

Π -Ware: An Embedded Hardware Description Language using Dependent Types

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Hardware design is hard(er)

- ▶ Strict(er) correctness requirements
 - You can't simply *update* a full-custom chip after production
 - Intel FDIV
 - Expensive verification / validation (up to 50% of development costs)
- ▶ Low-level details (more) important
 - Layout / area
 - Power consumption / fault tolerance



Hardware design is growing

- ▶ Moore's law will still apply for some time
 - We can keep packing more transistors into same silicon area
- ▶ **But** optimizations in CPUs display diminishing returns
 - Thus, more algorithms *directly* in hardware

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Hardware Description Languages

- ▶ All started in the 1980s
- ▶ *De facto* industry standards: VHDL and Verilog
- ▶ Were intended for *simulation*, not modelling or synthesis
 - *Unsynthesizable* constructs
 - Widely variable tool support

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Functional Programming

- ▶ Easier to *reason* about program properties
- ▶ Inherently *parallel* and *stateless* semantics
 - In contrast to imperative programming

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Functional Hardware Description

- ▶ A functional program describes a circuit
- ▶ Several *functional* Hardware Description Languages (HDLs) during the 1980s
 - For example, μ FP [Sheeran, 1984]
- ▶ Later, *embedded* hardware Domain-Specific Languages (DSLs)
 - For example, Lava (Haskell) [Bjesse et al., 1998]

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Embedded DSLs for Hardware

- ▶ Lava
- ▶ Limitations
 - Low level types
 - Not guaranteeing size match

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Dependently-Typed Programming

Dependently-Typed Programming (DTP) är en
programmationstechnik...

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Research Question

“What are the improvements that DTP can bring to hardware design?”

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Methodology

- ▶ Develop a hardware DSL, *embedded* in a dependently-typed language (Agda)
 - Called **Π -Ware**
 - allowing simulation, synthesis and verification

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Dependently-Typed Programming

► Types can depend on values

- Example: `data Vec (α : Set) : $\mathbb{N} \rightarrow$ Set where...`
- Compare with Haskell (GADT style):
`data List :: * -> * where...`

► Types of arguments can depend on *values of previous arguments*

- Ensure a “safe” domain
- `take : (m : \mathbb{N}) \rightarrow Vec α ($m + n$) \rightarrow Vec α m`

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Dependently-Typed Programming

- ▶ Type checking requires *evaluation* of functions
 - We want `Vec Bool (2 + 2)` to unify with `Vec Bool 4`
- ▶ Consequence: all functions must be *total*
- ▶ Termination checker ensures (heuristics)
 - Structurally-decreasing recursion
 - This passes the check:
`add : $\mathbb{N} \rightarrow \mathbb{N} \rightarrow \mathbb{N}$`
`add zero y = y`
`add (suc x') y = suc (add x' y)`
 - This does not:
`silly : $\mathbb{N} \rightarrow \mathbb{N}$`
`silly zero = zero`
`silly (suc n') = silly [n' /2]`

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Dependently-Typed Programming

- ▶ Dependent pattern matching can *rule out* impossible cases

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Dependently-Typed Programming

► Dependent pattern matching can *rule out* impossible cases

- Classic example: *safe head* function

$\text{head} : \text{Vec } \alpha \ (\text{suc } n) \rightarrow \alpha$

$\text{head } (x :: xs) = x$

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Dependently-Typed Programming

- ▶ Dependent pattern matching can *rule out* impossible cases

- Classic example: *safe head* function

$$\text{head} : \text{Vec } \alpha \ (\text{suc } n) \rightarrow \alpha$$
$$\text{head } (x :: xs) = x$$

- The **only** constructor returning $\text{Vec } \alpha \text{ (suc } n)$ is $_::_$

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Dependent types as logic

► Programming language / Theorem prover

- Types as propositions, terms as proofs [Wadler, 2014]

► Example:

- Given the relation (drawn triangle):

data $_ \leq _$: $\mathbb{N} \rightarrow \mathbb{N} \rightarrow$ **Set** **where**

$z \leq n$: $\forall \{n\} \rightarrow$ **zero** $\leq n$

$s \leq s$: $\forall \{m\ n\} \rightarrow m \leq n \rightarrow$ **suc** $m \leq$ **suc** n

- Proposition:

twoLEQFour : $2 \leq 4$

- Proof:

twoLEQFour = $s \leq s$ ($s \leq s$ $z \leq n$)

$s \leq s$ ($s \leq s$ ($z \leq n$: $0 \leq 4$) : $1 \leq 4$) : $2 \leq 4$

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Agda syntax for Haskell programmers

