Lista 1

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1.(a)
$$A+2B=\begin{pmatrix}1 & 0\\ 3 & 7\end{pmatrix}+2\begin{pmatrix}0 & 5\\ 2 & -2\end{pmatrix}=\begin{pmatrix}1 & 0\\ 3 & 7\end{pmatrix}+\begin{pmatrix}0 & 10\\ 4 & -4\end{pmatrix}=\begin{pmatrix}1 & 10\\ 7 & -11\end{pmatrix}$$

(b)
$$AB-BA=\begin{pmatrix}1 & 0\\ 3 & 7\end{pmatrix}\begin{pmatrix}0 & 5\\ 2 & -2\end{pmatrix}-\begin{pmatrix}0 & 5\\ 2 & -2\end{pmatrix}\begin{pmatrix}1 & 0\\ 3 & 7\end{pmatrix}=$$

$$\begin{pmatrix}1.0+0.2 & 1.5+0.-2\\ 3.0+7.2 & 3.5+7.-2\end{pmatrix}-\begin{pmatrix}0.1+5.3 & 0.0+5.7\\ 2.1-2.3 & 2.0-2.7\end{pmatrix}=$$

$$\begin{pmatrix}0 & 5\\ 14 & 1\end{pmatrix}-\begin{pmatrix}15 & 35\\ -4 & -14\end{pmatrix}=\begin{pmatrix}-15 & -30\\ 18 & 15\end{pmatrix}$$

(c) Não é possível calcular essa expressão, pois as matrizes C e D têm ordens distintas.

(d)
$$2D^t-3E^t=2\begin{pmatrix}-3 & 2 & 0\\ 1 & 1 & 4\\ -2 & 0 & 2\end{pmatrix}^t-3\begin{pmatrix}2 & 4 & -3\\ -1 & 0 & -4\\ -6 & 0 & -1\end{pmatrix}^t=2\begin{pmatrix}-3 & 1 & -2\\ 2 & 1 & 0\\ 0 & 4 & 2\end{pmatrix}-$$

$$3\begin{pmatrix}2 & 1 & -6\\ 4 & 0 & 0\\ -3 & -4 & -1\end{pmatrix}=\begin{pmatrix}-6 & 2 & -4\\ 4 & 2 & 0\\ 0 & 8 & 4\end{pmatrix}-\begin{pmatrix}6 & 3 & -12\\ 12 & 0 & 0\\ -9 & -12 & -3\end{pmatrix}=\begin{pmatrix}-12 & -1 & 8\\ -8 & 2 & 0\\ 9 & 20 & 7\end{pmatrix}$$

(e)
$$D^2+DE=\begin{pmatrix}-3 & 2 & 0\\ 1 & 1 & 4\\ -2 & 0 & 2\end{pmatrix}\begin{pmatrix}-3 & 2 & 0\\ 1 & 1 & 4\\ -2 & 0 & 2\end{pmatrix}+\begin{pmatrix}-3 & 2 & 0\\ 1 & 1 & 4\\ -2 & 0 & 2\end{pmatrix}\begin{pmatrix}2 & 4 & -3\\ -1 & 0 & -4\\ -6 & 0 & -1\end{pmatrix}=$$

$$\begin{pmatrix} -3.-3+2.1+0.-2 & -3.2+2.1+0.0 & -3.0+2.4+0.2 \\ 1.-3+1.1+4.-2 & 1.2+1.1+4.0 & 1.0+1.4+4.2 \\ -2.-3+0.1+2.-2 & -2.2+0.1+2.0 & -2.0+0.4+2.2 \end{pmatrix} +$$

$$\begin{pmatrix} -3.2+2.-1+0.-6 & -3.4+2.0+0.0 & -3.-3+2.-4+0.-1 \\ 1.2+1.-1+4.-6 & 1.4+1.0+4.0 & 1.-3+1.-4+4.-1 \\ -2.2+0.-1+2.-6 & -2.4+0.0+2.0 & -2.-3+0.-4+2.-1 \end{pmatrix} =$$

$$\begin{pmatrix} 11 & -4 & 8 \\ -10 & 3 & 12 \\ 2 & -4 & 4 \end{pmatrix} + \begin{pmatrix} -8 & -12 & 1 \\ -23 & 4 & -11 \\ -16 & -8 & 4 \end{pmatrix} = \begin{pmatrix} 3 & -16 & 9 \\ -33 & 7 & 1 \\ -14 & -12 & 8 \end{pmatrix}$$

