Proving Compiler Correctness with Dependent Types

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Source language, Target language

- Example source code (expression language):

```
Add (Val 1) (Add (Val 1) (Val 3))
```

- Example target code (for a stack machine):

```
PUSH 1 >> PUSH 1 >> PUSH 3 >> ADD >> ADD
```


Evaluation, execution

- An **eval** function gives the semantics for the **source** language
  - Denotational semantics
  - Maps terms to values
- An **exec** function gives the semantics for the "**machine**" language
  - For each instruction, an operation to perform on the machine state (stack)

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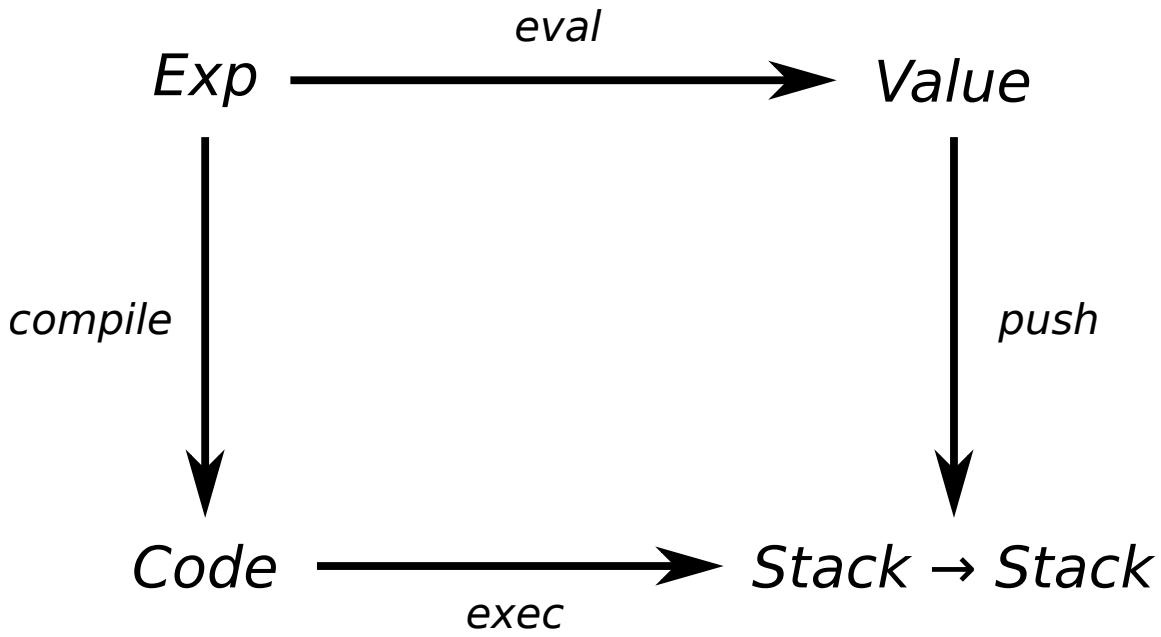
What does "correct" mean?

- Both semantics (before and after compilation) should be "equivalent"
- Compiling then executing must give the same result as direct evaluation



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Reference paper

- "A type-correct, stack-safe, provably correct expression compiler in Epigram"
  - James McKinna, Joel Wright
- Basic ideas and proofs, which we extended...

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Extending the source language

- More "realistic" languages have sharing constructs
- We wanted the "simplest possible" extension with sharing behaviour.
- **Chosen extension: if_then_else + sequencing**

```
if c then t else e >> common-suffix
```

- The "naïve" compile function will duplicate the suffix
- Having Bytecode defined as graph (structured graph) instead of tree would solve this problem
  - But proofs would be more complex

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What we ideally want

- Have a "smart" graph-based compiler, generating code which uses sharing
- Write the correctness proof only for the "dumb" compiler, have correctness **derived** for the smart version.

Reference paper

- "Proving Correctness of Compilers using Structured Graphs"
  - Patrick Bahr (visiting researcher)

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Our project's goals

- Integrating the best of both "reference" papers
- Our contributions:
  - (Simplest possible) language extension showing sharing behaviour.
  - Proof of correctness for the **stack-safe** "naïve" compiler
    - The one that just duplicates code.
  - A way to to lift this **stack-safe** "naïve" correctness proof
    - Into a proof concerning the more **efficient** compiler.

