Lista 4

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1.(a) $\tilde{M} = [\vec{A}|\vec{B}] = \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right) \longrightarrow \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{\tilde{A}} \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 2 \\ -1 \end{pmatrix}}_{\tilde{B}}$ solução: $\begin{cases} x = 2 \\ y = -1 \end{cases}$

(b) $\tilde{M} = [\vec{A}|\vec{B}] = \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \longrightarrow \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\tilde{A}} \underbrace{\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}}_{\tilde{B}}$

solução: $\begin{cases} w = 4 \\ x = 3 \\ y = 2 \\ z = 1 \end{cases}$

(c) $\tilde{M} = [\tilde{A}|\tilde{B}] = \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right) \longrightarrow \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}}_{\tilde{A}} \underbrace{\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}}_{\tilde{B}}$

Solução: $\begin{cases} w = 6 \\ x = 3 \\ y + z = 2 \end{cases}$ seja $z = \alpha ; \alpha \in \mathbb{R}$ $\begin{cases} w = 6 \\ x = 3 \\ y = 2 - \alpha \\ z = \alpha \end{cases} ; \alpha \in \mathbb{R}$

(d) $\tilde{M} = [\vec{A}|\vec{B}] = \left(\begin{array}{ccc|c} 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & 2 \end{array}\right) \longrightarrow \underbrace{\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \end{pmatrix}}_{\tilde{A}} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 1 \\ 2 \end{pmatrix}}_{\tilde{B}}$

solução: $\begin{cases} x + 3z = 1 \\ y - z = 2 \end{cases}$

Seja $z = \alpha ; \alpha \in \mathbb{R}$ $\begin{cases} x = 1 - 3\alpha \\ y = 2 + \alpha \\ z = \alpha \end{cases}$

(e)

$$\tilde{M}=[\tilde{A}|\tilde{B}]=\left(\begin{array}{cccc|c}1&0&0&-7&8\\0&1&0&3&2\\0&0&1&1&-5\end{array}\right)\rightarrow\underbrace{\begin{pmatrix}1&0&0&-7\\0&1&0&3\\0&0&1&1\end{pmatrix}}_{\tilde{A}}\underbrace{\begin{pmatrix}w\\x\\y\\z\end{pmatrix}}_{X}=\underbrace{\begin{pmatrix}8\\2\\-5\end{pmatrix}}_{\tilde{B}}$$

solução: $\begin{cases}w-7z=8\\x+3z=2\\y+z=-5\end{cases}$ seja $z=\alpha;\alpha\in\mathbb{R}$ $\begin{cases}w=8+7\alpha\\x=2-3\alpha\\y=-5-\alpha\\z=\alpha\end{cases},\alpha\in\mathbb{R}$

(f)

$$\tilde{M}=[\tilde{A}|\tilde{B}]=\left(\begin{array}{ccccc|c}1&-6&0&0&3&-2\\0&0&1&0&4&7\\0&0&0&1&5&8\\0&0&0&0&0&0\end{array}\right)\rightarrow\underbrace{\begin{pmatrix}1&-6&0&0&3\\0&0&1&0&4\\0&0&0&1&5\\0&0&0&0&0\end{pmatrix}}_{\tilde{A}}\underbrace{\begin{pmatrix}v\\w\\x\\y\\z\end{pmatrix}}_{X}=\underbrace{\begin{pmatrix}-2\\7\\8\\0\end{pmatrix}}_{\tilde{B}}$$

Solução: $\begin{cases}v-6w+3z=-2\\x+4z=7\\y+5z=8\end{cases}$ sejam $w=\alpha$ e $z=\beta;\alpha,\beta\in\mathbb{R}$

$$\begin{cases}v=-2+6\alpha-3\beta\\w=\alpha\\x=7-4\beta\\y=8-5\beta\\z=\beta\end{cases}$$

2.(a) $\begin{cases}3x-4y=1\\x+3y=9\end{cases}$ $A=\begin{pmatrix}3&-4\\1&3\end{pmatrix}$; $B=\begin{pmatrix}1\\9\end{pmatrix}$

Matriz ampliada: $M=[A|B]=\begin{pmatrix}3&-4&1\\1&3&9\end{pmatrix}\underset{\sim}{\ell_1\longleftrightarrow\ell_2}\begin{pmatrix}1&3&9\\3&-4&1\end{pmatrix}\underset{\sim}{\ell_2\leftarrow\ell_2-3\ell_1}$

