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Simplified Static Analysis of Large-Dimension Parallel Cable-Driven Robots

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Abstract— **This paper introduces a new simplified static analysis of parallel robots driven by inextensible cables of non-negligible mass. It is based on a known hefty cable static modeling which seems to have been overlooked in previous works on parallel cable-driven robots. This cable modeling is obtained from a well-known sagging cable modeling, known as the catenary, by assuming that cable sag is relatively small. The use of the catenary has been shown to lead to a non-linear set of equations describing the kinetostatic behavior of parallel robots driven by cables of non-negligible mass. On the contrary, the proposed simplified static analysis yields a linear relationship between (components of) the forces in the cables and the external wrench applied to the robot mobile platform. As a consequence, by means of the simplified static analysis, useful wrench-based analysis and design techniques devised for parallel robots driven by massless cables can now be extended to cases in which cable mass is to be accounted for.**

I. Introduction

Parallel cable-driven robots consist mainly of a mobile platform actuated in parallel by cables. The actuators are generally fixed to the ground and drive winches which control the position and orientation of the mobile platform by varying the cable lengths. Parallel cable-driven robots have several interesting characteristics such as reduced moving part mass and inertia which can be useful for high-speed pick-and-place applications [1]. Other interesting properties include the flexible nature of the cables and their low visual intrusion which can make parallel cable robots well adapted to force feedback haptic interfaces [2], [3] and rehabilitation systems [4]. In these latter applications, another useful property of parallel cable robots is their potentially large workspace. In fact, very long cables can be used which enables very wide workspaces. Thus, several studies on large-dimension cable-driven robots have been made [5]–[13].

In many previous studies, the cables are modeled as massless straight bodies. This assumption is usually valid for robots of reasonable size carrying light payloads, but for large-dimension cable-driven robots or for heavy payloads, such an assumption may be invalid [14]. In this latter reference, a well-known sagging cable modeling is used. This modeling takes cable mass and elasticity into account and is referred to as the *elastic catenary* [15]. Apart from studies using finite element methods [16], [17], the elastic catenary has been considered in several previous studies on large-dimension parallel cable-driven robots [9], [11], [18], [19]. As discussed in [9], the static analysis of parallel cable-driven robots based on the elastic catenary leads to a system of non-linear equations. In this system, the robot kinematics and the mobile platform static equilibrium are coupled. When cable elasticity is negligible, the problem is partially decoupled in the sense that the platform static equilibrium is described by a system of non-linear equations in which the cable lengths no longer appear [9]. A solution of this system being determined, each cable length can then be computed by means of a closed-form formula. However, even in this simpler case of inextensible cables, a system of non-linear equations is still involved in the static modeling when the catenary is considered. This notably prevents the application of efficient tools to evaluate wrench feasibility [20], [21].

In this paper, a new static analysis of parallel robots driven by inextensible cables of non-negligible mass is proposed. It is based on a known hefty cable static modeling [15] (Chapter 2) which seems to have been overlooked in previous works on parallel cable-driven robots. This latter cable modeling is valid as long as cable sagging is relatively small. The present work is mainly dedicated to n degree-of-freedom (DOF) underconstrained large-dimension parallel robots driven by m cables with $m \geq n$. Indeed, we believe that, in many situations, an inextensible cable of non-neglible mass is likely to be an appropriate modeling of the cables of these robots. Note that the term underconstrained is used here as opposed to fully constrained: The robots considered are driven by a number of cables equal to or greater than the number of DOF of their mobile platform but, in the considered working configurations, their mobile platform is not fully constrained by the cables. This type of parallel cable-driven robots is sometimes referred to as cable-suspended robots [22]–[24]. All their cable drawing points are generally located above their workspace so that the part of the workspace located below the platform is not cluttered by the cables.

The paper is organized as follows. Notations are introduced in Section II. The static analysis of parallel robots driven by inextensible cables of non-negligible mass, as presented in [9], is recalled in Section III. In section IV, a static modeling of a hefty inextensible cable is presented together with a criterion which ensures its validity. This modeling and the associated criterion are known [15] but, to the best of our knowledge, they have never been used in

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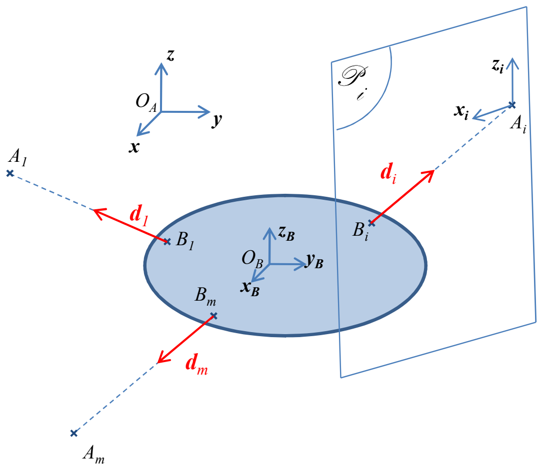

Fig. 1. Mobile platform and notations.