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Source

Source types:

$$\begin{aligned}&\mathsf{data}\ \mathsf{Ty}_s : \mathsf{Set}\ \mathsf{where}\\ &\quad \mathbb{N}_s : \mathsf{Ty}_s\\ &\quad \mathbb{B}_s : \mathsf{Ty}_s\end{aligned}$$

Source terms (snippet):

$$\begin{aligned}&\mathsf{data}\ \mathsf{Src} : (t : \mathsf{Ty}_s) \to (z : \mathsf{Size}_s) \to \mathsf{Set}\ \mathsf{where}\\ &\quad \mathsf{v}_s \quad : \forall\ \{t\} \to (v : \{\!|\ t\ |\!\}) \to \mathsf{Src}\ t\ 1\\ &\quad \_+_s\_ \quad : (e_1\ e_2 : \mathsf{Src}\ \mathbb{N}_s\ 1) \to \mathsf{Src}\ \mathbb{N}_s\ 1\end{aligned}$$

Denotational semantics (snippet):

$$\begin{aligned}&[\![\_]\!] : \{t : \mathsf{Ty}_s\}\ \{z : \mathsf{Size}_s\} \to (e : \mathsf{Src}\ t\ z) \to \mathsf{Vec}\ \{\!|\ t\ |\!\}\ z\\ &[\![\ \mathsf{v}_s\ v\ ]\!] \quad = [\ v\ ]\\ &[\![\ e_1\ +_s\ e_2\ ]\!] \quad = [\ [\![\ e_1\ ]\!]' + [\![\ e_2\ ]\!]'\ ]\end{aligned}$$

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Bytecode

Typed stack:

```
StackType : Set
StackType = List Ty_s
```

```
data Stack : StackType → Set where
  ε     : Stack []
  _▽_   : ∀ {t s'} → { t } → Stack s' → Stack (t :: s')
```

Typed bytecode (snippet):

```
data Bytecode : StackType → StackType → Set where
  SKIP : ∀ {s}       → Bytecode s s
  PUSH : ∀ {t s}     → (x : { t }) → Bytecode s (t :: s)
  ADD  : ∀ {s}       → Bytecode (ℕ_s :: ℕ_s :: s) (ℕ_s :: s)
```


Compiler correctness

$$\texttt{correct} : \{\texttt{t} : \texttt{Ty}_s\}\ \{\texttt{z} : \texttt{Size}_s\}\ (\texttt{e} : \texttt{Src t z}) \\ \rightarrow \texttt{exec (compile e)} \equiv [\![\ \texttt{e}\ ]\!]$$

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Tree fixpoints

Fixed Point for standard Functors
Fixed Point for indexed Functors

Bytecode Tree Representation

```
data Bytecode : StackType → StackType → Set where
  SKIP : ∀ {s}       → Bytecode s s
  PUSH : ∀ {t s}     → (x : ⌞ t ⌟) → Bytecode s (t :: s)
  ADD  : ∀ {s}       → Bytecode (ℕs :: ℕs :: s) (ℕs :: s)
```

'Bytecode' is isomorphic to 'HTree BytecodeF': We have: 'fromGraph . toGraph == id' And: 'toGraph . fromGraph == id'

Correctness on Trees

‘execT‘ executes ‘HTree‘ represented Bytecode

```
correctT : ∀ {t z s'} → (e : Src t z)
         → execT (compileT e) ≡ ⟦ e ⟧
```

The proof for ‘correctT‘ can be trivially lifted from ‘correct‘, because ‘Bytecode‘ is structurally the same as ‘HTree BytecodeF‘

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Graphs

```
data HGraph .. : ... -> Set where ...
```

‘HGraph‘ is like ‘HTree‘, but with additional constructors to represent shared subtrees
‘Bytecode‘ is not exactly isomorphic to ‘HGraph BytecodeF‘:
We have: ‘fromGraph . toGraph == id‘ But: ‘toGraph . fromGraph /= id‘
HGraph -¿ Bytecode -¿ HGraph loses sharing

Bytecode Graph Representation

'execG' executes 'HGraph' represented Bytecode

```
correctG : ∀ {t z s'} → (e : Src t z)
         → execG (compileG e) ≡ ⟦ e ⟧
```

Using machinery, we get this proof automatically from 'correctT'

Achieved

- Agda "framework" for deriving compiler correctness proofs
  - Compilers with **typed source** and **typed target**
  - Given correctness of a "naïve" compiler, derive correctness of "optimized" version
- Correctness proof for an expression language (with sequencing)
  - As "instance" of this framework

Agda limitations we faced

- Strict positivity requirement
  - When defining fixed point type operators
- Totality checker
  - When defining folds
- Type checking with positivity check disabled made debugging hard
  - Stack overflow, memory consumption

Yet to be done

- Sequence clause of "basic" (non-lifted) correctness proof
- Prove a final lemma to complete the lifting (fusion law)

Thank you!

Questions?

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