Lista 5

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1.(a) $\vec{BF} = \vec{BA} + \vec{AF} = -\vec{AB} + \vec{f} = \boxed{-\vec{b} + \vec{f}}$

(b) $\vec{AG} = \vec{AF} + \vec{FG} = \vec{f} + \vec{BC} = \vec{f} + \vec{BA} + \vec{AC} = \vec{f} - \vec{AB} + \vec{c} = \boxed{\vec{f} - \vec{b} + \vec{c}}$

(c) $\vec{AE} = \vec{BF} = -\vec{AB} + \vec{AF} = \boxed{-\vec{b} + \vec{f}}$

(d) $\vec{BG} = \vec{BC} + \vec{CG} = -\vec{AB} + \vec{AC} + \vec{BF} = -\vec{b} + \vec{c} - \vec{AB} + \vec{AF} = -\vec{b} + \vec{c} - \vec{b} + \vec{f} =$
$\boxed{-2\vec{b} + \vec{c} + \vec{f}}$

(e) $\vec{HB} = \vec{HE} + \vec{EA} + \vec{AB} = \vec{CB} + \vec{FB} + \vec{b} = \vec{b} - \vec{AC} + \vec{AB} - \vec{AF} + \vec{AB} = \vec{b} - \vec{c} + \vec{b} - \vec{f} + \vec{b} =$
$\boxed{3\vec{b} - \vec{c} - \vec{f}}$

(f) $\vec{AB} + \vec{FG} = \vec{b} + \vec{BC} = \vec{b} - \vec{AB} + \vec{AC} = \vec{b} - \vec{b} + \vec{c} = \boxed{\vec{c}}$

(g) $\vec{AD} + \vec{HG} = \vec{BC} + \vec{AB} = -\vec{AB} + \vec{AC} + \vec{b} = -\vec{b} + \vec{c} + \vec{b} = \boxed{\vec{c}}$

(h) $\vec{HF} + \vec{AG} - \vec{EF} = \vec{HE} + \vec{EF} + \vec{AB} + \vec{BC} + \vec{CG} - \vec{AB} = \vec{CB} + \vec{AB} + \vec{b} - \vec{AB} + \vec{AC} +$
$\vec{BF} - \vec{b} = \vec{b} - \vec{b} + \vec{b} - \vec{b} + \vec{c} - \vec{AC} + \vec{AB} - \vec{AB} + \vec{AF} = \vec{c} - \vec{c} + \vec{b} - \vec{b} + \vec{f} = \boxed{\vec{f}}$

(i) $2\vec{AD} - \vec{FG} - \vec{BH} + \vec{GH} = 2(\vec{BC}) - (\vec{BC}) - (\vec{BF} + \vec{FE} + \vec{EH}) + (\vec{BA}) = (\vec{BC}) - (-\vec{AB} +$
$\vec{AF} + \vec{BA} + \vec{BC}) - \vec{AB} = (-\vec{AB} + \vec{AC}) - (-\vec{b} + \vec{f} - \vec{b} - \vec{AB} + \vec{AC}) - \vec{b} = -\vec{b} + \vec{c} + \vec{b} - \vec{f} + \vec{b} +$
$\vec{AB} - \vec{AC} - \vec{b} = \vec{c} - \vec{f} + \vec{b} - \vec{c} = \boxed{\vec{b} - \vec{f}}$

2.(a) $\vec{DF} = \vec{DE} + \vec{DC} + \vec{DE} = \boxed{2\vec{DE} + \vec{DC}}$

(b) $\vec{DA} = \vec{DE} + \vec{DC} + \vec{DC} + \vec{DE} = \boxed{2\vec{DE} + 2\vec{DC}}$

(c) $\vec{DB} = \vec{DC} + \vec{DE} + \vec{DC} = \boxed{2\vec{DC} + \vec{DE}}$

(d) $\vec{DO} = \boxed{\vec{DE} + \vec{DC}}$

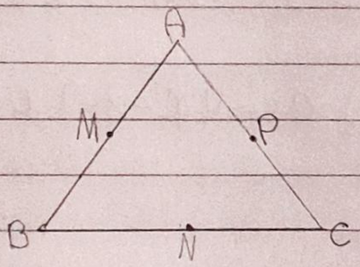
(e) $\vec{EC} = \boxed{\vec{DC} - \vec{DE}}$

(f) $\vec{EB} = \vec{DC} + \vec{DC} = \boxed{2\vec{DC}}$

(g) $\vec{OB} = \boxed{\vec{DC}}$

(h) $\vec{AF} = \boxed{-\vec{DC}}$

3.

