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Theory and Methodology

The Hamiltonian p-median problem *

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Abstract: This paper is concerned with a new mixed routing location problem embedding the p-median and the travelling salesman problems. The Hamiltonian p-median problem (HPMP) is formulated and several heuristics are proposed. Computational experience with a set of test problems is reported.

Keywords: Combinational optimisation, p-median, travelling salesman, Hamiltonian p-median

1. Introduction

This paper is concerned with a new mixed routing location problem embedding the p-median and the travelling salesman problems.

Let N denote a set of points associated with the location of customers and depots. The Hamiltonian p-median problem (HPMP) consists in selecting p depots from N in order to supply all customers. It is assumed that each customer is served from one, and only one, depot and that the customers assigned to a depot define an Hamiltonian circuit passing through the depot. The aim is to find out the p circuits that minimize total distribution costs.

In graph theory terms, if $G = (N, A)$ is a complete directed graph with cost d_{ij} associated with arc (i, j), this problem consists in obtaining the p Hamiltonian circuits with minimum total cost such that each vertex $i \in N$ belongs to one, and only one circuit. In other words, the goal is to determine the best cost partition of the graph into p-Hamiltonian circuits. Any vertex in each circuit may be selected as depot location for that particular circuit or route.

It is clear that a travelling salesman problem (TSP) is associated with each depot and its customers and that the HPMP can be seen as a set partitioning problem with a special cost function, that assigns to each partition the optimal value of the associated TSP.

The Hamiltonian p-median problem belongs to a class of routing–location problems that has started attracting research just recently, due to its strong combinatorial nature which implies great computational complexity (see Burness and White [5], Jacobsen and Madsen [12], Laporte and Norbert [14], Laporte et al. [15], Nambiar et al. [18], Branco et al. [1,3,4], Christofides et al. [7,8] and Drezner et al. [9]).

The problem proposed here arises in its own or as a subproblem in real world situations such as schools location, milk stations and depot location for different industrial and commercial purposes.

In Section 2, mathematical programming formulations for the HPMP are provided. Heuristic

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methods for solving the HPMP and our computational experience with a set of test problems with a number of customers ranging between 10 and 100 are given. The addition of capacity constraints to the HPMP is discussed in Section 4. Finally, Section 5 includes concluding comments and suggestions for further research.

2. Formulation of the HPMP

This problem accepts many different formulations, a number of which have been provided in Branco and Coelho [2]. Here we confine ourselves to two of those.

First we shall state the HPMP as a set partitioning problem. Let $P(N)$ denote the set of parts of N and F the subset of $P(N)$ formed by the elements of $P(N)$ except the empty set and all subsets of N with cardinality greater than $n-(p-1)$. Let q represent the cardinality of F.

Given a cost–distance matrix $D=[d_{ij}]$, then for any set of vertices $R_s \in F$ $(s=1,\dots,q)$ a cost C_s equal to the optimal cost of the travelling salesman problem on R_s can be assigned. In addition, define the cost of a partition P of N as the sum of costs of its elements.

The HPMP consists in finding the least cost partition with p elements chosen from F. Thus when considering the matrix $T=[t_{is}]$, defined as

$$t_{is}=\begin{cases}1 & \text{if vertex } i \in R_s, \\ & \quad (i=1,\dots,n;\ s=1,\dots,q) \\ 0 & \text{otherwise,}\end{cases}$$

and considering the binary variables

$$w_s=\begin{cases}1 & \text{if } R_s \text{ is part of the solution partition,} \\ & \quad (s=1,\dots,q) \\ 0 & \text{otherwise,}\end{cases}$$

the HPMP can be formulated as the following set partitioning problem:

$$\text{(P1)}\quad \min.\quad Z=\sum_s C_s w_s$$

$$\text{s.t.}\quad \sum_s t_{is} w_s = 1 \quad (i=1,\dots,n), \tag{1}$$

$$\sum_s w_s = p, \tag{2}$$

where equation (1) ensures that each vertex belongs to one, and only one, subset R_s in the partition and (2) states that the partition is formed by p subsets.