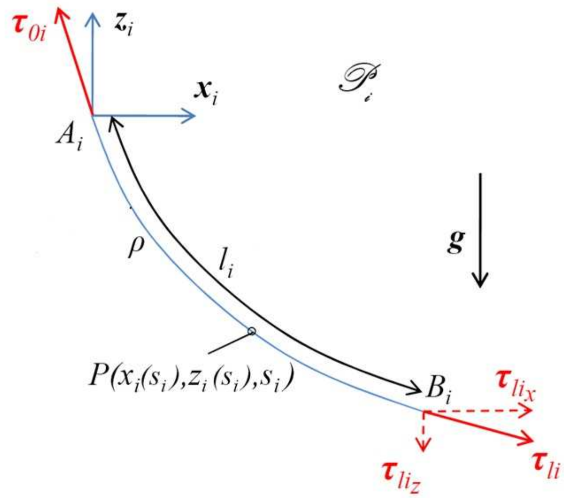

Fig. 2. Cable i lying in its vertical plane $\mathscr{P}_i$. In the figure, $\boldsymbol{\tau}_{li}$ is the force applied by the platform to the cable at its attachment point B_i. s_i the curvilinear abscissa that is the distance along the cable from A_i to P. The cable point P has coordinates x_i and z_i in the local frame $\mathscr{R}_i$.

the modeling of parallel cable-driven robots. The new static analysis is then introduced in Section V. Finally, Section VI presents an example of a large 6-DOF cable-suspended robot driven by 6 cables to which the proposed static analysis is applied. By comparing the results with those obtained by means of both the elastic catenary and the basic massless inextensible cable modeling, this example shows that the new static analysis introduced in this paper can be a valid useful alternative to previously proposed methods.

II. Notations

The pose $\boldsymbol{p}$ of the platform (position and orientation) is expressed in the global fixed frame $\mathscr{R}_A$ $(O_A, \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$. The position vectors of the cable drawing points A_i with respect to frame $\mathscr{R}_A$ are denoted $\boldsymbol{a}_i$. The points A_i are supposed to be fixed. The positions of the cable attachment points B_i on the platform are expressed in the frame $\mathscr{R}_B$ $(O_B, \boldsymbol{x}_B, \boldsymbol{y}_B, \boldsymbol{z}_B)$ attached to the platform, O_B being the platform mass center. In the frame $\mathscr{R}_B$, the position vector of B_i is denoted $\boldsymbol{b}_i$. Matrix $\boldsymbol{Q}$ is the rotation matrix from $\mathscr{R}_A$ to $\mathscr{R}_B$. Let us call $\boldsymbol{d}_i$ the unit vector along the chord A_iB_i—straight line segment—from B_i to A_i. The number of driving cables is denoted m whereas the number of DOF of the mobile platform is n.

Let us consider cable i which, under static loading conditions, lies in the vertical plane $\mathscr{P}_i$ containing A_i and B_i as shown in Fig. 1. A frame $\mathscr{R}_i$ $(A_i, \boldsymbol{x}_i, \boldsymbol{z}_i)$ is attached to this plane. This frame shares the vector $\boldsymbol{z}$ with the frame $\mathscr{R}_A$. Axis $\boldsymbol{x}_i$, orthogonal to $\boldsymbol{z}_i$, is chosen so that the x-coordinate B_{i_x} of B_i in frame $\mathscr{R}_i$ is positive. Thus, to transform $\mathscr{R}_A$ into the frame attached to the plane $\mathscr{P}_i$, one single rotation of angle γ_i around $\boldsymbol{z}$ is needed. The angle γ_i corresponds to the angle between the unit vectors $\boldsymbol{x}_i$ and $\boldsymbol{x}$ of frame $\mathscr{R}_i$ and $\mathscr{R}_A$, respectively.

III. Static analysis considering inextensible cables with non-negligible mass

In most previous works on large parallel cable-driven robots, the cable mass and elasticity are accounted for by means of the elastic catenary. The details and main consequences of this modeling are presented in [9]. In this latter paper, the simpler case of an inextensible cable with non-negligible mass is also dealt with and is reproduced in this section. This case yields the following two equations which give the coordinates B_{i_x} and B_{i_z} in frame $\mathscr{R}_i$ of the cable attachment point B_i.