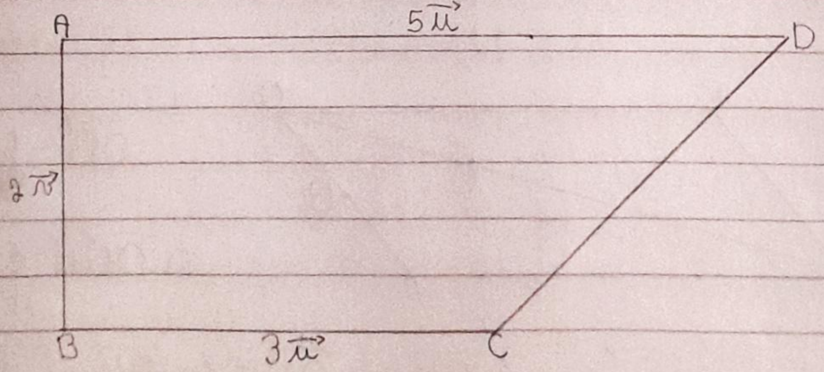


$$\vec{BP} = \boxed{-\vec{AB} + \frac{1}{2}\vec{AC}}$$

$$\vec{AN} = \vec{AP} + \vec{PN} = \frac{1}{2}\vec{AC} + \vec{MB} = \boxed{\frac{1}{2}\vec{AC} + \frac{1}{2}\vec{AB}}$$

$$\vec{CM} = \vec{CA} + \vec{AM} = \boxed{-\vec{AC} + \frac{1}{2}\vec{AB}}$$

4.(a)



$\vec{AD} = 5\vec{u}$

$\vec{AB} = 2\vec{v}$

$\vec{BC} = 3\vec{u}$

$$\vec{CD} = -\vec{BC} - \vec{AB} + \vec{AD} = -3\vec{u} - 2\vec{v} + 5\vec{u} = \boxed{2\vec{u} - 2\vec{v}}$$

$\vec{BD} = -\vec{AB} + \vec{AD} = \boxed{-2\vec{v} + 5\vec{u}}$ $\quad$ $\vec{CA} = -\vec{BC} - \vec{AB} = \boxed{-3\vec{u} - 2\vec{v}}$

(b) $\vec{AD} = 5\vec{u}$
$\vec{BC} = 3\vec{u}$
$\vec{AB} = 2\vec{v}$
$\vec{DC} = 2\vec{v} - 2\vec{u}$

Então $\begin{cases} \vec{AD} // \vec{BC} \\ \|\vec{AD}\| \neq \|\vec{BC}\| \end{cases}$

formando um trapézio

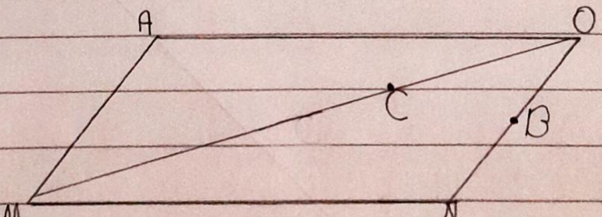
5. $\vec{DE} = \vec{DA} + \vec{AO} + \vec{OB} + \vec{BE} = -\vec{AD} - \vec{OA} + \vec{OB} + \vec{BE} = -\frac{1}{4}\vec{c} - \vec{a} + \vec{b} + \frac{5}{6}\vec{a} =$
$\boxed{-\frac{1}{6}\vec{a} + \vec{b} - \frac{1}{4}\vec{c}}$

6. $\vec{AC} = -\vec{OA} + \vec{OC} = -(\vec{a} + 2\vec{b}) + 5\vec{a} + x\vec{b} = -\vec{a} - 2\vec{b} + 5\vec{a} + x\vec{b} = 4\vec{a} + (x-2)\vec{b}$
$\vec{BC} = -\vec{OB} + OC = -(3\vec{a} + 2\vec{b}) + 5\vec{a} + x\vec{b} = -3\vec{a} - 2\vec{b} + 5\vec{a} + x\vec{b} = 2\vec{a} + (x-2)\vec{b}$

$\begin{vmatrix} 4 & (x-2) \\ 2 & (x-2) \end{vmatrix} = 0 \longrightarrow 4(x-2) - 2(x-2) = 0 \longrightarrow 4x - 8 - 2x + 4 = 0 \longrightarrow$
$2x - 4 = 0 \longrightarrow x = \frac{4}{2} \longrightarrow x = 2$

Para os vetores $\vec{AC}$ e $\vec{BC}$ serem linearmente dependentes, x tem que ser igual a 2.

