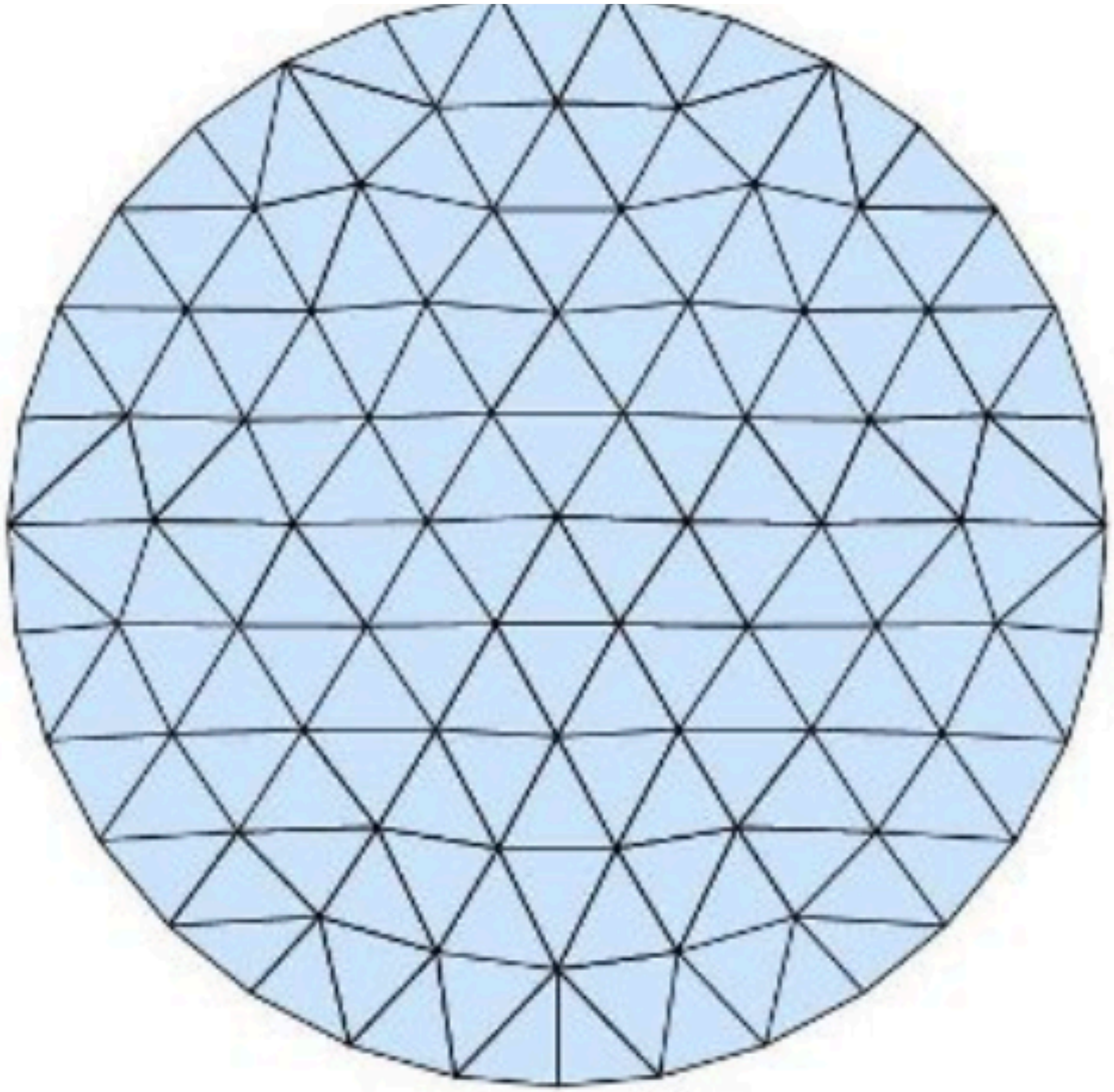
DD2370 Computational Methods for Electromagnetics