Let now define the following binary variables:

$$y_{ik}=\begin{cases}1 & \text{if vertex } i \text{ belongs to the Hamiltonian} \\ & \text{circuit } k, \\ & \quad (i=1,\dots,n;\ k=1,\dots,p) \\ 0 & \text{otherwise;}\end{cases}$$

$$x_{ijk}=\begin{cases}1 & \text{if vertex } i \text{ precedes vertex } j \text{ in} \\ & \text{circuit } k, \\ & \quad (i,\ j=1,\dots,n;\ k=1,\dots,p) \\ 0 & \text{otherwise.}\end{cases}$$

The HPMP can now be reformulated as

$$\text{(P2)}\quad \min.\quad Z=\sum_k \sum_{ij} d_{ij} x_{ijk}$$

$$\text{s.t.}\quad \sum_k y_{ik}=1 \quad (i=1,\dots,n), \tag{3}$$

$$\sum_i y_{ik} \geqslant 1 \quad (k=1,\dots,p), \tag{4}$$

$$\sum_i x_{ijk} = y_{jk} \quad (j=1,\dots,n;\ k=1,\dots,p), \tag{5}$$

$$\sum_j x_{ijk} = y_{ik} \quad (i=1,\dots,n;\ k=1,\dots,p), \tag{6}$$

$$\sum_{i,j \in S} x_{ijk} \leqslant |S|-s \quad \forall S \subset R_k:\ |S| \geqslant 1 \quad (k=1,\dots,p), \tag{7}$$

where $R_k=\{i\colon y_{ik}=1\}$ and $|S|$ denotes the cardinality of S. Constraint (3) ensures that each vertex is assigned to one circuit, constraint (4) sets up p circuits, equations (5) and (6) together with (3) establishes that each vertex is visited in one, and only one circuit, and finally, constraint (7) prevents the formation of subcircuits in each R_k. We note that this last formulation has some resemblance with the one given by Fisher and Jaikumar [11] for the vehicle routing problem. Both formulations, (P1) and (P2), refer to a direct graph. The adaptation to an undirected graph is straightforward [2].

3. Heuristic methods

The HPMP is an NP-complete problem (see Cerdeira [6]) and, therefore, it is natural that heuristic methods shall be used either to generate an initial solution or a near optimal one. A few heuristic algorithms will be outlined below. These procedures have been coded in FORTRAN 77 and run for several test problems in a UNIVAC 1100 computer. The codes assume that an undirected graph is provided and that single vertex circuits are excluded.

3.1. The clustering heuristic

The idea underpinning this method is to generate a partition of N into p clusters R_k, $k = 1, \ldots, p$, and to solve a TSP for each cluster.

The procedure starts by selecting p seed vertices from N. It makes sense to assume that the two vertices further apart will belong to different clusters. Thus, those two vertices are picked up to generate different clusters. Let S denote the set of vertices already included into a cluster and $N' = N - S$ the set of the free vertices. The next vertex selected to initiate a cluster is a vertex with maximum distance from S, until p clusters are formed. Next step consists in the selection of a second vertex to each cluster R_k which is accomplished by selecting the free vertex nearest to R_k. The vertices remaining free after these two steps are allocated to the p clusters by a minimal cost assignment.

Two different allocation cost definitions have been considered. Let k_1 and k_2 be the seed vertices for cluster R_k. The allocation cost C_{ik} of assigning vertex i to cluster R_k has been taken as

$$C_{ik} = d_{ik_1} + d_{ik_2} \quad \text{(version 1)},$$

or

$$C_{ik} = d_{ik_1} + d_{ik_2} - d_{k_1k_2} \quad \text{(version 2)}.$$

The last step is to solve the TSP associated with each one of the p clusters. A 3-optimal heuristic have been used but any TSP heuristic could be adopted. The clustering method has been tested over 22 test problems.

