

# $\Pi$ -Ware: An Embedded Hardware Description Language using Dependent Types

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# Chapter 1

## Introduction

Several factors have been causing an increasing demand for hardware acceleration of algorithms. On the one hand, Moore's law still holds for the near future [12], which means bigger circuits in the same wafer area. On the other hand, advances in Instruction-Level Paralellism (ILP) and microarchitecture of processors are starting to have diminishing returns [10].

Also, there is pressure to reduce the duration and cost of the circuit design process. These trends are at odds with the techniques and tooling used in the hardware design process, which have not experienced the same evolution as the ones for software development.

The design of an Application-Specific Integrated Circuit (ASIC) imposes strong requirements, specially with regards to correctness (functional and otherwise). Design mistakes which make their way into the manufacturing process can be very expensive, and errors that reach the consumer cost even more, as there's no such thing as "updating" a chip. One infamous example of such a bug reaching the consumer was the *FDIV* bug in Intel's Pentium chip, which might have costed the company over 400 million dollars [20].

In the software industry, especially in application domains requiring high correctness assurance (such as information security, machine control, etc.), functional programming techniques have long been used to improve productivity and reduce the need for extensive testing and debugging. These claims have been confirmed both in industrial settings [25] and in experiments [14].

Another advantage usually attributed to functional programming is an increased capacity to *reason* about your programs, specially to perform what is called *equational reasoning*. Equational reasoning can be used even as an optimization technique. For example, some libraries for array programming in Haskell take advantage of the following law involving map and function composition. The proof is derived by structural induction on the array or list, and by using equational reasoning with the definitions of the functions involved:

```
map f . map g == map (f . g)
```

Besides the growing usage of functional programming languages in several domain areas, functional techniques and constructs keep "penetrating" imperative languages with each new release. Some examples are:

- Apple's recently released Swift™ language, which features immutable data structures, first-class functions and type inference.

- Java's adoption of *generics* and – more recently – of lambda expressions<sup>1</sup>.
- The concept of *nullable type*<sup>2</sup> in C#, equivalent to Haskell's **Maybe**.
- Python's *generator expressions*, inspired by list comprehensions and lazy evaluation.

In a certain way, we can compare the power of the tools and techniques used nowadays in hardware design to the early days of software development. Of course there are inherent and fundamental differences between the two activities, but this comparison leads us to ask whether recent ideas from programming language research, specially those related to functional programming, can be used to improve hardware design.

Research trying to answer this broad question started already in the 1980s, with the work of Prof. Mary Sheeran and others [23], developing *functional* hardware description languages, such as  $\mu FP$  [22]. Later, a trend emerged of developing Embedded Domain-Specific Languages (EDSLs) for hardware description, *hosted* in purely functional languages, such as Haskell [17]. Prominent examples of this trend are the Lava [4] family, as well as ForSyDe [21].

A circuit description written in Lava – an EDSL hosted in Haskell – can look like the following:

```
toggle :: Signal Bool
toggle = let output = inv (latch output) in output
```

Even though pure functional languages (especially Haskell) are well-suited for implementing EDSLs, there is still room for improvement in the type-safety of the embedded languages. This can be done by hosting the EDSLs in a language supporting *dependent types*, as exemplified in the paper "The Power of Pi" [18].

A type system with *dependent types* can express strong properties about the programs written in it. Systems with dependent types can be seen as regular programming languages, but because of the expressive power of dependent types, we can also see them as *interactive theorem provers*.

Using dependent types, one can more easily bring together *specification* and *implementation*. The type signature of a function can give many more *guarantees* about its behaviour. A classical example when introducing Dependently-Typed Programming (DTP) is the type of statically-sized vectors, along with the safe **head** function, depicted in Listing 1.

```
data Vect ( $\alpha$  : Set) :  $\mathbb{N} \rightarrow$  Set where
   $\epsilon$       : Vect  $\alpha$  zero
  _ :: _ :  $\forall \{n\} \rightarrow \alpha \rightarrow$  Vect  $\alpha$   $n \rightarrow$  Vect  $\alpha$  (suc  $n$ )

head :  $\forall \{ \alpha \ n \} \rightarrow$  Vect  $\alpha$  (suc  $n$ )  $\rightarrow \alpha$ 
head (x :: xs) = x
```

Listing 1: Type of sized vectors and the safe **head** function.

In this example, the parameter to **head** cannot be empty – its size will be at least 1 (suc zero). This is expressed in the parameter's type: Vect  $\alpha$  (suc  $n$ ). When checking the

<sup>1</sup><http://docs.oracle.com/javase/tutorial/java/java00/lambdaexpressions.html>

<sup>2</sup><http://msdn.microsoft.com/en-us/library/1t3y8s4s.aspx>

totality of `head` (whether all cases are covered by pattern matching), the type checker will notice that the only constructor of `Vect` able to produce an element of type `Vect α` (`suc n`) is `_ :: _`.

In the safe `head` example, we made the specification more precise by constraining the type of an argument. We can also use dependent types to make the return type of a function more precise. The `group` function for sized vectors in *Agda*'s standard library has the following type:

$$\begin{aligned} \text{group} : \forall \{ \alpha : \text{Set} \} n k &\rightarrow (xs : \text{Vec } \alpha (n * k)) \\ &\rightarrow \exists \lambda (xss : \text{Vec } (\text{Vec } \alpha k) n) \rightarrow xs \equiv \text{concat } xss \end{aligned}$$

It receives as parameters, besides the vector to be "sliced", the number of slices desired ( $n$ ) and the size of each slice ( $k$ ). Notice that the size of the passed vector needs to match  $(n * k)$ . The return type of this function may be read as:

A vector ( $xss$ ) of size  $n$  (whose elements are vectors of size  $k$ ), such that  $xs \equiv \text{concat } xss$ .

Therefore, it returns a collection of groups "sliced" from the original vector, each with the requested size. Additionally, it returns a proof that concatenating all groups results in the original vector. This type serves as a *complete specification of correctness* for the function. Any function with this type is, by definition, a *correct grouping function*.

In a deep-embedded DSL for hardware there is usually a *type* representing circuits. One can imagine that it would be useful to design this type in such a way that as few of its elements as possible are *malformed*. Therefore, we want to make the typing of the circuits as strong as possible, to eliminate as many *classes of design mistakes* as possible.

Dependent type systems allow for easy expression of these *well-formedness* rules for circuits. One simple criterion of circuit *well-formedness* is, for example, that it contains no *floating* signals. That is, all of a circuit's internal components must have all their ports connected. Figure 1.1 shows an example of circuit violating this rule.

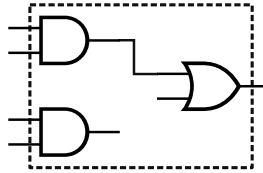


Figure 1.1: Malformed circuit with a *floating* signal.

In this M.Sc thesis we developed a hardware EDSL –  $\Pi$ -Ware – which enforces this and other well-formedness rules using dependent types.  $\Pi$ -Ware is a *deep-embedded* Domain-Specific Language (DSL) hosted in the *Agda* programming language, supporting simulation for combinational and synchronous sequential circuits. Furthermore, the user of  $\Pi$ -Ware can prove *expressive* properties about circuits in the logic of *Agda* itself (intuitionistic logic).

In Chapter 2 we will establish the questions, concepts, languages and tools from which we took inspiration to create  $\Pi$ -Ware. We will discuss the process of hardware design, functional languages for hardware, and how dependent types can enter the picture.

In Chapter 3 we dive into a detailed study of the  $\Pi$ -Ware EDSL itself. We provide a detailed account of how  $\Pi$ -Ware works currently, what design decisions were involved in its development, and how to use it. We give examples of circuits modelled in  $\Pi$ -Ware and proofs of properties involving these circuits.

Finally, in Chapter 4, we discuss what has been achieved by  $\Pi$ -Ware as it stands by comparing it with other hardware EDSLs, recent and past. We also clarify what desirable features remain to be implemented as future work, and on which conditions (if any) these implementation efforts depend.

## Chapter 2

# Background

### 2.1 Hardware Description

The hardware development process can be well understood by analyzing its similarities and differences to software development. Both hardware and software development usually "begin" with a high-level *specification* of the algorithm to be implemented. Also, both proceed by a series of translations, increasingly adding more details to the description.

However, the final targets of both hardware and software development differ: while in software the final artifact is machine code (sequence of instructions) for some architecture, in hardware the target is usually a *floorplan*, a spacially placed graph of logic gates and wires.

Also, the transformation steps in the software and hardware chains are different, as depicted in Figure 2.1. In this figure, the rectangular boxes represent transformations steps towards a more detailed description, while the ellipsoid shapes represent artifacts that are consumed/produced by each step.

