Π-Ware: An Embedded Hardware Description Language using Dependent Types

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Hardware Design
Functional Hardware

Research

Question

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Hardware design is hard(er)

- Strict(er) correctness requirements
 - You can't simply update a full-custom chip after production
 - Intel FDTV
 - Expensive verification / validation (up to 50% of development costs)
- ▶ Low-level details (more) important
 - Layout / area
 - Power consumption / fault tolerance

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Hardware design is growing

- ▶ Moore's law will still apply for some time
 - We can keep packing more transistors into same silicon area
- ▶ **But** optimizations in CPUs display diminishing returns
 - Thus, more algorithms directly in hardware

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Hardware Description Languages

- ▶ All started in the 1980s
- ▶ De facto industry standards: VHDL and Verilog
- ▶ Were intended for *simulation*, not modelling or synthesis
 - Unsynthesizable constructs
 - Widely variable tool support

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Functional Programming

- ▶ Easier to *reason* about program properties
- ▶ Inherently *parallel* and *stateless* semantics
 - · In contrast to imperative programming

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Functional Hardware Description

- A functional program describes a circuit
- Several functional Hardware Description Languages (HDLs) during the 1980s
 - For example, μ FP [Sheeran, 1984]
- ▶ Later, embedded hardware Domain-Specific Languages (DSLs)
 - For example, Lava (Haskell) [Bjesse et al., 1998]

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Embedded DSLs for Hardware

- ▶ Lava
- Limitations
 - Low level types
 - Not guaranteeing size match

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Dependently-Typed Programming (DTP) är en programmationstechnik...

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Research Question

"What are the improvements that DTP can bring to hardware design?"

Question



Methodology

- Develop a hardware DSL, embedded in a dependently-typed language (Agda)
 - Called **Π-Ware**
 - allowing simulation, synthesis and verification

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- Types can depend on values
 - Example: data Vec (α : Set) : N → Set where...
 - Compare with Haskell (GADT style):
 data List :: * -> * where...
- Types of arguments can depend on values of previous arguments
 - Ensure a "safe" domain
 - take : $(m : \mathbb{N}) \to \text{Vec } \alpha \ (m+n) \to \text{Vec } \alpha \ m$

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- ▶ Type checking requires *evaluation* of functions
 - We want Vec Bool (2 + 2) to unify with Vec Bool 4
- ▶ Consequence: all functions must be total
- ► Termination checker ensures (heuristics)
 - Structurally-decreasing recursion
 - This passes the check:

```
\begin{array}{ll} \mathrm{add} \,:\, \mathbb{N} \to \mathbb{N} \to \mathbb{N} \\ \mathrm{add} \,\, \mathrm{zero} & y = y \\ \mathrm{add} \,\, (\mathrm{suc} \,\, x') & y = \mathrm{suc} \,\, (\mathrm{add} \,\, x' \,\, y) \end{array}
```

· This does not:

```
\begin{array}{ll} \text{silly : } \mathbb{N} \to \mathbb{N} \\ \text{silly zero} &= \text{zero} \\ \text{silly (suc } n') &= \text{silly } \lfloor \ n' \ /2 \rfloor \end{array}
```

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Dependent pattern matching can rule out impossible cases

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▶ Dependent pattern matching can *rule out* impossible cases

• Classic example: safe head function

 $\mathsf{head}\,:\,\mathsf{Vec}\,\,\alpha\,\,(\mathsf{suc}\,\,n)\,\to\,\alpha$

 $\mathsf{head}\ (x :: xs) = x$

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- ▶ Dependent pattern matching can *rule out* impossible cases
 - Classic example: safe head function head : Vec α (suc n) $\rightarrow \alpha$

head (x :: xs) = x

• The **only** constructor returning $Vec \alpha$ (suc n) is $_::_$

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Depedent types as logic

- Programming language / Theorem prover
 - Types as propositions, terms as proofs [Wadler, 2014]
- ► Example:
 - Given the relation (drawn triangle):

```
data \_ \le \_ : \mathbb{N} \to \mathbb{N} \to \text{Set where}

z \le n : \forall \{n\} \to \text{zero} \le n

s \le s : \forall \{m \ n\} \to m \le n \to \text{suc } m \le \text{suc } n
```

• Proposition:

```
twoLEQFour : 2 \le 4
```

• Proof:

```
\begin{aligned} & twoLEQFour = s \leq s \ (s \leq s \ z \leq n) \\ s \leq s \ (s \leq s \ (z \leq n \ : \ 0 \leq 4) \ : \ 1 \leq 4) \ : \ 2 \leq 4 \end{aligned}
```

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Agda syntax for Haskell programmers