For problems 1, 2 and 3 the optimal solutions are known, their costs are respectively 194, 192 and 213. Problems 4 to 22 have an integer cost matrix corresponding to the minimal distances between points randomly generated in the plane

Table 1
Results for the clustering heuristic

Prob. no.	No. of customers	No. of depots	Cost		Time [a] (sec)	
			Version 1	Version 2	Version 1	Version 2
1	20	3	194	194	0.08	0.08
2	20	4	223	214	0.06	0.06
3	20	4	213	213	0.04	0.04
4	10	2	322	322	0.03	0.03
5	10	3	165	165	0.02	0.02
6	20	3	535	534	0.10	0.09
7	20	4	483	500	0.07	0.07
8	20	5	406	406	0.04	0.05
9	30	2	618	618	1.84	1.84
10	30	3	554	554	0.83	0.82
11	30	4	526	530	0.44	0.26
12	30	5	576	571	0.22	0.18
13	30	6	537	531	0.19	0.16
14	50	5	657	663	0.84	1.13
15	50	8	649	632	0.29	0.48
16	50	10	583	580	0.34	0.32
17	75	10	706	708	1.16	1.10
18	75	15	629	636	0.46	0.48
19	75	20	588	584	0.40	0.47
20	100	15	829	839	1.00	0.86
21	100	20	749	750	0.69	0.93
22	100	25	681	689	0.74	0.73

[a] CPU time excluding input–output in a UNIVAC 1100.

with coordinates in the interval [0, 100]. Results are shown in Table 1.

3.2. 3-Optimal method

This is a 3-optimal procedure for the HPMP. Analogously to other 3-optimal methods (Lin [16], Lin and Kernigham [17]) it starts with a feasible solution and changes involving three links are systematically tried in order to obtain a better solution, which will replace the previous one, until no improvements are possible. Since the outcome depends on the starting solution, the method can be re-started with different initial solutions and the best result chosen.

A feasible solution for the HPMP consists of n links defining p disjoint circuits. Removing three links will fall into one of the cases:

A–The three links are chosen from one circuit.

B–Two links are picked from a circuit and the other is selected from a different one.

C–Each link is chosen from a different circuit.

In case A, only one circuit is changed and therefore it reduces to the 3-optimal method for the TSP associated to that circuit.

In case B, four new feasible solutions modifying two circuits may be generated, as sketched in Figure 1. A fifth feasible solution is possible (see Figure 2) which involves changes in only one circuit and therefore is tackled by the TSP 3-optimal method on that circuit.

In case C, since the number of circuits is fixed no changes are possible (see Figure 3).

Starting from a feasible solution, having the TSP 3-optimal method applied to each circuit, then the search for a better solution is confined to the changes described in Figure 1. Whenever an improvement is found, the two new circuits are perturbed by a TSP 3-optimal procedure, but only the exchanges involving at least one of the 'new' links need to be tested until a better circuit is detected.

In our implementation, the circuits are numbered from 1 to p, two circuits are picked at a time and all feasible exchanges are tested for all pairs of circuits. Two different sequencing procedures were tried: in the first one, whenever a better solution is reached the search is re-initiated from circuit 1. In the second one, the search is re-initiated from the circuit coming immediately after the last one tested. In addition, two sub-