$$B_{i_x} = \frac{|\tau_{li_x}|}{\rho g}\left[sinh^{-1}\left(\frac{\tau_{li_z}}{\tau_{li_x}}\right) - sinh^{-1}\left(\frac{\tau_{li_z} - \rho g l_i}{\tau_{li_x}}\right)\right] \quad (1)$$

$$B_{i_z} = \frac{1}{\rho g}\left[\sqrt{\tau_{li_x}^2 + \tau_{li_z}^2} - \sqrt{\tau_{li_x}^2 + \left(\tau_{li_z} - \rho g l_i\right)^2}\right] \quad (2)$$

In these equations, ρ, g and l_i are respectively the linear density of the cable, the gravity acceleration and the length of cable i between points A_i and B_i. τ_{li_x} and τ_{li_z} are the components in the local cable frame $\mathscr{R}_i$ of the force $\boldsymbol{\tau}_{li}$ applied by the platform to the cable at its end point B_i. These notations are illustrated in Fig. 2. Note that $\tau_{li_x} \geq 0$ since, because of the choice of the direction of axis $\boldsymbol{x}_i$, $\tau_{li_x} < 0$ would mean that cable i is working in compression. Solving Eq. (1) for the length l_i, we obtain:

$$l_i = -\frac{1}{\rho g}\left[\tau_{li_x} sinh\left(sinh^{-1}\left(\frac{\tau_{li_z}}{\tau_{li_x}}\right) - \frac{\rho g}{|\tau_{li_x}|}B_{i_x}\right) - \tau_{li_z}\right] \quad (3)$$

Substituting this expression of l_i into Eq. (2) provides a *non-linear* equation involving the cable force components τ_{li_x} and τ_{li_z} but in which the cable length l_i does not appear:

$$\begin{aligned} B_{i_z} = \frac{|\tau_{li_x}|}{\rho g}\Bigg[&\sqrt{\left(\frac{\tau_{li_z}}{\tau_{li_x}}\right)^2 + 1} \\ &- cosh\left(sinh^{-1}\left(\frac{\tau_{li_z}}{\tau_{li_x}}\right) - \frac{\rho g}{|\tau_{li_x}|}B_{i_x}\right)\Bigg] \end{aligned} \quad (4)$$

Now, let us introduce the platform static equilibrium equations. At the reference point O_B of the platform, the wrench resulting from the force $-\boldsymbol{\tau}_{li}$ applied by cable i on the platform can be written:

$$-\boldsymbol{w}_{fi}\begin{bmatrix}\tau_{li_x}\\ \tau_{li_z}\end{bmatrix} = -\begin{bmatrix}\boldsymbol{q}_{i_1} & \boldsymbol{q}_{i_3}\\ \boldsymbol{Q}\boldsymbol{b}_i\times\boldsymbol{q}_{i_1} & \boldsymbol{Q}\boldsymbol{b}_i\times\boldsymbol{q}_{i_3}\end{bmatrix}_{n\times 2}\begin{bmatrix}\tau_{li_x}\\ \tau_{li_z}\end{bmatrix} \quad (5)$$

where $\boldsymbol{Q}$ and $\boldsymbol{b}_i$ are defined in Section II. The vectors $\boldsymbol{q}_{i_1}$ and $\boldsymbol{q}_{i_3}$ are two of the three column vectors of the rotation matrix between the global frame $\mathscr{R}_A$ and the local frame $\mathscr{R}_i$:

$$\boldsymbol{Q}_i = \begin{bmatrix} c\gamma_i & -s\gamma_i & 0\\ s\gamma_i & c\gamma_i & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}\boldsymbol{q}_{i_1} & \boldsymbol{q}_{i_2} & \boldsymbol{q}_{i_3}\end{bmatrix} \quad (6)$$

Adding the m cable wrenches given in Eq. (5), for $i = 1,\ldots,m$, to the external wrench $\boldsymbol{f_e}$ applied on the platform yields:

$$-\boldsymbol{W}_f\boldsymbol{f} + \boldsymbol{f_e} = 0 \quad (7)$$

where $\boldsymbol{f} = \begin{bmatrix}\tau_{l1_x} & \tau_{l1_z} & \cdots & \tau_{lm_x} & \tau_{lm_z}\end{bmatrix}^T_{2m\times 1}$, and $\boldsymbol{W}_f = \begin{bmatrix}\boldsymbol{w}_{f1} & \boldsymbol{w}_{f2} & \cdots & \boldsymbol{w}_{fm}\end{bmatrix}_{n\times 2m}$. Note that a minus sign appears in the platform static equilibrium equations (7) because the force $\boldsymbol{\tau}_{li}$ is defined as the force applied by the platform on the cable (as shown in Fig. 2).