- ► Liberal identifier lexing (Unicode everywhere)
 - $a\equiv b+c$ is a valid identifer, $a\equiv b+c$ an expression
 - · Actually used in Agda's standard library
 - And in Π-Ware: C, [c], ↓, ↑
- Mixfix notation
 - _[_]≔_ is the vector update function: v [# 3] ≔ true.
 - _[_]:=_ v (# 3) true ⇔ v [# 3] := true
- ▶ Almost nothing built-in
 - $_+_$: $\mathbb{N} \to \mathbb{N} \to \mathbb{N}$ defined in Data.Nat
 - if then else : Bool ightarrow lpha
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 ightarrow lpha defined in Data.Bool

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Agda syntax for Haskell programmers

- Implicit arguments
 - Don't have to be passed if Agda can guess it
 - Syntax: ε : $\{\alpha : \mathsf{Set}\} \to \mathsf{Vec} \; \alpha \; \mathsf{zero}$
- ▶ "For all" syntax: $\forall n \iff (n : _)$
 - Where _ means: guess this type (based on other args)
 - Example:
 - $\forall n \rightarrow \text{zero} \leq n$
 - data $\underline{\quad} \leq \underline{\quad} : \mathbb{N} \to \mathbb{N} \to \mathsf{Set}$
- ▶ It's common to combine both:
 - $\forall \{\alpha \ n\} \rightarrow \mathsf{Vec} \ \alpha \ (\mathsf{suc} \ n) \rightarrow \alpha \iff \{\alpha : _\} \{n : _\} \rightarrow \mathsf{Vec} \ \alpha \ n \rightarrow \alpha$

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Low-level circuits

- Structural representation
- Untyped but sized

```
data \mathbb{C}': \mathbb{N} \to \mathbb{N} \to \mathsf{Set}
data \mathbb{C}' where
     Nil : \mathbb{C}' zero zero
```

Gate : $(g\# : Gates\#) \rightarrow \mathbb{C}'$ ([in] g#) ([out] g#)

 $\rightarrow (f : \operatorname{Fin} o \rightarrow \operatorname{Fin} i) \rightarrow \mathbb{C}' i o$ Plug : $\forall \{i \ o\}$

$$\mathsf{DelayLoop} \,:\, (c \,:\, \mathbb{C}' \,\, (i \,+\, l) \,\, (o \,+\, l)) \,\, \{\mathsf{comb}' \,\, c\} \,\to\, \mathbb{C}' \,\, {\color{black} i \,\, o}$$

Syntax



Atoms

- ▶ How to carry values of an Agda type in *one* wire
- ▶ Defined by the Atomic type class in PiWare.Atom

```
record Atomic : Set<sub>1</sub> where field

Atom : Set
```

|Atom|−1 : N

 $n \rightarrow atom$: Fin (suc |Atom|-1) $\rightarrow Atom$ $atom \rightarrow n$: $Atom \rightarrow Fin$ (suc |Atom|-1)

inv-left : $\forall i \rightarrow atom \rightarrow n \ (n \rightarrow atom \ i) \equiv i$ inv-right : $\forall a \rightarrow n \rightarrow atom \ (atom \rightarrow n \ a) \equiv a$

```
|Atom| = suc |Atom|-1
Atom# = Fin |Atom|
```

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Atomic instances

- ► Examples of types that can be Atomic
 - Bool, std_logic, other multi-valued logics
 - Predefined in the library: PiWare.Atom.Bool
- First, define how many atoms we are interested in

$$|B|-1 = 1$$

 $|B| = suc |B|-1$

Friendlier names for the indices (elements of Fin 2)

```
pattern False# = Fz
pattern True# = Fs Fz
```

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Atomic instance (Bool)

▶ Bijection between $\{n \in \mathbb{N} \mid n < 2\}$ (Fin 2) and Bool

```
n \rightarrow B = \lambda { False# \rightarrow false; True# \rightarrow true } B \rightarrow n = \lambda { false \rightarrow False#; true \rightarrow True# }
```

▶ Proof that $n \rightarrow B$ and $B \rightarrow n$ are inverses

```
inv-left-B = \lambda { False# \rightarrow refl; True# \rightarrow refl; } inv-right-B = \lambda { false \rightarrow refl; true \rightarrow refl }
```

With all pieces at hand, we construct the instance

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Gates

- ▶ Circuits parameterized by collection of *fundamental gates*
- Examples:
 - {NOT, AND, OR} (BoolTrio)
 - {NAND}
 - · Arithmetic, Crypto, etc.
- ► The definition of what means to be such a collection is in PiWare.Gates.Gates