(f)

$$C^t A = \begin{pmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{pmatrix}^t \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} = \begin{pmatrix} -2 & 7 \\ 3 & -3 \\ -7 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} = \begin{pmatrix} -2+21 & 0+49 \\ 3-9 & 0-21 \\ -7-6 & 0-14 \end{pmatrix} =$$

$$\begin{pmatrix} 19 & 49 \\ -6 & -21 \\ -13 & -14 \end{pmatrix}$$

(g)

$$E-AC = \begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} \begin{pmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{pmatrix} = \begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix} -$$

$$\begin{pmatrix} -2+0 & 3-0 & -7-0 \\ -6+49 & 9-21 & -21-14 \end{pmatrix} = \begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix}_{3\times 3} - \begin{pmatrix} -2 & 3 & -7 \\ 43 & -12 & -35 \end{pmatrix}_{2\times 3}$$

Não está definida a matriz resultante.

(h)

$$F^tE=\begin{pmatrix}1\\-2\\0\end{pmatrix}^t\begin{pmatrix}2&4&-3\\-1&0&-4\\-6&0&-1\end{pmatrix}=\begin{pmatrix}1&-2&0\end{pmatrix}\begin{pmatrix}2&4&-3\\-1&0&-4\\-6&0&-1\end{pmatrix}=$$

$$\begin{pmatrix}2+2-0 & 4-0+0 & -3+8-0\end{pmatrix}=\begin{pmatrix}4&4&5\end{pmatrix}$$

(i)

$$BCF=\begin{pmatrix}0&5\\2&-2\end{pmatrix}\begin{pmatrix}-2&3&-7\\7&-3&-2\end{pmatrix}\begin{pmatrix}1\\-2\\0\end{pmatrix}=\begin{pmatrix}0+35&0-15&0-10\\-4-14&6+6&-14+4\end{pmatrix}\begin{pmatrix}1\\-2\\0\end{pmatrix}=$$

$$\begin{pmatrix}35&-15&-10\\-18&12&-10\end{pmatrix}\begin{pmatrix}1\\-2\\0\end{pmatrix}=\begin{pmatrix}35+30-0\\-18-24-0\end{pmatrix}=\begin{pmatrix}65\\-42\end{pmatrix}$$

2.(a) $A_{2\times3}B_{3\times4}=C_{2\times4}$ $B_{3\times4}A_{2\times3}$ não está definido

(b) $A_{4\times1}B_{1\times2}=C_{4\times2}$ $B_{1\times2}A_{4\times1}$ não está definido

(c) $A_{1\times2}B_{3\times1}$ não está definido $B_{3\times1}A_{1\times2}$ está definido

(d) $A_{5\times2}B_{2\times3}=C_{5\times3}$ $B_{2\times3}A_{5\times2}$ não está definido

(e) $A_{4\times4}B_{3\times3}$ não está definido $B_{3\times3}A_{4\times4}$ não está definido

(f) $A_{4\times2}B_{2\times4}=C_{4\times4}$ $B_{2\times4}A_{4\times2}$ está definido

(g) $A_{2\times1}B_{1\times3}=C_{2\times3}$ $B_{1\times3}A_{2\times1}$ não está definido

 (h) $A_{2\times2}B_{2\times2}=C_{2\times2}$ $B_{2\times2}A_{2\times2}$ está definido

3.(a) $A=\begin{pmatrix} 3.1-2.1 & 3.1-2.2 & 3.1-2.3 \\ 3.2-2.1 & 3.2-2.2 & 3.2-2.3 \end{pmatrix}=\begin{pmatrix} 3-2 & 3-4 & 3-6 \\ 6-2 & 6-4 & 6-6 \end{pmatrix}$