Equations (4) for $i = 1,\ldots,m$ and Eq. (7) together provide a system of $m+n$ equations. Being given the mobile platform pose, the coordinates B_{i_x} and B_{i_z} of each attachment point B_i as well as the matrix $\boldsymbol{W}_f$ are known. Then, for a given external wrench $\boldsymbol{f_e}$, the unkowns of the system of $m+n$ equations (4)-(7) are the $2m$ cable force components gathered in vector $\boldsymbol{f}$. When $m = n$, the system has as many variables as equations whereas, when $m > n$, the number of variables is greater than the number of equations and the system is underconstrained. Note that the underconstrained case is out of the scope of this paper as further work is required to deal with the infinite number of solutions.

Finally, $\boldsymbol{f}$ being determined, the cable lengths l_i are given directly by Eq. (3). This partial decoupling makes the problem slightly simpler to solve compared to the situation in which the cable elasticity is not neglected [9]. *However, it remains that the system to be solved to find the cable force components τ_{li_x} and τ_{li_z} is non-linear* because of Eq. (4).

IV. Another model of an inextensible hefty cable

The elastic catenary considered in [9] to analyze robots driven by cables of non-negligible mass is drawn from [15]. In this latter reference, another hefty cable modeling is also discussed. It is obtained from the elastic catenary by considering the cables to be inextensible and by means of some simplifications which are valid as long as the cable profile remains relatively close to the chord A_iB_i. This simplified modeling should therefore be of interest to analyze parallel cable-driven robots since cable sag is, on the one hand, generally required to be limited but, on the other hand, inevitable when the cable mass is non-negligible. To the best of our knowledge, this cable model has however never been applied to the analysis of large parallel cable-driven robots.

By making some simplifications to the differential equations governing a hefty inextensible cable static behavior, Irvine obtains the following *parabolic cable profile equation* [15]:

$$z = \frac{-\rho g L_i}{2\tau_{li_x}B_{i_x}}x(B_{i_x}-x) + x\tan\beta_{0i} \quad (8)$$

where x and z are the coordinates of a point P on the cable and L_i is the length of the chord A_iB_i. As shown in Fig. 3, β_{0i} is the angle between the horizontal and the chord ($\beta_{0i} = \tan^{-1}\left(B_{i_z}/B_{i_x}\right)$). Equation (8) is useful because it provides an explicit description of the cable profile.

Then, let us consider the sag h, which is the signed vertical distance between the chord and the cable. Since $z = x\tan\beta_{0i} + h$, its expression results directly from Eq. (8) (we intentionally omit subscript i in z, x and h).

$$h = \frac{-\rho g L_i}{2\tau_{li_x}B_{i_x}}x(B_{i_x}-x) \quad (9)$$

The maximal sag d_i, as presented in Fig. 3, is obtained when the derivative dh/dx is null:

$$\frac{dh}{dx} = \frac{-\rho g L_i}{2\tau_{li_x}B_{i_x}}(B_{i_x}-2x) = 0 \quad (10)$$

that is to say when $x = B_{i_x}/2$. Substituting this expression of x into Eq. (9), the maximal sag d_i is found to be equal to:

$$d_i = \frac{-\rho g B_{i_x} L_i}{8\tau_{li_x}} \quad (11)$$

According to [15], the condition for Eq. (8) to be a valid description of the profile of an inextensible hefty cable involves the ratio of sag d_i to span B_{i_x} and can be written:

$$\frac{|d_i|}{B_{i_x}} \leq \frac{1}{8} \quad (12)$$

In the latter equation, ratios of $1/5$ or $1/4$ also ensure validity of Eq. (8) but the value of $1/8$ is one of convenience since, according to Eq. (11), Eq. (12) is equivalent to:

$$\frac{\tau_{li_x}}{\rho g L_i} \geq 1 \quad (13)$$

Eq. (13) means that the horizontal component of the force applied to the cable at point B_i must be greater than the weight of the cable assimilated to the straight line segment between A_i and B_i (the chord). Equivalently, Eq. (12) is verified when the cable sag is limited.

V. A simplified static analysis

A. Wrench applied by a cable on the platform

The main idea now is to use the explicit parabolic cable profile equation presented in the previous section in order to get a linear relationship between the components τ_{li_x} and τ_{li_z} of the force $\boldsymbol{\tau}_{li}$ applied to cable i at its attachment point B_i. In fact, in Eq. (8), taking the derivative of z with respect to x, the equations of the tangents to the cable profile at points A_i and B_i can be written as:

$$z = \left(\frac{-\rho g L_i}{2\tau_{li_x}} + \tan\beta_{0i}\right)x \quad (14)$$