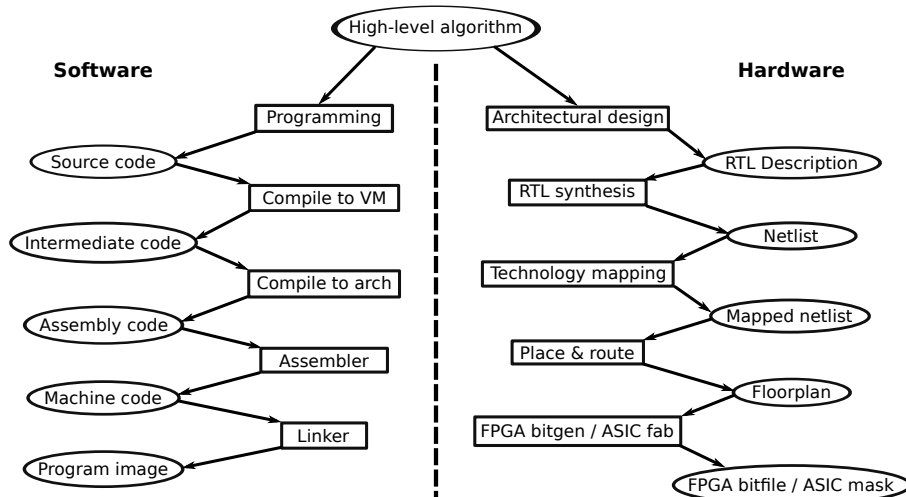


Figure 2.1: Software and Hardware refinement chains.

The first (highest) two levels in the hardware implementation flow are usually described using so-called Hardware Description Languages (HDLs), usually Verilog or VHDL. Nowadays they are used by hardware engineers to write both behavioural specifications of circuits (topmost ellipsis in Figure 2.1) as well as Register-Transfer Level (RTL) descriptions. However, these languages were originally designed for *simulation* purposes, and there are several problems that arise when using them to model hardware *architecture* and behaviour.

First of all, only a *subset* of these languages can be used for *synthesis* (actually deriving a netlist and floorplan). Although there is a standard [1] defining a *synthesizable subset* of VHDL, tools differ greatly in the level of support.

To further complicate the matter, this synthesizable subset is not *syntactically segregated*. One example of complex requirement for synthesizability of VHDL is: "in a process, every output must be assigned a value for every possible combination of input values". In Listing 2 we have an example process violating this requirement (the `if` statement lacks an `else` branch, and the `y` output has no assigned value when `sel` is different than zero).

```
process(sel, a, b)
begin
    if (sel = '00') then
        y <= a;
    end if;
end process;
```

Listing 2: Unsynthesizable VHDL process.

Another significant difference between hardware and software development is the level of *automatization* in the chains (Figure 2.1). Usually, all transformation steps in the software chain are automatic. On the hardware chain, the *crucial* "Architectural design" step is mostly manual.

The complex task of making explicit the space-time trade-offs lays in the hands of the hardware designer: they must decide how much parallelism to use, how to pipeline the processing steps of the algorithm, taking care to not generate data hazards, etc.

Recently, tools have been developed for this first step – called High-Level Synthesis (HLS). They usually take *behavioural VHDL*, C or even C++ as input, and produce RTL code. However, current HLS-based hardware implementation chains still face two main problems:

- The input languages (VHDL, C) are not expressive enough.
- Specification, verification and synthesis are all done with different tools and different languages.
  - Even behavioural VHDL can be considered practically a different language than Register-Transfer Level VHDL.

Functional programming languages have been touted as a solution to both of these problems. A functional program is a more abstract and expressive specification of an algorithm than a C function or VHDL entity. Also, functional languages could be used to *both* write the specification of a circuit's behaviour, as well as its Register-Transfer Level description.

## 2.2 Functional Hardware Description

In the beginning of the 1980s, many researchers were trying to use functional programming languages to design and reason about hardware circuits. These developments were happening at the same time when the VHSIC Hardware Description Language (VHDL) was being designed and standardized. Even though VHDL and Verilog ended up "winning" and becoming the *de facto* industry standards, it is still useful to take a look at the ideas behind these early HDLs, as some of them inspired current approaches.

The idea was then to come up with new functional languages, *specialized* to do hardware description. One of the prominent early examples in this set is  $\mu\text{FP}$  [22], which was in turn inspired by John Backus's FP [3]. In contrast to VHDL, which was later adapted to synthesis in an ad-hoc way,  $\mu\text{FP}$  was designed since the beginning to have both interpretations:

**Behavioural** Each "primitive" circuit as well as each combining form (higher-order function) has an attached *functional semantics*, used in simulation.

**Geometric** Each combining form has a typical geometric interpretation. For example, sequential composition of two circuits  $c_1$  and  $c_2$  will result in a floorplan in which  $c_2$  is placed *adjacent to*  $c_1$  and connected to it by the required wires.

The  $\mu\text{FP}$  language is an *extension* of Backus's FP, and contained only one extra combining form:  $\mu$ . The  $\mu$  combining form is responsible for the creation of functions (circuits) with internal state. According to the seminal  $\mu\text{FP}$  paper [22]:

The meaning of  $\mu f$  is defined in terms of the meaning of  $f$ . The functional out hides the state so that while  $M(f)$  maps a sequence of input-state pairs to a sequence of output-state pairs,  $\text{out}(M(f))$  just maps a sequence of inputs to a sequence of outputs. For a given cycle, the next output and the next state depend on the current input and the current state.

One very simple example of a circuit with internal state is a shift register. In Figure 2.2, we can see the structure of this circuit. Each of the dotted boxes represents a  $\mu\text{FP}$  expression. The smaller box denotes a combinational circuit (with 2 inputs and 2 outputs) that swaps its inputs. The bigger box corresponds to the application of the  $\mu$  combinator to the smaller one. It adds the indicated latch and a feedback loop, creating a stateful circuit.

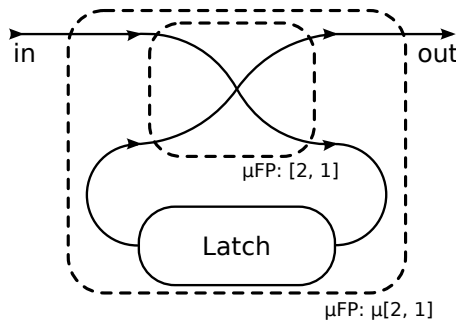


Figure 2.2: *Shift register*: a simple example of sequential circuit in  $\mu\text{FP}$ .



By being a conservative extension of Backus's FP, almost all algebraic laws of FP also hold for  $\mu$ FP. Furthermore, the  $\mu$  combining form has useful algebraic laws of its own. The most notable of these laws states that the composition of two " $\mu$ -wrapped" functions can be converted to a single " $\mu$ -wrapper".

In other words: a circuit with several, localized, memory elements can be converted into a circuit with a single centralized memory bank and a combinational block. If we apply this rewrite rule from right to left, we can see it as a form of "optimization", in which we start with a single memory bank and refine the design towards one in which memory elements sit closer to the sub-circuits using them.

Two of the core ideas of  $\mu$ FP served as inspiration for  $\Pi$ -Ware:

**Double interpretation** As in  $\mu$ FP, each circuit and circuit combinator in  $\Pi$ -Ware has two distinct semantics: they can be simulated (functional semantics) or synthesized to a netlist (geometric semantics).

**Single sequential constructor** As in  $\mu$ FP, there is only *one way* in  $\Pi$ -Ware to construct a sequential circuit. Also, the same law regarding the *state-introducing constructor* holds in  $\Pi$ -Ware, but because we use Agda as host language, this meta-theoretical property can be proven *in the same language* in which circuits are described.

## 2.2.1 Embedded Functional Hardware Description

With the growing popularity and advocacy [13] of *embedded DSLs*, a trend emerged of also having embedded HDLs. Functional programming languages were a natural fit for hosting DSLs. In particular, the Haskell programming language proved to be a popular choice of host, due to features such as lazy evaluation and a highly-customizable syntax.

Some examples of highly-successful DSLs implemented in Haskell are:

- Attoparsec <sup>1</sup> (monadic parsing)
- Accelerate <sup>2</sup> (data-parallel computing)
- Esqueleto <sup>3</sup> (SQL queries)
- Diagrams <sup>4</sup> (2D and 3D vector graphics)

Also, some of the most popular and powerful Embedded Hardware Description Languages (EHDLs) are hosted by Haskell. Let us now present some of these EHDLs and discuss some of their limitations, which will lead to the question of how to solve them using dependent types.

### Lava

Perhaps the most popular family of hardware DSLs embedded in Haskell is the *Lava* family. Lava's first incarnation [4] was developed at Chalmers University of Technology and Xilinx, and used a shallow-embedding in Haskell, along with a monadic description style to handle naming and sharing. These circuit monads were parameterized, and

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<sup>1</sup><https://github.com/bos/attoparsec>

<sup>2</sup><https://github.com/AccelerateHS/accelerate/wiki>

<sup>3</sup><https://github.com/prowdsponsor/esqueleto>

<sup>4</sup><http://projects.haskell.org/diagrams/>

by using different instances, different interpretations could be given to a single circuit description, such as simulation, synthesis, or model checking.

The design of Lava suffered significant changes later on, abandoning the monadic interface and adopting an observable sharing solution based on reference equality [8]. Currently, Lava has several dialects, such as *Chalmers-Lava*, *Xilinx-Lava*, *York-Lava* and *Kansas-Lava*. We base our examples in *Chalmers-Lava*<sup>5</sup>, which can be said to be the "canonical" dialect, and also the most actively developed.