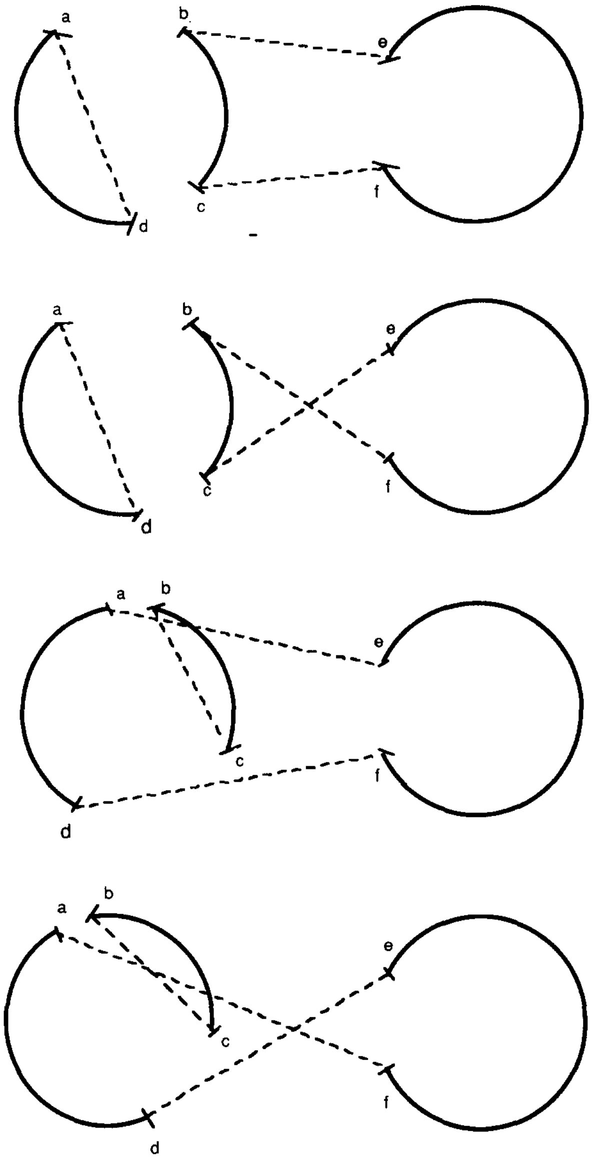

Figure 1. New feasible solutions (case B)

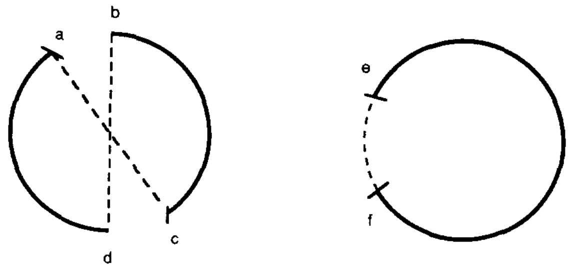

Figure 2. Feasible solution changing only one circuit (case B)

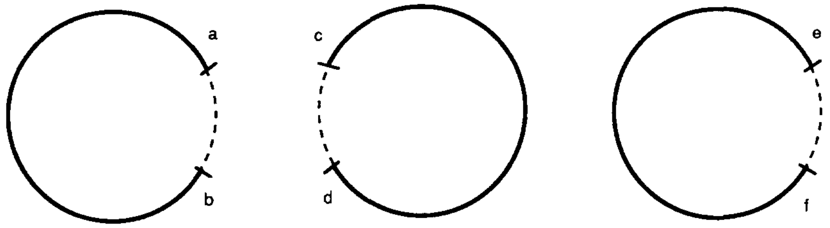

Figure 3. Unique feasible solution (case C)

routines for the TSP 3-optimal have been tested, one due to Fenel [10] and another expressly coded for insertion in our algorithm. Better performances have been achieved with this last subroutine and the second sequencing procedure.

The 3-optimal heuristic has been tested over the previous 22 test problems using as starting feasible solution the one obtained from the clustering heuristic. Also, randomly generated initial feasible solutions were tested for problems 1–13, 15, 16, and 8 aditional problems numbered from 23 to 30. Problems 25 to 30 are real problems referring to several Portuguese communities. Results are provided in Tables 2 and 3.

In Table 2, we may observe that the best 3-optimal solution comes from version 1 in 23% of the cases and this percentage grows to 32% for version 2. Furthermore, the clustering heuristic provides 3-optimal solutions in 18% of the cases.

Confronting Tables 2 and 3, we conclude that for problems 7, 10, 11, 12, 15 and 16 it was worthwhile both in terms of cost and time to start with a feasible solution generated by the clustering method, as it is the case for problems 1, 3, 5 and 6

Table 2
Results for the 3-optimal heuristic with initial feasible solution generated by the clustering method

Prob. no.	No. of customers	No. of depots	Cost		Time [a] (sec.)	
			Version 1	Version 2	Version 1	Version 2
1	20	3	194	194	0.23	0.23
2	20	4	202	199	0.47	0.23
3	20	4	213	213	0.19	0.19
4	10	2	236	236	0.11	0.11
5	10	3	165	165	0.06	0.06
6	20	3	415	394	1.74	1.69
7	20	4	387	387	0.91	0.81
8	20	5	406	406	0.20	0.20
9	30	2	471	471	11.27	11.27
10	30	3	448	448	5.24	5.24
11	30	4	453	411	2.14	2.98
12	30	5	394	458	4.93	2.02
13	30	6	404	404	1.98	1.48
14	50	5	503	517	43.99	16.71
15	50	8	490	506	12.38	18.24
16	50	10	469	468	30.26	9.62
17	75	10	586	577	49.23	71.80
18	75	15	540	545	25.42	15.38
19	75	20	522	522	9.20	12.20
20	100	15	673	669	70.30	81.72
21	100	20	657	644	33.72	25.87
22	100	25	621	627	18.99	20.08

[a] CPU time excluding input–output in a UNIVAC 1100.

