

Random Threshold Networks with Controllable Activity



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Abstract

In this work we study the dynamics of random threshold networks with both excitatory and inhibitory links. We find the balance between excitatory and inhibitory links to be a key parameter in the dynamics. By varying the fraction of excitatory links F_+ , we are able to control the stable activity A_∞ of the network. We also analyze the influence of the average degree K on A_∞ , and conclude that the dynamics is independent of degree distribution for high K . We develop a mean-field approximation of the dynamics, to provide a practical tool. In the second part of this work, we propose a minimal model of an adaptive threshold network. With it, we are able to control the stable activity of a network, and of individual groups within the same network. This adaptive model can be extended in order to generate networks with controllable activity and specific topologies.

Definitions

- Dynamical state of node i

$$\sigma_i(t+1) = \begin{cases} 1 & \text{if } \sum_{j=1}^n w_{ij} \sigma_j(t) > h \\ 0 & \text{if } \sum_{j=1}^n w_{ij} \sigma_j(t) \leq h \end{cases}$$

- Network activity $A(t) = \frac{1}{n} \sum_i \sigma_i(t)$

- Sensitivity λ

- Chaotic: $\lambda > 1$
- Critical: $\lambda = 1$
- Ordered: $\lambda < 1$

Parameters:

- K : average degree
- F_+ : fraction of links $w_{ij} = +1$
- h : activation threshold
- α : evolutionary parameter

Activity and Criticality

Connections in the brain come in two basic types: excitatory and inhibitory. Here we study the effect of a varying balance between excitatory and inhibitory connections on a simple dynamical model with all-or-nothing firing, the Random Threshold Network (RTN) model.

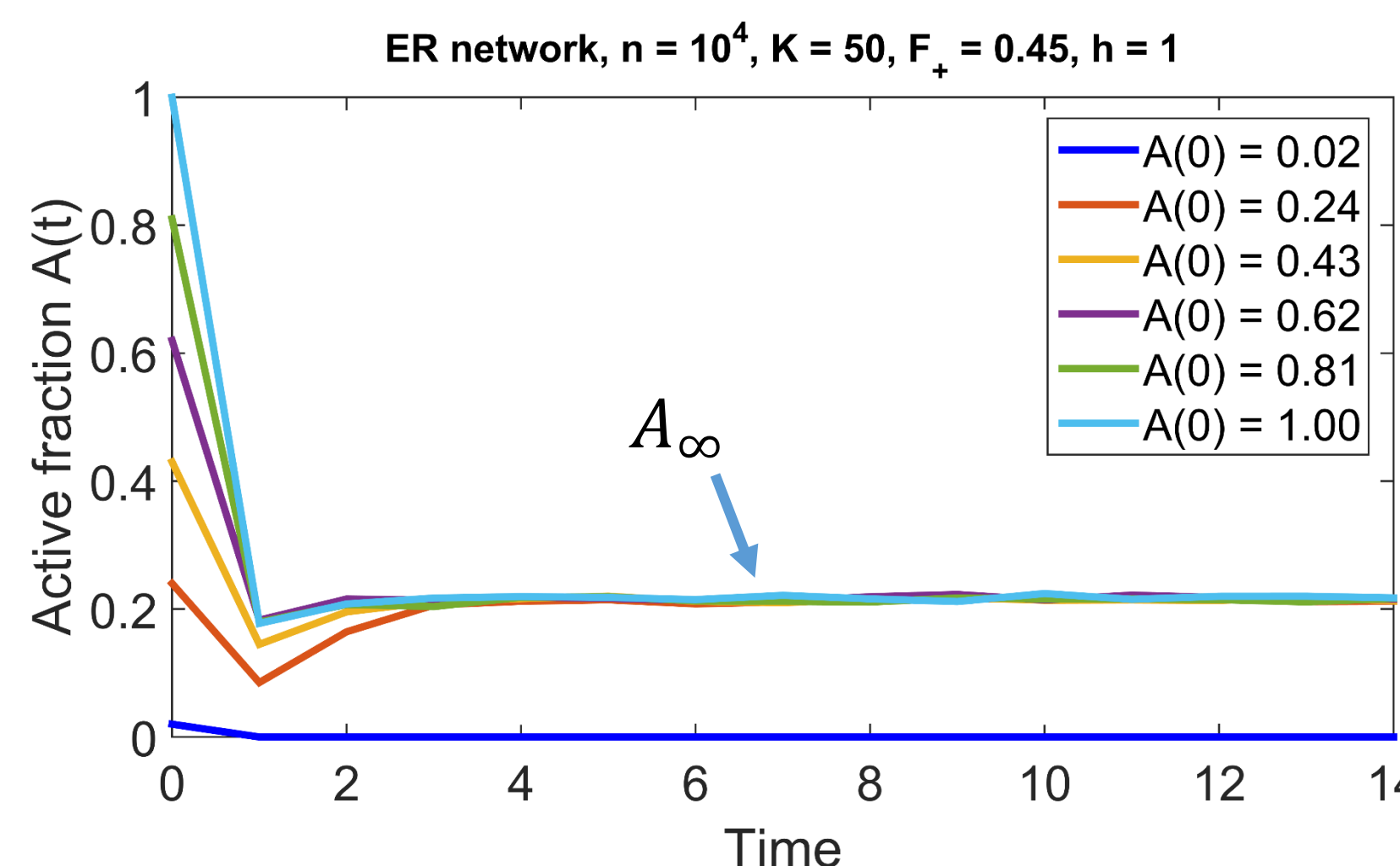


Fig. 1: Example of network activity of a RTN.

