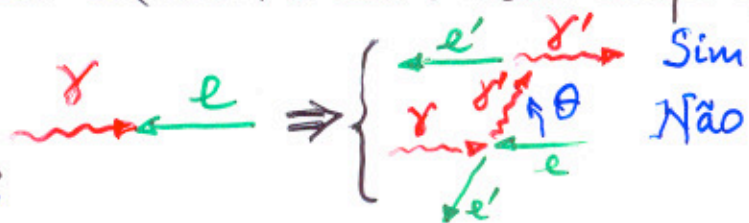


## Distribuição angular de Compton

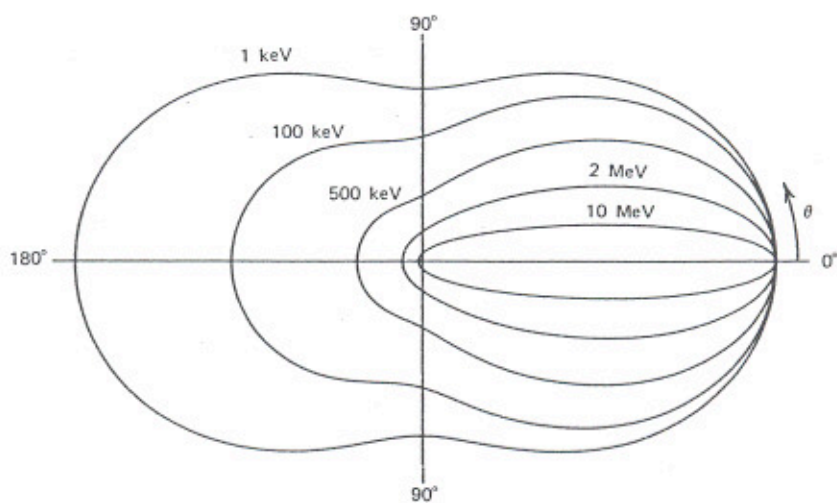
A dependência angular do fóton difundido na interação de Compton é dada pela expressão de Klein-Nishina, obtida no quadro da Mecânica Quântica: há supressão a grandes ângulos devido à conservação da helicidade do fóton:



$$\frac{d\sigma}{d\Omega} = Zr_0^2 \left( \frac{1}{1 + \alpha(1 - \cos \theta)} \right)^2 \left( \frac{1 + \cos^2 \theta}{2} \right) \left( 1 + \frac{\alpha^2(1 - \cos \theta)^2}{(1 + \cos^2 \theta)[1 + \alpha(1 - \cos \theta)]} \right)$$

em que  $\alpha = \frac{E_\gamma}{m_e c^2}$  e  $r_0 = \frac{e^2}{m_e c^2} = 2.818 \text{ fm}$   
(raio clássico do  $e^-$ )

Graficamente:



A medida que  $E_\gamma \ll m_e c^2$ , a dependência angular torna-se mais simétrica e, no limite das baixas energias, integrando em  $\theta$ , obtém-se:

$$\sigma_C = \int \frac{d\sigma}{d\Omega} d\Omega = r_0^2 \int_0^\pi 2\pi \sin \theta \frac{1 + \cos^2 \theta}{2} d\theta = \frac{8}{3} \pi r_0^2$$

a seção eficaz de Thomson.  $= 0,665 \text{ barn} \equiv \sigma_{TK}$

# EXPERIMENT 9.1

## The Measurement of $E_{\gamma'}$ vs $\theta$

### Discussion

The Compton Scattering apparatus that will be used for this experiment is shown in fig. 9.4. The distance  $R_1$  in the figure is about 20 cm. For the purpose of this discussion, let us assume that the  $^{137}\text{Cs}$  source has an activity of 10 mCi. The intensity ( $I$ ) of photons at the aluminum scattering cylinder is then:

$$I = \frac{10 \times 10^{-3} \text{ Ci} \times 3.7 \times 10^{10} \text{ d/s}}{4 \pi (20 \text{ cm})^2} \quad (10)$$

$$= 7.36 \times 10^4 \text{ } \gamma\text{'s/cm}^2\text{s}$$

As we will see, this intensity of  $\gamma$ 's at the scattering sample will give a  $\gamma'$  counting rate in the movable detector that is barely adequate to measure a pulse height spectrum in a time of 5 minutes. The point here is that the  $^{137}\text{Cs}$  needs to have an activity of from 10 to 30 mCi for us to be able to complete this experiment in a two hour laboratory period.