Table 3
Results for 3-optimal heuristic with initial feasible solution randomly generated

Prob. no.	No. of customers	No. of depots	Cost (sec.)	Time [a]
1	20	3	194	2.15
2	20	4	192	1.94
3	20	4	213	1.56
4	10	2	236	0.39
5	10	3	165	0.24
6	20	3	394	2.58
7	20	4	390	4.02
8	20	5	383	3.02
9	30	2	465	7.77
10	30	3	462	9.12
11	30	4	455	6.72
12	30	5	397	9.71
13	30	6	393	5.53
15	50	8	527	36.99
16	50	10	479	30.57
23	16	4	39	0.57
24	6	2	31	0.02
25	10	2	1605	0.20
26	16	4	1874	0.85
27	20	4	1742	1.10
28	20	5	1798	0.63
29	30	3	2218	10.02
30	30	6	2261	7.84

[a] CPU time excluding input–output in a UNIVAC 1100.

Table 4
Results for the shrinking heuristic

Prob. no.	No. of customers	No. of depots	Cost	Time [a] (sec.)
1	20	3	194	0.20
2	20	4	199	0.17
3	20	4	213	0.14
4	10	2	236	0.05
5	10	3	165	0.04
6	20	3	420	0.22
7	20	4	403	0.20
8	20	5	397	0.17

[a] CPU time including output and referring to the shrinking phase in a UNIVAC 1100.

just in terms of time. However, a randomly generated initial feasible solution provides the best performance with problems 2, 8, 9 and 13.

3.3. *The shrinking heuristic*

The 2-matching problem reported in [15] is a relaxation of the Hamiltonian p-median problem. Moreover examining 3-optimal solutions we realize that most of them arise from shrinking 2-matching ones. This suggests another heuristic method, as follows:

1. Start solving the 2-matching problem with same cost matrix.
2. Shrink connected components of the 2-matching solution until a feasible HPMP solution is obtained.
3. Perturb the solution by the 3-optimal algorithm.

In step 2, the total number of existing circuits is reduced by one in each iteration. Thus, if m denotes the number of circuits in a 2-matching solution, the algorithm will require $m-p$ iterations. Since $\lfloor\frac{1}{2}n\rfloor$, the greatest integer less or equal to $\frac{1}{2}n$, is an upper bound for m, $\lfloor\frac{1}{2}n\rfloor-p$ is an upper bound on the total number of iterations.

Starting from the 2-matching solution, a shrinking penalty defined as the minimal cost of merging two circuits by removing a link from each one and joining the resulting Hamiltonian chains, is evaluated for each pair of circuits. The pair of circuits that offers the minimal shrinking penalty is merged into one circuit. Shrinking penalties between the new circuit and all others are computed and the process repeated until a feasible HPMP is obtained.

At present, this heuristic has been tested only over the first 8 test problems. Note that for these problems, no further improvement where achieved in step 3, thus we may assert that the solutions obtained after step 2 were already 3-optimal ones. Results are given in Table 4. According to these results, the method seems very promising, but further tests are required.

3.4. *The spanning walk heuristic*

A spanning walk heuristic has been proposed by Karp [13] for the Travelling Salesman Problem.

In Karp's method it is assumed that the n cities are scattered in a rectangular region of the plane, which will be subdivided in 2^k subrectangles, each containing at most α cities. The TSP in each subrectangle is solved. The union of 2^k subtours forms a spanning walk through the n cities [1]. The spanning walk is then transformed in a tour over

[1] We recall that a spanning walk is an undirected and connected multigraph where all vertices are of even degree.

all cities, with cost less or equal to that of the spanning walk.

