DD2370 Computational Methods for Electromagnetics

Weighted Residual Formulation of Finite Element Method

Stefano Markidis, KTH Royal Institute of Technology

Problem to be Solved

- We start by giving the general recipe for how to solve a differential equation by the FEM. The equation is written as

$$L[f] = s$$

- where ***L* is an operator**, *s* the source, and ***f* the unknown function** to be computed in the region Ω.

1. Discretization

- Subdivide the solution domain into cells, or ***elements***. For example, a 2D domain can be subdivided into triangles or quadrilaterals.

2. Approximation of solution with Basis Function

Approximate the solution by an expansion in a finite number of *basis functions*

$$f(\boldsymbol{r}) \approx \sum_{i=1}^{n} f_i \varphi_i(\boldsymbol{r})$$

Where **f_i are (unknown) coefficients** multiplying the basis functions ϕ_i.

The basis functions ϕ_i are generally **low-order polynomials that are nonzero only in a few adjacent elements**.

3. Form Residual

Form the **residual**

$$\boxed{r = L[f] - s}$$

which we want to make as small as possible.

In general, **it will not be zero pointwise**, but we require it to be zero in the so-called weak sense by setting a **weighted average of it to zero.**

4. Introduce Weighting Functions – Galerkin Method

- Choose *test*, or *weighting*, functions w_i, i = 1,2,…,n (as many as there are unknown coefficients) for weighting the residual *r*.
- Often, the weighting functions are the same as the basis functions
  - $w_i = \phi_i$, and this method is then called **Galerkin's method**.

5. Solve the equation of Weighted Residual

- Set the weighted residuals to zero and solve for the unknowns f ; i.e., solve the **set of equations**

$$\langle w_i , r \rangle = \int_{\Omega} w_i \, r \, d\Omega = 0, \; i = 1, 2, \ldots, n$$

“Finite Element”

- In mathematical definitions, the term *finite element* usually refers to an element (e.g., a triangle) together with a polynomial space defined in this element (e.g., the space of linear functions) and a set of **degrees of freedom** defined on this space (e.g., **the values of the linear functions in the corners (nodes) of the triangle**).
- This definition is seldom used in electrical engineering, where one tends to focus on the basis functions used to expand the solution instead.