**Caution:** Sources of this activity should be loaded into the lead housing by the instructor using tongs, or special handling rods, that are provided by the  $^{137}\text{Cs}$  source manufacturer. Once the source has been loaded in its housing, it can safely be used by the students following the procedures outlined by the instructor.

### Experimental Procedure

1. Use the techniques that were outlined in Experiment 2 NaI(Tl) spectroscopy to calibrate the MCA. The  $^{137}\text{Cs}$  gamma should appear in approximately channel 350. Other sources that can be used for this calibration are  $^{22}\text{Na}$ ,  $^{65}\text{Zn}$ ,  $^{60}\text{Co}$  and  $^{57}\text{Co}$ . Plot the energy vs channel number from the data points.
2. Determine the resolution of the detector for all of the gamma lines for the above source that is used.
3. Place the  $\frac{1}{2}$  inch diameter Aluminum Scattering Rod in the scattering position. Set the movable detector at an angle of 20 degrees and accumulate a spectra for a period of time long enough to obtain a  $(\Sigma-\beta)$  of 3000 counts in the photopeak. Record the time, calculate  $(\Sigma-\beta/t)$  and enter this figure in Table 9.1. From the calibra-

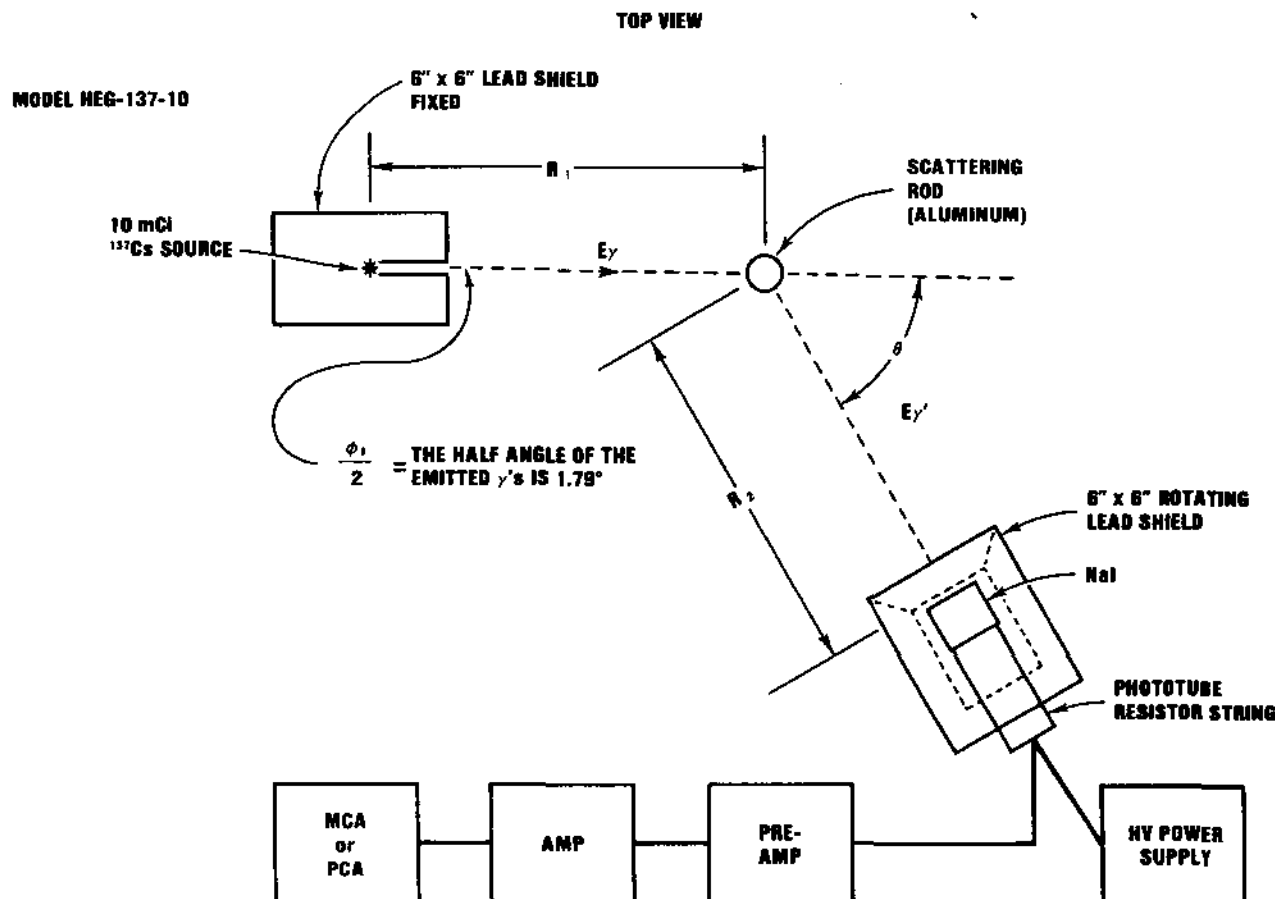


Figure 9.4. Plan view of Compton scattering apparatus showing the electronics for measuring  $E_{\gamma'}$  vs  $\theta$ .

tion curve determine  $E_{\gamma'}$  and record it in the table. Repeat the measurement for the rest of the angles in the data table.

From eq. (8), calculate  $E_{\gamma'}$  for all of the angles listed in Table 9.1. Fill in  $1/E_{\gamma'}$  (theory) and the  $(1-\cos\theta)$  entry in the table. On a piece of linear graph paper, plot  $1/E_{\gamma'}$  (theory) y axis vs  $(1-\cos\theta)$ . From eq. (8), it can be seen:

$$\frac{1}{E_{\gamma'}} = \frac{1}{E_{\gamma}} + \frac{1}{m_0 c^2} (1-\cos\theta) \quad (10)$$

This equation is of the form:

$$y = mx + b \quad (11)$$

where

$$y = \frac{1}{E_{\gamma'}}$$

$$m = \frac{1}{m_0 c^2} = \frac{1}{0.511 \text{ MeV}} = 1.96 \text{ MeV}^{-1}$$

$$b = \frac{1}{E_{\gamma}} = 1.51 \text{ MeV}^{-1}$$

The "theory" plot of  $1/E_{\gamma'}$  vs  $(1-\cos\theta)$  should therefore be a straight line.

*Use this function to make a fit to the measured  $(1/E_{\gamma'})$  points with the estimated error bars for  $1/E_{\gamma'}$ .*

Figure 9.5 shows typical experimental results.

It should be noticed from the experimental data (on fig. 9.5) that the fundamental rest mass ( $m_0$ ) can be determined from the graph. Instead of plotting the straight line theory on the figure, fit a straight line to your experimental points. The experimental slope thus obtained is  $(1/m_0 c^2)$ . This simple experiment thus yields one of the most fundamental quantities in nuclear science "the rest mass of the electron."

Table 9.1 Data Table for Compton Scattering:  $E_{\gamma'}$  vs  $\theta$ , and  $(d\sigma/d\Omega)_{\text{Compton}}$

Run No.	Peak Channel	$\theta$ Degrees	$E_{\gamma'}$ (Calculated)	$E_{\gamma'}$ (Measured)	$\left(\frac{\Sigma\beta}{t}\right)$	$\left(\frac{d\sigma}{d\Omega}\right)$ Measured	$\left(\frac{d\sigma}{d\Omega}\right)$ Theory	$\frac{1}{E_{\gamma'}}$ MeV <sup>-1</sup> Theory	$(1-\cos\theta)$
1		20							
2		40							
3		60							
4		80							
5		100							
6		120							
7		140							
8		160							