Following the same idea we develop a similar heuristic to solve the HPMP in the plane. The main difference is that p circuits must be built, which requires that the spanning walk will be subdivided into p connected components ($p \leqslant 2^k$).

Now, let us define as critical points the vertices in the spanning walk with degree greater than 2. Let X represent the set of critical points and $\bar{A}$ the subset of links from the spanning walk with at least one extreme in X. B_s, $s = 1, \ldots, 2^k$, will denote the set of vertices in rectangle s.

The subdivision of the spanning walk is performed iteratively establishing a new connected component for iteration.

An iteration starts selecting from $\bar{A}$ the most expensive link, say a. Assume that link a belongs to subrectangle s. Select from this subrectangle a link adjacent to a, say b. Removing links a and b from subrectangle s will generate two vertices of odd degree. In order to create a new connected component keeping the spanning walk structure, these vertices are joined by the link defined by them. Next, sets $\bar{A}$ and X are updated, and the process is repeated until p spanning walks have been built. The transformation of the spanning walks into circuits follows Karp's method.

More precisely, the algorithm for the subdivision of the spanning walk is as follows.

Initialization: $m = 1$. For each vertex i ($i = 1, \ldots, n$) compute its degree g_i.

Step 1. Select the link $a \in \bar{A}$ with greatest cost. Let $a = [i, j]$. Moreover consider s such that $i, j \in B_s$.

Test: Do both extreme vertices of a belong to X?

Yes – go to Step 3.

No – go to Step 2.

Step 2. Since one of the extreme vertices of a is a critical point, assume that it is vertex j. If $|B_s| = 2$ set $\bar{A} = \bar{A} - \{a\}$ and return to Step 1.
Otherwise, set $j^* = j$; $i^* = 0$, remove links $a = [i, j]$ and $b = [l, j]$ where $l \in B_s$, add link $c = [i, l]$ to the spanning walk, and if $l \in X$, set $\bar{A} = \bar{A} \cup \{c\}$. Go to Step 4.

Step 3. If $|B_s| = 2$, let $b = [j, i]$ and remove links a and b from the spanning walk. Update $j^* = j$; $i^* = i$. Go to Step 4.
Otherwise ($|B_s| > 2$), compute

$$\mu_i = d_{tj} - d_{it}, \quad \mu_j = d_{t'i} - d_{jt'},$$

where $t, t' \in B_s$ and $[i, t]$ and $[j, t']$ are links adjacents to a.

Let $\mu_{j^*} = \min\{\mu_i, \mu_j\}$. Remove from the spanning walk in B_s the two links incident at j^*. One of them is $a = [i, j]$. Let the other be $b = [l, j^*]$, $l \in B_s$. Add link $c = [i, l]$ or $c = [j, l]$, if $j^* = j$ ($l = t'$) or $j^* = i$ ($l = t$); respectively set $i^* = 0$ and $\bar{A} = \bar{A} \cup \{c\}$ and go to the next step.

Step 4. (i) Set $\bar{A} = \bar{A} - \{a\} - \{b\}$.

(ii) Update $g_{j^*} = g_{j^*} - 2$. If $g_{j^*} > 2$ proceed to (iii).

Otherwise since j^* was a critical point, it must be $g_{j^*} = 2$. Update $X = X - \{j^*\}$.
Consider links $e_1 = [v_1, j^*]$ and $e_2 = [v_2, j^*]$ incident at j^*. If $v_1 \notin X$ set $\bar{A} = \bar{A} - \{e_1\}$. If $v_2 \notin X$, set $\bar{A} = \bar{A} - \{e_2\}$.

(iii) If $i^* = 0$ set $j^* = i^*$; $i^* = 0$, and return to (ii).

Otherwise proceed to Step 5.

Step 5. $m = m + 1$. If $m < p$ go to Step 1.
Otherwise, stop. The spanning walk has been subdivided into p spanning walks.

For illustrative purposes, let us consider an example for an HPMP with $n = 19$ and $p = 3$. If we choose $\alpha = 4$, then $k = 3$ and the original rectangle is subdivided into 8 rectangles. Figure 4 gives the subdivision and the resulting spanning walk. Circled points represent the critical points and the links marked are those of $\bar{A}$.