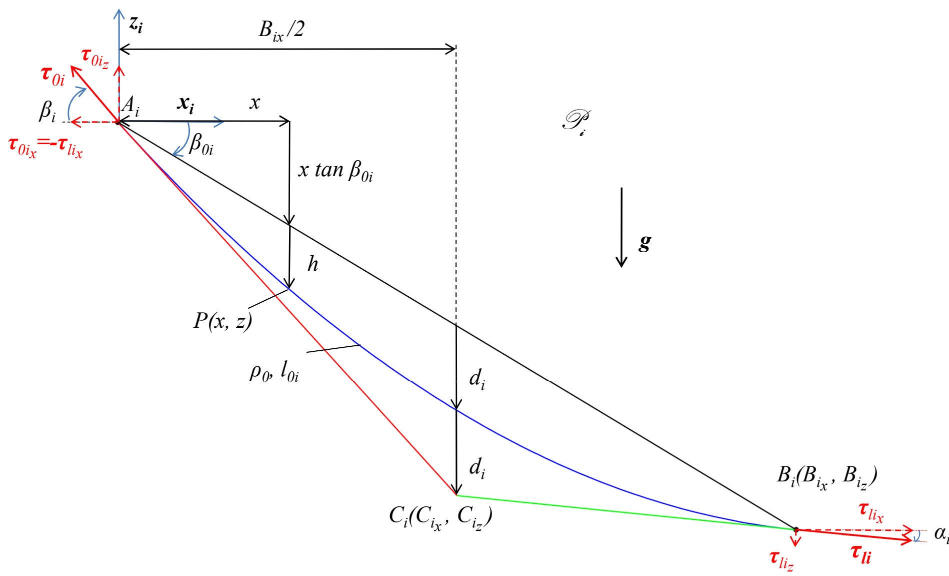

Fig. 3. Profile of cable i in plane $\mathscr{P}_i$ and notations associated with the simplified hefty cable model.

and

$$z = \left(\frac{\rho g L_i}{2\tau_{li_x}} + \tan\beta_{0i}\right)x - \frac{\rho g L_i B_{i_x}}{2\tau_{li_x}} \tag{15}$$

respectively. From Eq. (15), the slope of the tangent to the cable profile at point B_i is:

$$\tan\alpha_i = \frac{\tau_{li_z}}{\tau_{li_x}} = \frac{\rho g L_i}{2\tau_{li_x}} + \tan\beta_{0i} \tag{16}$$

where α_i is the angle between the horizontal and the direction of the force $\boldsymbol{\tau}_{li}$, as shown in Fig. 3. Thereby, we obtain the desired linear relationship between τ_{li_z} and τ_{li_x}:

$$\tau_{li_z} = \frac{\rho g L_i}{2} + \tau_{li_x}\tan\beta_{0i} \tag{17}$$

The force applied by cable i to the platform, whose components in the global fixed frame $\mathscr{R}_A$ are denoted F_x, F_y and F_z, can then be written as a function of the horizontal component τ_{li_x} only:

$$\left[\begin{array}{ccc} F_x & F_y & F_z \end{array}\right]^T = -\left(\boldsymbol{r}_i \tau_{li_x} + \boldsymbol{f}_i\right) \tag{18}$$

where $\boldsymbol{r}_i = \left[\begin{array}{ccc} \cos\gamma_i & \sin\gamma_i & \tan\beta_{0i} \end{array}\right]^T$, and $\boldsymbol{f}_i = \left[\begin{array}{ccc} 0 & 0 & \rho_0 g L_i/2 \end{array}\right]^T$. The negative sign in Eq. (18) comes from the fact that $\boldsymbol{\tau}_{li}$ is the force applied by the platform to cable i.

From Eq. (18), the wrench applied by cable i on the platform at its reference point O_B is:

$$-\left(\boldsymbol{w}_{x_i}\tau_{li_x} + \boldsymbol{f}_{cab_i}\right) \tag{19}$$

where

$$\boldsymbol{w}_{x_i} = \left[\begin{array}{c} \boldsymbol{r}_i \\ \boldsymbol{Q}\boldsymbol{b}_i \times \boldsymbol{r}_i \end{array}\right] \tag{20}$$

and

$$\boldsymbol{f}_{cab_i} = \left[\begin{array}{c} \boldsymbol{f}_i \\ \boldsymbol{Q}\boldsymbol{b}_i \times \boldsymbol{f}_i \end{array}\right] \tag{21}$$

B. Static equilibrium of the platform

From the expression (19) of the wrench applied by cable i, the static equilibrium of the platform of a parallel robot driven by m inextensible hefty cables is given by:

$$-\left(\boldsymbol{W}_x \boldsymbol{\tau}_x + \boldsymbol{f}_{cab}\right) + \boldsymbol{f}_e = \boldsymbol{0} \tag{22}$$

where, as in Section III, $\boldsymbol{f}_e$ is the external wrench applied to the platform and where the components of vector $\boldsymbol{\tau}_x$ are the horizontal components τ_{li_x} of the forces $\boldsymbol{\tau}_{li}$:

$$\boldsymbol{\tau}_x = \left[\begin{array}{cccc} \tau_{l1_x} & \tau_{l2_x} & \cdots & \tau_{lm_x} \end{array}\right]^T \tag{23}$$