In Lava, circuits are written as normal Haskell functions, by using pattern matching, function application and local naming. One restriction is that the types of the arguments are constructed by the `Signal` type constructor. For example, a negation gate in Lava would have the following type signature:

```
inv :: Signal Bool -> Signal Bool
```

Another restriction is that all circuit inputs must be *uncurried*, i.e., even though the *circuit* has several inputs, the *function* modeling that circuit must have only one argument, with all inputs in a *tuple*. A NAND gate in Lava would look like this:

```
nand2 :: (Signal Bool, Signal Bool) -> Signal Bool
nand2 (a, b) = inv (and2 (a, b))
```

Finally, due to the instances provided for the `Signal` type, the only way to aggregate bits in Lava is by using tuples and lists. Using only these structures we forego some type safety that Haskell *could* provide. For example, an n-bit binary (ripple-carry) adder in Lava has the following description:

```
adder :: (Signal Bool, ([Signal Bool], [Signal Bool]))
      -> ([Signal Bool], Signal Bool)

adder (carryIn, ([], [])) = ([], carryIn)
adder (carryIn, (a:as, b:bs)) = (sum:sums, carryOut)
  where (sum, carry) = fullAdd (carryIn, (a, b))
        (sums, carryOut) = adder (carry, (as, bs))
```

In this circuit description, there is an *expectation* that both inputs have the same size. When this expectation is not met, a *run-time* error will occur during simulation. This happens because the given definition is *partial*: the cases for `(carryIn, ([], b:bs))` and `(carryIn, (a:as, []))` are left undefined.

This limitation of Lava can be solved by dependent types, namely by using statically-sized vectors. Another limitation of Lava is related to the way in which it solves the observable sharing question: in order to detect sharing and cycles, the equality over the `Signal` type is defined as an equality over the references used. Therefore, comparisons of signals "created" in different sites will fail, even though their values are the same. For example, the following expressions will evaluate to `False`:

```
test1 :: Bool
test1 = low == low

test2 :: Bool
test2 = simulate adder (low, ([low], [low])) == low
```

<sup>5</sup><https://hackage.haskell.org/package/chalmers-lava2000>

This limitation is also not present in  $\Pi$ -Ware, due to the way in which we describe circuits in a structural fashion, completely avoiding the need to deal with observable sharing.

## ForSyDe

Another example of EHDL in Haskell is ForSyDe [21]. ForSyDe and Lava differ substantially in description style and internal workings, and a more complete comparison of the two can be found in the final report [19] of the experimentation project conducted as preparation to this thesis.

In ForSyDe, the central concepts are those of *process* and *signal*. Whereas in Lava only *synchronous* sequential circuits can be described – that is also the case in  $\Pi$ -Ware – in ForSyDe processes can belong to synchronous, asynchronous and continuous *models of computation*.

In the synchronous model of computation (which we studied more deeply), process constructors take a combinational function (called *process function* in ForSyDe jargon) and turn it into a synchronous sequential circuit (for example a state machine).

Instead of just using (smart) constructors of a certain *circuit datatype* to build process functions, ForSyDe relies on *Template Haskell* to *quote* regular Haskell syntax into an Abstract Syntax Tree (AST) – which is then further processed by ForSyDe. For example, the description of an adder in ForSyDe would look like the following:

```
adderFun :: ProcFun (Int8 -> Int8 -> Int8)
adderFun = $(newProcFun [d|
    adderFun' :: Int8 -> Int8 -> Int8
    adderFun' x y = x + y
|])

adderProc :: Signal Int8 -> Signal Int8 -> Signal Int8
adderProc = zipWithSY "adderProcess" adderFun
```

Notice how the `adderFun` *process function* is built from a regular Haskell function (`adderFun'`), which is then *quoted* (by the `[d|` quasi-quoter), processed by `newProcFun` and finally *spliced* back into place. The (combinational) process function is then *lifted* into the synchronous sequential setting by the `zipWithSY` *process constructor*, which just "zips" the inputs signals (streams) by applying the given process function *pointwise*.

Not any Haskell function can be quoted and processed by `newProcFun` and turned into a ForSyDe process function: the argument and return types of a `ProcType` must belong to the `ProcType` type class. Instances of this class are provided only for:

**Primitive types** `Int`, `Int8`, `Int16`, `Int32`, `Bool`, `Bit`

**Enumerated types** User-defined enumerations, with derived instances for `Data` and `Lift`

**Containers** Tuples and fixed-length vectors (`Data.Param.FSVec`), holding a type of the above two categories and unrestrictedly nested.

Under certain conditions, ForSyDe is able to generate VHDL netlists from the system description. In order for a ForSyDe system description to be *synthesizable*, all *process functions* need to comply with some extra requirements:

**Pointed notation** Declarations with point-free notation are not accepted

**Single-clause** To be synthesizable, the body of a *process function* cannot have multiple clauses, and it cannot have `let` or `where` blocks. This essentially forbids recursion inside *process functions*. Pattern matching is only possible by using the `case` construct.

Despite all these limitations, ForSyDe still provides a more typed approach to hardware description than Lava, with perhaps its most distinctive feature being the usage of fixed-length vectors. In the parameterized-data package page on *Hackage* <sup>6</sup>, the authors admit that the library's function is to provide "type-level computations and *emulate dependent types*". Therefore it is not a stretch to assume that using *actual* dependent types could improve on the ideas proposed by ForSyDe.

## Hawk and Clash

Finally, in this short review of functional hardware EDSLs, there are two more alternatives to be mentioned: Hawk and Clash.

Hawk [16] is a Haskell-hosted EDSL with a different target than the ones presented until now: instead of modelling circuits, it intends to model *microarchitectures*. Therefore, it has a higher level of abstraction than Lava or even ForSyDe.

Models in Hawk are executable, and *shallow-embedded*. However, Hawk uses a technique similar to the original Lava version [4] in order to also allow for symbolic evaluations: The descriptions are "parameterized" by type classes (in Hawk's case they are `Instruction`, `Boolean`, etc.), and by providing different instances, different *interpretations* are achieved.

The authors of Hawk mention some shortcomings of Haskell which they met, among which:

- Haskell's `List` datatype doesn't quite match the intended semantics for Hawk's `Signals`: the preferred semantics for `Signal` should be a truly infinite, coinductive *stream*. The authors mention a parallel effort of them to embed Hawk in the Isabelle theorem prover, which achieved the desired semantics. In  $\Pi$ -Ware, we make use of *coinductive streams* [15] to model the inputs and outputs of synchronous sequential circuits, exactly as the authors of Hawk desired.
- The type class system of Haskell is limited: the authors mention the desire to be able to explicitly provided specific instances at specific sites, and also the desire to use *views* on datatypes. Both features are available on *Agda*.

Finally, we would like to mention Clash [2]. Even though it is not (strictly speaking) an *embedded* DSL, it has very powerful features.

Clash was developed at the University of Twente, as an independent DSL, i.e, with a compiler of its own. However, its source language is strongly based on Haskell, and Clash's compiler reuses several pieces of machinery from Haskell's toolchain (specifically, GHC), in order to perform its term-rewriting and supercompilation.

The circuit models in Clash are higher-order, polymorphic functional programs, and they can be synthesized to VHDL netlist. In this sense, Clash serves as a good *target* with which to compare hardware EDSL development efforts: it is not – in itself – embedded, but has the same goals.

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<sup>6</sup><http://hackage.haskell.org/package/parameterized-data-0.1.5>

## 2.3 Dependently-Typed Programming

### 2.3.1 Type systems

A type system, in the context of programming languages, serves the purpose of grouping values, so that meaningless and potentially undesirable operations are avoided. For example, in some dynamic languages, a string and an integer can be "added", because the language runtime first *implicitly casts* the integer to a string, effectively performing string concatenation. This *implicit* behaviour can cause all sorts of programmer mistakes to go undetected. It is also hard to formulate properties about addition *in general* when all sorts of coercions can happen at runtime. To define addition sensibly, therefore, a type system can help by banning all programs that would try to add incompatible values.

Type systems can have very different properties and be implemented in very different ways. Some of the ways in which type systems can be categorized are [7]:

- Weak vs. strong
- Static vs. dynamic
- Polymorphism (parametric, ad-hoc, subtyping)

The most basic form of abstraction – values depending on values (the concept of functions) – is supported in practically all type systems. In some systems, the result type of functions can also depend on the *types* of the arguments; this property is called *polymorphism*. There are two kinds of polymorphism relevant to our discussion:

- In *parametric polymorphism*, functions are written without mentioning any specific type, and can therefore be applied to any instantiation of the type variable. A typical example of a parametric polymorphic function is obtaining the length of a list, where the same definition works for any type of element in the list.
- In *ad-hoc polymorphism*, the behaviour of a function varies with the type of the inputs. In Haskell, ad-hoc polymorphism is implemented in the *type class system*. A typical example of an ad-hoc polymorphic function is a comparison-based sorting algorithm in which – depending on the type of the elements in the collection – different comparison operators are be used.

Polymorphism adds *expressivity* and makes a type system stronger, in the sense that it allows for more precise specifications. For example, if we want to implement a swap function for pairs in Haskell, we could do start by specifying the type of the function in a non-polymorphic way:

```
swap' :: (Int , Int) -> (Int , Int)
```

There are several definitions satisfying the above type which do not swap the elements. For example, one "wrong" implementation would output a constant pair:

```
swap' _ = (1 , 1)
```

Notice how the argument is ignored (we use the "don't care" pattern). To *rule out* this class of wrong implementations, we could make the type polymorphic:

```
swap'' :: (a , a) -> (a , a)
```

Now any "constant" definition will *not anymore fit the type specified*. This because the only way to get an element of the *polymorphic type* `a` is to use what is in the parameter passed to the function. However, we could still write a wrong functions with this type, for example the identity:

```
swap'' (x , y) = (x , y)
```

This is possible because our type does not *yet* reflect a precise enough specification of `swap`. On the type we have now, the types of both elements in the pair are the same. If we lift this artificial restriction, we get the type signature which *fully specifies* `swap`.

```
swap :: (a , b) -> (b , a)
```

Now, *any total definition with this type* is a correct definition of `swap`. Because of the way in which we use type variables (`a` and `b`) in the signature, there is only one possible implementation of `swap`:

```
swap (x , y) = (y , x)
```

Going one step further, some type systems support types that depend on other types, these are called *type operators* or *type-level functions*. In Haskell, this form of abstraction is implemented as regular parameterized type constructors (`List`, `Maybe`, etc.) and as *indexed type families*.

