DD2370 Computational Methods for Electromagnetics

Boundary Conditions for Open Regions

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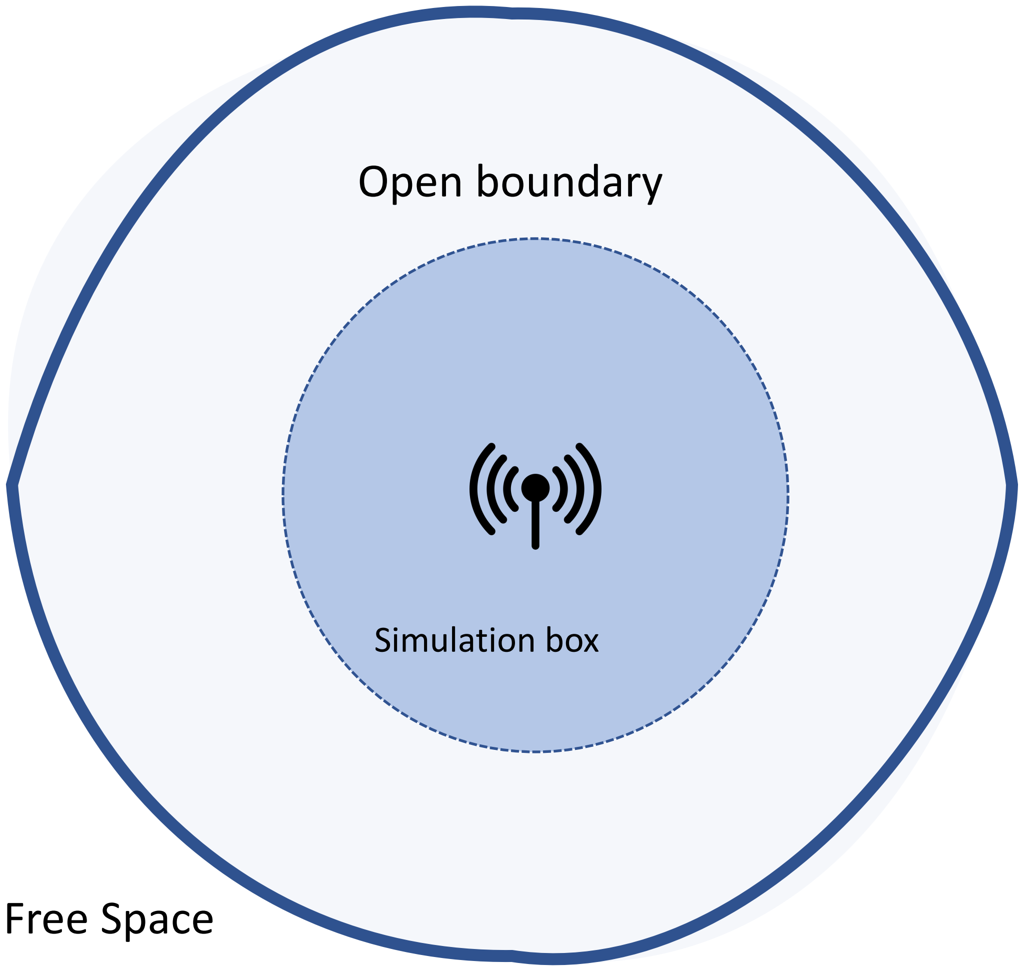
Motivation for Open Boundary Conditions

The FDTD is often applied to <u>microwave problems</u> such as calculation of:

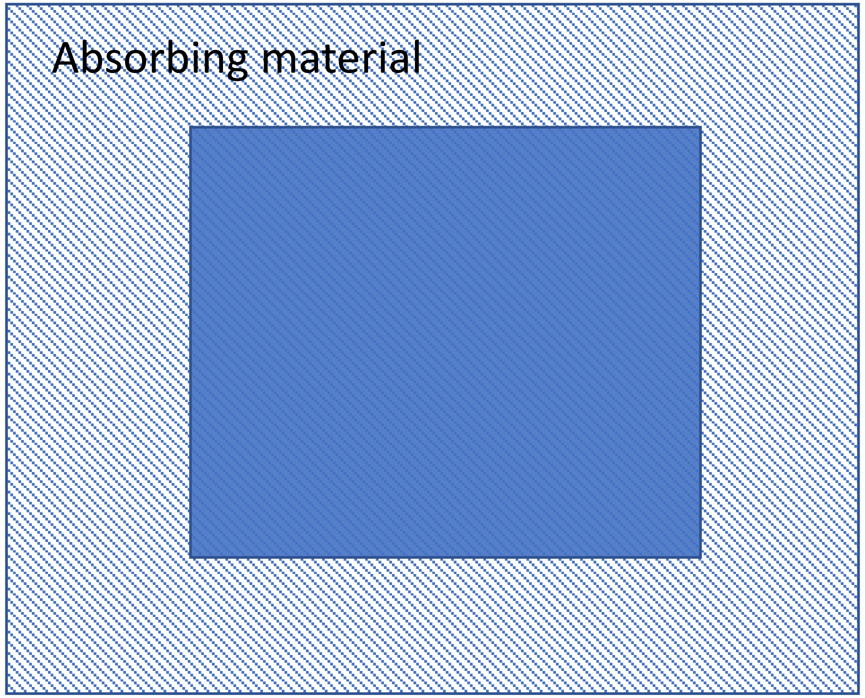
- Radiation patterns from antennas
- Radar cross sections (RCS)