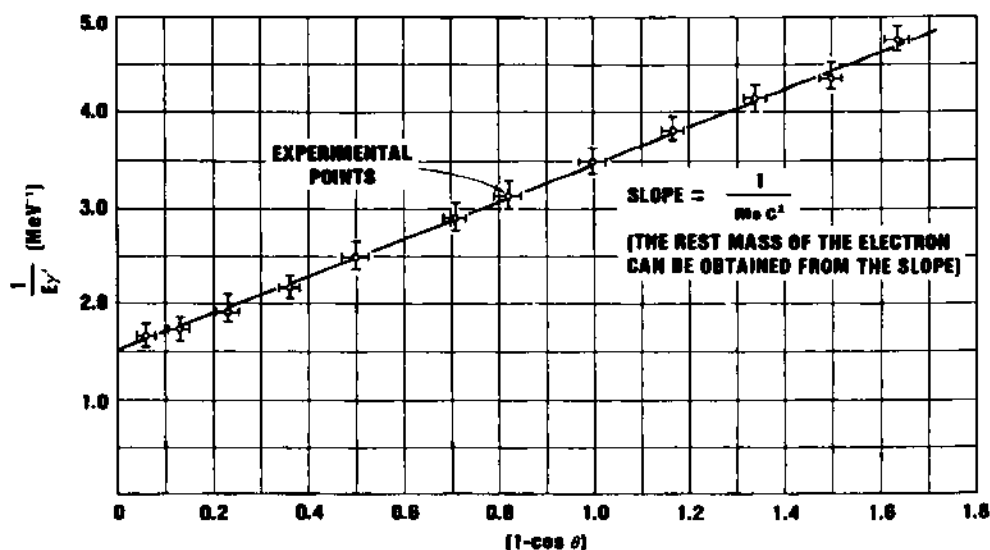


Figure 9.5. Experimental verification of the Compton scattering equations (see eq. 10).

**EXPERIMENT 9.2**

# The Differential Cross Section for Comptons Scattering

## Discussion

The theoretical scattering cross-section for the Compton Interaction can be written (see refs. 2 and 3):

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{r_0^2}{2} \left[ \frac{1 + \cos^2\theta}{[1 + \gamma(1 - \cos\theta)]^2} \right] \quad (12)$$

$$\times \left[ 1 + \frac{\gamma^2 (1 - \cos\theta)^2}{(1 + \cos^2\theta) [1 + \gamma(1 - \cos\theta)]} \right] \frac{\text{cm}^2}{\text{Sr}}$$

where

$$\gamma = \frac{E_\gamma}{m_0 c^2} = 1.29 \text{ for the } 0.662 \text{ E}_\gamma \text{ from } ^{137}\text{Cs}$$

and

$$r_0 = 2.82 \times 10^{-13} \text{ cm (classical electron radius)}$$

Figure 9.6 shows a plot of eq. (12) as a function of  $\theta$ . In this experiment, the measured data points from Table 9.1 will be compared to this theoretical expression.

**Exercise A** The data for this experiment was collected in experiment 9.1. It was recorded in Table 9.1 as  $(\Sigma - \beta)/t$  for  $E_{\gamma'}$ .

Figure 9.7 shows  $E_{\gamma'}$  at  $\theta = 60$  degrees. At larger angles  $E_{\gamma'}$  decreases in accordance with eq. (10). At 130 degrees  $E_{\gamma'}$  would be 211 keV. The values of  $(\Sigma - \beta)$  recorded in Table 9.1 have to be corrected for the intrinsic peak efficiency of the detector. The student should review Experiment 2 for this procedure. The corrected sum is given by:

$$\left(\frac{\Sigma - \beta}{t}\right)_{\text{corrected}} = \frac{1}{\epsilon_p} \left(\frac{\Sigma - \beta}{t}\right) \quad (13)$$

where  $\epsilon_p$  is the intrinsic peak efficiency for  $E_{\gamma'}$ .