The RTN dynamics stabilize quickly around $A(t \rightarrow \infty) = A_\infty$. It doesn't depend on initial activation $A(0)$, provided that $A(0)$ is larger than a (usually low) minimum value. The dynamics is not frozen. This behavior (and the value of A_∞) does not depend on the network topology for high K .

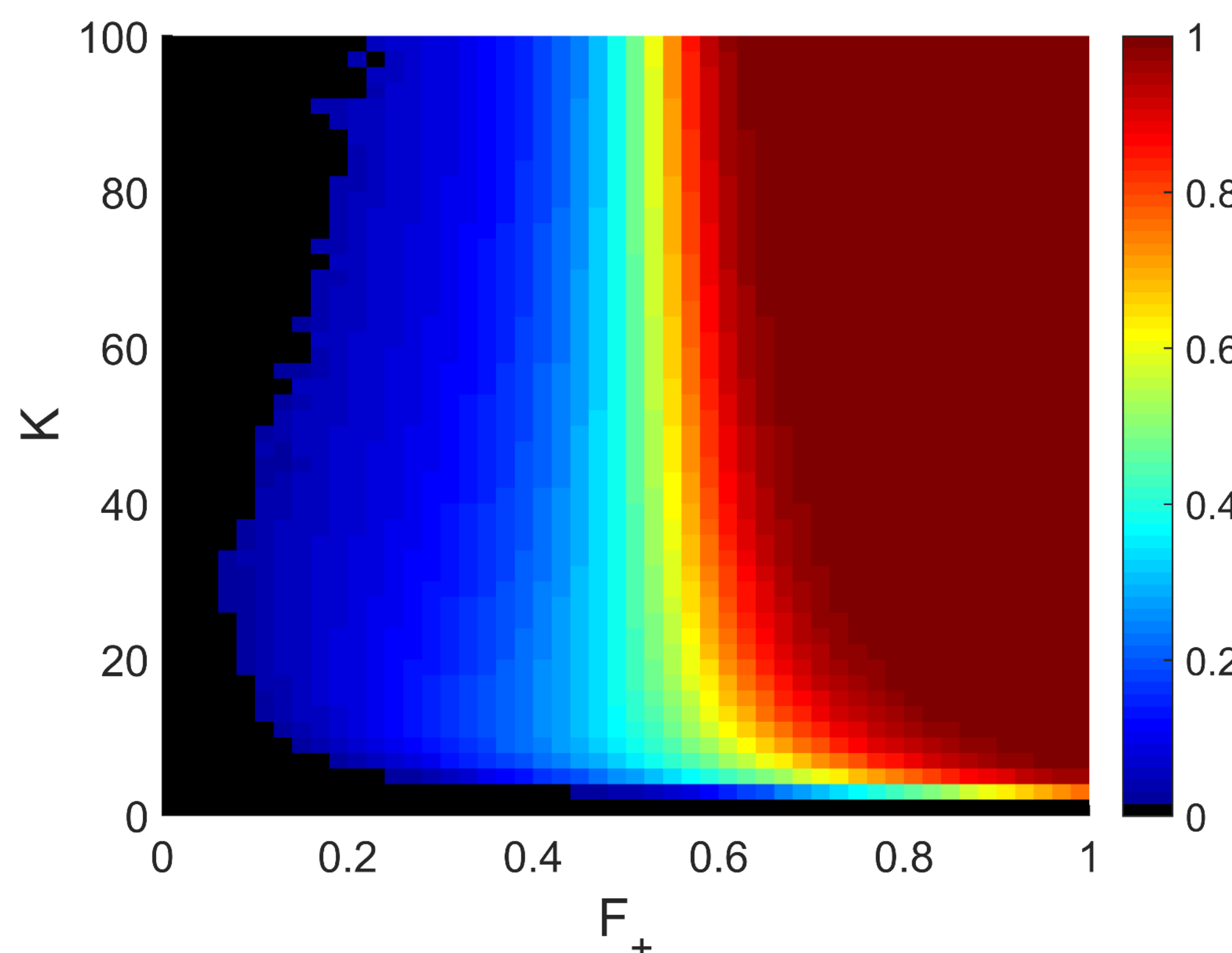


Fig. 2: Average activity A_∞ as a function of K and F_+ , for $h = 0$. In the black region networks can't maintain activity.

By varying F_+ , we are able to control the activity of the network A_∞ within almost the entire range $A_\infty \in [0,1]$. It depends on K , but not on network size.