- These problems involve *<u>open regions</u>*
  - the computational **domain extends to infinity**
- It is not feasible to discretize an infinite region
  - Special boundary conditions can be applied to terminate the computational region.



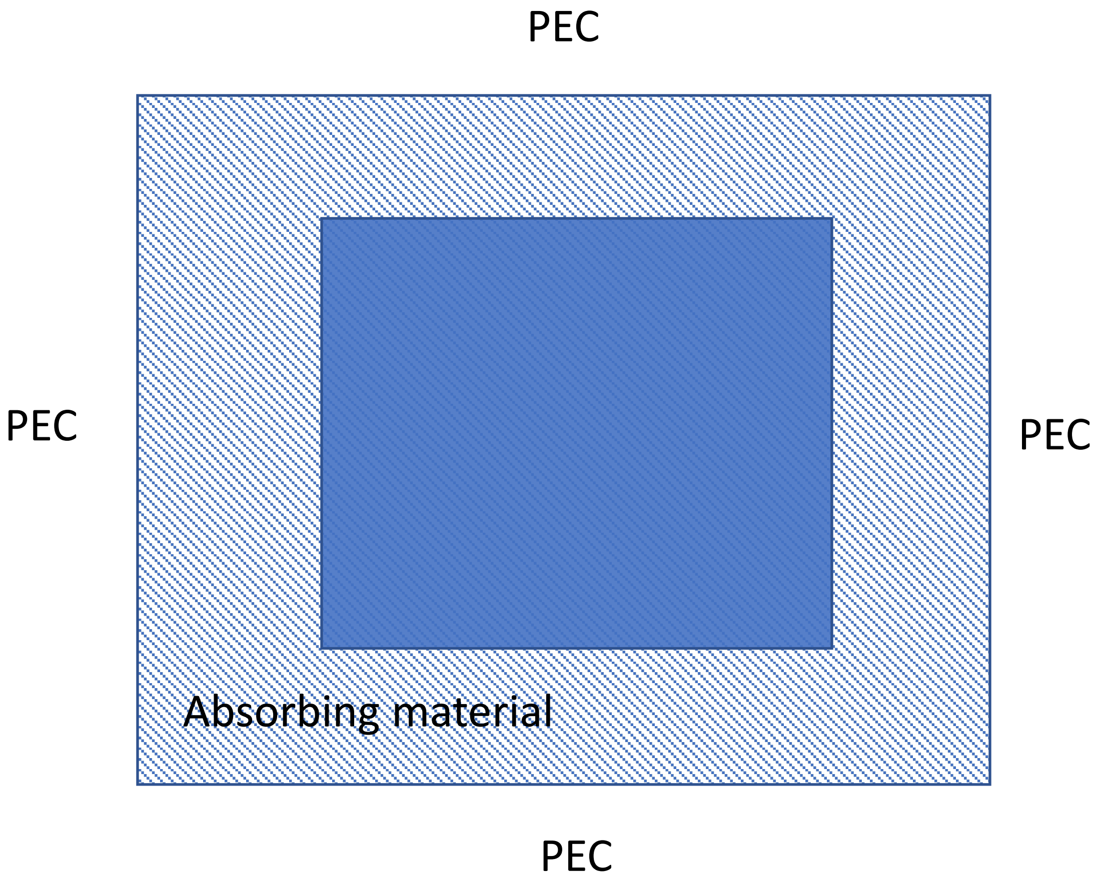
Absorbing Boundary Conditions

Such boundary conditions serve to absorb outgoing waves, and are called ***absorbing boundary conditions*** (ABC).