$A=\begin{pmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \end{pmatrix}$

(b) $B=\begin{pmatrix} 2.1+1 & 1^2-2 & 1^2-3 \\ 2^2-1 & 2.2+2 & 2^2-3 \\ 3^2-1 & 3^2-2 & 2.3+3 \end{pmatrix}=\begin{pmatrix} 2+1 & 1-2 & 1-3 \\ 4-1 & 4+2 & 4-3 \\ 9-1 & 9-2 & 6+3 \end{pmatrix}$

$B=\begin{pmatrix} 3 & -1 & -2 \\ 3 & 6 & 1 \\ 8 & 7 & 9 \end{pmatrix}$

(c) $C=\begin{pmatrix} 1^1 & 2^1 & 3^1 & 4^1 \end{pmatrix}=\begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}$

(d) $D=\begin{pmatrix} 1^2+1^2 & 2.1.2 & 2.1.3 & 2.1.4 \\ 2.2.1 & 2^2+2^2 & 2.2.3 & 2.2.4 \\ 2.3.1 & 2.3.2 & 3^2+3^2 & 2.3.4 \\ 2.4.1 & 2.4.2 & 2.4.3 & 4^2+4^2 \end{pmatrix}=\begin{pmatrix} 1+1 & 4 & 6 & 8 \\ 4 & 4+4 & 12 & 16 \\ 6 & 12 & 9+9 & 24 \\ 8 & 16 & 24 & 16+16 \end{pmatrix}$

$D=\begin{pmatrix} 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 6 & 12 & 18 & 24 \\ 8 & 16 & 24 & 32 \end{pmatrix}$

4.(a)

$$[BA]_{23} = \left[\begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -17 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ -2 & 3 & 2 \\ 1 & 4 & 5 \end{pmatrix}\right]_{23} = \begin{bmatrix} 1-0+3 & 2-0+12 & 1+0+15 \\ 2+2+4 & 4+3+16 & 2-2+20 \\ -3+2-17 & -6+3-68 & -3-2-85 \end{bmatrix}_{23} =$$

$$\begin{bmatrix} 4 & 14 & 16 \\ 8 & 23 & 20 \\ -18 & -71 & -90 \end{bmatrix}_{23} = \boxed{20}$$

(b)

$$[AB]_{23} = \left[\begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 2 \\ 1 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -17 \end{pmatrix}\right]_{23} = \begin{bmatrix} 1+4-3 & 0-2-1 & 3+8-17 \\ -2-6-6 & 0+3-2 & -6-12-34 \\ 1+8-15 & 0-4-5 & 3+16-85 \end{bmatrix}_{23} =$$

$$\begin{bmatrix} 2 & -3 & -6 \\ -14 & 1 & -52 \\ -6 & -9 & -66 \end{bmatrix}_{23} = \boxed{-52}$$

(c) $[B^2]_{31} = \left[\begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -17 \end{pmatrix}\begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ 3 & -1 & -17 \end{pmatrix}\right]_{31} = \begin{bmatrix} 1+0-9 & 0-0-3 & 3+0-51 \\ 2-2-12 & 0+1-4 & 6-4-68 \\ -3-2+51 & 0+1+17 & -9-4+289 \end{bmatrix}_{31} =$

$\begin{bmatrix} -8 & -3 & -48 \\ -12 & -3 & -66 \\ 46 & 18 & 276 \end{bmatrix}_{31} = \boxed{46}$

(d) $tr(A) = a_{11} + a_{22} + a_{33} = 1 + (-3) + 5 = \boxed{3}$

(e) $tr(B^t) = [B^t]_{11} + [B^t]_{22} + [B^t]_{33} = 1 + (-1) + (-17) = \boxed{-17}$

$B^t = \begin{pmatrix} 1 & 2 & -3 \\ 0 & -1 & -1 \\ 3 & 4 & -17 \end{pmatrix}$

(f) $tr(A-B) = [A-B]_{11} + [A-B]_{22} + [A-B]_{33} = 0 + (-2) + 22 = \boxed{20}$

$A-B = \begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 2 \\ 1 & 4 & 5 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -17 \end{pmatrix} = \begin{pmatrix} 0 & 2 & -2 \\ -4 & -2 & -2 \\ 4 & 5 & 22 \end{pmatrix}$