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The Gates type class

```
W: \mathbb{N} \to Set
W = Vec Atom
 record Gates: Set where
   field
        |Gates| : N
        |\mathsf{in}| |\mathsf{out}| : \mathsf{Fin} |\mathsf{Gates}| \to \mathbb{N}
                      : (g : Fin | Gates|)
        spec
                          \rightarrow (W (|in| g) \rightarrow W (|out| g))
    Gates# = Fin |Gates|
```

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Gates instances

- ► Example: PiWare.Gates.BoolTrio
- ► First, how many gates are there in the library |BoolTrio| = 5
- ▶ Then the friendlier names for the indices

```
pattern FalseConst# = Fz

pattern TrueConst# = Fs Fz

pattern Not# = Fs (Fs Fz)

pattern And# = Fs (Fs (Fs Fz))

pattern Or# = Fs (Fs (Fs (Fs Fz)))
```

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Gates instance (BoolTrio)

▶ Defining the *interfaces* of the gates

```
|in| FalseConst# = 0
|in| TrueConst# = 0
|in| Not# = 1
|in| And# = 2
|in| Or# = 2
```

|out| = 1

▶ And the specification function for each gate

```
\begin{array}{lll} \operatorname{spec-false} & = [ \ \operatorname{false} \ ] \\ \operatorname{spec-true} & = [ \ \operatorname{true} \ ] \\ \operatorname{spec-not} & (x :: \varepsilon) & = [ \ \operatorname{not} \ x \ ] \\ \operatorname{spec-and} & (x :: y :: \varepsilon) & = [ \ x \land y \ ] \\ \operatorname{spec-or} & (x :: y :: \varepsilon) & = [ \ x \lor y \ ] \end{array}
```

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Gates instance (BoolTrio)

Mapping each gate index to its respective specification

```
specs-BoolTrio FalseConst# = spec-false

specs-BoolTrio TrueConst# = spec-true

specs-BoolTrio Not# = spec-not

specs-BoolTrio And# = spec-and

specs-BoolTrio Or# = spec-or
```

With all pieces at hand, we construct the instance

```
BoolTrio: Gates
BoolTrio = record { |Gates| = |BoolTrio| ; |in| = |in| ; |out| = |out| ; spec = specs-BoolTrio }
```

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High-level circuits

- ▶ User is not supposed to describe circuits at low level (\mathbb{C}')
- ► The high level circuit type (ℂ) alloes for typed circuit interfaces
 - The input and output indices are Agda types

```
data \mathbb{C} (\alpha \beta : Set) {i j : \mathbb{N}} : Set where

Mk\mathbb{C} : {\{s\alpha : \psi \forall \forall \alpha \{i\}\}\}} {\{s\beta : \psi \forall \forall \beta \{j\}\}\}}

\rightarrow \mathbb{C}' i j \rightarrow \mathbb{C} \alpha \beta \{i\} \{j\}
```

- ► MkC takes:
 - Low level description (ℂ¹)
 - Information on how to synthesize elements of lpha and eta
 - Passed as instance arguments

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Synthesizable

- ▶ \#W↑ type class (pronounced Synthesizable)
 - Describes how to *synthesize* a given Agda type (α)
 - Two fields: from element of α to a word and back

```
record \Downarrow W \Uparrow (\alpha : Set) \{i : \mathbb{N}\} : Set where constructor <math>\Downarrow W \Uparrow [\_, \_] field \Downarrow : \alpha \to W i \Uparrow : W i \to \alpha
```

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₩M↑ instances

- ▶ Any *finite* type can have such an instance
- ▶ Predefined in the library: Bool; _x_; _⊎_; Vec
- Example: instance for products (_x_)

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Synthesizable

▶ Both fields \$\\$\\$ and \$\\$\\$ should be inverses of each other

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Circuit semantics

- ▶ Synthesis semantics: produce a netlist
 - Tool integration / implement in FPGA or ASIC.
- Simulation semantics: execute a circuit.
 - · Given circuit model and inputs, calculate outputs
- ▶ Other semantics possible:
 - · Timing analysis, power estimation, etc.
 - This possibility guided Π-Ware's development

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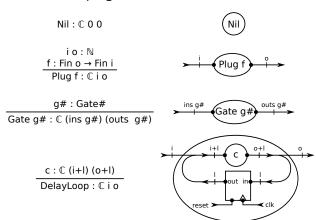
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Synthesis

- ▶ Netlist: directed graph where nodes are *gates* and edges are buses
- Work-in-progress



Semantics



Simulation

- Combinational
- ► Sequential

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Problems

▶ Definition of [_] blocks reduction

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Summary

▶ Π-Ware is...

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Current limitations

- ▶ Problem with proofs (definition of [_])
- ▶ Proofs on (infinite) Streams
- ▶ Bla

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▶ Proof by reflection for finite cases

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Thank you!

Questions?



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Future work

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