$\begin{pmatrix}1&3&9\\0&-13&-26\end{pmatrix}\underset{\sim}{\ell_2\leftarrow\ell_2/13}\begin{pmatrix}1&3&9\\0&-1&-2\end{pmatrix}\underset{\sim}{\ell_1\leftarrow\ell_1+3\ell_2}\begin{pmatrix}1&0&3\\0&-1&-2\end{pmatrix}\underset{\sim}{\ell_2\leftarrow\ell_2/(-1)}$

$\left(\begin{array}{cc|c}1&0&3\\0&1&2\end{array}\right) = \tilde{M} = [\tilde{A}|\tilde{B}]$ (SPD)

conferindo: $3.3-4.2=1$
$9-8$ e $3+3.2=9$
$3+6=9$ e

Sistema equivalente: $\tilde{A}X=\tilde{B}$

$\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}3\\2\end{pmatrix}$ solução: $\begin{cases}x=3\\y=2\end{cases}$

(b) $\begin{cases}5x+8y=34\\10x+16y=50\end{cases}$ $A=\begin{pmatrix}5&8\\10&16\end{pmatrix}$; $B=\begin{pmatrix}34\\50\end{pmatrix}$

Matriz ampliada: $M=[A|B]=\begin{pmatrix}5&8&34\\10&16&50\end{pmatrix} \underset{\sim}{\ell_2\leftarrow \ell_2/2} \begin{pmatrix}5&8&34\\5&8&25\end{pmatrix} \underset{\sim}{\ell_2\leftarrow \ell_2-\ell_1}$

$\begin{pmatrix}5&8&34\\0&0&-9\end{pmatrix} \underset{\sim}{\ell_2\leftarrow \ell_2/(-9)} \begin{pmatrix}5&8&34\\0&0&1\end{pmatrix} \underset{\sim}{\ell_1\leftarrow \ell_1/5} \left(\begin{array}{cc|c}1&\frac{8}{5}&\frac{34}{5}\\0&0&1\end{array}\right) = \tilde{M}=[\tilde{A}|\tilde{B}]$

Roché-Capelli:
$\left.\begin{array}{l}\text{posto de } M=2\\ \text{posto de } A=1\end{array}\right\}\neq$ conclusão: (SI) sem solução

(c) $\begin{cases}x+2y=5\\2x-3y=-4\end{cases}$ $A=\begin{pmatrix}1&2\\2&-3\end{pmatrix}$; $B=\begin{pmatrix}5\\-4\end{pmatrix}$

$M=[A|B]=\begin{pmatrix}1&2&5\\2&-3&-4\end{pmatrix} \underset{\sim}{\ell_2\leftarrow \ell_2-2\ell_1} \begin{pmatrix}1&2&5\\0&-7&-14\end{pmatrix} \underset{\sim}{\ell_2\leftarrow \ell_2/7} \begin{pmatrix}1&2&5\\0&-1&-2\end{pmatrix}$

$\underset{\sim}{\ell_1\leftarrow \ell_1+2\ell_2} \begin{pmatrix}1&0&1\\0&-1&-2\end{pmatrix} \underset{\sim}{\ell_2\leftarrow \ell_2/(-1)} \begin{pmatrix}1&0&1\\0&1&2\end{pmatrix} = \tilde{M}=[\tilde{A}|\tilde{B}]$

$\left.\begin{array}{l}\text{posto de } A=2\\ \text{posto de } M=2\end{array}\right\} = \text{(SP)}$

Grau de liberdade: $2-2=0$ (SPD)

Sistema equivalente: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ solução: $\begin{cases} x = 1 \\ y = 2 \end{cases}$

conferindo: $1+2.2=5$ $\quad 2.1-3.2=-4$

$1+4=5$ e $\quad 2-6=-4$ e

(d) $\begin{cases} 3x+2y-5z=8 \\ 2x-4y-2z=-4 \\ x-2y-3z=-4 \end{cases}$ $A=\begin{pmatrix} 3 & 2 & -5 \\ 2 & -4 & -2 \\ 1 & -2 & -3 \end{pmatrix}$; $B=\begin{pmatrix} 8 \\ -4 \\ -4 \end{pmatrix}$