Moreover, the matrix $\boldsymbol{W}_x$ depends only on the mobile platform pose and is defined as:

$$\boldsymbol{W}_x = \left[\begin{array}{cccc} \boldsymbol{w}_{x_1} & \boldsymbol{w}_{x_2} & \cdots & \boldsymbol{w}_{x_m} \end{array}\right]_{n\times m} \tag{24}$$

and

$$\boldsymbol{f}_{cab} = \sum_{i=1}^{m} \boldsymbol{f}_{cab_i} \tag{25}$$

For parallel cable-driven robots having as many cables as DOF ($m = n$), the system of equations (22) with unknowns $\boldsymbol{\tau}_x$ is square. Then, for a given external wrench $\boldsymbol{f}_e$ and for a non-singular matrix $\boldsymbol{W}_x$, its single solution is:

$$\boldsymbol{\tau}_x = \boldsymbol{W}_x^{-1}\left(\boldsymbol{f}_e - \boldsymbol{f}_{cab}\right) \tag{26}$$

With a massless inextensible cable model, for a given pose, if one tension is negative, the static equilibrium of the platform cannot be realized with all cables tensed. In the formulation of the platform static equilibrium proposed in this paper, Eq. (22), vector $\boldsymbol{\tau}_x$ is not directly composed of the cable tensions but contains the horizontal components τ_{li_x} of the forces $\boldsymbol{\tau}_{li}$. However, vector $\boldsymbol{\tau}_x$ given by Eq. (26) is an admissible solution if all the components τ_{li_x} are nonnegative.

Indeed, if one of these components is negative, it means that at least one cable is working in compression which is not considered admissible. In summary, as in the case of a massless cable model, there must exist nonnegative solutions $\boldsymbol{\tau}_x$ to Eq. (22) for the platform static equilibrium to be feasible. In the case of n-DOF robots driven by $m > n$ cables, the system (22) has more unknowns than equations and one has to deal with the selection of a tension distribution or the determination of the actual one. This relevant issue is, however, out of the scope of this paper. Finally, note that τ_{li_x} being determined, the component τ_{li_z} of the cable force is directly given by Eq. (17).

To conclude this section, let us emphasize the main useful property of the mobile platform static equilibrium equations introduced in Eq. (22). They provide a linear relationship between the cable force components τ_{li_x} and the external wrench $\boldsymbol{f}_e$. Contrary to the static analysis presented in Section III in which a non-linear system of equations must be solved to find τ_{li_x} and τ_{li_z}, recent works [25] show that the linear relationship of Eq. (22) allows the extension to large parallel robot driven by cables of non-negligible mass of some useful existing tools, e.g. [20], [21]. This extension is made possible by Eq. (22) whose form is similar to the usual linear relationship between tensions (in massless cables) and platform wrench [26]. Finally, note that this extension is relevant if the validity criteria of Eq. (12) or Eq. (13) is verified and if the cables can be considered inextensible.

C. Maximal cable tension

Contrary to a massless cable model, the tension along a cable is not constant when the cable mass is considered. In fact, the tension along the cable, defined as the norm of the force in the cable, is given by the following equation [9]:

$$\tau(s_i) = \sqrt{\tau_{li_x}^2 + \left(\tau_{li_z} + \rho g (s_i - l_i)\right)^2} \tag{27}$$

in which l_i is the cable length and s_i is the curvilinear abscissa of the current cable point P as illustrated in Fig. 2.

From equation (27), we can obtain the maximal tension in the cable. To simplify the discussion and since this work has been done with underconstrained parallel cable robots in mind, henceforth, the cable drawing points A_i are supposed to be located above the mobile platform for all the poses in the robot workspace. Then, the cable attachment points B_i is always located below A_i and, apart from special configurations, $\tau_{li_z} < 0$ since the platform exerts a downward force on the cable. Consequently, according to Eq. (27), the maximal cable tension is obtained for $s_i = 0$ that is at point A_i which is the highest cable point. This maximal cable tension is thus the norm of the force $\boldsymbol{\tau}_{0i}$ applied to the cable at A_i

$$\tau_{imax} = \tau_{0i} = \sqrt{\tau_{0i_x}^2 + \tau_{0i_z}^2} \tag{28}$$

where τ_{0i_x} and τ_{0i_z} are the horizontal and vertical components of $\boldsymbol{\tau}_{0i}$ as shown in Fig. 3. According to Eq. (27) with $s_i = 0$, we have:

$$\tau_{imax} = \tau_{0i} = \sqrt{\tau_{li_x}^2 + \left(\tau_{li_z} - \rho g l_i\right)^2} \tag{29}$$

Note that this latter expression of the maximal tension in cable i is valid for the various cable modeling: elastic catenary, inextensible hefty cable model of Section III and the model introduced in Section IV. However, since the values computed for τ_{li_x} and τ_{li_z} change according to the model considered, the maximal tensions computed will also be different for these various models.