The last step then in our "ladder" of type system expressiveness is *dependent types*.

### 2.3.2 Dependent types

A *dependent type* is a type that depends on a value. A typical example of dependent type is the type of *dependent pairs*, in which the *type* of the second element depends on the *value* of the first:

```
record  $\Sigma$  ( $\alpha$  : Set) ( $\beta$  :  $\alpha \rightarrow$  Set) : Set where
  constructor _,_
  field
    fst :  $\alpha$ 
    snd :  $\beta$  fst
```

In a dependent type system, we can also have functions in which the return *type* depends on the *value* of a parameter. These functions belong to the so-called *dependent function space*.

For example, we can imagine having a `take` function for vectors which makes use of this possibility to have a more precise specification. First of all, when indexing or obtaining a prefix from a vector, we need to ensure that the index (or amount to be extracted) is within bounds. That is, we cannot `take` more elements than the size of the vector. The type signature of `take` is shown in Listing 3.

```
take : { $\alpha$  : Set} { $n$  :  $\mathbb{N}}$  ( $k$  :  $\mathbb{N}$ )  $\rightarrow$  Vec  $\alpha$  ( $k + n$ )  $\rightarrow$  Vec  $\alpha$   $k$ 
```

Listing 3: A "size-safe" prefix-taking function for sized vectors.

In the signature of `take`, a dependent function space is used both to restrict the *domain* and to express a desired property of the *result*. Notice how the vector parameter

has a type ( $\text{Vec } \alpha (k + n)$ ) which *depends on the value* of the first (explicit) parameter:  $k$ . Also, the result type is restricted to  $\text{Vec } \alpha k$ .

Finally, we observe that the type of `take` is a *necessary but not sufficient condition* for what we would intuitively conceive as being a "correct" prefix-taking function. There are "wrong" implementations which still would have this type, for example returning a constant vector of size  $k$ . This type, however, provides many more *static guarantees*: all implementations violating bounds and producing incorrectly-sized results are ruled out *by the type checker at compile-time*.

Until now, we have approached languages with dependent types as a way to help with *programming*. Specifically, we gave examples on how using dependent types in function's type signatures can rule out incorrect implementations *by design*. Dependently-typed languages can also be seen from a *logical* point of view.

This remark – that a programming language (with its type system) can also be seen as a logic – goes back to the early days of computing, and is known by several names, among which "Curry-Howard isomorphism" and "propositions as types" [24]. It initially was "discovered" as a connection between the simply-typed lambda calculus and intuitionistic propositional logic. As decades went by, this correspondence was found to be much more general, and several connections were drawn between diverse typed lambda calculi and logics.

The system of dependent types which we use in this thesis (the one used by *Agda*) corresponds to *intuitionistic predicate logic*.

Even though there are logics connected to other – less powerful – typed lambda calculi, the one used in *Agda* is expressive enough to radically improve its area of use. In a language with dependent types, we can not only write *programs*, but also *proofs*.

For example, in *Agda*, we can model the less-than-or-equal order relation using an *indexed data family* 4, where the indices are the usual Peano naturals.

```
data _ ≤ _ : ℕ → ℕ → Set where
  z ≤ n : {n : ℕ} → zero ≤ n
  s ≤ s : {m n : ℕ} → m ≤ n → suc m ≤ suc n
```

Listing 4: Order relation ( $\leq$ ) over naturals, as an *Agda* indexed data family.

Having defined this relation, we can then prove facts which involve it. For example, one trivial fact is that two is smaller-than-or-equal to four, which has the trivial proof shown in Listing 5.

```
twoLEQFour : 2 ≤ 4
twoLEQFour = s ≤ s (s ≤ s z ≤ n)
```

Listing 5: A very simple proof involving the  $\leq$  relation over natural numbers.

Following the "propositions as types" isomorphism, the type of `twoLEQFour` is a logic statement, and the expression to the right-hand side of the equals sign (the body of the function) is a *proof* of this statement. The proof is built with  $z \leq n$  as its "basis", the fact that `zero` is lesser-than-or equal to any number. Then  $s \leq s$  is applied twice, producing, respectively,  $1 \leq 4$  and finally  $2 \leq 4$ .

Of course more "interesting" and general statements can be proved. For example, the fact that the  $\leq$  relation is *transitive* is stated and proved in Listing 6.

```

≤ trans : { a b c : ℕ } → a ≤ b → b ≤ c → a ≤ c
≤ trans z ≤ n _ = z ≤ n
≤ trans (s ≤ s ab') (s ≤ s bc') = s ≤ s (≤ trans ab' bc')

```

Listing 6: Proof that the  $\leq$  relation is transitive.

As proofs are first-class citizens, they can be passed as arguments to functions, and returned as well. This provides yet another powerful way of expressing requirements of function arguments, and showing that the result returned satisfies some desired property.

For example, we have already shown (in Listing 3) how to have a safe version of a prefix-taking function for statically-sized arrays: by using a specially-designed type for the desired amount. Now we have another way to express the same requirement: we can explicitly require a proof argument be passed, guaranteeing that the amount to be "taken" is *less than or equal* to the array size.

```

take' : { α : Set } { n : ℕ } ( m : ℕ ) { p : m ≤ n } → Vec α n → Vec α m

```

This capability for precisely defining and enforcing requirements and invariants make dependently-typed programming languages very good candidates for hosting EDSLs. In the paper "The power of Pi" [18], three examples are shown of how to embed DSLs in Agda and get corresponding *Domain-Specific Type Systems* (equipped with desirable properties) "for free".

The examples in "The power of Pi" – specially the embedding of *Cryptol*, a low-level cryptographic DSL – served as inspiration, and lead us to investigate how dependently-typed programming can benefit hardware design, the main question targeted by this project.

## 2.4 Hardware and dependent types

We are aware of three attempts in the literature of bringing dependent types to improve hardware design. Each of them has provided us with specific insights and ideas, some of which  $\Pi$ -Ware incorporates. Let us now briefly review these works, and highlight the most important contributions of each.

The paper "Constructing Correct Circuits" [5], from 2007, gives a clear example of how dependent types can *tie together* specification and implementation. In this paper, the authors give a mapping between the usual, Peano-based, naturals and binary numbers, which they then use to build a (ripple-carry) binary adder which is *correct by construction*.

This approach is significantly different than the one taken in  $\Pi$ -Ware. In the paper by Brady, McKinna & Hammond [5], the functional specification of a circuit is carried *in the type* of the circuit. This, according to the authors, means that "a type correct program is a constructive proof that the program meets its specification".

In  $\Pi$ -Ware, on the other hand, a circuit's type only express its input and output types, and give some *structural well-formedness guarantees* (correct sizing). The development of  $\Pi$ -Ware was already underway as this paper first came to my knowledge. Even so, the decision of keeping circuits and proofs apart (though in the same language) still seems to have been the right one; mainly because of the requirement that circuits be synthesizable to VHDL.



### 2.4.1 Coquet

The most significant source of inspiration for the design of  $\Pi$ -Ware was Coquet [6], a hardware DSL embedded in the Coq language<sup>7</sup>. Coquet and  $\Pi$ -Ware share some very important characteristics and differ on several others, with these differences caused not only by the different host languages used (Coq and Agda), but also by different implementations chosen for some concepts.

First of all, the similarities. Coquet's circuit datatype utilizes the same concept of *structural* description as  $\Pi$ -Ware, with a "fundamental" gate constructor and constructors for structural combination. Also, the idea of dedicating one specific constructor for building "rewiring" circuits was inspired by Coquet's **Plug**. The **Circuit** datatype of Coquet is shown on Listing 7.

```
Context {tech : Techno}
Inductive Circuit : Type -> Type -> Type :=
| Atom : forall {n m : Type} {Hfn : Fin n} {Hfm : Fin m},
      techno n m -> Circuit n m

| Plug : forall {n m : Type} {Hfn : Fin n} {Hfm : Fin m} (f : m -> n),
      Circuit n m

| Ser : forall {n m p : Type},
      Circuit n m -> Circuit m p -> Circuit n p

| Par : forall {n m p q : Type},
      Circuit n p -> Circuit m q -> Circuit (n + m) (p + q)

| Loop : forall {n m p : Type},
      Circuit (n + p) (n + p) -> Circuit n m
```

Listing 7: Coquet's **Circuit** datatype.

Some differences between Coquet's **Circuit** type and that of  $\Pi$ -Ware (detailed in Section 3.1) are, however, immediately noticeable:

**Indices are Types, instead of just sizes** In  $\Pi$ -Ware, the core circuit datatype is indexed by two natural numbers, representing the *sizes* of the circuit's input and output. In Coquet, the indices are arbitrary types, *constrained* by the **Fin** type class. The definition of **Fin** makes the synthesis of Coquet's circuits *interfaces* to VHDL harder. Before deciding to use natural number indices on  $\Pi$ -Ware, another alternative for generalization was considered, detailed in Section 4.2.1.