$M=[A|B]=\begin{pmatrix} 3 & 2 & -5 & 8 \\ 2 & -4 & -2 & -4 \\ 1 & -2 & -3 & -4 \end{pmatrix} \begin{matrix} \ell_2 \xleftarrow[\sim]{} \ell_2/2 \\ \\ \ell_3 \longleftrightarrow \ell_1 \end{matrix} \begin{pmatrix} 1 & -2 & -3 & -4 \\ 1 & -2 & -1 & -2 \\ 3 & 2 & -5 & 8 \end{pmatrix} \begin{matrix} \ell_2 \xleftarrow[\sim]{} \ell_2-\ell_1 \\ \\ \ell_3 \longleftarrow \ell_3-3\ell_1 \end{matrix}$

$\begin{pmatrix} 1 & -2 & -3 & -4 \\ 0 & 0 & 2 & 2 \\ 0 & 8 & 4 & 20 \end{pmatrix} \begin{matrix} \ell_2 \xleftarrow[\sim]{} \ell_2/2 \\ \\ \ell_3 \longleftarrow \ell_3/4 \end{matrix} \begin{pmatrix} 1 & -2 & -3 & -4 \\ 0 & 0 & 1 & 1 \\ 0 & 2 & 1 & 5 \end{pmatrix} \ell_1 \xleftarrow[\sim]{} \ell_1+\ell_3$

$\begin{pmatrix} 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 2 & 1 & 5 \end{pmatrix} \ell_2 \xleftrightarrow[\sim]{} \ell_3 \begin{pmatrix} 1 & 0 & -2 & 1 \\ 0 & 2 & 1 & 5 \\ 0 & 0 & 1 & 1 \end{pmatrix} \ell_2 \xleftarrow[\sim]{} \ell_2-\ell_3 \begin{pmatrix} 1 & 0 & -2 & 1 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

$\begin{matrix} \ell_1 \xleftarrow[\sim]{} \ell_1+2\ell_3 \\ \\ \ell_2 \longleftarrow \ell_2/2 \end{matrix} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) = \tilde{M} = [\tilde{A}|\tilde{B}]$

$\left.\begin{matrix} \text{posto de } A=3 \\ \text{posto de } M=3 \end{matrix}\right\} = (SP)$ grau de liberdade: $3-3=0$ (SPD)

Sistema equivalente: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

Solução: $\begin{cases} x = 3 \\ y = 2 \\ z = 1 \end{cases}$

conferindo:

$3.3+2.2-5.1=8$
$9+4-5=8$
$13-5=8$ ✓

$2.3-4.2-2.1=-4$
$6-8-2=-4$
$-2-2=-4$ ✓

$3-2.2-3.1=-4$
$3-4-3=-4$
$-1-3=-4$ ✓

(e) $\begin{cases} 2x-6y=-4 \\ x+3y=1 \\ 4x+12y=2 \end{cases}$ $\quad A=\begin{pmatrix} 2 & -6 \\ 1 & 3 \\ 4 & 12 \end{pmatrix}$; $B=\begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix}$

$$M=[A|B]=\begin{pmatrix} 2 & -6 & -4 \\ 1 & 3 & 1 \\ 4 & 12 & 2 \end{pmatrix} \begin{matrix} l_1 \longleftrightarrow l_2 \\ \sim \\ l_3 \longleftarrow l_3/2 \end{matrix} \begin{pmatrix} 1 & 3 & 1 \\ 2 & -6 & -4 \\ 2 & 6 & 1 \end{pmatrix} \begin{matrix} l_2 \longleftarrow l_2-2l_1 \\ \sim \\ l_3 \longleftarrow l_3-2l_1 \end{matrix} \begin{pmatrix} 1 & 3 & 1 \\ 0 & -12 & -6 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\begin{matrix} l_1 \longleftarrow 2l_1 \\ \sim \\ l_2 \longleftarrow l_2/2 \end{matrix} \begin{pmatrix} 2 & 6 & 2 \\ 0 & -6 & -3 \\ 0 & 0 & -1 \end{pmatrix} \begin{matrix} l_1 \longleftarrow l_1+l_2 \\ \sim \\ l_3 \longleftarrow -l_3 \end{matrix} \begin{pmatrix} 2 & 0 & -1 \\ 0 & -6 & -3 \\ 0 & 0 & 1 \end{pmatrix} \begin{matrix} l_1 \longleftarrow l_1+l_3 \\ \sim \\ l_2 \longleftarrow l_2/3 \end{matrix}$$