D. Inverse kinematics

Solving the inverse kinematics consists here in determining the length of each cable for a given pose of the platform. Once the platform static equilibrium of Eq. (22) is solved, the horizontal components τ_{li_x} are known for all cables. It is then possible to determine the cable lengths l_i by integrating a cable length element over the interval $[0, B_{i_x}]$ of x. Using a symbolic computation software, we obtain:

$$\begin{aligned} l_i &= \int_0^{B_{i_x}} \sqrt{1 + \left(\frac{dz}{dx}\right)^2} dx \\ &= B_{i_x} \frac{c_{1i} k_{1i} - c_{2i} k_{2i} + \ln\left(\frac{c_{1i} + k_{1i}}{c_{2i} + k_{2i}}\right)}{2 r_i}, \end{aligned}$$

where $r_i = \frac{\rho g L_i}{\tau_{li_x}}$, $k_{1i} = \tan\beta_{0i} + {}^{r_i}/_2$, $k_{2i} = \tan\beta_{0i} - {}^{r_i}/_2$, $c_{1i} = \sqrt{1 + k_{1i}^2}$, $c_{2i} = \sqrt{1 + k_{2i}^2}$ and where dz/dx has been obtained from the expression of the cable profile given in Eq. (8).

VI. An Example

Let us consider a large-dimension 6-DOF cable-suspended robot driven by 6 cables. We have chosen a parallel cable-suspended robot having as many cables as DOF so that the results are independent of the choice of a particular distribution of cable forces which is not unique for n-DOF robot using m cables with $m > n$. The geometric characteristics of the robot are presented in Tab. I. They are similar to those of the Robocrane [5], *i.e.*, the robot is composed of a triangular-shaped base with 2 drawing points at each vertex. The platform is also triangular with 2 attachment points at each vertex.

TABLE I

6-DOF parallel robot driven by 6 cables: Coordinates of the drawing points in the global frame $\mathscr{R}_A$ and of the cable attachment points on the platform in the local frame $\mathscr{R}_B$.

	x (m)	y (m)	z (m)		x (m)	y (m)	z (m)
$\boldsymbol{a_1}$	-6.982	-4	8	$\boldsymbol{b_1}$	-0.693	0.4	0
$\boldsymbol{a_2}$	-6.982	-4	8	$\boldsymbol{b_2}$	0	-0,8	0
$\boldsymbol{a_3}$	6.982	-4	8	$\boldsymbol{b_3}$	0	-0.8	0
$\boldsymbol{a_4}$	6.982	-4	8	$\boldsymbol{b_4}$	0.693	0.4	0
$\boldsymbol{a_5}$	0	8	8	$\boldsymbol{b_5}$	0.693	0.4	0
$\boldsymbol{a_6}$	0	8	8	$\boldsymbol{b_6}$	-0.693	0.4	0

The mass of the unloaded platform is 50 kg and the maximal payload is 450 kg, which gives a total maximum-loaded platform of 500 kg. The cable used to withstand these payloads is a steel cable with a diameter of 10 mm. The corresponding linear density ρ_0 is 343 g/m. For a cylindrical desired workspace with a radius of 2.5 m and of 5 m high, we have computed the values of the ratio appearing in the left-hand side of Eq. (13) to verify the validity of the simplified hefty cable model of Section IV. The results in the worst case (minimal payload and maximal cable lengths) are presented in Fig. 4. The figure shows the minimal value of the validity ratio of the 6 cables in the plane $z = 0$ m and for the reference (null) orientation of the unloaded platform. It shows that the ratio is over 1, and thus that the simplified hefty cable model is valid, in almost all the desired workspace.

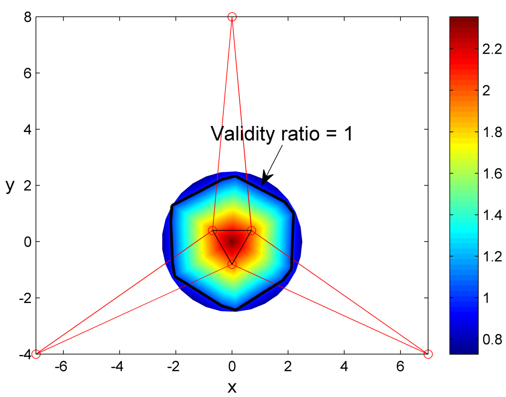

Fig. 4. Value of the validity ratio over the plane $z = 0$ and for a null orientation of a 50 kg platform. The region delimited by the black polygon shown in the figure corresponds to validity ratio values greater than one.