Introduction to FEM

Stefano Markidis, KTH Royal Institute of Technology

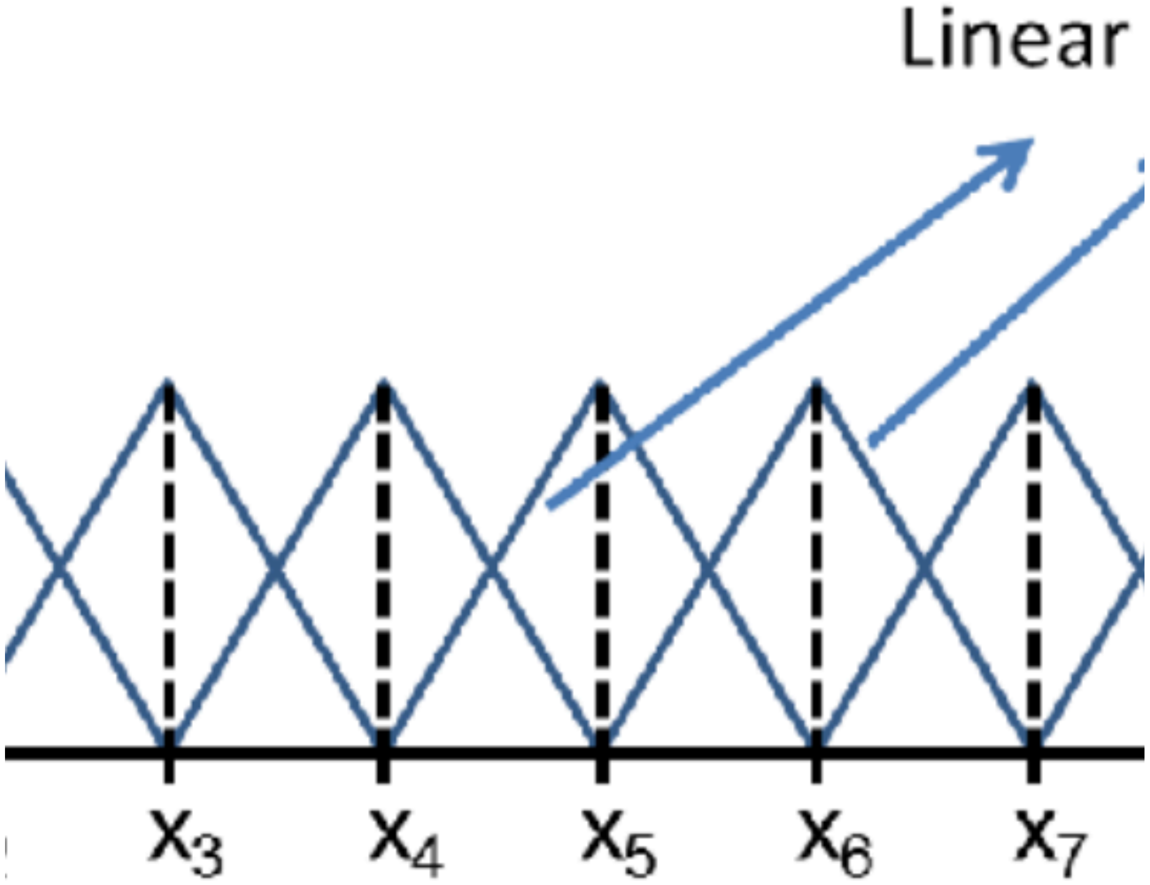
Finite-Element Method in CEM

- In the FEM, the domain is subdivided into small subdomains of simple shape and, as an initial example, we may consider a **circular domain** in two dimensions that is subdivided into **triangular subdomains**.
- The collection of triangular subdomains cover the original circular domain and the triangular subdomains do not overlap each other.
    - Thus, the circular boundary is approximated by a polygon that consists of the outermost edges of the triangular subdomains.