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & -2 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{matrix} l_1 \longleftarrow l_1/2 \\ \sim \\ l_2 \longleftarrow l_2+l_3 \end{matrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{matrix} l_2 \longleftarrow l_2/(-2) \\ \sim \end{matrix} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) = \tilde{M}=[\tilde{A}|\tilde{B}]$$

$\left.\begin{matrix} \text{posto de } A = 2 \\ \text{posto de } M = 3 \end{matrix}\right\} \neq$

Conclusão: (SI) sem solução

(f) $\begin{cases} x+2y-z=2 \\ 2x-y+3z=9 \\ 3x+3y-2z=3 \end{cases}$ $\quad A=\begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 3 & -2 \end{pmatrix}$; $B=\begin{pmatrix} 2 \\ 9 \\ 3 \end{pmatrix}$

$$M=[A|B]=\begin{pmatrix}1&2&-1&2\\2&-1&3&9\\3&3&-2&3\end{pmatrix}\begin{matrix}l_2\leftarrow l_2-2l_1\\ \sim \\ l_3\leftarrow l_3-3l_1\end{matrix}\begin{pmatrix}1&2&-1&2\\0&-5&5&5\\0&-3&1&-3\end{pmatrix}\begin{matrix}l_2\leftarrow l_2/(-5)\\ \sim\end{matrix}$$

$$\begin{pmatrix}1&2&-1&2\\0&1&-1&-1\\0&-3&1&-3\end{pmatrix}\begin{matrix}l_1\leftarrow l_1-2l_2\\ \sim \\ l_3\leftarrow l_3+3l_2\end{matrix}\begin{pmatrix}1&0&1&4\\0&1&-1&-1\\0&0&-2&-6\end{pmatrix}\begin{matrix}l_3\leftarrow l_3/(-2)\\ \sim\end{matrix}$$

$$\begin{pmatrix}1&0&1&4\\0&1&-1&-1\\0&0&1&3\end{pmatrix}\begin{matrix}l_1\leftarrow l_1-l_3\\ \sim \\ l_2\leftarrow l_2+l_3\end{matrix}\left(\begin{array}{ccc|c}1&0&0&1\\0&1&0&2\\0&0&1&3\end{array}\right)=\tilde{M}=[\tilde{A}|\tilde{B}]$$

$\left.\begin{matrix}\text{posto de } A=3\\ \text{posto de } M=3\end{matrix}\right\}$ = (SP) grau de liberdade: $3-3=0$ (SPD)

Sistema equivalente: $\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$

Solução: $\begin{cases}x=1\\y=2\\z=3\end{cases}$ conferindo:

$1+2.2-3=2$
$1+4-3=2$
$5-3=2$ ✓

$2.1-2+3.3=9$
$2-2+9=9$
$0+9=9$ ✓

$3.1+3.2-2.3=3$
$3+6-6=3$
$9-6=3$ ✓

(g) $\begin{cases}x+3z=-8\\2x-4y=-4\\3x-2y-5z=26\end{cases}$ $A=\begin{pmatrix}1&0&3\\2&-4&0\\3&-2&-5\end{pmatrix}$; $B=\begin{pmatrix}-8\\-4\\26\end{pmatrix}$

$$M=[A|B]=\begin{pmatrix}1&0&3&-8\\2&-4&0&-4\\3&-2&-5&26\end{pmatrix}\begin{matrix}\ell_2\leftarrow \ell_2-2\ell_1\\ \sim \\ \ell_3\leftarrow \ell_3-3\ell_1\end{matrix}\begin{pmatrix}1&0&3&-8\\0&-4&-6&12\\0&-2&-14&50\end{pmatrix}\begin{matrix}\ell_2\leftarrow \ell_2/2\\ \sim \\ \ell_3\leftarrow \ell_3/2\end{matrix}$$

$$\begin{pmatrix}1&0&3&-8\\0&-2&-3&6\\0&-1&-7&25\end{pmatrix}\begin{matrix}\ell_2\leftarrow \ell_2-2\ell_3\\ \sim \\ \ell_3\leftarrow -\ell_3\end{matrix}\begin{pmatrix}1&0&3&-8\\0&0&11&-44\\0&1&7&-25\end{pmatrix}\begin{matrix}\ell_2\leftarrow \ell_2/11\\ \sim \\ \end{matrix}\begin{pmatrix}1&0&3&-8\\0&0&1&-4\\0&1&7&-25\end{pmatrix}\begin{matrix}\ell_2\longleftrightarrow \ell_3\\ \sim \\ \end{matrix}$$