7.



$\vec{OB} = \frac{1}{n}\vec{ON}$

$\vec{OC} = \frac{1}{1+n}\vec{OM}$

$\vec{BC} = \vec{BO} + \vec{OC} = -\vec{OB} + \vec{OC} = -\frac{1}{n}\vec{ON} + \frac{1}{1+n}\vec{OM} = \frac{1}{1+n}\vec{OM} - \frac{1}{n}\vec{ON}$

$\vec{CA} = \vec{CM} + \vec{MA} = (1 - \frac{1}{1+n})\vec{OM} + \vec{NO} = -\vec{ON} + (1 - \frac{1}{1+n})\vec{OM} = \frac{n}{1+n}\vec{OM} - \vec{ON}$

tilibra

$$\begin{vmatrix} \frac{1}{1+n} & -\frac{1}{n} \\ \frac{n}{1+n} & -1 \end{vmatrix} = -\frac{1}{1+n} - \left(-\frac{1}{n} \cdot \frac{n}{1+n}\right) = -\frac{1}{1+n} - \left(-\frac{n}{n(1+n)}\right) =$$

$$-\frac{1}{1+n} + \frac{n}{n+n^2} = -\frac{1}{1+n} + \frac{n}{n(1+n)} = -\frac{1}{1+n} + \frac{1}{1+n} = 0 \text{ (vetores LD)}$$

Se $\overrightarrow{BC}$ e $\overrightarrow{CA}$ são palalelos, então os pontos A, B e C são colineares.

8. $\begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} = -4 - 1 = -5 \ (\neq 0)$ então são LI, sendo uma base para o plano.

9.a) $\left|\begin{matrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{matrix}\right|\begin{matrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{matrix} = -1+1-(1+1) = 0-2 = -2 \ (\neq 0)$ então também são LI

b) $\vec{t} = a\vec{u} + b\vec{v} + c\vec{w}$

$\vec{u} + \vec{t} = a\vec{u} + b\vec{v} + c\vec{w} + \vec{u} = (a+1)\vec{u} + b\vec{v} + c\vec{w}$

$\vec{v} + \vec{t} = a\vec{u} + b\vec{v} + c\vec{w} + \vec{v} = a\vec{u} + (b+1)\vec{v} + c\vec{w}$

$\vec{w} + \vec{t} = a\vec{u} + b\vec{v} + c\vec{w} + \vec{w} = a\vec{u} + b\vec{v} + (c+1)\vec{w}$

$\left|\begin{matrix} a+1 & b & c \\ a & b+1 & c \\ a & b & c+1 \end{matrix}\right|\begin{matrix} a+1 & b \\ a & b+1 \\ a & b \end{matrix} \neq 0 \longrightarrow (a+1)(b+1)(c+1) + bca + cab - [ac(b+1) + bc(a+1) + ab(c+1)]$

$\neq 0 \longrightarrow abc + ab + ac + a + bc + b + c + 1 + 2abc - [abc + ac + abc + bc + abc + ab] \neq 0 \longrightarrow 3abc + ab + ac + bc + a + b + c + 1 - [3abc + ab + ac + bc] \neq 0 \longrightarrow \cancel{3abc} + \cancel{ab} + \cancel{ac} + \cancel{bc} + a + b + c + 1 - \cancel{3abc} - \cancel{ab} - \cancel{ac} - \cancel{bc} \neq 0$

$\longrightarrow a+b+c+1 \neq 0 \longrightarrow a+b+c \neq -1$ então, para que os vetores sejam LI, $a+b+c$ tem que ser diferente de -1.