**Lack of a "sum" constructor** Both Coquet and  $\Pi$ -Ware have a "parallel composition" constructor which, given two circuits as arguments (with inputs respectively  $\alpha$  and  $\beta$ ), will build a circuit with the *product*  $\alpha \times \beta$  as its input.  $\Pi$ -Ware, however, has an extra "sum" constructor, building a circuit which is capable of "deciding" which one of the two subcircuits to apply to an input depending on a *tag*. This is useful for implementing, for example, generic multiplexers.

<sup>7</sup>Referring to Coq as a language is a misnomer, as it is an interactive theorem prover that uses different languages for defining terms, interactive commands, and user-defined proof tactics

**Single fundamental gate vs. gate library** Coquet's fundamental constructor (**Atom**) is parameterized by the `techno` instance, which gives the description of **one** technology-dependent gate (NAND, NOR, etc.).  $\Pi$ -Ware circuit descriptions, however, are parameterized by a *gate library*, with an attached specification function to each of its elements, as detailed in Section 3.2.2.

Another similarity between Coquet and  $\Pi$ -Ware is the usage of *data abstraction*, embodied in Coquet by the **Iso** type class and in  $\Pi$ -Ware by `Synthesizable`, described in more detail in Section 3.2.4.

The goal of data abstraction is to be able to express the specification of circuits in more user-friendly (high-level) types, while the circuits themselves are just described in terms of sizes and wires.

Finally, some of the the more fundamental differences between  $\Pi$ -Ware and Coquet are:

**Relational vs. functional semantics** Coquet defines a *relational* semantics for circuits, that is, the semantics of a circuit is a datatype defined by recursion on circuit structure.  $\Pi$ -Ware, on the other hand, has an executable *functional semantics*<sup>8</sup>, in which each circuit is mapped to an Agda function over sequences of bits (or other types).

**Semantics of sequential circuits**  $\Pi$ -Ware, in contrast to Coquet, makes a clear distinction between *purely combinational* and *possibly sequential* circuits. We maintain the invariant that *purely combinational circuits never contain loops*. Also, in  $\Pi$ -Ware, the only constructor which builds loops also adds a *delay element*<sup>9</sup>, eliminating the need to consider an extra "sequential fundamental gate", as in Coquet. The functional semantics of sequential circuits in  $\Pi$ -Ware is defined as a *causal stream function* (detailed in Section 3.3.2), while Coquet uses regular functions over streams.

---

<sup>8</sup>another term commonly used in the literature is *denotational* semantics

<sup>9</sup>in hardware terms, a *flip-flop*

## Chapter 3

# The $\Pi$ -Ware library

$\Pi$ -Ware is an EHDL that allows for circuit description (modelling), simulation, reasoning (proving correctness or other properties), and synthesis to netlists. In this chapter we will describe in detail how the  $\Pi$ -Ware library is organized, what principles are behind some of the most important design decisions taken in its development, and how to use  $\Pi$ -Ware to model, simulate and reason about circuit behaviour.

The reader is assumed to be familiar with the *Agda* programming language, as this is the language in which  $\Pi$ -Ware is embedded. An introductory-level knowledge of dependent type theory in general is also appreciated.

### 3.1 Circuit Syntax

The most basic activity allowed by  $\Pi$ -Ware is the *description* of circuits: as already mentioned briefly in the introduction,  $\Pi$ -Ware is *deeply* embedded, which means there is a *datatype* whose values are circuits.

A deep embedding was chosen in order to allow for semantics other than execution (simulation). Particularly, the possibility of *synthesizing* circuit models was a requirement kept in mind throughout the whole development.

The circuit datatype ( $\mathbb{C}'$ ) is the most *fundamental* of the whole library. It is defined as an dependent inductive family, indexed by two natural numbers, as shown in Listing 8.

```
data  $\mathbb{C}'$  where
  Nil      :  $\mathbb{C}'$  zero zero
  Gate     : (g# : Gates#) →  $\mathbb{C}'$  (ins g#) (outs g#)
  Plug     :  $\forall \{i\ o\} \rightarrow (f : \text{Fin } o \rightarrow \text{Fin } i) \rightarrow \mathbb{C}'\ i\ o$ 

  DelayLoop :  $\forall \{i\ o\ l\} (c : \mathbb{C}'\ (i + l)\ (o + l)) \{p : \text{comb}'\ c\} \rightarrow \mathbb{C}'\ i\ o$ 

   $\_ \gg' \_$     :  $\forall \{i\ m\ o\} \rightarrow \mathbb{C}'\ i\ m \rightarrow \mathbb{C}'\ m\ o \rightarrow \mathbb{C}'\ i\ o$ 
   $\_ |' \_$     :  $\forall \{i_1\ o_1\ i_2\ o_2\} \rightarrow \mathbb{C}'\ i_1\ o_1 \rightarrow \mathbb{C}'\ i_2\ o_2 \rightarrow \mathbb{C}'\ (i_1 + i_2)\ (o_1 + o_2)$ 
   $\_ | + ' \_$  :  $\forall \{i_1\ i_2\ o\} \rightarrow \mathbb{C}'\ i_1\ o \rightarrow \mathbb{C}'\ i_2\ o \rightarrow \mathbb{C}'\ (\text{succ } (i_1 \sqcup i_2))\ o$ 
```

Listing 8: The core circuit type ( $\mathbb{C}'$ ) of  $\Pi$ -Ware.

A *structural representation* of a circuit is achieved by the constructors of  $\mathcal{C}'$ . This is in stark contrast with the the description style used in the Lava family, for example. In Lava, the constructors of the circuit datatype represent solely logic (or arithmetic) gates, and metalanguage (Haskell) constructs such as application, tupling and local naming are used to represent sequencing, parallel composition, loops and sharing. In  $\Pi$ -Ware, on the other hand, explicit constructors represent these combinations, avoiding the need to implement some form of *observable sharing* [11].

The indices of  $\mathcal{C}'$  represent, respectively, the *size* of the circuit's input and output. This can be thought of as the number of "wires" entering (resp. leaving) that circuit. Notice that the representation of inputs and outputs used in  $\mathcal{C}'$  is untyped and unstructured. However, there is another – higher-level – circuit datatype ( $\mathcal{C}$ ), which adds a layer of typing *on top* of  $\mathcal{C}'$ , and constitutes the actual intended *interface* between the user (hardware designer) and  $\Pi$ -Ware. This abstraction layer will be discussed in more detail on Section 3.2.3.

To better understand the reasoning behind the design of the low-level  $\mathcal{C}'$  datatype, its constructors can be considered to belong to one of three categories:

**Fundamental:** These construct predefined, or "atomic" circuits, with no sub-components.

- **Nil:** The *empty circuit*. Performs no computation and has neither inputs nor outputs. It is mainly useful as a "base case" when building large circuits with recursive definitions.
- **Gate:** Constructs a chosen circuit among those provided by a *gate library* passed as parameter to the **Circuit** module.
- **Plug:** Constructs a "rewiring" circuit. Performs no computation, but can be used to apply permutations, associativity and to duplicate or discard wires.

**Structural:** Represent ways in which smaller circuits can be interconnected to build a bigger one.

- $c_1 \gg c_2$ : Sequential composition. Given  $c_1$  and  $c_2$ , connects the output of  $c_1$  into the input of  $c_2$ .
- $c_1 | c_2$ : Parallel composition. Splits the input into two parts, connected to  $c_1$  and  $c_2$ , and rejoins the outputs of  $c_1$  and  $c_2$  into a single output.
- $c_1 | + c_2$ : Tagged branching. Based on the value of a *tag* given in one of the input wires, feed the remaining input wires into *either*  $c_1$  or  $c_2$ .

**Delay:** The **DelayLoop** constructor belongs to a category of its own. Its purpose is to construct a state-holding circuit given a purely combinational circuit as argument.

These descriptions are just a rough summary of the *synthesis semantics* of  $\Pi$ -Ware, that is, how each circuit constructor creates a netlist, given netlists as arguments. The precise *definitions* of the synthesis semantics is given in Section 3.3. The same section also contains a detailed definition and explanation of a simulation semantics for  $\Pi$ -Ware circuits, both purely combinational and sequential ones.

### 3.1.1 Design discipline enforced by circuit constructors

The several constructors of  $\mathcal{C}'$  "calculate" the port sizes of the circuits they construct, based on the sizes of the circuits given as arguments. These calculations implement *structural well-formedness* rules for circuits. One example of such rule can be seen in the case of parallel composition:

$$\_||'\_ : \forall \{i_1\ o_1\ i_2\ o_2\} \rightarrow \mathcal{C}'\ i_1\ o_1 \rightarrow \mathcal{C}'\ i_2\ o_2 \rightarrow \mathcal{C}'\ (i_1 + i_2)\ (o_1 + o_2)$$

In this case, the input (resp. output) size of the composed circuit equals the *sum* of the input (resp. output) sizes of the constituent sub-circuits. Other such rules are also imposed by  $\_||'\_$ ,  $\_||+\'_\$  and **Plug**. Together, all of them ensure:

**No floating wires** Circuit sizes need to match in order for the usage of a constructor to be type-correct. In particular, no circuit *input* wires are ever left *disconnected*. Figure 3.1 illustrates an example of a situation banned by the sizing rule present in the type of  $\_||'\_$ .