(g) $tr(AB) = [AB]_{11} + [AB]_{22} + [AB]_{33} = 2 + 1 + (-66) = \boxed{-63}$

$AB = \begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 2 \\ 1 & 4 & 5 \end{pmatrix}\begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -17 \end{pmatrix} = \begin{pmatrix} 1+4-3 & 0-2-1 & 3+8-17 \\ -2-6-6 & 0+3-2 & -6-12-34 \\ 1+8-15 & 0-4-5 & 3+16-85 \end{pmatrix} =$

$\begin{pmatrix} 2 & -3 & -6 \\ -14 & 1 & -52 \\ -6 & -9 & -66 \end{pmatrix}$

5.(a) $2X + A = 3B + C$

$2X = 3B + C - A$

$\frac{1}{2}(2X) = \frac{1}{2}(3B + C - A)$

$X = \frac{3}{2}B + \frac{1}{2}C - \frac{1}{2}A$

$$X = \frac{3}{2}\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} = \begin{pmatrix} \frac{6}{2} & \frac{3}{2} \\ \frac{12}{2} & \frac{9}{2} \end{pmatrix} + \begin{pmatrix} 0 & \frac{2}{2} \\ \frac{1}{2} & 0 \end{pmatrix} - \begin{pmatrix} \frac{-1}{2} & \frac{7}{2} \\ \frac{2}{2} & \frac{6}{2} \end{pmatrix} =$$

$$\begin{pmatrix} \frac{6}{2} & \frac{5}{2} \\ \frac{13}{2} & \frac{9}{2} \end{pmatrix} - \begin{pmatrix} -\frac{1}{2} & \frac{7}{2} \\ \frac{2}{2} & \frac{6}{2} \end{pmatrix} = \begin{pmatrix} \frac{7}{2} & -\frac{2}{2} \\ \frac{11}{2} & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} \frac{7}{2} & -1 \\ \frac{11}{2} & \frac{3}{2} \end{pmatrix}$$

(b) $Y + A = \frac{1}{2}(B - C)^t$

$Y = \frac{1}{2}(B - C)^t - A$

$$Y = \frac{1}{2}\left[\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} - \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}\right]^t - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} = \begin{bmatrix} 2 & -1 \\ 3 & 3 \end{bmatrix}^t - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} =$$

$$\begin{pmatrix} 2 & 3 \\ -1 & 3 \end{pmatrix} - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ -3 & -3 \end{pmatrix}$$

(c) $3X + A = B - X$

$3X + X = B - A$

$4X = B - A$

$\frac{1}{4}(4X) = \frac{1}{4}(B - A)$

$X = \frac{1}{4}B - \frac{1}{4}A$

$$\rightarrow X = \frac{1}{4}\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} - \frac{1}{4}\begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} = \begin{pmatrix} \frac{2}{4} & \frac{1}{4} \\ \frac{4}{4} & \frac{3}{4} \end{pmatrix} - \begin{pmatrix} -\frac{1}{4} & \frac{7}{4} \\ \frac{2}{4} & \frac{6}{4} \end{pmatrix} =$$

$$\begin{pmatrix} \frac{3}{4} & \frac{-6}{4} \\ \frac{2}{4} & \frac{-3}{4} \end{pmatrix} = \begin{pmatrix} \frac{3}{4} & -\frac{3}{2} \\ \frac{1}{2} & -\frac{3}{4} \end{pmatrix}$$

(d) $\begin{cases} X+Y=3A \\ X-Y=2B+C \end{cases}$

$2X = 3A + 2B + C$

$\frac{1}{2}(2X) = \frac{1}{2}(3A+2B+C)$

$X = \frac{3}{2}A + B + \frac{1}{2}C$

$X = \frac{3}{2}\begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} =$

$\begin{pmatrix} -\frac{3}{2} & \frac{21}{2} \\ \frac{6}{2} & \frac{18}{2} \end{pmatrix} + \begin{pmatrix} \frac{4}{2} & \frac{2}{2} \\ \frac{8}{2} & \frac{6}{2} \end{pmatrix} + \begin{pmatrix} 0 & \frac{2}{2} \\ \frac{1}{2} & 0 \end{pmatrix} =$