10.a) $\overrightarrow{AB} = B - A = (1,0,-1) - (1,3,2) = \boxed{(0,-3,-3)}$

$\overrightarrow{BC} = C - B = (1,1,0) - (1,0,-1) = (1-1, 1, 1) = \boxed{(0,1,1)}$

$\overrightarrow{CA} = A - C = (1,3,2) - (1,1,0) = \boxed{(0,2,2)}$

b) $\overrightarrow{AB} + \frac{2}{3}\overrightarrow{BC} = (0,-3,-3) + \frac{2}{3}(0,1,1) = (0,-3,-3) + (0, \frac{2}{3}, \frac{2}{3}) =$

$\boxed{\left(0, -\frac{7}{3}, -\frac{7}{3}\right)}$

c) $C + \frac{1}{2}\overrightarrow{AB} = (1,1,0) + \frac{1}{2}(0,-3,-3) = (1,1,0) + \left(0, -\frac{3}{2}, -\frac{3}{2}\right) = \boxed{\left(1, -\frac{1}{2}, -\frac{3}{2}\right)}$

d) $A - 2\overrightarrow{BC} = (1,3,2) - 2(0,1,1) = (1,3,2) + (0,-2,-2) = \boxed{(1,1,0)}$

11. a) $\{(2,3),(0,2)\}$ $\begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} = 4 - 0 = 4\ (\neq 0)$ vetores LI

b) $\{(3,0),(-2,0)\}$ $\begin{vmatrix} 3 & 0 \\ -2 & 0 \end{vmatrix} = 0 + 0 = 0$ esses vetores não são LI

c) $\{(2,3,4),(0,3,3)\}$ são LD, se $\exists\, \alpha \in \mathbb{R}$, tal que, $(2,3,4) = \alpha\,(0,3,3)$

$\begin{cases} 2 = 0 \\ 3 = \alpha 3 \\ 4 = \alpha 3 \end{cases}$ $\nexists\, \alpha \in \mathbb{R} \mid (2,3,4) = \alpha\,(0,3,3)$ então, os vetores são LI

d) $\{(1,-1,2),(1,1,0),(1,-1,1)\}$ $\left|\begin{array}{ccc|cc} 1 & -1 & 2 & 1 & -1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 \end{array}\right| = 1 - 2 - (2 - 1) = -1 - 1 = -2$ $(\neq 0)$

vetores LI

e) $\{(1,-1,1),(-1,2,1),(-1,2,2)\}$ $\left|\begin{array}{ccc|cc} 1 & -1 & 1 & 1 & -1 \\ -1 & 2 & 1 & -1 & 2 \\ -1 & 2 & 2 & -1 & 2 \end{array}\right| = 4 + 1 - 2 - (-2 + 2 + 2) =$ $3 - (2) = 1\ (\neq 0)$

vetores LI

f) $\{(1,0,1),(0,0,1),(2,0,5)\}$ $\left|\begin{array}{ccc|cc} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 5 & 2 & 0 \end{array}\right| = 0$ esses vetores não são LI

12. a) Sejam $x, y \in \mathbb{R}$

$x\vec{u} + y\vec{v} = \vec{w}$

$x(2,-1) + y(1,-1) = (1,1)$

$\begin{cases} 2x + y = 1 \ldots (1) \\ -x - y = 1 \ldots (2) \end{cases}$ $\quad (1)+(2): x = 2$

$-2 - y = 1 \longrightarrow y = -3$ $\quad 2\vec{u} - 3\vec{v} = \vec{w}$

Então, considerando os vetores $\vec{u}$ e $\vec{v}$ como uma base, as coordenadas de $\vec{w}$ são $(2,-3)$.

b) Sejam $x, y, z \in \mathbb{R}$

$x\vec{a} + y\vec{b} + z\vec{c} = \vec{Z}$

$x(1,1,1) + y(0,1,1) + z(1,1,0) = (1,2,3)$

$\begin{cases} x + z = 1 \\ x + y + z = 2 \\ x + y = 3 \end{cases}$

$3 + z = 2 \longrightarrow z = -1$

$x - 1 = 1 \longrightarrow x = 2$

$2 + y - 1 = 2 \longrightarrow y = 1$ $\quad 2\vec{a} + \vec{b} - \vec{c} = \vec{Z}$

Então, considerando os vetores $\vec{a}, \vec{b}$ e $\vec{c}$ como uma base, as coordenadas de $\vec{Z}$ são $(2,1,-1)$.

