Modelos Computacionais em Economia

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Bounds Estimation [Adams, 2020]

We are interested in using the data to estimate the bounds on the policy parameter of interest (we are limited to estimating a range of values).

Why not average treatment effect (ATE)?

Public state colleges free: if everyone changes from not attending college to attending college, the ATE predicts what would happen. If attending college has the same effect on everyone, then the ATE provides useful information.

What does the ATE tell us will happen to those that are **newly encouraged** to go to college? **Not that much**.

Bounds Estimation [Adams, 2020]

Two implications:

- **First case:** the data allows the ATE to be estimated, but we would prefer to know its distribution. In general, **we cannot estimate this distribution**. We can, however, bound it.

  Kolmogorov bounds: Soviet mathematician, Andrey Kolmogorov.
- **Second case:** the data does not allow the ATE to be estimated. We are unwilling to make the non-credible assumptions necessary to estimate the ATE.

  Charles Manski:
  [Manski, 1990] *range of estimates is better than providing precisely estimated nonsense* (Manski's natural bounds).

Bounds Estimation [Adams, 2020]

Potencial Outcomes

You have been tasked by the **AOC 2028** campaign to estimate the likely impact of a proposal to make state public universities tuition free.

How many more people will attend college once it is made free?
You need to estimate the **treatment effect** of college.

Two possible outcomes:

- The income the individual receives if they attend college ($y_i(1)$).
- The income they would receive if they did not attend college ($y_i(0)$).

$$y_i(x_i) = a + b_i x_i + v_i$$

where $x_i \in \{0, 1\}$ and the tratment effect is represented by b_i. For each individual,

$$b_i = y_i(1) - y_i(0)$$

Bounds Estimation [Adams, 2020]

Simulation of Impossible Data (counter-factual outcomes)

These counter-factual outcomes are called potential outcomes [Rubin, 1974].

```
set.seed(123456789)
N <- 200
a <- 2
b <- rnorm(N,mean=2,sd=3)
# this creates variation in the slope with an average
# effect of 2.
x0 <- rep(0,N) # creates a vector of zeros
x1 <- rep(1,N)
u <- rnorm(N)
y <- a + b*cbind(x0,x1) + u
# y is a matrix, [a + u, a + b + u]
```

Bounds Estimation [Adams, 2020]

Simulation of Impossible Data (counter-factual outcomes)

Distribution of the Treatment Effect: density functions, means and cumulative distribution functions of the two potential outcomes.

```
par(mfrow=c(2,1)) # creates a simple panel plot
par(mar=c(2,4,0.5,0.5)) # adjusts margins between plot
plot(density(y[,1]),type="l",lwd=5,xlim=range(y),
ylab="density",main="")
lines(density(y[,2]),lwd=2)
abline(v=colMeans(y),lwd=c(5,2))
legend("topright",c("No College","College"),lwd=c(5,2)
```

Assuming that this simulated data represented real data, **should AOC 2028 use these results as evidence for making college free?**

A concern is that the **two distributions overlap**. Moreover, the cumulative **distributions functions cross**. There may be individuals in the data who are actually better off if $x = 0$.

Bounds Estimation [Adams, 2020]

Simulation of Impossible Data (counter-factual outcomes)

Distribution of the Treatment Effect: density and cumulative distribution function for the difference in outcome if the individual attended college and if the individual did not.

```
plot(ecdf(y[,1]), xlim=range(y),main="",do.points=FALS
lwd=5,xlab="y")
lines(ecdf(y[,2]),lwd=2,do.points=FALSE)
# ecdf empirical cumulative distribution function.
```

The treatment effect **varies across individuals** (heterogeneous) and the effect of college may either **increase or decrease income**, depending on the individual.