$\begin{pmatrix} \frac{1}{2} & \frac{23}{2} \\ \frac{14}{2} & \frac{24}{2} \end{pmatrix} + \begin{pmatrix} 0 & \frac{2}{2} \\ \frac{1}{2} & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & \frac{24}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & 12 \end{pmatrix}$

$X+Y=3A$

$Y=3A-X$

$Y = 3\begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} - \begin{pmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & \frac{24}{2} \end{pmatrix} = \begin{pmatrix} -3 & 21 \\ 6 & 18 \end{pmatrix} - \begin{pmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & \frac{24}{2} \end{pmatrix} = \begin{pmatrix} -\frac{6}{2} & \frac{42}{2} \\ \frac{12}{2} & \frac{36}{2} \end{pmatrix} - \begin{pmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & \frac{24}{2} \end{pmatrix} =$

$\begin{pmatrix} -\frac{7}{2} & \frac{17}{2} \\ -\frac{3}{2} & \frac{12}{2} \end{pmatrix} = \begin{pmatrix} -\frac{7}{2} & \frac{17}{2} \\ -\frac{3}{2} & 6 \end{pmatrix}$

$X = \begin{pmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & 12 \end{pmatrix}$; $Y = \begin{pmatrix} -\frac{7}{2} & \frac{17}{2} \\ -\frac{3}{2} & 6 \end{pmatrix}$

6. $A^2 = 2A$

$$A^2 = \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix} = \begin{pmatrix} 1+\frac{1}{x}.x & \frac{1}{x}+\frac{1}{x} \\ x+x & x.\frac{1}{x}+1 \end{pmatrix} = \begin{pmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{pmatrix}$$

$$2A = 2\begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix} = \begin{pmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{pmatrix}$$ Logo, $A^2 = 2A$

$$A^3 = \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix} = \begin{pmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix} = \begin{pmatrix} 2+\frac{2}{x}.x & \frac{2}{x}+\frac{2}{x} \\ 2x+2x & 2x.\frac{1}{x}+2 \end{pmatrix} =$$

$$\begin{pmatrix} 4 & \frac{4}{x} \\ 4x & 4 \end{pmatrix}$$ $A^3 = 4A$

$$A^4 = A^3A = \begin{pmatrix} 4 & \frac{4}{x} \\ 4x & 4 \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix} = \begin{pmatrix} 4+\frac{4}{x}.x & \frac{4}{x}+\frac{4}{x} \\ 4x+4x & 4x.\frac{1}{x}+4 \end{pmatrix} = \begin{pmatrix} 8 & \frac{8}{x} \\ 8x & 8 \end{pmatrix}$$

$A^4 = 8A$

$$A^1 = \begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix}$$ $A^1 = 1A$

$\boxed{A^n = 2^{n-1}A}$

7.(a) $A(B+C)$

$AB+AC=X+Y$

(b) $B^t A^t$

$(AB)^t=X^t$

(c) $C^t A^t$

$(AC)^t=Y^t$

(d) $(ABA)C$

$(AB)(AC)=XY$

8.(a) $A=A^t$

$$\begin{pmatrix}4 & x+2\\ 2x-3 & x+1\end{pmatrix}=\begin{pmatrix}4 & 2x-3\\ x+2 & x+1\end{pmatrix}$$

$4=4$

$x+2=2x-3$

$x-2x=-3-2$

$-x=-5$

$\boxed{x=5}$

$$A=\begin{pmatrix}4 & 5+2\\ 2.5-3 & 5+1\end{pmatrix}=\begin{pmatrix}4 & 7\\ 7 & 6\end{pmatrix}$$

(b) $-B=B^t$

$$\begin{pmatrix}0 & 4 & -2\\ -x & 0 & -1-z\\ -y & -2z & 0\end{pmatrix}=\begin{pmatrix}0 & x & y\\ -4 & 0 & 2z\\ 2 & 1-z & 0\end{pmatrix}$$

$\boxed{x=4}$ $\boxed{y=-2}$

$-2z=1-z$

$-2z+z=1$

$-z=1$

$\boxed{z=-1}$

9. $$3\begin{pmatrix}x & y\\ z & t\end{pmatrix}=\begin{pmatrix}x & 6\\ -1 & 2t\end{pmatrix}+\begin{pmatrix}4 & x+y\\ z+t & 3\end{pmatrix}$$

$$\begin{pmatrix}3x & 3y\\ 3z & 3t\end{pmatrix}=\begin{pmatrix}x+4 & x+y+6\\ -1+z+t & 2t+3\end{pmatrix}$$

