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Critério de Leibniz

$$\begin{cases} a_n > 0 \\ \lim\limits_{n \to +\infty} u_n = 0 \\ a_{n+1} < a_n \end{cases}$$

Como a série é de termos positivos, $\forall n \in \mathbb{N}: a_n > 0$

$$\lim_{n \to +\infty} \frac{2n}{n^3+3n^2+4} + \frac{1}{n^3+3n^2+4} =$$

$$= \lim_{n \to +\infty} \frac{2n}{n^3} + \frac{1}{n^3} = \lim_{n \to +\infty} \frac{2}{n^2} + \frac{1}{n^3} = 0+0 = 0$$

$a_{n+1} < a_n \Leftrightarrow a_2 < a_1$

$$\frac{2 \times 2}{2^3 + 3 \times 2^2 + 4} + \frac{1}{2^3 + 3 \times 2^2 + 4} = \frac{4}{8+12+4} + \frac{1}{8+12+4} = \frac{5}{24}$$

$$\frac{2 \times 1 + 1}{1^3 + 3 \times 1^2 + 4} = \frac{3}{1+3+4} = \frac{3}{8} \qquad \frac{5}{24} < \frac{3}{8}$$

Logo a série é absolutamente convergente.

ii) $\sum\limits_{n=1}^{+\infty} \frac{n^n}{2^n \times n!}$

Pelo critério d'Alembert

$$\lim_{n \to +\infty} \frac{\frac{(n+1)^{n+1}}{2^{n+1} \times (n+1)!}}{\frac{n^n}{2^n \times n!}} =$$

$$= \lim_{n \to +\infty} \frac{(n+1)^{n+1}}{2^{n+1} \times (n+1)!} \times \frac{2^n \times n!}{n^n} =$$

$$= \lim_{n \to +\infty} \frac{(n+1)^n \times \cancel{(n+1)}}{\cancel{2^n} \times 2 \times \cancel{(n+1)} \times \cancel{n!}} \times \frac{\cancel{2^n} \times \cancel{n!}}{n^n}$$

$$= \lim_{n \to +\infty} \frac{(n+1)^n}{2n^n} = \lim_{n \to +\infty} \frac{1}{2} \times \left(\frac{n+1}{n}\right)^n = \frac{1}{2} \times \lim_{n \to +\infty} \left(1 + \frac{1}{n}\right)^n =$$

$$= \frac{1}{2} \times e^1 = \frac{e}{2}$$, convergente simples.