Bounds Estimation [Adams, 2020]

Average Treatment Effect (ATE)

Outside of our impossible data we cannot observe the difference in potential outcomes. **How can we measure the average of something we cannot observe?**

Its derivation: the mean of the difference is equal to the difference of the means.

```
# mean of the difference vs difference of the means
mean(y[,2]-y[,1]) == mean(y[,2]) - mean(y[,1])
```

Averages are linear operators such that we can can write out the expected difference in potential outcomes by the Law of Total Expectations.

$$\mathbb{E}(Y_1 - Y_0) = \int_{y_0} \int_{y_1} (y_1 - y_0) f(y_1|y_0) f(y_0) dy_1 dy_0$$

Bounds Estimation [Adams, 2020]

Average Treatment Effect (ATE)

Manipulating the conditional expectations [Rubin, 1974]:

$$
\begin{aligned}
\mathbb{E}(Y_1 - Y_0) &= \int_{y_0} \left(\int_{y_1} y_1 f(y_1|y_0) dy_1 - y_0 \right) f(y_0) dy_0 \\
&= \int_{y_0} \left(\int_{y_1} y_1 f(y_1|y_0) dy_1 \right) f(y_0) dy_0 - \int_{y_0} y_0 f(y_0) dy_0 \\
&= \int_{y_1} y_1 f(y_1) dy_1 - \int_{y_0} y_0 f(y_0) dy_0 \\
&= \mathbb{E}(Y_1) - \mathbb{E}(Y_0)
\end{aligned} \tag{1}
$$

We have an estimate of the average treatment effect. But can we estimate the average potential outcome?

Bounds Estimation [Adams, 2020]

ATE and Do Operators [Pearl and Mackenzie, 2018]

The **expected potential outcome** if $X = 1$ is assumed to be equal to the **expected outcome conditional** on $do(X) = 1$.

$$\mathbb{E}(Y_1) = \mathbb{E}(Y|do(X) = 1)$$

By "do" we mean that this is the expected outcome if individuals in the data faced a policy which forced the treatment $X = 1$.

Expected potential outcome of a treatment may **not be equal** to expected outcomes in a particular treatment.
In general, $E(Y|do(X) = 1) \neq= E(Y|X = 1)$. The second term is observed in data.

Bounds Estimation [Adams, 2020]

ATE and Do Operators [Pearl and Mackenzie, 2018]

We can write down the **expected outcome conditional on the do operator** by the Law of Total Expectations.

$$\begin{aligned} \mathbb{E}(Y|do(X)=1) = \mathbb{E}(Y|do(X)=1, X=0)Pr(X=0) \\ + \mathbb{E}(Y|do(X)=1, X=1)Pr(X=1) \end{aligned} \tag{2}$$

- We observe the probabilities.
- We assume that $\mathbb{E}(Y|do(X)=1, X=1) = \mathbb{E}(Y|X=1)$ (the expected outcome for people assigned to a treatment will be the same as if there was a policy that assigned them to the same treatment).
- We do not observe in the data is $\mathbb{E}(Y|do(X)=1, X=0)$

Bounds Estimation [Adams, 2020]

ATE and Unconfoundedness

Assumption 1. Unconfoundedness:
$\mathbb{E}(Y|do(X) = x, X = x) = \mathbb{E}(Y|do(X) = x, X = x')$.
The expected outcome of the policy does not vary with treatment observed in the data. This assumption may be **reasonable if we have data from an ideal randomized controlled trial**.

The assumption implies:

$$\begin{aligned}\mathbb{E}(Y|do(X) = 1)\mathbb{E}(Y|do(X) = 1, X = 0)&Pr(X = 0)\\ + \mathbb{E}(Y|do(X) = 1, X = 1)&Pr(X = 1)\\ = \mathbb{E}(Y|do(X) = 1, X = 1)&Pr(X = 0)\\ + \mathbb{E}(Y|do(X) = 1, X = 1)&Pr(X = 1)\\ = \mathbb{E}(Y|X = 1)&\end{aligned} \tag{3}$$

The implication is that we can estimate the average of the potential outcomes or each treatment.

