Lista 2

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1.(a) $\det A = \begin{vmatrix} 2 & 1 \\ -4 & 3 \end{vmatrix} = 1(-1)^{1+2}|-4| + 2(-1)^{1+1}|3| = -1(-4) + 2(3) = 4 + 6 = \boxed{10}$

(b) $\det B = \begin{vmatrix} \sqrt{2} & 3\sqrt{6} \\ 2 & \sqrt{3} \end{vmatrix} = \sqrt{2}(-1)^{1+1}|\sqrt{3}| + 2(-1)^{2+1}|3\sqrt{6}| = \sqrt{2}\,\sqrt{3} + (-2)3\sqrt{6} =$

$\sqrt{6} - 6\sqrt{6} = \boxed{-5\sqrt{6}}$

(c)
$\det C = \begin{vmatrix} 1 & 0 & 2 \\ 5 & -1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 1(-1)^{3+1}\begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} = 1.-1(-1)^{2+1}|2| = 1.2 = \boxed{2}$

(d)
$\det D = \begin{vmatrix} -2 & 1 & -1 \\ 1 & 5 & 4 \\ -3 & 4 & 2 \end{vmatrix} = -2(-1)^{1+1}\begin{vmatrix} 5 & 4 \\ 4 & 2 \end{vmatrix} + 1(-1)^{1+2}\begin{vmatrix} 1 & 4 \\ -3 & 2 \end{vmatrix} - 1(-1)^{1+3}\begin{vmatrix} 1 & 5 \\ -3 & 4 \end{vmatrix} =$

$-2[4(-1)^{2+1}|4| + 2(-1)^{2+2}|5|] - 1[1(-1)^{1+1}|2| - 3(-1)^{2+1}|4|] - 1[1(-1)^{1+1}|4| - 3(-1)^{2+1}|5|] =$

$-2[-4.4 + 2.5] - 1[1.2 + 3.4] - 1[1.4 + 3.5] = -2[-16 + 10] - 1[2 + 12] - 1[4 + 15] =$

$-2.(-6) - 14 - 19 = 12 - 33 = \boxed{-21}$

(e)
$\det E = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & 5 \\ 2 & -1 & 2 \end{vmatrix} = -2(-1)^3\begin{vmatrix} 1 & 5 \\ 2 & 2 \end{vmatrix} = -2[1(-1)^2|2| + 2(-1)^3|5|] = -2[1.2 - 2.5] =$

$-2[2 - 10] = -2[-8] = \boxed{16}$

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(f)

$$\det F = \begin{vmatrix} 3 & -1 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} = 3(-1)^2 \begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = 3\left[1(-1)^2 \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} + 1(-1)^4 \begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix}\right] =$$

$$3\left[1.1(-1)^2 |-1| + 1.1(-1)^4 |-1|\right] = 3\left[1.(-1) + 1.(-1)\right] = 3[-2] = \boxed{-6}$$

(g)

$$\det G = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 5 & 1 & 2 & 5 & 3 \\ 1 & 2 & \sqrt{3} & 0 & 0 \\ 10 & -3 & 6 & 1 & 0 \\ -2 & -3 & 0 & 0 & 0 \end{vmatrix} = 1(-1)^2 \begin{vmatrix} 1 & 2 & 5 & 3 \\ 2 & \sqrt{3} & 0 & 0 \\ -3 & 6 & 1 & 0 \\ -3 & 0 & 0 & 0 \end{vmatrix} = 1.3(-1)^5 \begin{vmatrix} 2 & \sqrt{3} & 0 \\ -3 & 6 & 1 \\ -3 & 0 & 0 \end{vmatrix} =$$

$$-3.1(-1)^5 \begin{vmatrix} 2 & \sqrt{3} \\ -3 & 0 \end{vmatrix} = 3.(-3)(-1)^3 |\sqrt{3}| = \boxed{9\sqrt{3}}$$

(h)

$$\det H = \begin{vmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 \end{vmatrix} = 1(-1)^8 \begin{vmatrix} 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 \\ 0 & -2 & 0 & 0 \end{vmatrix} = 1.2(-1)^6 \begin{vmatrix} 3 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & -2 & 0 \end{vmatrix} =$$

$$2.3(-1)^2 \begin{vmatrix} 0 & -2 \\ -2 & 0 \end{vmatrix} = 6.(-2)(-1)^3 |-2| = 6.2(-2) = \boxed{-24}$$