Figure 3.1: Example of circuit banned by the type of  $\_||'\_$ .

**No short-circuits** The **Plug** constructor, the only one which can be used for "rewiring" purposes, has a type which forbids connecting multiple sources to the same load. Its argument is a *function from outputs to inputs*. In this way, definitions connecting multiple inputs to the same output are banned, as they are *not* functions from outputs to inputs. Also, when a plug is used between two sub-circuits  $c_1$  and  $c_2$ , a definition in which an *input* of  $c_2$  would be left *disconnected* is disallowed by Agda, as such a definition would not be *total*. The diagram on Figure 3.2 represents such a banned situation:



Figure 3.2: This circuit cannot be constructed because  $f_p$  is not *total*.

### Purely combination vs. possibly sequential

Another piece of information "carried" together with circuits is whether it is *purely combinational*, i.e., has no internal state, no notion of *history*, and the value of its current outputs depend only on the value of its current inputs.

This information is represented by a metalanguage truth value, an Agda **Set** which can be either  $\top$  (the **Set** with one value – "top") or  $\perp$  (the **Set** with no values – "bottom").

```

comb' Nil          = T
comb' (Gate _)     = T
comb' (Plug _)     = T
comb' (DelayLoop _) = ⊥
comb' (c1 >> c2)   = comb' c1 × comb' c2
comb' (c1 |' c2)    = comb' c1 × comb' c2
comb' (c1 |+' c2)   = comb' c1 × comb' c2

```

Listing 9: Predicate telling whether a low-level circuit is purely combinational.

The predicate `comb'` over low-level ( $\mathbb{C}'$ ) circuits defines when is a circuit purely combinational. The code for `comb'` is shown on Listing 9.

Notice that the fundamental constructors `Nil`, `Gate` and `Plug` *always* build purely combinational circuits, while `DelayLoop` always produces circuits with internal state. The structural constructors `(_)>>_`, `_|'_` and `_|+'_` yield a purely combinational circuit only when *both* of its argument are themselves purely combinational. This is expressed by using the product type constructor `_ × _`, which in the `Set` level corresponds to a logical conjunction.

This categorization of circuits as stateful or stateless has one main goal: To avoid the evaluation of *combinational loops*. Their presence in a circuit is almost always a design mistake, and by combining the `DelayLoop` constructor with the `comb'` predicate we guarantee that:

- The only way to create a loop in a circuit (`DelayLoop`) also introduces a delay.
- Combinational evaluation (ignoring past inputs) only happens when safe.

## 3.2 Abstraction Mechanisms

Several of the definitions (modules, datatypes, functions) of  $\Pi$ -Ware are parameterized in order to provide for better code reuse. The library allows for a choice of the type of data carried in individual wires, the types that serve as inputs and outputs to circuits, and the set of fundamental gates from which circuits are built.

In this section we present how this flexibility is achieved, which requirements are imposed upon the parameters in each case of parameterization. Also, the motivation behind the chosen requirements is presented, based on the general goals of  $\Pi$ -Ware as well as the wish to impose as few constraints as possible.

### 3.2.1 Atoms

In EDSLs of the current Lava family [8], only values of type `Bool` and `Int` can be carried by the "wires", and circuits can only operate on inputs and outputs which are tuples or lists of these basic types.

There is a little more flexibility in ForSyDe [21], where the `ProcType` type class determines what values of what types can be used as inputs and outputs of *process functions* (combinational circuits). ForSyDe has predefined instances of `ProcType` for fixed-width numeric types (`Int8`, `Int16`, etc.) and `Bool`, while relying on *Template Haskell* to generate instances for user-defined *enumerations* at compile time.

$\Pi$ -Ware's approach is similar to ForSyDe's, in that it shares the same goal (to "carry" in the wires any type that belongs to a type class). However,  $\Pi$ -Ware does not use any metaprogramming, and uses dependent types to ensure that the provided instances satisfy desired properties. The type class `Atomic` (shown in Listing 10) expresses in  $\Pi$ -Ware what are the requirements for a type to be carried in the wires of a circuit.

```
record Atomic : Set1 where
  field
    Atom      : Set
    |Atom| - 1 : ℕ
    n → atom  : Fin (suc |Atom| - 1) → Atom
    atom → n  : Atom → Fin (suc |Atom| - 1)

    inv - left : ∀ i → atom → n (n → atom i) ≡ i
    inv - right : ∀ a → n → atom (atom → n a) ≡ a

    |Atom| = suc |Atom| - 1
    Atom#  = Fin |Atom|
```

Listing 10: The `Atomic` type class.

The fields `Atom` and `|Atom| - 1` determine, respectively, the Agda `Set` to be used as basis and *how many elements* of this `Set` are to be used as atoms (the number of atoms is actually `|Atom| = suc |Atom| - 1`).

Then there are the fields `n → atom` and `atom → n`, which define a *bijection* between the `Atom` type and `Fin |Atom|` (naturals from `zero` to `|Atom|`). Finally, the two last fields in `Atomic` are proofs that `n → atom` and `atom → n` are inverses of each other, therefore proving that they are also bijections.

### Instance for Bool

Included with  $\Pi$ -Ware there is already a predefined instance of `Atomic` for `Bool` (the boolean type in Agda's standard library). Knowing the meaning of each field, the definitions comprising the instance are mostly easy to understand. First, we start by defining the cardinality of the set of values we are interest in (`B` is used as synonym for `Bool`):

```
|B| - 1 = 1
|B| = suc |B| - 1
```

Now, before defining the bijections that enumerate the set of atoms, we give more convenient *names* to the elements of `Fin |B|` used in the enumeration by using an Agda feature known as *pattern synonyms*:

```
pattern False# = Fz
pattern True#  = Fs Fz
pattern Absurd# n = Fs (Fs n)
```

The usage of the `Absurd#` pattern becomes clearer when we take a look at the definitions of the bijections themselves:

```

n → B : Fin |B| → B
n → B = λ { False# → false; True# → true; (Absurd# ()) }

B → n : B → Fin |B|
B → n = λ { false → False#; true → True# }

```

In the case of the `Absurd#` pattern, there is no need to provide a right-hand side to the equation, as the Agda typechecker can verify that there is no  $x$  such that  $Fs (Fs x)$  belongs to `Fin 2`.

Finally, there are the proofs stating that  $n \rightarrow B$  has a two-sided inverse (namely,  $B \rightarrow n$ ), and, therefore, it is a bijection. The proofs use simple case analysis (and reflexivity).

```

inv - left - B : ∀ i → B → n (n → B i) ≡ i
inv - left - B = λ { False# → refl; True# → refl; (Absurd# ()) }

inv - right - B : ∀ b → n → B (B → n b) ≡ b
inv - right - B = λ { false → refl; true → refl }

```

### Other possibly interesting instances

Even though currently `Bool` (from the Agda standard library) is the only type for which there is already a predefined instance of `Atomic`, other types could be interestingly used as atoms. For example:

**Three-valued logic types** Representing the values TRUE, FALSE and UNKNOWN

**IEEE1164 nine-valued logic** Standardized logic used widely in industry as a VHDL/Verilog type. Besides the usual FALSE and TRUE logical values (represented respectively by '0' and '1') it accomodates also the following others:

- 'Z': *High impedance*, means output is not being driven.
- 'X': *Strong drive, unknown logic value*.
- 'W': *Weak drive, unknown logic value*.
- 'L': *Weak drive, logic zero*.
- 'H': *Weak drive, logic one*.
- 'U': *Uninitialized*. Used as default initial value in simulation.
- '-': *Don't care*.

### 3.2.2 Gate library

Besides being parameterized by the type of `Atoms` that can be carried in their wires,  $\Pi$ -Ware low-level circuit descriptions ( $\mathbb{C}'$ ) are also parameterized by a *library of fundamental gates* upon which circuits are built.

There are several interesting possible gate libraries which could be used to describe circuits, among which:

- *Functionally complete* sets of boolean functions such as {NAND}, {NOR} or {NOT, AND, OR} (predefined in the  $\Pi$ -Ware library).



- Arithmetic primitives over boolean vectors, such as  $\{+_-, *_-\}$ .
- A logic-arithmetic library together with some purpose-specific primitives, for example cryptographic and Digital Signal Processing (DSP) functions.

A gate library is defined as an element of the `Gates` record type, shown in Listing 11.

```

W : ℕ → Set
W = Vec Atom

record Gates : Set where
  field
    |Gates| - 1 : ℕ
    ins outs    : Fin (suc |Gates| - 1) → ℕ
    spec        : (g : Fin (suc |Gates| - 1)) → (W (ins g) → W (outs g))

|Gates| = suc |Gates| - 1
Gates#  = Fin |Gates|

```

Listing 11: Definition of a gate library: the `Gates` record.

The first field of the `Gates` record ( $|Gates| - 1$ ) determines *how many* gates the library has, and consequently also the *the type used as gate index* (or *gate identifier*), which is `Gates# = Fin |Gates|`.

For each gate identifier, the functions `ins` and `outs` determine, respectively, the size of the input and output of that gate; therefore together they determine the gate's *interface*.

Finally, the `spec` field determines the *functional specification* of a given gate identifier. Notice how the whole `Gates` module is parameterized by an instance of `Atomic`. In this way, the `spec` function yields, given a gate identifier, a function over *words* (`Atom` vectors) of the correct length. This functional specification of each fundamental gate is used for the *simulation semantics*.