$3x=x+4$

$3x-x=4$

$2x=4$

$\boxed{x=2}$

$3y=2+y+6$

$3y-y=8$

$2y=8$

$\boxed{y=4}$

$3t=2t+3$

$3t-2t=3$

$\boxed{t=3}$

$3z=-1+z+3$

$3z-z=2$

$2z=2$

$\boxed{z=1}$

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10. (a) $AA^t = A^tA = I_n$

$$RR^t = \begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix} =$$

$$\begin{pmatrix} \cos^2\theta + \operatorname{sen}^2\theta & -\cos\theta\operatorname{sen}\theta + \operatorname{sen}\theta\cos\theta \\ -\operatorname{sen}\theta\cos\theta + \cos\theta\operatorname{sen}\theta & \operatorname{sen}^2\theta + \cos^2\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$R^tR = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix} =$$

$$\begin{pmatrix} \cos^2\theta + \operatorname{sen}^2\theta & \cos\theta\operatorname{sen}\theta - \operatorname{sen}\theta\cos\theta \\ \operatorname{sen}\theta\cos\theta - \cos\theta\operatorname{sen}\theta & \operatorname{sen}^2\theta + \cos^2\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Logo, $R(\theta)$ é ortogonal

(b) $AA^t = I_n$

$$\begin{pmatrix} 1 & 0 & x \\ 0 & \frac{1}{\sqrt{2}} & y \\ 0 & \frac{1}{\sqrt{2}} & z \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ x & y & z \end{pmatrix} =$$

$$\begin{pmatrix} 1+0+x^2 & 0+0+xy & 0+0+xz \\ 0+0+yx & 0+\frac{1}{2}+y^2 & 0+\frac{1}{2}+yz \\ 0+0+zx & 0+\frac{1}{2}+zy & 0+\frac{1}{2}+z^2 \end{pmatrix} = \begin{pmatrix} 1+x^2 & xy & xz \\ yx & \frac{1}{2}+y^2 & \frac{1}{2}+yz \\ zx & \frac{1}{2}+zy & \frac{1}{2}+z^2 \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$x^2+1=1$

x^2-0

$\boxed{x=0}$

$\frac{1}{2}+y^2=1$

$y^2=\pm\frac{1}{2}$

$\boxed{y=\pm\frac{1}{\sqrt{2}}}$

$z^2+\frac{1}{2}=1$

$z^2=\pm\frac{1}{2}$

$\boxed{z=\pm\frac{1}{\sqrt{2}}}$

$A^t A = I_n$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ x & y & z \end{pmatrix} \begin{pmatrix} 1 & 0 & x \\ 0 & \frac{1}{\sqrt{2}} & y \\ 0 & \frac{1}{\sqrt{2}} & z \end{pmatrix} =$$

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$$\begin{pmatrix} 1+0+0 & 0+0+0 & x+0+0 \\ 0+0+0 & 0+\frac{1}{2}+\frac{1}{2} & 0+\frac{1}{\sqrt{2}}y+\frac{1}{\sqrt{2}}z \\ x+0+0 & 0+y\frac{1}{\sqrt{2}}+z\frac{1}{\sqrt{2}} & x^2+y^2+z^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & x \\ 0 & 1 & \frac{1}{\sqrt{2}}(y+z) \\ x & \frac{1}{\sqrt{2}}(y+z) & x^2+y^2+z^2 \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad x=0 \quad \begin{matrix} x^2+z^2+y^2=1 \\ 0+z^2+y^2=1 \end{matrix} \quad \begin{matrix} \frac{1}{\sqrt{2}}(y+z)=0 \\ (y+z)=0 \end{matrix}$$

Então: $x=0;\ y=\frac{1}{\sqrt{2}}$ e $z=-\frac{1}{\sqrt{2}}$ ou $x=0;\ y=-\frac{1}{\sqrt{2}}$ e $z=\frac{1}{\sqrt{2}}$

Conclusão: $x=0$ e $y=-z$