In order to evaluate the errors resulting from the simplifications of the elastic catenary, we compute the cable lengths and cable force components τ_{li_x} and τ_{li_z} across the desired workspace and compare the results for the two cable models, elastic catenary and simplified hefty cable model of Section IV. We compare the cable tensions by considering the maximal relative error on maximal cable tensions. This relative error is defined as follows:

$$\varepsilon_{\tau_{max}}(\boldsymbol{p}) = \max_{i=1..m} \left(\frac{\tau'_{imax} - \tau_{imax}}{\tau_{imax}} \right), \tag{30}$$

where τ'_{imax} is the maximal tension in cable i obtained with the simplified hefty cable model and τ_{imax} the maximal tension in cable i for the elastic catenary model. These maximal tensions are computed by means of Eq. (29). Figure 5 shows this maximal relative error in the plane $z = 0$ for a null orientation and with the unloaded platform.

This figure shows that the maximal relative error is small as it is always lower than 3.3% and is less than 1.5% when the validity ratio is verified. Note that when the maximal payload is considered, the maximal relative error is lower than 0.05% in the plane $z = 0m$ and lower than 0.07% in the plane $z = 5m$. These small errors show that the static analysis proposed in this paper can be accurate enough to be applied to large dimension parallel cable-driven robots.

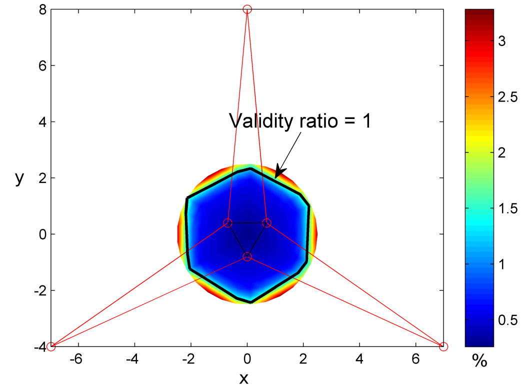

Fig. 5. Maximal relative error (%) on maximal cable tension over the plane $z = 0$ and for a null orientation of a 50 kg platform (simplified hefty cable model versus elastic catenary model)

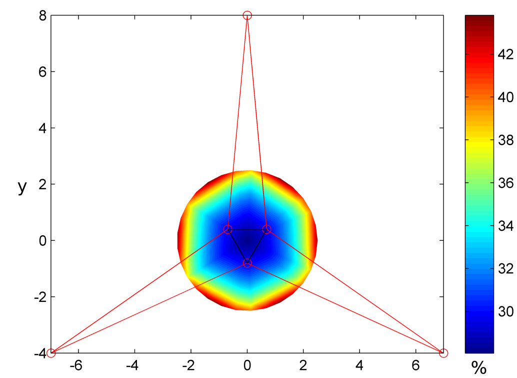

Fig. 6. Maximal relative error (%) on maximal cable tension over the plane $z = 0$ and for a null orientation of a 50 kg platform (massless model versus elastic catenary model)

Finally, when comparing the massless model with the elastic catenary model, the maximal relative error is far much larger and can reach more than 40% as illustrated in Fig. 6. This shows that taking the cable mass into accout for the robot considered in this example is necessary.

About the errors on the cable lengths, the use of the simplified hefty model is almost equivalent to the use of the catenary model assuming inextensible cables. Both models lead to a small maximal relative error of about 0.1% with respect to the elastic catenary model. Considering massless cables would have led to a relative error of up to 3%, which remains reasonable compared to the error on the cable tensions.

VII. Conclusion

This paper proposed a new simplified static analysis of parallel robots driven by inextensible cables of non-negligible mass. It is based on a known simplified hefty cable static modeling which, to the best of our knowledge, has never been applied to parallel cable-driven robots. This simplified cable modeling provides an explicit expression of the profile of a cable subjected to the combined action of its own weight and of forces applied to its extremities. As shown in the paper, this leads to an original formulation of the mobile platform static equilibrium. This formulation is similar to the one obtained with a massless cable model in the sense that a linear relationship between (components of) the forces in the cables and the external wrench applied to the platform is obtained. Thereby, the extension of wrench-based analysis and design techniques devised for parallel robots driven by massless cables can be extended to cases in which cable mass is to be accounted for.

The static analysis introduced in this paper is valid under some assumptions and has been derived by considering the cables as being inextensible. An example has been used to show that these assumptions can be realistic. But clarifying the range of validity of the static analysis proposed in the paper is necessary and is part of our future works. Finally, other of our future works include the study of large underconstrained parallel robots driven by a number of cables greater than their number of DOF, the main issue being the distribution of forces among the cables which is not unique even for given platform external loading conditions.

VIII. ACKNOWLEDGMENTS

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