### Assumed correctness

Later, in Section 3.4, we will give a precise definition for what is considered the *functional correctness* of a circuit; its *compliance* to a certain functional specification.

Usually, the proof of functional correctness for a circuit is built using, in some way, the correctness proofs of its subcomponents. However, fundamental gates have no subcomponents, so their correctness is assumed. By definition, the simulation semantics of a fundamental gate is equal to the `spec` function given for it in the gate library definition.

### Relation to synthesis

Even though fundamental gates have no subcomponents *at the  $\Pi$ -Ware level*, they are not in any way directly synthesizable to VHDL. To convert a  $\Pi$ -Ware circuit to VHDL, each fundamental gate in the library being used needs to have a corresponding excerpt of VHDL code representing it.

This feature is not integrated in the current version of  $\Pi$ -Ware, but assuming the existence of a hierarchy of datatypes representing VHDL code, and specifically the existence of a type `VHDLEntity`, we could have an extra field in the `Gates` record, as such:

```
netlist : (g : Fin (suc |Gates| - 1)) → VHDLEntity (ins g) (out g)
```

The `VHDLEntity` is also parameterized by the input and output sizes of the circuit, tying the  $\Pi$ -Ware fundamental gate interface with the VHDL abstract syntax.

### Instance for BoolTrio

One specific instance of gate library is already "shipped" with  $\Pi$ -Ware: the usual set of boolean gates {NOT, AND, OR}, together with the constants {FALSE, TRUE}. The first definitions establish that this library contains five gates:

```
|BoolTrio| - 1 = 4
|BoolTrio|     = suc |BoolTrio| - 1
```

Then, as already done in the instance of `Atomic`, we define some *pattern synonyms* to make the gate identifiers easier to read:

```
pattern FalseConst# = Fz
pattern TrueConst#  = Fs Fz
pattern Not#        = Fs (Fs Fz)
pattern And#        = Fs (Fs (Fs Fz))
pattern Or#         = Fs (Fs (Fs (Fs Fz)))
pattern Absurd# n   = Fs (Fs (Fs (Fs (Fs n))))
```

The next step is to define the *interface* of each gate on the library. All gates have exactly one output, while the number of inputs vary from 0 to 2.

```
ins outs : Fin |BoolTrio| → ℕ
ins = λ { FalseConst# → 0; TrueConst# → 0; Not# → 1; And# → 2; Or# → 2; (Absurd# ()) }
outs _ = 1
```

Finally, the last piece of information needed to define the gate library is the functional specification of each gate.

```
spec : (g : Fin |BoolTrio|) → (W (ins g) → W (outs g))
spec FalseConst# = const [ false ]
spec TrueConst#  = const [ true  ]
spec Not#        = λ { (x :: ε) → [ not x ] }
spec And#        = λ { (x :: y :: ε) → [ x ∧ y ] }
spec Or#         = λ { (x :: y :: ε) → [ x ∨ y ] }
spec (Absurd# ())
```

Notice that the gates use `Bool` from Agda's standard library as their `Atom` type. Also the logic operators from the standard library (`Data.Bool`) are used in the specification. With all the necessary pieces of the puzzle in hand, the definition of the `BoolTrio` gate library is as follows:

```
BoolTrio : Gates
BoolTrio = record {
  |Gates| - 1 = |BoolTrio| - 1
; ins       = ins
; outs      = outs
; spec      = spec
}
```

### 3.2.3 High-level circuit datatype

When presenting the low-level circuit datatype ( $\mathbb{C}'$ ) on Section 3.1, it was mentioned that the representation using  $\mathbb{C}'$  is untyped and unstructured, i.e., all circuit inputs and outputs at that level are just sequences of **Atoms** (for example, bits).

However, the user of  $\Pi$ -Ware is not supposed to actually model circuits using the  $\mathbb{C}'$  datatype. There is a layer of abstraction sitting on top of  $\mathbb{C}'$ , and circuits at this higher level have inputs and outputs with Agda types. This layer is coded as the  $\mathbb{C}$  datatype, shown in Listing 12.

```
data C (α β : Set) {i j : ℕ} : Set where
  MkC : (⟦ sα : ↓ W ↑ α {i} ⟧ ⟦ sβ : ↓ W ↑ β {j} ⟧) → C' i j → C α β {i} {j}
```

Listing 12: High-level circuit datatype ( $\mathbb{C}$ ).

First of all, notice the *lack of a prime symbol in the name of the datatype*. This is a general naming convention: whenever there are two versions of a definition, one at a low level of abstraction and one higher, the low-level gets the same name of the high-level one, with a prime symbol as suffix.

In contrast to the low-level ( $\mathbb{C}'$ ) circuit type,  $\mathbb{C}$  is parameterized by two Agda **Sets**, and has only one constructor - **MkC** - which "packs" the low-level circuit description with the information on how to convert between the input/output Agda types (resp.  $\alpha$  and  $\beta$ ) and the appropriately-sized *words* used in the low level. This conversion information is coded in the  $\downarrow W \uparrow$  type class (pronounced *Synthesizable*), which is covered in more detail on Section 3.2.4.

Furthermore, the **PiWare.Circuit** module defines also *smart constructors* for  $\mathbb{C}$ , corresponding to each of the constructors at the low level. One example is parallel composition:

```
_|_| : ∀ {α i γ k β j δ l} → (⟦ sα : ↓ W ↑ α {i} ⟧ ⟦ sγ : ↓ W ↑ γ {k} ⟧
    → ⟦ sβ : ↓ W ↑ β {j} ⟧ ⟦ sδ : ↓ W ↑ δ {l} ⟧)
    → C α γ {i} {k} → C β δ {j} {l} → C (α × β) (γ × δ) {i + j} {k + l}
_|_| ⟦ sα ⟧ ⟦ sγ ⟧ ⟦ sβ ⟧ ⟦ sδ ⟧ (MkC c₁) (MkC c₂) =
  MkC (⟦ ↓ W ↑ × ⟧ ⟦ sα ⟧ ⟦ sβ ⟧ ⟦ ↓ W ↑ × ⟧ ⟦ sγ ⟧ ⟦ sδ ⟧) (c₁ |' c₂)
```

Using a *gate library* together with these smart constructors, one can model hardware circuits operating over Agda types. One very simple example is a XOR circuit (shown in Listing 13), built using the **BoolTrio** gate library mentioned before. The block diagram in Figure 3.3 illustrates the architecture of such circuit.



Figure 3.3: Architecture of a sample XOR gate.

```

⊥C TC : C T B
⊥C = MkC (Gate FalseConst#)
TC = MkC (Gate TrueConst#)

¬C : C B B
¬C = MkC (Gate Not#)

∧C ∨C : C (B × B) B
∧C = MkC (Gate And#)
∨C = MkC (Gate Or#)

⊔C : C (B × B) B
⊔C = pForkx
    >> (¬C || pid >> ∧C) || (pid || ¬C >> ∧C)
    >> ∨C

```

Listing 13: Example of a XOR gate built with the BoolTrio library.

Again in this example, as customary, we renamed `Bool` to `B`, and `pid` is the *identity plug*. The identity plug can be substituted in the netlist by a bus (with the adequate sizes), and is defined (together with other useful plugs) in the `PiWare.Plugs` module.

### 3.2.4 The Synthesizable type class

The "connection" between the low-level (`C'`) and high-level (`C`) circuit types is done by the `↓ W ↑` type class (pronounced Synthesizable). Instances of the `↓ W ↑` class define a mapping between a given Agda type and an appropriately-sized *word* type. Word types (`W n`, for some  $n$ ) are *synthesizable* to VHDL vectors, thus the name of the class. Listing 14 shows the definition of `↓ W ↑`:

```

W : ℕ → Set
W = Vec Atom

record ↓ W ↑ (α : Set) {i : ℕ} : Set where
  constructor ↓ W ↑ [, _]
  field
    ↓ : α → W i
    ↑ : W i → α

open ↓ W ↑ [ ... ]

```

Listing 14: The `↓ W ↑` (Synthesizable) type class.

The type class has two parameters: the type which it encodes ( $\alpha$ ), and the size of the *word type* to which  $\alpha$  corresponds ( $i$ ). Notice that the size parameter is passed *implicitly*, because in some occasions Agda might be able to automatically infer (by unification) this value.

One can imagine that all *finite types* (types with finitely many elements) can be made synthesizable under the definition of `↓ W ↑`, given a certain encoding scheme. We do

not give a proof of this statement, but one way of doing it would be to translate Agda datatype definitions into a sum-of-products view and then use a standardized encoding scheme (for example, some form of *Church encoding*).

$\Pi$ -Ware does *not* provide facilities for mapping arbitrary Agda datatypes into a sum-of-products view (we believe this is the domain of a *generic programming* library, not of a hardware EDSL). However,  $\Pi$ -Ware *does* provide predefined instances of  $\Downarrow \mathbb{W} \Uparrow$  for units ( $\mathbf{T}$ ), booleans ( $\mathbf{Bool}$ ), products ( $\_ \times \_$ ), vectors ( $\mathbf{Vec}$ ) and sums ( $\_ \uplus \_$ ), in order to facilitate working with complex types when modelling hardware.