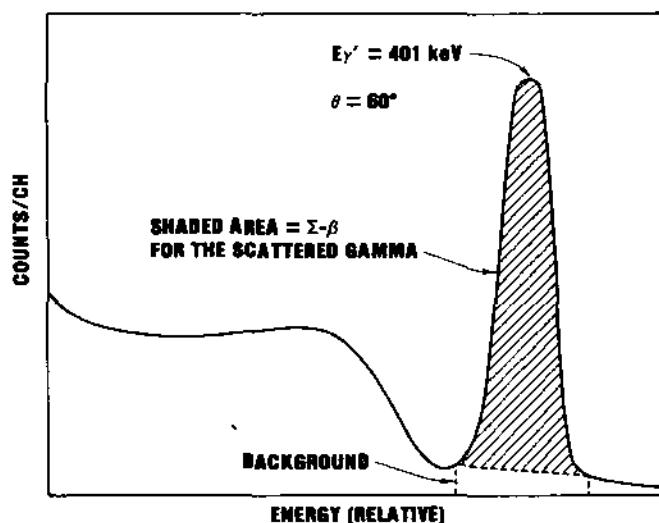


Figure 9.7. NaI(Tl) pulse height spectrum of Compton scattered gammas at  $\theta = 60$  degrees from  $^{137}\text{Cs}$ . (Note: At  $\theta = 0$  degrees, the scattered peak would have the full energy of the  $^{137}\text{Cs}$  source 662 keV).

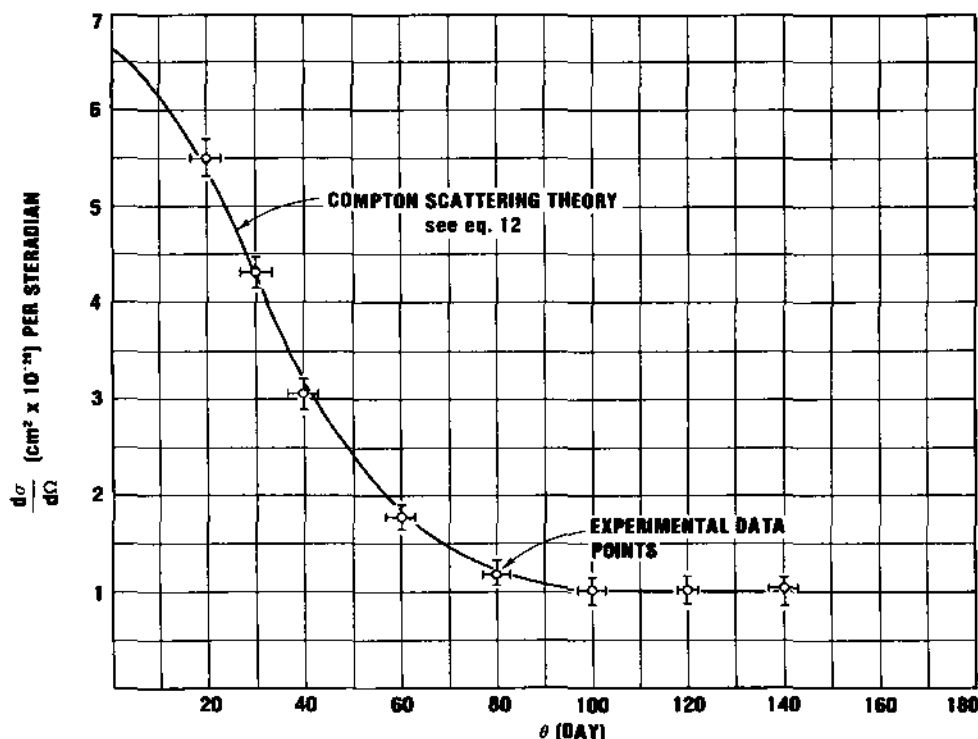
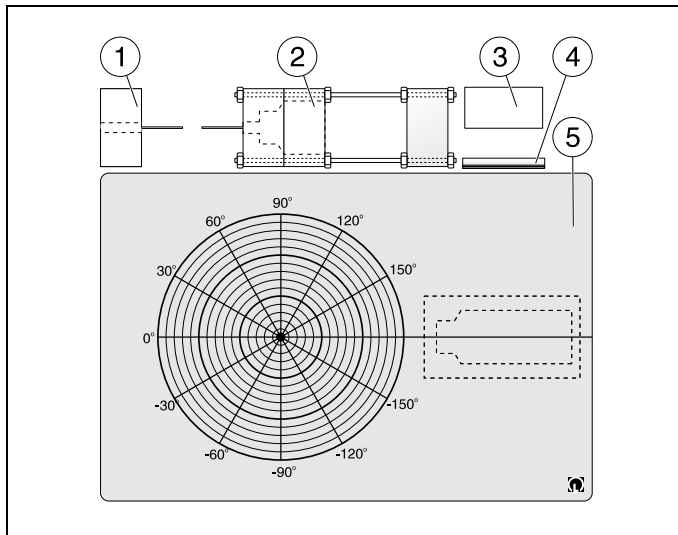