- ▶ Liberal identifier lexing (Unicode **everywhere**)
 - $a \equiv b + c$ is a valid identifier, $a \equiv b + c$ an expression
 - Actually used in Agda's standard library
 - And in Π -Ware: \mathbb{C} , $\llbracket c \rrbracket$, \Downarrow , \Uparrow
- ▶ *Mixfix* notation
 - $_[_]_ := _$ is the vector update function: $v \ [\# \ 3 \] := \text{true}$.
 - $_[_]_ \ v \ (\# \ 3) \ \text{true} \iff v \ [\# \ 3 \] := \text{true}$
- ▶ Almost nothing built-in
 - $_+__ : \mathbb{N} \rightarrow \mathbb{N} \rightarrow \mathbb{N}$ defined in **Data.Nat**
 - $\text{if_then_else_} : \text{Bool} \rightarrow \alpha \rightarrow \alpha \rightarrow \alpha$ defined in **Data.Bool**

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Agda syntax for Haskell programmers

- Implicit arguments

- Don't have to be passed if Agda can **guess** it
- Syntax: $\epsilon : \{ \alpha : \text{Set} \} \rightarrow \text{Vec } \alpha \text{ zero}$

► “For all” syntax: $\forall n \iff (n : _)$

- Where `_` means: guess this type (based on other args)
- Example:
 - $\forall n \rightarrow \text{zero} \leq n$
 - `data < : ℕ → ℕ → Set`

- ▶ It's common to combine both:

- $\forall \{ \alpha \ n \} \rightarrow \text{Vec } \alpha \ (\text{succ } n) \rightarrow \alpha \iff$
 $\{ \alpha : _ \} \{ n : _ \} \rightarrow \text{Vec } \alpha \ n \rightarrow \alpha$

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Low-level circuits

- Structural representation
- Untyped but *sized*

data $\mathbb{C}' : \mathbb{N} \rightarrow \mathbb{N} \rightarrow \text{Set}$

data \mathbb{C}' where

Nil : $\mathbb{C}' \text{ zero zero}$

Gate : $(g\# : \text{Gates\#}) \rightarrow \mathbb{C}' (|\text{in}| g\#) (|\text{out}| g\#)$

Plug : $\forall \{i\ o\} \rightarrow (f : \text{Fin } o \rightarrow \text{Fin } i) \rightarrow \mathbb{C}' i\ o$

DelayLoop : $(c : \mathbb{C}' (i + l) (o + l)) \{\text{comb}'\ c\} \rightarrow \mathbb{C}' i\ o$

$_ \gg' _ : \mathbb{C}' i\ m \rightarrow \mathbb{C}' m\ o \rightarrow \mathbb{C}' i\ o$

$_ |' _ : \mathbb{C}' i_1\ o_1 \rightarrow \mathbb{C}' i_2\ o_2 \rightarrow \mathbb{C}' (i_1 + i_2) (o_1 + o_2)$

$_ |+' _ : \mathbb{C}' i_1\ o \rightarrow \mathbb{C}' i_2\ o \rightarrow \mathbb{C}' (\text{suc } (i_1 \sqcup i_2))\ o$

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Atoms

- ▶ How to carry values of an Agda type in *one* wire
- ▶ Defined by the **Atomic** type class in **PiWare.Atom**

record Atomic : Set₁ **where**

field

Atom : Set

|Atom|−1 : ℕ

n→atom : Fin (suc |Atom|−1) → Atom

atom→n : Atom → Fin (suc |Atom|−1)

inv-left : $\forall i \rightarrow \text{atom} \rightarrow n \ (n \rightarrow \text{atom} \ i) \equiv i$

inv-right : $\forall a \rightarrow n \rightarrow \text{atom} \ (\text{atom} \rightarrow n \ a) \equiv a$

|Atom| = suc |Atom|−1

Atom# = Fin |Atom|

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Atomic instances

- ▶ Examples of types that can be **Atomic**
 - Bool, std_logic, other multi-valued logics
 - Predefined in the library: **PiWare.Atom.Bool**
- ▶ First, define how many atoms we are interested in

|B| - 1 = 1

|B| = suc **|B|** - 1

- ▶ Friendlier names for the indices (elements of **Fin 2**)

pattern **False#** = Fz

pattern **True#** = Fs Fz

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Atomic instance (Bool)

- Bijection between $\{n \in \mathbb{N} \mid n < 2\}$ (Fin 2) and Bool

$$n \rightarrow B = \lambda \{ \text{False\#} \rightarrow \text{false}; \text{True\#} \rightarrow \text{true} \}$$
$$B \rightarrow n = \lambda \{ \text{false} \rightarrow \text{False\#}; \text{true} \rightarrow \text{True\#} \}$$