Bounds Estimation [Adams, 2020]

ATE and Simulated Data

Consider we only see one outcome and one treatment for each individual:

```
X <- runif(N) < 0.3 # treatment assignment
Y <- (1-X)*y[,1] + X*y[,2]
# outcome conditional on treatment
```

We can make the unconfoundedness assumption then we can estimate the average treatment effect. Our new data satisfies the assumption because the **assignment to treatment is random**.

```
mean(Y[X==1]) - mean(Y[X==0])
```

the true average treatment effect is 2. Our estimate is 2.43. What changes could you make to the simulated data that would increase the accuracy of the estimate?

Bounds Estimation [Adams, 2020]

Kolmogorov Bounds

It may be useful for policy makers to know something about the **joint distribution of potential outcomes** or the **distribution of the treatment effect**. **We cannot estimate** them. However, we can bound these distributions.

The difference of two random variables with known marginals could be bounded [Fan and Park, 2010]:

Teorem 1. *Kolmogorov's Conjecture.* Let $\beta_i = y_i(1) - y_i(0)$ denote the treatment effect and F denote its distribution. Let F_0 denote the distribution of outcomes for treatment $(x = 0)$ and F_1 denote the distribution of outcomes for treatment $(x = 1)$. Then $F^L(b) \leq F(b) \leq F^U(b)$, where

$$F^L(b) = \max\{\max_y F_1(y) - F_0(y - b), 0\}$$

and

$$F^U(b) = 1 + \min\{\min_y F_1(y) - F_0(y - b), 0\}$$

Bounds Estimation [Adams, 2020]

Kolmogorov Bounds in R

```r
FL <- function(b, y1, y0) {
f <- function(x) -(mean(y1 < x) - mean(y0 < x - b))
# note the negative sign as we are maximizing
# (Remember to put it back!)
a <- optimize(f, c(min(y1,y0),max(y1,y0)))
return(max(-a$objective,0))
}
FU <- function(b, y1, y0) {
f <- function(x) mean(y1 < x) - mean(y0 < x - b)
a <- optimize(f, c(min(y1,y0), max(y1,y0)))
return(1 + min(a$objective,0))
}
```

Bounds Estimation [Adams, 2020]

Kolmogorov Bounds in R

```
K <- 50
min_diff <- min(y[,1]) - max(y[,2])
max_diff <- max(y[,1]) - min(y[,2])
delta_diff <- (max_diff - min_diff)/K
y_K <- min_diff + c(1:K)*delta_diff
plot(ecdf(y[,2] - y[,1]), do.points=FALSE,lwd=3,main="
lines(y_K,sapply(y_K, function(x) FL(x,y[,2],y[,1])),
lty=2,lwd=3)
lines(y_K,sapply(y_K, function(x) FU(x,y[,2],y[,1])),
lty=3,lwd=3)
abline(v=0,lty=2,lwd=3)
```

Bounds Estimation [Adams, 2020]

Do "Nudges" Increase Saving?

Can policies or products be provided that “nudge” individuals to make better decisions?
[Ashraf et al., 2006] use a field experiment to determine the effectiveness of a commitment savings account.

- There are three treatment groups:
  1) The first group is offered the commitment savings account at no extra cost or savings;
  2) The second group is provided information on the value of savings;
  3) The third is a control.

Bounds Estimation [Adams, 2020]

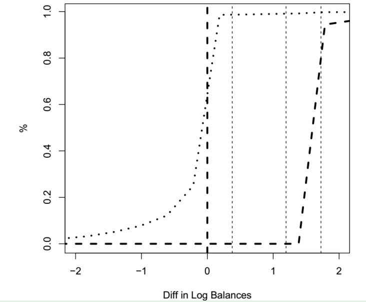
Field Experiment Data

```
require(readstata13)
# this data set was saved with Stata version 13.
x <- read.dta13("seedanalysis_011204_080404.dta")
index_na <- is.na(rowSums(cbind(x$treatment,
x$balchange,x$marketing)))==0
x1 <- x[index_na,]
bal_0 <- x1[x1$treatment==0 & x1$marketing==0,]$balchange
bal_1 <- x1[x1$treatment==1 & x1$marketing==0,]$balchange
# we are just going to look at the people who did not receive
# the marketing information.
# These people are split between those that received
# the account
# (treatment = 1), and those that did not (treatment = 0).
# balchange - measure their balance changed in a year.
lbal_0 <- log(bal_0 + 2169)
lbal_1 <- log(bal_1 + 2169)
# the distribution of balances is very skewed.
mean(bal_1) - mean(bal_0)
```

Bounds Estimation [Adams, 2020]

Field Experiment Data

However, it is not clear if everyone benefits and how much benefit these accounts provide.