Figure 9.6. Theoretical Compton scattering cross-section vs angle.

11/96-Pr-Sf-



## Gebrauchsanweisung Instruction Sheet

559 800

## Gerätesatz Comptonstreuung Apparatus Set Compton

Fig. 1

Der Gerätesatz Comptonstreuung dient zur qualitativen und quantitativen Beobachtung des Comptoneffektes. Die Comptonstreuung beleuchtet insbesondere den Teilchencharakter der  $\gamma$ -Strahlung und spielt daher als grundlegendes Experiment der Quantenmechanik eine wichtige Rolle im Dualismus Welle - Teilchen. Mit dem experimentellen Aufbau lassen sich die Wellenlängenänderungen der gestreuten  $\gamma$ -Strahlung in Abhängigkeit vom Steuwinkel sowie deren Intensität bestimmen. Damit ist dann auch der differentielle Wirkungsquerschnitt der Messung zugänglich.

The Compton scattering equipment set makes possible qualitative and quantitative observation of the Compton effect. Compton scattering illustrates the particle character of  $\gamma$  radiation particularly well, and thus plays an important role as an introductory experiment to quantum mechanics and the dualism of wave and particle. The experiment setup allows you to measure the changes in the wavelength of the scattered  $\gamma$  radiation as a function of the scattering angle and intensity. This in turn permits determination of the differential effective cross-section.

### 1 Gerätebeschreibung, Lieferumfang, technische Daten (siehe Fig. 1)

- ① Probenhalter Øi 12 mm, mit Kollimator
- ② Detektorhalter mit Bleiabschirmung
- ③ bewegliche Zusatzabschirmung
- ④ Aluminiumstreuer
- ⑤ Experimentierplatte mit Winkelskala und Polarkoordinaten

- Zu ① Probenhalter als Bleiziegel mit zentrischer Bohrung Øi 12 mm, zur Aufnahme der radioaktiven Präparate vorbereitet; zusätzliches Sackloch, zur Aufnahme eines der mitgelieferten Stahlstifte als Richtungszeiger für die Kollimatorbohrung
- Zu ② Detektorhalter mit Bleiabschirmung ergibt definierte Eintrittsrichtung der  $\gamma$ -Strahlung, ausgelegt auf den Szintillationszähler (559 90) in Verbindung mit der Detektor-Ausgangsstufe (559 91), zusätzliches Sackloch, zur Aufnahme eines der mitgelieferten Stahlstifte als Richtungsanzeiger für das Detektorfenster
- Zu ③ bewegliche Zusatzabschirmung, Bleiziegel dient insbesondere bei kleinen Streuwinkeln und geringen Abständen Quelle - Streuer - Detektor zur Reduzierung der Intensität der ungestreuten  $\gamma$ -Strahlung.
- Zu ④ Aluminiumstreuer, zylindrischer Reinaluminiumstab als Streuzentrum.

