

# Proving Compiler Correctness with Dependent Types

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## Introduction

- Context/Terminology
- Compiler correctness
- Sharing
- Goals

## Implementation (code)

- Basic correctness
- Lifting to sharing setting

## Conclusions



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## Source language, Target language

- ▶ Example source code (expression language):  
Add (Val 1) (Add (Val 1) (Val 3))
- ▶ Example target code (for a stack machine):  
PUSH 1 >> PUSH 1 >> PUSH 3 >> ADD >> ADD

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## Evaluation, execution

- ▶ An **eval** function gives the semantics for the **source** language
  - Denotational semantics
  - Maps terms to values
- ▶ An **exec** function gives the semantics for the “**machine**” language
  - For each instruction, an operation to perform on the machine state (stack)

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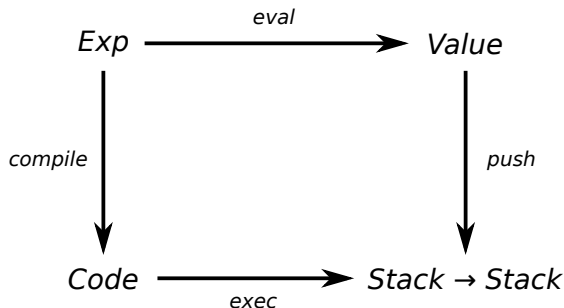
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# What does "correct" mean?

- ▶ Both semantics (before and after compilation) should be "equivalent"
- ▶ Compiling then executing must give the same result as direct evaluation



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## Reference paper

- ▶ "A type-correct, stack-safe, provably correct expression compiler in Epigram"
  - James McKinna, Joel Wright
- ▶ Basic ideas and proofs, which we extended. . .

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## Extending the source language

- ▶ More "realistic" languages have sharing constructs
- ▶ We wanted the "simplest possible" extension with sharing behaviour.

- Chosen extension: `if_then_else` + sequencing

```
if c then t else e >> common-suffix
```

- ▶ The "naïve" compile function will duplicate the suffix
- ▶ Having Bytecode defined as graph (structured graph) instead of tree would solve this problem
  - But proofs would be more complex

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## Reference paper

- ▶ "Proving Correctness of Compilers using Structured Graphs"
  - Patrick Bahr (visiting researcher)

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## Our project's goals

- ▶ Integrating the best of both “reference” papers
- ▶ Our contributions:
  - (Simplest possible) language extension showing sharing behaviour.
  - Proof of correctness for the **stack-safe** “naïve” compiler
    - The one that just duplicates code.
  - A way to lift this **stack-safe** “naïve” correctness proof
    - Into a proof concerning the more **efficient** compiler.

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# Source

Source types:

`data Tys : Set where`

`Ns : Tys`

`Bs : Tys`

Source terms (snippet):

`data Src : (t : Tys) → (z : Sizes) → Set where`

`vs : ∀ {t} → (v : { t }) → Src t 1`

`_+_s_ : (e1 e2 : Src Ns 1) → Src Ns 1`

Denotational semantics (snippet):

$\llbracket \_ \rrbracket : \{t : \text{Ty}_s\} \{z : \text{Size}_s\} \rightarrow (e : \text{Src } t \ z) \rightarrow \text{Vec } \{ t \} \ z$

$\llbracket v_s \ v \rrbracket = [v]$

$\llbracket e_1 \ +_s \ e_2 \rrbracket = [\llbracket e_1 \rrbracket' + \llbracket e_2 \rrbracket']$

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# Bytecode

Typed stack:

$\text{StackType} : \text{Set}$

$\text{StackType} = \text{List } \text{Ty}_s$

**data**  $\text{Stack} : \text{StackType} \rightarrow \text{Set}$  **where**

$\epsilon : \text{Stack } []$

$\_ \nabla \_ : \forall \{t\ s'\} \rightarrow \{t\} \rightarrow \text{Stack } s' \rightarrow \text{Stack } (t :: s')$

Typed bytecode (snippet):

**data**  $\text{Bytecode} : \text{StackType} \rightarrow \text{StackType} \rightarrow \text{Set}$  **where**

$\text{SKIP} : \forall \{s\} \rightarrow \text{Bytecode } s\ s$

$\text{PUSH} : \forall \{t\ s\} \rightarrow (x : \{t\}) \rightarrow \text{Bytecode } s\ (t :: s)$

$\text{ADD} : \forall \{s\} \rightarrow \text{Bytecode } (\mathbb{N}_s :: \mathbb{N}_s :: s)\ (\mathbb{N}_s :: s)$

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# Tree fixpoints

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# Bytecode Tree Representation

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# Correctness on Trees

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## Yet to be done

- ▶ Sequence clause of “basic” (non-lifted) correctness proof
- ▶ Proof a final lemma to complete the lifting (fusion law)

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Thank you!

Questions?