Upper and lower bounds on the **distribution of difference in log balances** between the treatment and the control. Marks at 1,000, 5,000 and 10,000 using the transformation above.

We have estimated the treatment effect of being "assigned" to a commitment account. But **people are not lab rats.**

Bounds Estimation [Adams, 2020]

Manski Bounds

[Manski, 1990] argues that many of the assumptions underlying standard econometrics are *ad hoc* and unjustified.
... Econometrician and the policy maker may have different views on the reasonableness of assumptions.
Confounded Model:

$$y_i(x_i) = a + b_i x_i + v_{1i}$$

Our model with income, attends college and some unobserved characteristic. The treatment effect is represented by b_i and this may vary across individuals.

Bounds Estimation [Adams, 2020]

Manski Bounds

This time, the value of the policy variable is also determined by the unobserved characteristic that determines income.

$$x_i^* = f + cv_{1i} + dz_i + v_{2i}$$

$$x_i = \begin{cases} 1 & \text{if } x_i^* > 0 \\ 0 & \text{otherwise} \end{cases}$$

Latent variables (x_i^*) are variables that are not directly observed but are rather inferred (through a mathematical model) from other variables that are observed.

So, if v_{1i} is large and the parameter c is positive, then y_i will tend to be larger when x_i is 1 and lower when x_i is 0.

Bounds Estimation [Adams, 2020]

Simulation of Manski Bounds

```
set.seed(123456789)
N <- 200
c <- 2
d <- 4
f <- -1
Z <- round(runif(N))
u_2 <- rnorm(N)
x_star <- f + c*u + d*Z + u_2
X <- x_star > 0 # treatment assignment
# outcome conditional on treatment
Y <- (1-X)*y[,1] + X*y[,2]
mean(Y[X==1]) - mean(Y[X==0])
```

Our estimate is not close to the true value of 2. Try running OLS of y on x. What do you get? In economics we call this a **selection problem**.

Bounds Estimation [Adams, 2020]

Bounding the Average Treatment Effect

$$ATE = \mathbb{E}(Y|do(X) = 1) - \mathbb{E}(Y|do(X) = 0)$$

By the Law of Total Expectation,

$$\begin{aligned} ATE = &\mathbb{E}(Y|do(X) = 1, X = 1)Pr(X = 1) + \\ &\mathbb{E}(Y|do(X) = 1, X = 0)Pr(X = 0) \\ &-(\mathbb{E}(Y|do(X) = 0, X = 1)Pr(X = 1) + \\ &\mathbb{E}(Y|do(X) = 0, X = 0)Pr(X = 0)) \end{aligned} \tag{4}$$

We observe the outcome of the policy that sends the individuals to college for the group that actually goes to college. If we assume that their outcome are the same, we can substitute the observed values into the equation.

$$\begin{aligned} ATE = &Pr(X = 1)(E(Y|X = 1) - E(Y|do(X) = 0, X = 1)) \\ &+ Pr(X = 0)(E(Y|do(X) = 1, X = 0) - E(Y|X = 0)) \end{aligned} \tag{5}$$

Bounds Estimation [Adams, 2020]

Natural Bounding the Average Treatment Effect

An expectation is bounded by the smallest possible value and the largest possible value. The **Natural Bounds** are created by replacing the unknown values with the smallest (largest) values they could be.