Solution expressed as low-order polynomials

- The field solution is expressed in terms of a low-order polynomial (for example a linear polynomial) on each of these subdomains
    - we have a ***piecewise*** **low-order polynomial** representation of the field.
- In general, such a representation of the field is **not flexible enough to be able to exactly fulfill the differential equation** and its boundary conditions in a pointwise manner.
    - The FEM relaxes this requirement slightly and, instead, it attempts to find a field solution that fulfills the differential equation and its boundary conditions in an **averaged sense (= weak form)**.

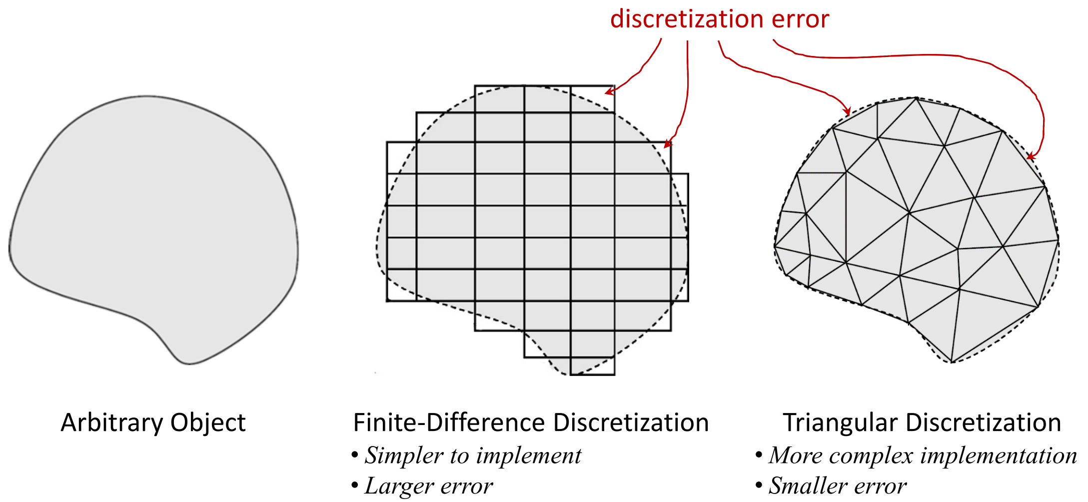


“Weak-form” Solution

- There are different approaches to how to construct this relaxed fulfillment of the differential equation and its boundary condition and two important methods are
    1. To **set the weighted average of the residual to zero** and
    2. To exploit a variational method to find a stationary point of a quadratic form.
- For a FEM that works correctly, the approximate solution tends to the exact solution as the size of the subdomains tends to zero and, consequently, the number of subdomains tend to infinity.

Advantages of FEM

- A very strong point of the FEM **is its ability to deal with complex geometries**.
  - Typically, this is done using **unstructured grids**, which are commonly referred to as (unstructured) meshes. These meshes may consist of triangles in two dimensions and tetrahedra in three dimensions.
- Unstructured meshes with, for instance, tetrahedra allow good representations of curved objects, **which are hard to represent on the Cartesian grids used by finite difference methods.**



Disadvantages of FEM

- A disadvantage of the FEM, compared to the FDTD, is that explicit formulas for updating the fields in time-domain simulations cannot be derived in the general case.
- Instead, **a linear system of equations has to be solved in order to update the fields**.
- Consequently, provided that the same number of cells are used for the two methods, **the FEM requires more computer resources, both in terms of CPU time and memory.**

Discretizing Differential and Integral Equations

- Normally, the FEM is used to **solve differential equations**. However, it is also possible to apply the FEM to integral equations, where the unknown field is part of the integrand.
  - In CEM, the **FEM applied to integral equations** is referred to as the ***Method of Moments*** and this technique is discussed in the next module of this course.