13. a) $\{(1, m-1, m), (m, 2n, 4)\}$ são LD, se, e somente se, $\exists \alpha \in \mathbb{R} \mid$

$(1, m-1, m) = \alpha (m, 2n, 4)$

$\begin{cases} \alpha m = 1 \longrightarrow \alpha = \frac{1}{m} \\ \alpha 2n = m - 1 \longrightarrow \alpha = \frac{m-1}{2n} \\ \alpha 4 = m \longrightarrow \alpha = \frac{m}{4} \end{cases}$

$\frac{1}{m} = \frac{m}{4} \longrightarrow m^2 = 4 \longrightarrow m = \pm 2$

$\frac{1}{2} = \frac{2-1}{2n} \longrightarrow 2n = 2(2-1) \longrightarrow 2n = 4 - 2 \longrightarrow n = \frac{2}{2} \longrightarrow n = 1$

$\frac{-1}{2} = \frac{-2-1}{2n} \longrightarrow -2n = 2(-2-1) \longrightarrow -2n = -4-2 \longrightarrow n = \frac{-6}{-2} \longrightarrow$

$n = 3$

Para que os vetores sejam LD, $m = 2$ e $n = 1$ ou $m = -2$ ou $n = 3$

13. b) $\{(1, m, n+1), (m, n+1, 8)\}$ são LD se, e somente se, $\exists \alpha \in \mathbb{R} \mid$
$\alpha(1, m, n+1) = (m, n+1, 8)$

$$\begin{cases} \alpha = m \\ \alpha m = n+1 \\ \alpha(n+1) = 8 \end{cases}$$

$\alpha^2 = n+1$

$\alpha^3 = 8 \longrightarrow \alpha = \sqrt[3]{8} \longrightarrow \alpha = 2$

$2(n+1) = 8 \longrightarrow 2n+2 = 8 \longrightarrow 2n = 6 \longrightarrow n = 3$

$m = \alpha \longrightarrow m = 2$

Para que os vetores sejam LD, $m = 2$ e $n = 3$

14. $\left|\begin{matrix} m & -1 & m^2+1 \\ m^2+1 & m & 0 \\ m & 1 & 1 \end{matrix}\right| \begin{matrix} m & -1 \\ m^2+1 & m \\ m & 1 \end{matrix} = 0 \longrightarrow m^2 + (m^2+1)^2 - [m^2(m^2+1) - m^2+1] = 0 \longrightarrow m^2 + m^4 + 2m^2 + 1 - [m^4 + m^2 - m^2 + 1] = 0 \longrightarrow \cancel{m^2} + \cancel{m^4} + 2m^2 + 1 - \cancel{m^4} - \cancel{m^2} + m^2 + 1 = 0 \longrightarrow 3m^2 + 2 = 0 \longrightarrow m^2 = -\frac{2}{3}$

$\nexists m \in \mathbb{R} \mid 3m^2 + 2 = 0$

15. a) $\left|\begin{matrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{matrix}\right| \begin{matrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{matrix} = 1 - (1-1) = 1 - 0 = 1 \ (\neq 0)$ os vetores são LI

b) $\vec{f_1} = (1,1,0)_B = \vec{e_1} + \vec{e_2}$
$\vec{f_2} = (1,0,1)_B = \vec{e_1} + \vec{e_3}$
$\vec{f_3} = (1,1,-1)_B = \vec{e_1} + \vec{e_2} - \vec{e_3}$

$$\begin{pmatrix} \vec{f}_1 \\ \vec{f}_2 \\ \vec{f}_3 \end{pmatrix}^t = \left[\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} \vec{e}_1 \\ \vec{e}_2 \\ \vec{e}_3 \end{pmatrix}\right]^t \longrightarrow \begin{pmatrix} \vec{f}_1 & \vec{f}_2 & \vec{f}_3 \end{pmatrix} = \begin{pmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix} = \begin{pmatrix} \vec{e}_1 \cdot \vec{e}_2 & \vec{e}_3 \end{pmatrix} \begin{pmatrix} 12 \\ 9 \\ -4 \end{pmatrix} =$$

$(12, 9, -4)_B$

c) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ $\quad \text{cof} = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 0 & -1 \\ 0 & -1 & -1 \end{bmatrix}$ $\quad \text{adj} = \begin{vmatrix} 1 & -1 & 0 \\ -1 & 0 & -1 \\ 1 & -1 & -1 \end{vmatrix}$

$$\vec{v} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 0 & -1 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 2-3 \\ -2-7 \\ 2-3-7 \end{pmatrix} = \begin{pmatrix} -1 \\ -9 \\ -8 \end{pmatrix} = (-1, -9, -8)_C$$