- $\underline{Y}$ the lower bound.
- $\overline{Y}$ the upper bound.

$$\overline{ATE} = (\mathbb{E}(Y|X=1) - \underline{Y})Pr(X=1) + (\overline{Y} - \mathbb{E}(Y|X=0))Pr(X=0)$$

$$\underline{ATE} = (\mathbb{E}(Y|X=1) - \overline{Y})Pr(X=1) + (\underline{Y} - \mathbb{E}(Y|X=0))Pr(X=0)$$

Bounds Estimation [Adams, 2020]

Natural Bounding with Simulated Data

```
PX1 = mean(X==1)
PX0 = mean(X==0)
EY_X1 = mean(Y[X==1])
EY_X0 = mean(Y[X==0])
minY = min(Y)
maxY = max(Y)
# ATE upper bound
(EY_X1 - minY)*PX1 + (maxY - EY_X0)*PX0
# ATE lower bound
(EY_X1 - maxY)*PX1 + (minY - EY_X0)*PX0
```

These bounds are wide.

Bounds Estimation [Adams, 2020]

Are Natural Bounds Useless?

What can we take away from this information? Make stronger assumptions?

1) [Manski, 1990] calls "lure of incredible certitude".

2) We don't learn anything from the data. We learn that the effect of a policy $do(X) = 1$ cannot have a larger effect than 8.

3) There may be assumptions and data that are reasonable and allow tighter bounds.

Bounds Estimation [Adams, 2020]

Bounds with Exogenous Variation

We **need variation such that the effect of the policy doesn't change across different subsets of the data, but the bounds do**.

Assumption 2. $E(Y|do(X)=1, Z=z) - E(Y|do(X)=0, Z=z)$
$= E(Y|do(X)=1, Z=z') - E(Y|do(X)=0, Z=z')$ for all z, z'.
[Manski, 1990] calls it a level-set assumption. Thus the new bounds are the intersection of these estimated bounds.

$$\begin{aligned} \overline{ATE} = \min\{&(\mathbb{E}(Y|X=1, Z=1) - \underline{Y})Pr(X=1|Z=1) \\ &+ (\overline{Y} - \mathbb{E}(Y|X=0, Z=1)Pr(X=0|Z=1), \\ &(\mathbb{E}(Y|X=1, Z=0) - \underline{Y})Pr(X=1|Z=1) \\ &+ (\overline{Y} - \mathbb{E}(Y|X=0, Z=0))Pr(X=0, Z=0)\} \end{aligned} \tag{6}$$

Bounds Estimation [Adams, 2020]

Bounds with Exogenous Variation

We **need variation such that the effect of the policy doesn't change across different subsets of the data, but the bounds do**.

Assumption 2. $E(Y|do(X) = 1, Z = z) - E(Y|do(X) = 0, Z = z)$
$= E(Y|do(X) = 1, Z = z') - E(Y|do(X) = 0, Z = z')$ for all z, z'.
[Manski, 1990] calls it a level-set assumption. Thus the new bounds are the intersection of these estimated bounds.

$$\begin{aligned} \underline{ATE} = \max\{&(\mathbb{E}(Y|X=1,Z=1) - \underline{Y})Pr(X=1|Z=1) \\ &+ (\underline{Y} - \mathbb{E}(Y|X=0,Z=1)Pr(X=0|Z=1), \\ &(\mathbb{E}(Y|X=1,Z=0) - \overline{Y})Pr(X=1|Z=1) \\ &+ (\underline{Y} - \mathbb{E}(Y|X=0,Z=0))Pr(X=0,Z=0)\} \end{aligned} \tag{7}$$

Bounds Estimation [Adams, 2020]

Exogenous Variation in Simulated Data

```
EY_X1Z1 = mean(Y[X==1 & Z==1])
EY_X1Z0 = mean(Y[X==1 & Z==0])
EY_X0Z1 = mean(Y[X==0 & Z==1])
EY_X0Z0 = mean(Y[X==0 & Z==0])
PX1_Z1 = mean(X[Z==1]==1)
PX1_Z0 = mean(X[Z==0]==1)
PX0_Z1 = mean(X[Z==1]==0)
PX0_Z0 = mean(X[Z==0]==0)
# ATE upper bound
min((EY_X1Z1 - minY)*PX1_Z1 + (maxY - EY_X0Z1)*PX0_Z1,
(EY_X1Z0 - minY)*PX1_Z0 + (maxY - EY_X0Z0)*PX0_Z0)

# ATE lower bound
max((EY_X1Z1 - maxY)*PX1_Z1 + (minY - EY_X0Z1)*PX0_Z1,
(EY_X1Z0 - maxY)*PX1_Z0 + (minY - EY_X0Z0)*PX0_Z0)
```

Bounds Estimation [Adams, 2020]

Bounds with Monotonicity

Can the bounds be tighter with some economics? We observe the cases where $do(X) = x$ and $X = x$ match. We don't observe the cases where they don't match. However, **we can use the observed cases to bound the unobserved cases**.

