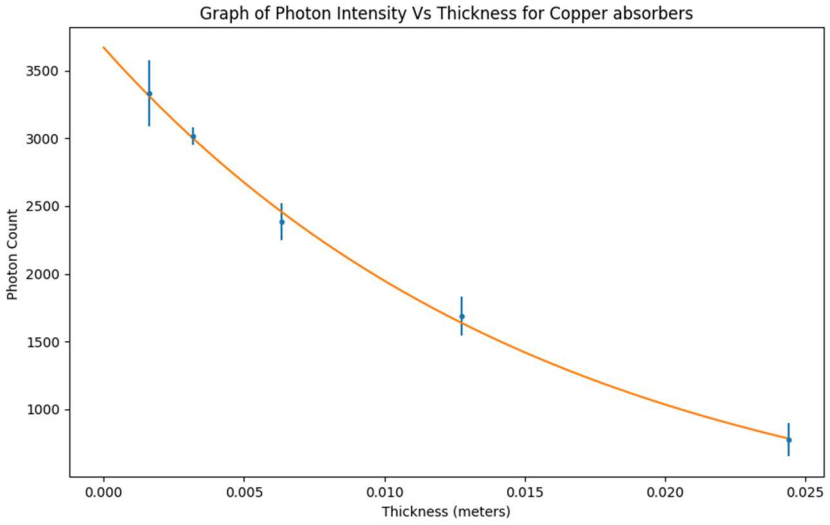

Figure 3: Graph of Photon Count vs Thickness for Copper

The above is an example of exponential decay relationship between the Photon Intensity and the thickness of the absorber used. The exponential decay makes intuitive sense when we're reminded that the photons being detected are the photons that make it through the absorber without interacting with any of the electrons. Therefore, the thicker the absorber, the larger the distance the photon must travel through in the absorber and the more likely it is to interact with an electron, whereas if it does it will be absorbed by the electron and likely re-emitted at an angle such that it will hit the lead tunnel and be absorbed without making it to the photon detector at the end of the tunnel. An exponential decay like the above was found for each of the 5 materials tested, and the parameter k from equation 6 was found with a respective fitting uncertainty given by the python script.

By comparing equation 3 and equation 6 we see the total cross-section (σ_{tot}) can be found from dividing the k by the atomic density (n) of the respective material ($\sigma_{tot} = \frac{k}{n}$). The results of this calculation are shown in the table below:

Table 2: Total cross-section values and uncertainties for each absorber material

Material	Carbon	Aluminum	Copper	Lead	Bismuth
σ_{tot}	1.12×10^{-28}	3.51×10^{-28}	7.47×10^{-28}	3.33×10^{-27}	3.75×10^{-27}
Uncertainty in σ_{tot}	4.09×10^{-30}	4.06×10^{-29}	2.58×10^{-29}	4.26×10^{-28}	4.49×10^{-29}

This data was fitted against the following equation[2] which gives the relationship between the total cross section and the Compton cross section based on the atomic number of the absorber material:

$$\sigma_{tot} = a \times Z^{4.2} + bZ \quad (8)$$

Where Z is the atomic number of each material and a and b were the parameters to be fitted against, with b representing the final value for the Compton cross section.

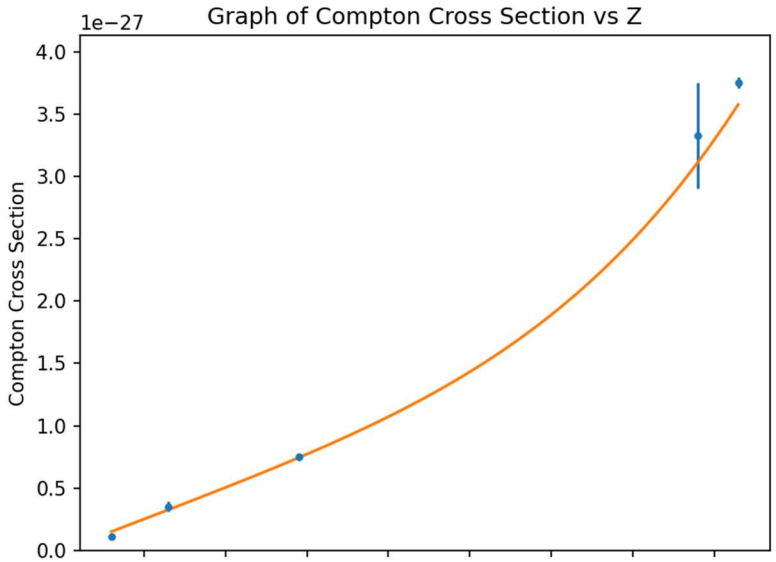

Figure 4: Final Graph of Compton Cross Section Vs Atomic Number (Z)

Unfortunately, for this last equation due to the small amount of data available the python script couldn't find a fit. However, after manually adjusting the parameters based on an initial educated guess, I was able to find a good fit which resulted in a σ value of $2.5 \times 10^{-29} \pm 1.5 \times 10^{-30}$. By plugging this into the analytical integral of the Klein-Nishina formula (equation 4) and using simple uncertainty propagation[7] through division and multiplication I was able to find a value for r_e. Once again, propagating the uncertainty and plugging r_e into equation 5 I got a value for the fine structure constant α of 0.007209 ± 0.0003604, which is within a 1.2% error compared to its expected value.

Sources of Error

The main sources of error in this experiment were due to forward scattering, absorber impurities, and the effect of electron binding energies. Forward scattering refers to photons that interacted with an electron from the absorber through Compton Scattering but were scattered at a small enough angle that they were able to reach the photomultiplier tube. This effect is expected to be small, but in a more elaborate experiment this could be corrected by calculating the angle at which scattered photons would still be detected and calculating the probability that a photon would be scattered at that angle and then adequately discounting it from the final photon count.

Another slight correction could be made due to absorber impurities. Although the absorbers are used in calculations as if they were perfect materials made of a single element some of the absorbers contain a small percentage of impurities which affect the computations[2] for σ_{pe}. This effect is most notable for Lead, which is known to contain ~3-5% of Sb.

Lastly, a small source of error was due to the assumption that the electrons that the photons interact with are free, this correction is small for most interactions as the binding energy of the electrons is negligible compared to the energy of the gamma rays. However, the inner-most electrons of Lead and Bismuth have larger binding energies that may result in a small source of error.

Conclusions

My final value for the fine structure constant α of 0.007209 ± 0.0003604 agrees with the theoretical value within uncertainty. The main issue with the python script not being able to find a working fit was that I wasn't able to get a precise fit uncertainty for the final σ value and by extension for α. However, I extrapolated to find a conservative uncertainty value based on the average uncertainties of the σ_{tot} values (6.3%) and the residual between the σ_{tot} values and my manually fitted function (5%). The fine structure constant is one of the fundamental parameters of Quantum Electrodynamics and the Standard Model of Particle Physics, so making precision measurements of α is a critical step for the field of experimental physics, and experiments like this that corroborate its known value have their own significance as well.

Appendix

References:

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[4] *Stanford Advanced Optical Ceramics Laboratory*. Stanford. (n.d.). Retrieved November 29, 2022, from http://web.stanford.edu/group/scintillators/scintillators.html

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[7] Lindberg, V. (n.d.). *Part I of a manual on Uncertainties, Graphing, and the Vernier Caliper*. Uncertainties and Error Propagation. Retrieved November 29, 2022, from https://www.geol.lsu.edu/jlorenzo/geophysics/uncertainties/Uncertaintiespart2.html