One interesting instance to look at is the one for Agda's standard library vectors ( $\mathbf{Vec}$ ), shown in Listing 15.

```

 $\Downarrow \mathbf{W} \Uparrow \mathbf{-Vec} : \forall \{ \alpha \ i \ n \} \rightarrow \{ \{ s \alpha : \Downarrow \mathbf{W} \Uparrow \alpha \{ i \} \} \} \rightarrow \Downarrow \mathbf{W} \Uparrow (\mathbf{Vec} \ \alpha \ n)$ 
 $\Downarrow \mathbf{W} \Uparrow \mathbf{-Vec} \{ \alpha \} \{ i \} \{ n \} \{ \{ s \alpha \} \} = \Downarrow \mathbf{W} \Uparrow [ \text{down} , \text{up} ]$ 
  where down :  $\mathbf{Vec} \ \alpha \ n \rightarrow \mathbf{W} \ (n * i)$ 
        down v = v >>=  $\Downarrow$ 

  up :  $\mathbf{W} \ (n * i) \rightarrow \mathbf{Vec} \ \alpha \ n$ 
  up w = mapv  $\Uparrow$  (proj1 $ group n i w)
```

Listing 15: Predefined instance of  $\Downarrow \mathbb{W} \Uparrow$  for fixed-length vectors.

In the `down` definition, the *bind* ( $>>=$ ) operator is defined as `concat ∘ map`, i.e., first each element of the vector is serialized (using the passed instance for  $\alpha$ ), then all the words (each with size  $i$ ) are concatenated, giving a total size of  $(n * i)$ .

To convert a serialized  $\mathbf{Vec}$  back `up`, first the given word is *grouped* into  $n$  smaller words (each of size  $i$ ), which is possible because the size of the passed word is  $(n * i)$ . Then, ignoring the proof object returned by `group` (which is in the second position of a pair, therefore we take `proj1`), we map the `up` instance of  $\alpha$  over each of the smaller words, obtaining in this way an element of type  $\mathbf{Vec} \ \alpha \ n$ .

### Proof of bijection

It is very desirable that the pair of fields "down" ( $\Downarrow$ ) and "up" ( $\Uparrow$ ), for any instance of  $\Downarrow \mathbb{W} \Uparrow$ , are inverses of each other, that is:

$$\begin{aligned} \forall (x : \alpha) &\rightarrow \Uparrow (\Downarrow x) \equiv x \\ \forall (w : \mathbb{W} \ i) &\rightarrow \Downarrow (\Uparrow w) \equiv w \end{aligned}$$

We indeed tried to embed these proofs also as methods of the  $\Downarrow \mathbb{W} \Uparrow$  class. However, when trying to write the proofs for the included instances, we faced some difficulties (especially in the sum case) and left this endeavour for future work (discussed in more detail in Section 4.2.1).

### Synthesizable instance for sums

The instance of  $\Downarrow \mathbb{W} \Uparrow$  for sums (disjoint unions) is the most complex among all predefined instances included with  $\Pi$ -Ware. This complexity comes mostly from the need to *distinguish* which set is concerned when converting a word "up" ( $\Uparrow$ ). Assuming that we want  $\Downarrow$  and  $\Uparrow$  to form a bijection (no information may be lost), let's first calculate how many atoms are necessary to represent a sum.

With a given encoding for set  $\alpha$  ( $s\alpha : \mathbb{W} i$ ) and one for  $\beta$  ( $s\beta : \mathbb{W} j$ ), it is clear that at least  $i \sqcup j$  atoms are necessary to encode  $\alpha \uplus \beta$ ; the size of a sum is at least the maximum among the sizes of its summands. Furthermore, it is necessary to somehow encode the choice (whether the sum was built from the left ( $\text{inj}_1$ ) or the right ( $\text{inj}_2$ )). This requires at least one more atom, bringing the size to  $\text{succ}(i \sqcup j)$ .

The predefined instance for sums included with  $\Pi$ -Ware is shown on Listing 16.

```

↓ W ↑ -⊕ : ∀ {α i β j} → (n m p : Atom#) {d : n ≠ m}
  → { sα : ↓ W ↑ α {i} } { sβ : ↓ W ↑ β {j} } → ↓ W ↑ (α ⊕ β) {succ(i ⊔ j)}
↓ W ↑ -⊕ {α} {i} {β} {j} n m p { sα } { sβ } = ↓ W ↑ [ down , up ]
where down : α ⊕ β → W (succ(i ⊔ j))
      down = [ (λ a → (n → atom n) :: padFst i j (n → atom p) (↓ a))
              , (λ b → (m → atom m) :: padSnd i j (m → atom p) (↓ b)) ]

up : W (succ(i ⊔ j)) → α ⊕ β
up = map⊕ ↑ ↑ ∘ untag

```

Listing 16: Predefined instance of  $\downarrow \mathbb{W} \uparrow$  for sums.

Besides the encodings of the summands (named  $s\alpha$  and  $s\beta$ , given as *instance arguments*), this instance is also passed some extra parameters: three *atom indices* ( $n$ ,  $m$  and  $p$ ) and a proof that  $n$  and  $m$  are different. The atom index  $n$  gives the `Atom` to be used in case the sum is built from the left ( $\text{inj}_1$ ), and  $m$  for the case when it is built from the right ( $\text{inj}_2$ ). They need to be different in order for the choice information not to be lost when synthesizing a sum. At last, the  $p$  parameter identifies the atom used for padding.

In the `down` function, the `[_,_] sum destructor` is used, and is passed two functions: one to operate at the left case and one at the right. They prepend to the output word an atom identified by either  $n$  or  $m$  and pad either  $a$  or  $b$  to fit  $i \sqcup j$ .

In the `up` conversion, first the `tag` is used to transform the input word into a sum of words ( $\mathbb{W} i \uplus \mathbb{W} j$ ), then the `map⊕` function is used to apply either the `↑` instance for  $\alpha$  or for  $\beta$ , giving the desired result type of  $\alpha \uplus \beta$ .

### Recursive instance search

To model synthesizable datatypes, we used the Agda way of implementing Haskell's type classes, namely, to code a type class as a record type, in which each field corresponds to a method. In this analogy, instances are just elements of the record type, and class constraints are coded in Agda by passing *instance arguments* [9] to the constrained function. Arguments passed in this way will be searched among the identifiers available in scope, in a type-directed fashion, similar to how the mechanism of instance search works in Haskell.

However, there was (until recently) a big drawback to Agda's instance search implementation: it was not recursive. There were some arguments for this decision in the seminal paper, most notably that having fully recursive instance resolution would "expose an additional computational model" and that the instance search mechanism would need to "perform a reasonably powerful automated proof search".

During the last years, since the publication of the paper on instance arguments [9], demand has grown for recursive instance search in Agda, and in a recent contribution

to the Agda codebase by Guillaume Brunerie <sup>1</sup>, costs and benefits were again weighted, and a reasonably efficient implementation was released in Agda version 2.4.0.2 (current version at the time of writing this report).

The impact of this change for  $\Pi$ -Ware is immense. Recursive instance resolution removes the need for a great deal of boilerplate code. For example, before the advent of recursive resolution, modelling the XOR circuit on Listing 13 required the user to manually construct instances of  $\Downarrow \mathbb{W} \Uparrow$  for more than 5 types ( $\mathbb{B} \times \mathbb{B}$ ,  $(\mathbb{B} \times \mathbb{B}) \times \mathbb{B}$ ,  $\mathbb{B} \times (\mathbb{B} \times \mathbb{B})$ ,  $(\mathbb{B} \times (\mathbb{B} \times \mathbb{B})) \times (\mathbb{B} \times \mathbb{B})$ , etc.). Now all of this work is done by Agda itself. On the other hand, the typechecking time grew a little, and this could be a problem when modelling big circuits, as discussed in Section 4.2.1.

### 3.3 Circuit Semantics

Functional semantics for simulation...

- Two types of evaluation: combinational and sequential
- Combinational eval requires a proof that the circuit contains no loops  
Eval of a fundamental gate is just its *definitional behaviour*
- For sequential circuits we use a *causal stream semantics*  
Current output depends on the current input and (possibly) on the past inputs  
*Different* than plain Stream functions
- There's also an eval function which allows the circuit to be viewed as function from Stream to Stream

#### 3.3.1 Combinational evaluation

#### 3.3.2 Sequential evaluation

Initially we tried to write a semantics yielding simply a function over streams.

But this is not quite a good semantic model for a synchronous sequential circuits, as a stream function is allowed to "look into future values" in order to produce the output stream, while a sequential circuit can only use information about its past inputs when calculating the next output.

Therefore, causal streams...

**Causal Streams**

### 3.4 Proving circuit properties

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<sup>1</sup><https://github.com/agda/agda/commit/7ca60fd60aec1b73d9f14aaa6e1aeef184bcca79>

# Chapter 4

## Conclusions

### 4.1 Discussion and Related Work

- Reiterate what was achieved
- Compare to other EDSLs for similarities and differences

### 4.2 Future work

#### 4.2.1 Current limitations

- What problems *II-Ware* still has.  
With suggestions for how to solve them, if possible
- Synthesizable instances are not proven bijections
- Big circuits consume too much time and memory to typecheck  
Who to blame?  
Instance search  
Combinational proofs
- Circuit interfaces are described by *just a size*  
Simple, but generates ugly VHDL  
All circuits have one big vector input and one output  
Could we make the interface indices more *structured*?

#### 4.2.2 Future research

- More broadly, considering *II-Ware*, what would be the ``next steps".



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