In the simulated data a higher unobserved term is associated with a greater likelihood of choosing treatment $x = 1$.

Assumption 3. Monotonicity.
$E(Y|do(X) = 1, X = 1) \geq E(Y|do(X) = 1, X = 0)$ and
$E(Y|do(X) = 0, X = 0) \leq E(Y|do(X) = 0, X = 1)$

Bounds Estimation [Adams, 2020]

Bounds with Monotonicity

In particular, the upper bound can be adjusted down.

$$\overline{\mathbb{E}(Y|do(X) = 1)} = \mathbb{E}(Y|X = 1)$$

$$\underline{\mathbb{E}(Y|do(X) = 0)} = \mathbb{E}(Y|X = 0)$$

The monotonicity assumption implies that forcing everyone into treatment $x = 1$ ($x = 0$) cannot lead to better (worse) expected outcomes than the outcomes we observe given the treatment.

$$\overline{ATE} = (\overline{Y} - \mathbb{E}(Y|X = 0))Pr(X = 0)$$

$$\underline{ATE} = (\mathbb{E}(Y|X = 1) - \overline{Y})Pr(X = 1)$$

Bounds Estimation [Adams, 2020]

Bounds with Monotonicity in Simulated Data

```
# ATE upper bound
(maxY - EY_X0)*PX0

# ATE lower bound
(EY_X1 - maxY)*PX1
```

Imposing Assumption 3 on the simulated data allows us to tighten the bounds. They reduce down to $[-3.65, 3.77]$. Remember the true average in the simulated data is 2.

Bounds Estimation [Adams, 2020]

More Guns, Less Crime?

The US states are a "laboratory of democracy."
[Aneja et al., 2011]:**The Impact of Right-to-Carry Laws and the NRC Report: Lessons for the Empirical Evaluation of Law and Policy.**

```
library(foreign)
# the data is standard Stata format, the library foreign
# allows this data to be imported.
x <- read.dta("UpdatedStateLevelData-2010.dta")
Y <- X <- Z <- NULL
```

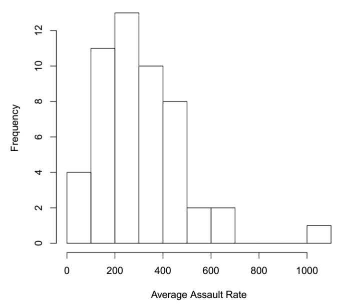
Bounds Estimation [Adams, 2020]

More Guns, Less Crime?

```
# the loop will create variables by adding to the vectors
for (i in 2:length(unique(x$state))) {
  # length measures the number of elements in the object.
  state = sort(unique(x$state))[i]
  # note the first state is "NA"
  X <- c(X,sum(x[x$state==state,]$shalll, na.rm = TRUE) > 0)
  # determines if a state has an RTC law at
  # some point in time.
  # na.rm tells the function to ignore NAs
  Y <- c(Y,mean(x[x$state==state & x$year > 1990,]$rataga,
  na.rm = TRUE))
  # determines the average rate of aggrevated assualt for the
  # state post 1990.
  Z <- c(Z,mean(x[x$state==state & x$year > 1990,]$area,
  na.rm = TRUE) > 53960)
  # determines the physical area of the state
  # Small state = 0, large stage = 1
  # print(i)
}
```

Bounds Estimation [Adams, 2020]

More Guns, Less Crime?



The average aggravated assault rate per state in the post 1990 years. It shows that rate per 100,000 is between 0 and 600 for the most part.