2.(a)

$$\det(A+B) = \left|\begin{pmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{pmatrix} + \begin{pmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{pmatrix}\right| = \begin{array}{ccc} & (\div 2) & (\div 2) \\ \end{array} \begin{vmatrix} 7 & -2 & 14 \\ 3 & 2 & 10 \\ 4 & -8 & 2 \end{vmatrix} = 2.2 \begin{vmatrix} 7 & -1 & 7 \\ 3 & 1 & 5 \\ 4 & -4 & 1 \end{vmatrix}\begin{matrix} 7 & -1 \\ 3 & 1 \\ 4 & -4 \end{matrix} =$$

$$4[7-20-84-(28-140-3)] = 4[-97-(-115)] = 4[18] = \boxed{72}$$

(b)

$$\det(AB)=\left|\begin{pmatrix}3&-5&7\\4&2&8\\1&-9&6\end{pmatrix}\begin{pmatrix}4&3&7\\-1&0&2\\3&1&-4\end{pmatrix}\right|=\begin{vmatrix}12+5+21&9-0+7&21-10-28\\16-2+24&12+0+8&28+4-32\\4+9+18&3-0+6&7-18-24\end{vmatrix}=$$

$$(\div 2)\begin{vmatrix}38&16&-17\\38&20&0\\31&9&-35\end{vmatrix}=\begin{vmatrix}38&16&-17\\19&10&0\\31&9&-35\end{vmatrix}2=2\begin{vmatrix}38&16&-17\\19&10&0\\31&9&35\end{vmatrix}\begin{matrix}38&16\\19&10\\31&9\end{matrix}=$$

$$\begin{array}{r}38\\ \times 35\\ \hline 190\\ +1140\\ \hline 1330\end{array}\qquad\begin{array}{r}171\\ \times 17\\ \hline 1197\\ +1710\\ \hline 2904\end{array}$$

$$2[13300+0-2904-(-5270+0+10640)]=$$

$$2[10396-(5370)]=2[5026]=\boxed{10052}$$

$$\begin{array}{r}13300\\ -2904\\ \hline 10396\end{array}\qquad\begin{array}{r}10640\\ -5270\\ \hline 05370\end{array}\qquad\begin{array}{r}10396\\ -5370\\ \hline 05026\end{array}$$

$$\begin{array}{r}310\\ \times 17\\ \hline 2170\\ +3100\\ \hline 5270\end{array}\qquad\begin{array}{r}35\\ \times 19\\ \hline 315\\ +350\\ \hline 665\\ \times 16\\ \hline 3990\\ +6650\\ \hline 10640\end{array}$$

(c)

$$\det(B^tA^t)=\det(AB)^t=\det(AB)=\boxed{10052}$$

(d)

$$\det(2A-3C+B)=\left|2\begin{pmatrix}3&-5&7\\4&2&8\\1&-9&6\end{pmatrix}-3\begin{pmatrix}2&3&-1\\6&9&-2\\8&12&-3\end{pmatrix}+\begin{pmatrix}4&3&7\\-1&0&2\\3&1&-4\end{pmatrix}\right|=\left|\begin{pmatrix}6&-10&14\\8&4&16\\2&-18&12\end{pmatrix}-\right.$$

$$\left.\begin{pmatrix}6&9&-3\\18&27&-6\\24&36&-9\end{pmatrix}+\begin{pmatrix}4&3&7\\-1&0&2\\3&1&-4\end{pmatrix}\right|=\left|\begin{pmatrix}0&-19&17\\-10&-23&22\\-22&-54&21\end{pmatrix}+\begin{pmatrix}4&3&7\\-1&0&2\\3&1&-4\end{pmatrix}\right|=\begin{vmatrix}4&-16&24\\-11&-23&24\\-19&-53&17\end{vmatrix}(\div 4)=$$

$$\left|\begin{array}{ccc|cc} 1 & -4 & 6 & 1 & -4 \\ -11 & -23 & 24 & -11 & -23 \\ -19 & -53 & 17 & -19 & -53 \end{array}\right| = -391+1824+3498-(2622-1272+748) =$$

$$-391+5322-(3370-1272)=4931-2098=\boxed{2833}$$

17×23: $51 + 340 = 391$

96×19: $864 + 960 = 1824$

$5322 - 391 = 4931$; $4931 - 2098 = 2833$

$2622 + 748 = 3370$; $3370 - 1272 = 2098$

$1824 + 3498 = 5322$

17×44: $68 + 680 = 748$

24×53: $72 + 1200 = 1272$

66×53: $198 + 3300 = 3498$

114×23: $342 + 2280 = 2622$

(e)

$\det(AC^t) = \det A . \det C^t = \det A . \det C = 66.0 = \boxed{0}$

$$\det A = \left|\begin{array}{ccc|cc} 3 & -5 & 7 & 3 & -5 \\ 4 & 2 & 8 & 4 & 2 \\ 1 & -9 & 6 & 1 & -9 \end{array}\right| = 36-40-252-(14-216-120) = -256+322) = 66$$

$$\det C = \begin{array}{ccc} (\div 2) & (\div 3) & \\ \end{array}$$

$$\det C = \begin{vmatrix} 2 & 3 & -1 \\ 6 & 9 & 2 \\ 8 & 12 & -3 \end{vmatrix} = 2.3 \left|\begin{array}{ccc|cc} 1 & 1 & -1 & 1 & 1 \\ 3 & 3 & -2 & 3 & 3 \\ 4 & 4 & -3 & 4 & 4 \end{array}\right| = 6[-9-8-12-(-12-8-9)] =$$

$$6[-29-(-29)] = 6[0] = 0$$

3. (a) $\det(A^t) = \det(A) = \boxed{-2}$ (b) $\det(5A) = 5^n \det(A) = 5^4.(-2) = 625.(-2) = \boxed{-1250}$