$$\begin{pmatrix}1&0&3&-8\\0&1&7&-25\\0&0&1&-4\end{pmatrix}\begin{matrix}\ell_1\leftarrow \ell_1-3\ell_3\\ \sim \\ \ell_2\leftarrow \ell_2-7\ell_3\end{matrix}\begin{pmatrix}1&0&0&4\\0&1&0&3\\0&0&1&-4\end{pmatrix}=\tilde{M}=[\tilde{A}|\tilde{B}]$$

$$\left.\begin{matrix}\text{posto de } A=3\\ \text{posto de } M=3\end{matrix}\right\}=(SP) \text{ grau de liberdade: } 3-3=0\ (SPD)$$

Sistema equivalente: $\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}4\\3\\-4\end{pmatrix}$ solução: $\begin{cases}x=4\\y=3\\z=-4\end{cases}$

conferindo: $4+3.-4=-8$ $\quad 2.4-4.3=-4$ $\quad 3.4-2.3-5.-4=26$

$4+(-12)=-8$ e $\quad 8-12=-4$ e $\quad 12-6+20=26$

$6+20=26$ e

(h) $\begin{cases}x+2y+3z=10\\3x+4y+6z=23\\2x+2y+3z=13\end{cases}$ $A=\begin{pmatrix}1&2&3\\3&4&6\\2&2&3\end{pmatrix}$; $B=\begin{pmatrix}10\\23\\13\end{pmatrix}$

$$M=[A|B]=\begin{pmatrix}1&2&3&10\\3&4&6&23\\2&2&3&13\end{pmatrix}\begin{matrix}\ell_2\leftarrow \ell_2-3\ell_1\\ \sim \\ \ell_3\leftarrow \ell_3-2\ell_1\end{matrix}\begin{pmatrix}1&2&3&10\\0&-2&-3&-7\\0&-2&-3&-7\end{pmatrix}\begin{matrix}\ell_1\leftarrow \ell_1+\ell_2\\ \sim \\ \ell_3\leftarrow \ell_3-\ell_2\end{matrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & -3 & -7 \end{pmatrix} \underset{\ell_2 \leftrightarrow \ell_3}{\overset{\ell_1 \leftarrow 7\ell_1}{\sim}} \begin{pmatrix} 7 & 0 & 0 & 21 \\ 0 & -2 & -3 & -7 \\ 0 & 0 & 0 & 0 \end{pmatrix} \underset{\ell_2 \leftarrow \ell_2/(-2)}{\overset{\ell_1 \leftarrow \ell_1/7}{\sim}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & \frac{3}{2} & \frac{7}{2} \\ 0 & 0 & 0 & 0 \end{array}\right)$$

posto de $A = 2$
posto de $M = 2$ } = (SP) grau de liberdade: $3 - 2 = 1$ (SPI)

Sistema equivalente:
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ \frac{7}{2} \\ 0 \end{pmatrix}$$

Solução: $\begin{cases} x = 3 \\ y + \frac{3}{2} z = \frac{7}{2} \end{cases}$ Seja $z = \lambda; \lambda \in \mathbb{R}$ $\begin{cases} x = 3 \\ y = \frac{7}{2} - \frac{3}{2}\lambda \\ z = \lambda \end{cases}$

(i) $\begin{cases} x - 3y + 4z - w = 2 \\ 2x - y + 3z - 2w = 19 \end{cases}$ $A = \begin{pmatrix} 1 & -3 & 4 & -1 \\ 2 & -1 & 3 & -2 \end{pmatrix}$ $B = \begin{pmatrix} 2 \\ 19 \end{pmatrix}$

$$M = [A|B] = \begin{pmatrix} 1 & -3 & 4 & -1 & 2 \\ 2 & -1 & 3 & -2 & 19 \end{pmatrix} \overset{\ell_2 \leftarrow \ell_2 - 2\ell_1}{\sim} \begin{pmatrix} 1 & -3 & 4 & -1 & 2 \\ 0 & 5 & -5 & 0 & 15 \end{pmatrix} \overset{\ell_2 \leftarrow \ell_2/5}{\sim}$$

$$\begin{pmatrix} 1 & -3 & 4 & -1 & 2 \\ 0 & 1 & -1 & 0 & 3 \end{pmatrix} \overset{\ell_1 \leftarrow \ell_1 + 3\ell_2}{\sim} \left(\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 11 \\ 0 & 1 & -1 & 0 & 3 \end{array}\right)$$

posto de $A = 2$
posto de $M = 2$ } = (SP) grau de liberdade: $4 - 2 = 2$ (SPI)