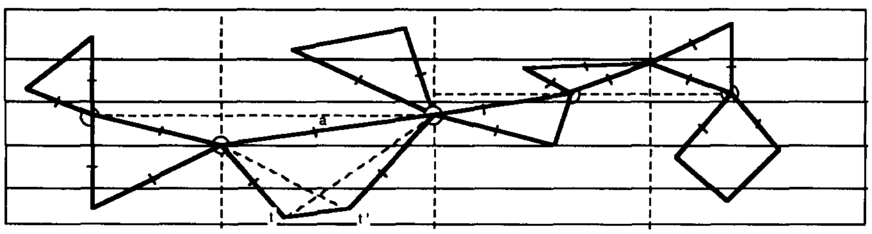

Figure 4. The spanning walk heuristic ($m = 1$)

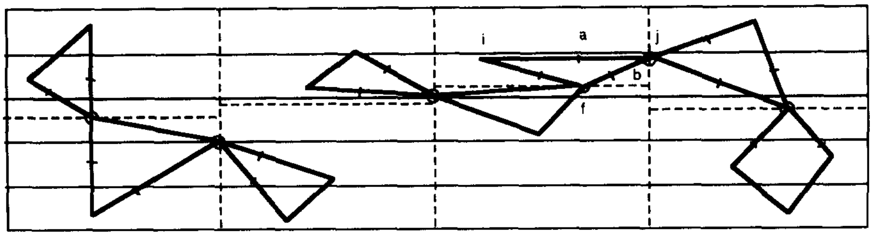

Figure 5. The spanning walk heuristic ($m = 2$)

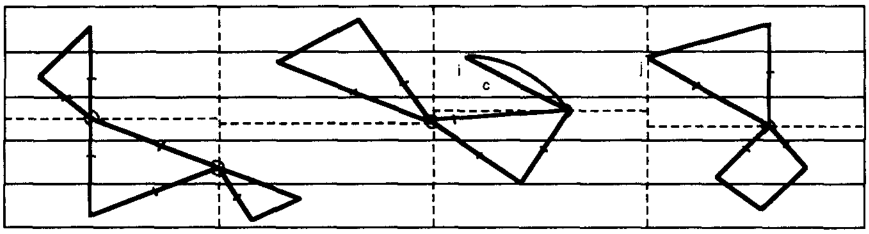

Figure 6. The spanning walk heuristic ($m = 3$)

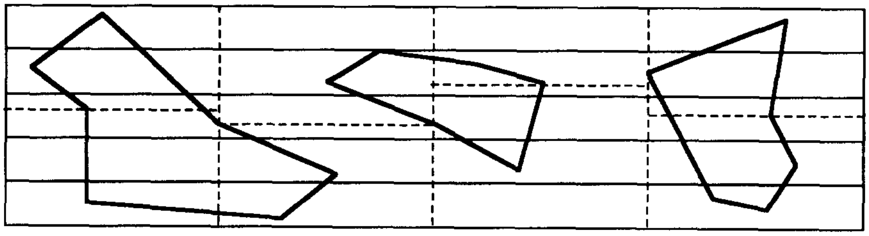

Figure 7. The spanning walk heuristic – Final solution

Assume that the most expensive link in $\bar{A}$ is $a = [i, j]$. Since both i and j are critical points Step 3 applies. Assume further that $\mu_i = d_{tj} - d_{it} > \mu_j = d_{ti} - d_{jt'}$. Then, $\mu_{j*} = \mu_j$, $j^* = j$, and $b = [j, t']$. Removing links a and b leaves vertices i and t' with odd degree, thus link $c = [i, t']$ is added and the situation plotted in Figure 5 is obtained.

Assume next that $a = [i, j]$ (see Figure 5) is selected from $\bar{A}$ since i is not a critical point, Step 2 applies. By removing links a and b, vertices i and f have odd degree and therefore link $c = [i, f]$ is added as shown in Figure 6.

Note that vertice j is no longer a critical point and link c is used twice. Thus the removal of link c would leave i with degree zero.