(c) $\det(A^6) = [\det(A)]^6 = [-2]^6 = \boxed{64}$ (d) $\det(A^{-1}) = \dfrac{1}{\det(A)} = \boxed{-\dfrac{1}{2}}$

4.(a) $\begin{vmatrix} a & b & c \\ d & e & f \\ 5g & 5h & 5i \end{vmatrix} = 5 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5.(-3) = \boxed{-15}$

(b) $\begin{vmatrix} a & b & \overset{(\div -2)}{-2c} \\ 3d & 3e & -6f \\ g & h & -2i \end{vmatrix} = -2 \begin{vmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{vmatrix} (\div 3) = -2.3 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -6.(-3) = \boxed{18}$

(c) $\begin{vmatrix} -a & -b & -c \\ g & h & i \\ -d & -e & -f \end{vmatrix} \begin{matrix} (\div -1) \\ \\ (\div -1) \end{matrix} = (-1)(-1) \begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix} \ell_2 \longleftrightarrow \ell_3 = -1 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} =$

$-1.(-3) = \boxed{3}$

(d) $\begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix} \ell_1 \longleftrightarrow \ell_2 = -1 \begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix} \ell_2 \longleftrightarrow \ell_3 = -1(-1) \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} =$

$1(-3) = \boxed{-3}$

(e) $\begin{vmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{vmatrix} \ell_2 \longleftarrow \ell_2 - \ell_1 = \begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g & h & i \end{vmatrix} (\div 2) =$

$2 \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2(-3) = \boxed{-6}$

(f) $\begin{vmatrix} ka+a & kb+b & ck+c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} ka & kb & kc \\ d & e & f \\ g & h & i \end{vmatrix} (\div k) =$

$$-3+k\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -3+k(-3) = \boxed{-3k-3}$$

5.

$$\det(A) = \begin{vmatrix} 5 & 4 & 20 & 1 \\ 4 & 6 & 20 & -4 \\ -5 & -7 & -30 & 9 \\ 3 & -6 & -30 & 12 \end{vmatrix} \begin{matrix} \\ (\div 2) \\ \\ (\div 3) \end{matrix} = 2.3 \begin{array}{c} \begin{matrix} & & (\div 10) & \end{matrix} \\ \begin{vmatrix} 5 & 4 & 20 & 1 \\ 2 & 3 & 10 & -2 \\ -5 & -7 & -30 & 9 \\ 1 & -2 & -10 & 4 \end{vmatrix} \end{array} =$$

$$6.10\begin{vmatrix} 5 & 4 & 2 & 1 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 9 \\ 1 & -2 & -1 & 4 \end{vmatrix} \ell_1 \leftarrow \ell_1 + 2\ell_4 = 60\begin{vmatrix} 7 & 0 & 0 & 9 \\ 2 & 3 & 1 & -2 \\ -5 & -7 & -3 & 9 \\ 1 & -2 & -1 & 4 \end{vmatrix} =$$

$$60\left[7(-1)^2\begin{vmatrix} 3 & 1 & -2 \\ -7 & -3 & 9 \\ -2 & -1 & 4 \end{vmatrix} + 9(-1)^5\begin{vmatrix} 2 & 3 & 1 \\ -5 & -7 & -3 \\ 1 & -2 & -1 \end{vmatrix}\right] = 60\left[7\begin{vmatrix} 3 & 1 & -2 \\ -7 & -3 & 9 \\ -2 & -1 & 4 \end{vmatrix}\begin{matrix} 3 & 1 \\ -7 & -3 \\ -2 & -1 \end{matrix} - 9\begin{vmatrix} 2 & 3 & 1 \\ -5 & -7 & -3 \\ 1 & -2 & -1 \end{vmatrix}\begin{matrix} 2 & 3 \\ -5 & -7 \\ 1 & -2 \end{matrix}\right] =$$

$$60\{7[-36-18-14-(-12-27-28)]-9[14-9+10-(-7+12+15)]\} = 60\{7[-68-(-67)]-9[15-(20)]\} = 60\{7[-1]-9[-5]\} =$$

$$60\{-7+45\} = 60.38 = \boxed{2280}$$

$$\begin{array}{r} {}^{4} \\ 38 \\ \times\ 60 \\ \hline 2280 \end{array}$$

6. (a)

$$\begin{vmatrix} 4 & 6 & x \\ 7 & 4 & 2x \\ 5 & 2 & -x \end{vmatrix} = -128 \longrightarrow \begin{vmatrix} 4 & 6 & x \\ 7 & 4 & 2x \\ 5 & 2 & -x \end{vmatrix}\begin{matrix} 4 & 6 \\ 7 & 4 \\ 5 & 2 \end{matrix} = -128 \longrightarrow$$

$$-16x+60x+14x-(20x+16x-42x) = -128 \longrightarrow$$

$$58x-(-6x) = -128 \longrightarrow 64x = -128 \longrightarrow \boxed{x=-2}$$