Sistema equivalente:

$$\begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 3 \end{pmatrix}$$

solução: $\begin{cases} w+y-z=11 \\ x-y=3 \end{cases}$

Sejam $y=\alpha$ e $z=\beta$; $\alpha,\beta \in \mathbb{R}$

$$\begin{cases} w = 11-\alpha+\beta \\ x = 3+\alpha \\ y=\alpha \\ z=\beta \end{cases} ; \alpha,\beta\in\mathbb{R}$$

3.(a)

$$M=[A|B]=\begin{pmatrix} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{pmatrix} \begin{matrix} l_2 \leftarrow l_2 - l_1 \\ \sim \\ l_3 \leftarrow l_3 - l_1 \end{matrix} \begin{pmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & -2 & -1 & 0 & -5 \end{pmatrix} \begin{matrix} l_1 \leftarrow l_1 - 2l_2 \\ \sim \\ l_3 \leftarrow l_3 + 2l_2 \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & 9 & 1 & 10 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -7 & 0 & -7 \end{pmatrix} \begin{matrix} l_3 \leftarrow l_3/(-7) \\ \sim \end{matrix} \begin{pmatrix} 1 & 0 & 9 & 1 & 10 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{matrix} l_1 \leftarrow l_1 - 9l_3 \\ \sim \\ l_2 \leftarrow l_2 + 3l_3 \end{matrix}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right) = \tilde{M} = [\tilde{A}|\tilde{B}]$$

$\left.\begin{matrix} \text{posto de } A = 3 \\ \text{posto de } M = 3 \end{matrix}\right\} =$ (SP)

grau de liberdade: $4-3=1$ (SPI)

Sistema equivalente: $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

Solução: $\begin{cases} w+z=1 \\ x=2 \\ y=1 \end{cases}$ seja $z=\alpha$; $\alpha\in\mathbb{R}$ $\begin{cases} w=1-\alpha \\ x=2 \\ y=1 \\ z=\alpha \end{cases}$

(b) $M = \begin{pmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{pmatrix} \begin{matrix} \ell_3 \leftarrow \ell_3 - \ell_1 \\ \sim \\ \end{matrix} \begin{pmatrix} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & -1 \end{pmatrix} \begin{matrix} \ell_1 \leftarrow \ell_1 + \ell_3 \\ \sim \\ \ell_3 \leftarrow 2\ell_3 \end{matrix}$

$\begin{pmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -2 & -2 & 4 & -2 \end{pmatrix} \begin{matrix} \ell_2 \leftarrow \ell_2 + \ell_3 \\ \sim \\ \ell_3 \leftarrow \ell_3/(-2) \end{matrix} \begin{pmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 1 & -2 & 1 \end{pmatrix} \begin{matrix} \ell_2 \leftrightarrow \ell_3 \\ \sim \\ \end{matrix}$

$\begin{pmatrix} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 & 1 \end{pmatrix} \begin{matrix} \ell_1 \leftarrow \ell_1 + 2\ell_3 \\ \sim \\ \ell_2 \leftarrow \ell_2 + \ell_3 \end{matrix} \begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & -1 & 1 & 1 \end{pmatrix} \begin{matrix} \ell_3 \leftarrow -\ell_3 \\ \sim \\ \end{matrix}$

$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right) = \tilde{M} = [\tilde{A} | \tilde{B}]$

posto de A = 3 }
posto de M = 3 } = (SP)

grau de liberdade: 4-3=1 (SPI)

Sistema equivalente: $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

Solução: $\begin{cases} w + z = 1 \\ x - z = 2 \\ y - z = -1 \end{cases}$ seja $z = \alpha$; $\alpha \in \mathbb{R}$ $\begin{cases} w = 1 - \alpha \\ x = 2 + \alpha \\ y = -1 + \alpha \\ z = \alpha \end{cases}$