The Perfectly Matched Layer to Damp Waves

- The *perfectly matched layer* (PML) was invented by Berenger
- The PML is a layer of <u>artificial material surrounding the computational region</u>
  - designed to damp waves propagating in the normal direction
  - If the waves are sufficiently damped out in the absorbing layer, very little reflection will occur at this PEC surface
- The region is then **terminated by a PEC**.
- The **thicker the absorbing layer** is, the more efficient is the damping

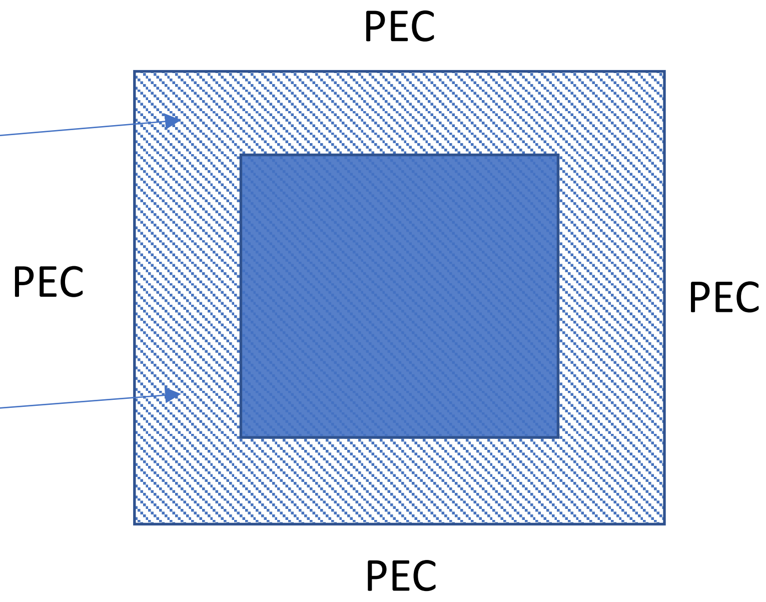


Absorbing Layer – Electric and Magnetic Conductivities

The basic idea behind the method is to introduce both an electric conductivity and a magnetic conductivity in the absorbing layer

$$\epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} + \sigma \boldsymbol{E} = \nabla \times \boldsymbol{H},$$

$$\mu_0 \frac{\partial \boldsymbol{H}}{\partial t} + \sigma^* \boldsymbol{H} = -\nabla \times \boldsymbol{E}$$



Wave Impedance in the Absorbing Material

We define a wave impedance as the ratio of the transversal electric and magnetic fields for the artificial material

$$Z_{PML} = \left(\frac{\mu_0 + \boxed{\sigma^*/j\omega}}{\epsilon_0 + \boxed{\sigma/j\omega}} \right)^{1/2}$$

For a wave that is normally incident on such a layer, the wave reflection coefficient is

$$\Gamma_0 = \frac{Z_0 - Z_{PML}}{Z_0 + Z_{PML}}$$

Where $Z_0 \equiv \sqrt{\mu_0/\epsilon_0}$

Free space impedance

Setting-up Zero Reflection in the PML

if the magnetic and electric conductivities are related as

$$\boxed{\frac{\sigma^*}{\mu_0} = \frac{\sigma}{\epsilon_0}} \Rightarrow \cancel{\Gamma_0 = \frac{Z_0 - Z_{PML}}{Z_0 + Z_{PML}}}$$

we get $Z_{PML} = Z_0$, and there is no reflection *at any frequency*.

Oblique Incidence - PML

- For oblique incidence and it is harder to avoid reflection
- Berenger found a trick that achieves this.
- We split each component of E and H into two parts.
- For example:
  - $E_x = E_{xy} + E_{xz}$ according to the direction of the curl operator that contributes to $\partial E/\partial t$
- Then, one uses nonzero σ and σ* only for the derivative in the direction normal to the absorbing layer.

Oblique Incidence - Example

- As an example, let us assume that the PML has ***z*** as the normal direction.
- The two equations for Ex and Ey are split into four
- The evolution equation for E_z is not modified for a layer with ***z*** as normal.
- The layer modifies the propagation only in the *z*-direction, normal direction of the PML, not in the tangential directions x and y.
  - Therefore, no reflection occurs even for waves obliquely incident on the PML

$$\epsilon \frac{\partial E_{xy}}{\partial t} = \frac{\partial (H_{zx} + H_{zy})}{\partial y},$$

$$\epsilon \frac{\partial E_{xz}}{\partial t} = -\frac{\partial (H_{yz} + H_{yx})}{\partial z} - \sigma_z E_{xz},$$

$$\epsilon \frac{\partial E_{yz}}{\partial t} = \frac{\partial (H_{xy} + H_{xz})}{\partial z} - \sigma_z E_{yz},$$

$$\epsilon \frac{\partial E_{yx}}{\partial t} = -\frac{\partial (H_{zx} + H_{zy})}{\partial x}.$$