Bounds Estimation [Adams, 2020]

ATE of RTC Laws under Unconfoundedness

Comparing the average rate of aggravated assault in states with RTC laws to states without RTC laws:

```
EY_X1 <- mean(Y[X==1])
EY_X0 <- mean(Y[X==0])
EY_X1 - EY_X0
```

Unconfoundedness is not a reasonable assumption. We are interested in **estimating the average effect of implementing an RTC law**. We are **not interested in the average rate of assaults conditional on the state having an RTC law**.

Bounds Estimation [Adams, 2020]

ATE of RTC Laws under Unconfoundedness

Natural Bounds on ATE of RTC Laws: We cannot observe the effect of RTC laws for states that do not have RTC laws. We could assume that the assault rate lies between 0 and 100,000 (which it does).

```
PX0 <- mean(X==0)
PX1 <- mean(X==1)
minY <- 0
minY <- min(Y)
maxY <- 100000
# ATE upper bound
(EY_X1 - minY)*PX1 + (maxY - EY_X0)*PX0
# ATE lower bound
(EY_X1 - maxY)*PX1 + (minY - EY_X0)*PX0
maxY <- max(Y)
# ATE upper bound
(EY_X1 - minY)*PX1 + (maxY - EY_X0)*PX0
# ATE lower bound
(EY_X1 - maxY)*PX1 + (minY - EY_X0)*PX0
```

Bounds Estimation [Adams, 2020]

Bounds on ATE of RTC Laws with Exogenous Variation (the average treatment effect of implementing an RTC law must be the same irrespective of the physical size of the state)

```r
PX1_Z1 <- mean(X[Z==1]==1)
PX1_Z0 <- mean(X[Z==0]==1)
PX0_Z1 <- mean(X[Z==1]==0)
PX0_Z0 <- mean(X[Z==0]==0)
EY_X1Z1 <- mean(Y[X==1 & Z==1])
EY_X1Z0 <- mean(Y[X==1 & Z==0])
EY_X0Z1 <- mean(Y[X==0 & Z==1])
EY_X0Z0 <- mean(Y[X==0 & Z==0])
# a NaN error below maybe due to a typo above.
# NaN error may occur because the vector is all NAs
# ATE upper bound
min((EY_X1Z1 - minY)*PX1_Z1 + (maxY - EY_X0Z1)*PX0_Z1,
(EY_X1Z0 - minY)*PX1_Z0 + (maxY - EY_X0Z0)*PX0_Z0)
# ATE lower bound
max((EY_X1Z1 - maxY)*PX1_Z1 + (minY - EY_X0Z1)*PX0_Z1,
(EY_X1Z0 - maxY)*PX1_Z0 + (minY - EY_X0Z0)*PX0_Z0)
```

Bounds Estimation [Adams, 2020]

Bounds on ATE of RTC Laws with Monotonicity

Would it be reasonable to use the monotonicity assumption above (Assumption 3)? Let's assume that

Assumption 4. $E(Y|do(X) = 1, X = 1) \leq E(Y|do(X) = 1, X = 0)$ and $E(Y|do(X) = 0, X = 0) \geq E(Y|do(X) = 0, X = 1)$

We assume that states that currently have RTC laws will also tend to have lower levels of aggravated assault. Moreover, forcing states that don't currently have RTC laws will not reduce the expected aggravated assaults below that level.

Bounds Estimation [Adams, 2020]

Bounds on ATE of RTC Laws with Monotonicity

Assumption 4 implies

$$\underline{\mathbb{E}(Y|do(X)=1)} = \mathbb{E}(Y|X=1)$$

$$\overline{\mathbb{E}(Y|do(X)=0)} = \mathbb{E}(Y|X=0)$$

Plugging these into the bounds on the ATE we have the following bounds on the effect of the RTC laws.

```
# ATE upper bound
(EY_X1 - minY)*PX1

# ATE lower bound
(minY - EY_X0)*PX0
```

These bounds are substantially tighter. So, the slogan may be more accurately stated as "**more guns, more or less crime.**"

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