- Proof that $n \rightarrow B$ and $B \rightarrow n$ are inverses

$$\text{inv-left-B} = \lambda \{ \text{False\#} \rightarrow \text{refl}; \text{True\#} \rightarrow \text{refl}; \}$$
$$\text{inv-right-B} = \lambda \{ \text{false} \rightarrow \text{refl}; \text{true} \rightarrow \text{refl} \}$$

- With all pieces at hand, we construct the instance

$$\begin{aligned} \text{Atomic-B} = \text{record} \{ & \text{Atom} = B \\ & ; |\text{Atom}|-1 = |B|-1 \\ & ; n \rightarrow \text{atom} = n \rightarrow B \\ & ; \text{atom} \rightarrow n = B \rightarrow n \\ & ; \text{inv-left} = \text{inv-left-B} \\ & ; \text{inv-right} = \text{inv-right-B} \} \end{aligned}$$

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The Gates type class

$W : \mathbb{N} \rightarrow \text{Set}$

$W = \text{Vec Atom}$

record Gates : Set where

field

|Gates| : \mathbb{N}

|in| |out| : Fin |Gates| $\rightarrow \mathbb{N}$

spec : (g : Fin |Gates|)
 $\rightarrow (W (|in| g) \rightarrow W (|out| g))$

Gates# = Fin |Gates|

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Gates instances

- ▶ Example: `PiWare.Gates.BoolTrio`
- ▶ First, how many gates are there in the library

`|BoolTrio| = 5`

- ▶ Then the friendlier names for the indices

```
pattern FalseConst# = Fz
pattern TrueConst#  = Fs Fz
pattern Not#        = Fs (Fs Fz)
pattern And#        = Fs (Fs (Fs Fz))
pattern Or#         = Fs (Fs (Fs (Fs Fz)))
```

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Gates instance (BoolTrio)

- ▶ Defining the *interfaces* of the gates

```
[in] FalseConst# = 0
```

```
|in| TrueConst# = 0
```

```
|in| Not# = 1
```

$$|in|_{And\#} = 2$$

|in| Or# = 2

$$|out| = 1$$

- And the specification function for each gate

```
spec=false == [ false ]
```

```
spec-true      = [ true  ]
```

$$\text{spec-not} \quad (x :: \varepsilon) \quad = \text{[not } x \text{]}$$

spec-and $(x :: y :: \varepsilon) = [x \wedge y]$

spec-or $(x :: y :: \varepsilon) = [x \vee y]$

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Gates instance (BoolTrio)

- Mapping each gate index to its respective specification

specs-BoolTrio FalseConst# = spec-false

specs-BoolTrio TrueConst# = spec-true

specs-BoolTrio Not# = spec-not

specs-BoolTrio And# = spec-and

specs-BoolTrio Or# = spec-or

- With all pieces at hand, we construct the instance

BoolTrio : Gates

```
BoolTrio = record { |Gates| = |BoolTrio|  
                  ; |in|    = |in|  
                  ; |out|   = |out|  
                  ; spec    = specs-BoolTrio }
```

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High-level circuits

- ▶ User is not supposed to describe circuits at low level (\mathbb{C}')
- ▶ The high level circuit type (\mathbb{C}) allows for *typed* circuit interfaces
 - The input and output indices are Agda types

```
data C (α β : Set) {i j : ℕ} : Set where
  MkC : { [ sα : ↓W↑ α {i} ] [ sβ : ↓W↑ β {j} ] }
        → C' i j → C α β {i} {j}
```

- ▶ MkC takes:
 - Low level description (\mathbb{C}')
 - Information on how to *synthesize* elements of α and β
 - Passed as *instance arguments*

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Synthesizable

- ▶ $\Downarrow W \Uparrow$ (pronounced Synthesizable)
 - $W\ n = \text{Vec}\ \alpha\ n$
- ▶ Example: $\Downarrow W \Uparrow\ (\alpha \times \beta)$

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Synthesis

- ▶ Work-in-progress
- ▶ **Atom** and **Gates** with VHDL *abstract syntax*

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Simulation

- ▶ Combinational
- ▶ Sequential

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Problems

- Definition of $\llbracket _ \rrbracket$ blocks reduction

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► Π-Ware is...

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Current limitations

- ▶ Problem with proofs (definition of $\llbracket_ \rrbracket$)
- ▶ Proofs on (infinite) **Streams**
- ▶ Bla

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Future work

- ▶ Proof by reflection for finite cases

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Thank you!

Questions?



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