Table 5
Results for the spanning walk heuristic

Prob. no.	No. of customers	No. of depots	α	k	No. of sub-rectangles	Cost	Time [a] (sec.)
4	10	2	4	2	4	253	–
5	10	3	4	2	4	244	–
6	20	3	6	2	4	509	–
6	20	3	4	3	8	560	–
7	20	4	6	2	4	486	–
7	20	4	4	3	8	580	0.36
10	30	3	6	3	8	589	0.38
10	30	3	4	4	16	536	0.63
11	30	4	6	3	8	567	0.42
12	30	5	6	3	8	587	0.44

[a] CPU time in a UNIVAC 1100. A '–' means that the CPU time has not been registered.

In Figure 6, we have already 3 spanning walks. Next, each spanning walk is transformed into a circuit.

Figure 7 gives a 3 tour solution obtained by applying a procedure identical to that reported in [13].

The spanning walk heuristic is yet in test phase. However, some preliminary results for problems 4, 5, 6, 7, 10, 11 and 12 with different α values are available. These results are reported in Table 5. Nevertheless, confronting Table 5 with Tables 1–4, it seems clear that costs are higher than those previously obtained, even though we cannot be conclusive before additional tests have been carried out.

Table 6
Results for the 3-optimal heuristic adapted to capacitated HPMP

Prob. no.	No. of customers	No. of depots	Capacity bounds		Cost	Time [a] (sec.)
			H	h		
31	10	3	60	20	339	0.31
32	10	3	80	25	553	0.12
33	20	4	100	30	922	0.40
34	20	5	90	30	622	1.60
35	20	4	90	20	1005	0.36
36	30	5	90	30	904	5.98
37	30	5	100	20	581	4.56
38	30	5	100	35	1191	1.63
39	30	4	150	35	535	5.91
40	50	6	140	50	827	24.82
41	50	7	120	40	1104	33.67
42	50	8	100	40	1908	4.54

[a] CPU time in a UNIVAC 1100.

4. The capacitated Hamiltonian *p*-median problem

In practical situations, we may come across the need of considering upper and lower bounds to the capacity of each depot.

Let $f_i \geqslant 0$ denote the demand associated to city i, and h and H lower and upper bounds on depot capacity, respectively.

The formulations given previously may be adjusted to the capacitated problem straightforwardly.

In (P1) we must include the following inequalities:

$$\sum_i t_{is} f_i w_s \leqslant H \quad (s = 1, \ldots, q), \tag{8}$$

$$\sum_i t_{is} f_i w_s \geqslant h \quad (s = 1, \ldots, q), \tag{9}$$

while in (P2) constraints are to be added as follows:

$$\sum_i f_i y_{ik} \leqslant H \quad (k = 1, \ldots, p), \tag{10}$$

$$\sum_i f_i y_{ik} \geqslant h \quad (k = 1, \ldots, p). \tag{11}$$

The 3-optimal heuristic has been adapted to the capacitated problem. An initial feasible solution satisfying capacity constraints is required. A better solution in cost terms becomes effective only if it satisfies the capacity constraints. Otherwise, it is dropped and the 3-optimal search proceeds.

From problems 5, 7, 8, 11, 12 and 15 and different lower and upper bounds, we built test problems 31–42 for the capacitated HPMP. Results are given in Table 6. Comparing Tables 3 and 6, we may observe that solutions with higher costs are attained requiring less computational time. This is completely justified since the number of feasible solutions decreases when capacity constraints are added.

5. Concluding comments

A new routing–location problem has been presented. Computational procedures are in a developing stage and require much further work. However, with the work already done, we are able to solve reasonably large problems.

The best performance was achieved by a combined version of the clustering and 3-optimal heuristics. With respect to the shrinking heuristic, promising preliminary results have been achieved within low computational time. Higher cost solutions have been attained with the spanning walk heuristic, but this method works faster than the 3-optimal one. Thus, it seems appropriate for large scale problems. However, to be conclusive, additional tests must be carried out.

The clustering heuristic can be modified, using different assignment costs and perhaps this will provide better solutions.

Other heuristics may be developed, namely one based upon our formulation (P2).

These are some topics for further research in the HPMP.

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