### 1 Equipment description, scope of delivery, technical data; (see Fig. 1)

- ① Sample holder, i.d. 12 mm, with collimator
- ② Detector holder with lead shielding
- ③ Moveable additional shielding
- ④ Aluminum scatterer
- ⑤ Experiment panel with angular scale and polar coordinates

- ① Sample holder as lead block with centered hole, 12 mm dia., for accommodating radioactive preparations. Additional blind hole for inserting one of the steel pins (included in scope of delivery) as directional indicator for collimator hole.
- ② Detector holder with lead shielding for defined direction of incoming  $\gamma$  radiation, designed for the scintillation counter (559 90) in conjunction with the detector output stage (559 91). Additional blind hole which for inserting one of the steel pins (included in scope of delivery) as directional indicator for detector window.
- ③ Moveable additional shielding: lead block serves to reduce the intensity of unscattered  $\gamma$  radiation, particularly for small scattering angles and short distances between source, scatterer and detector.
- ④ Aluminum scatterer, cylindrical pure aluminum rod as center of scattering.

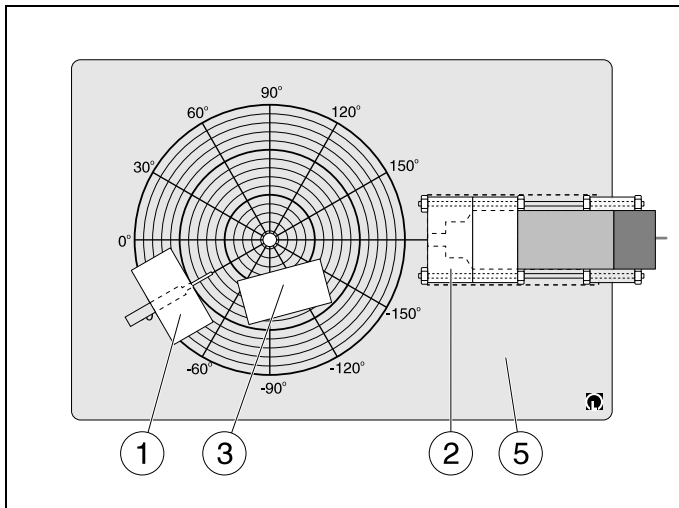


Fig. 2

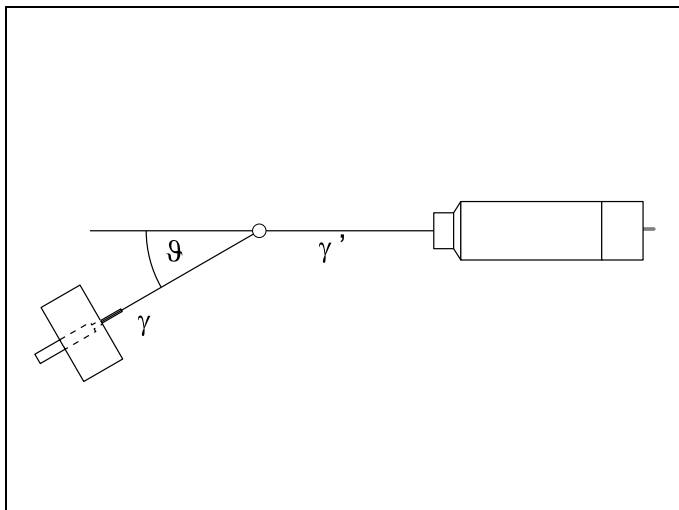


Fig. 3

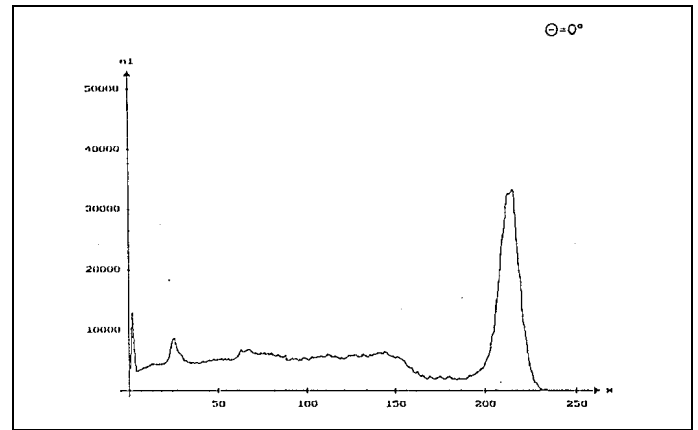


Fig. 4.1

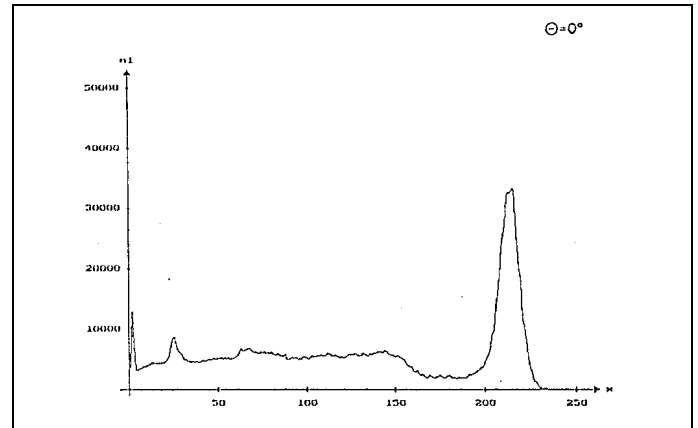


Fig. 4.2

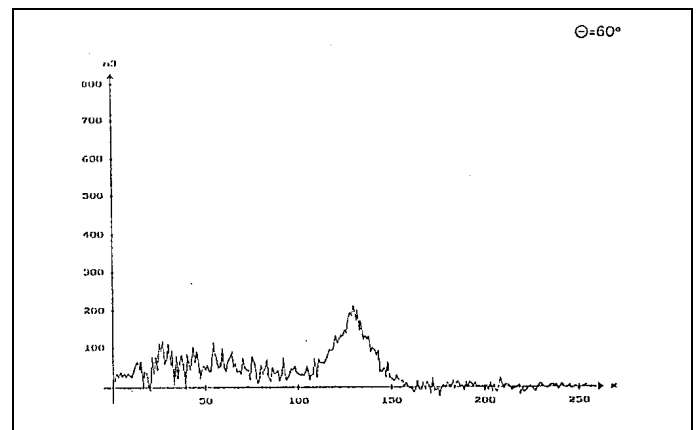


Fig. 4.3

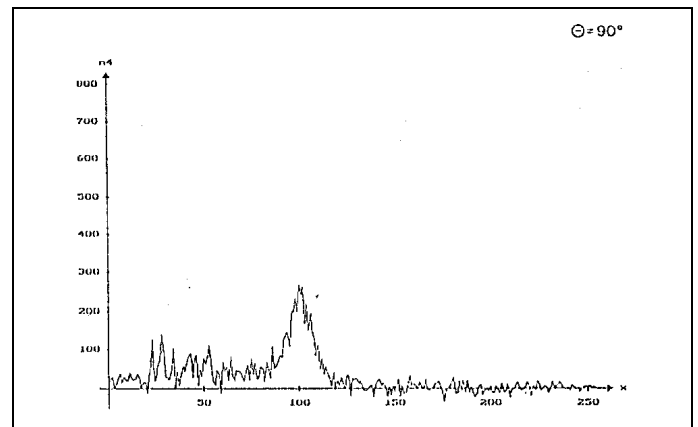


Fig. 4.4

Fig. 4  
Mit VKA-CASSY aufgenommene Spektren

Spectra recorded with MCA-CASSY

Fig. 4.1  
 $^{137}\text{Cs}$ -Spektrum bei  $\vartheta = 0^\circ$

$^{137}\text{Cs}$  spectrum at  $\vartheta = 0^\circ$

Fig. 4.2 - 4.4  
Differenzspektren zum Compton-Effekt mit  $^{137}\text{Cs}$  (559 809) für verschiedene Winkel  $\vartheta$ , ermittelt aus den mit und ohne Aluminium aufgenommenen Spektren

Differential spectra for Compton effect with  $^{137}\text{Cs}$  (559 809) for various angles  $\vartheta$ , determined from the spectra recorded with and without aluminium