Another important observable is the sensitivity λ . It measures the perturbation spreading on the dynamics. In the critical state, a perturbation can take a long time to disappear[1,2].

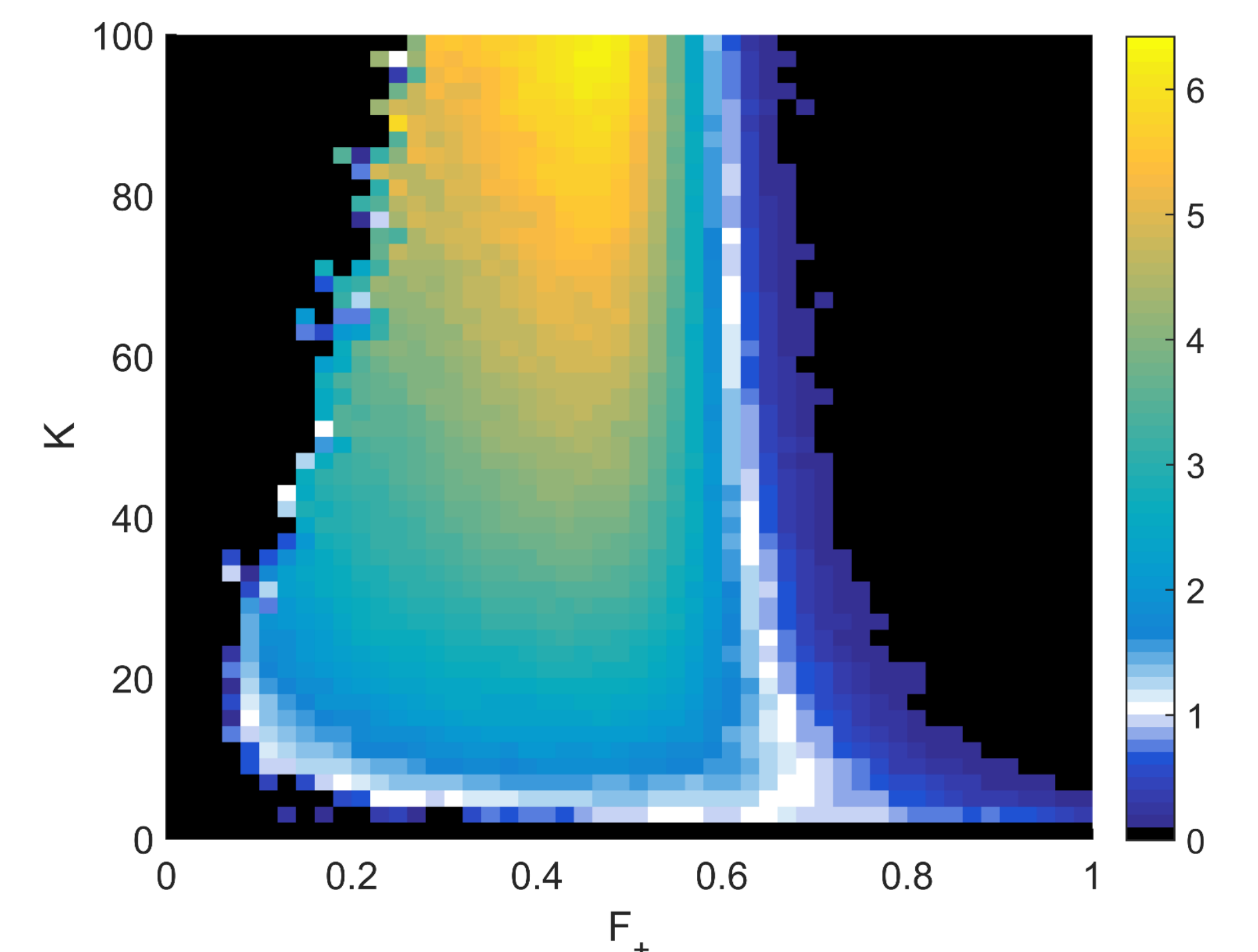


Fig. 3: Network sensitivity λ as a function of K and F_+ for $h = 0$. The white region ($\lambda = 1$) corresponds to the critical state.

Instead of a critical point, we have a critical curve ($\lambda = 1$) that depends non-linearly on A_∞ . For a fixed F_+ or K , this means that activity and sensitivity are not independent.

For a fixed K , the balance F_+ necessary for a desired activity A_∞ can be calculated from

$$F_+ = I_{A_\infty}^{-1} \left(\frac{KA_\infty + h + 3/2}{2}, \frac{KA_\infty - h + 1/2}{2} \right)$$

where $z = I_X^{-1}(a, b)$ is the inverse incomplete beta function. The sensitivity λ is given by

$$\lambda = K \sum_{m=h}^{K-1} \binom{K-1}{m} A_\infty^m (1-A_\infty)^{K-1-m} F_+^{(m+h)/2} (1-F_+)^{(m-h)/2} \gamma_h(m, F_+)$$

where $\gamma_h(m, F_+) = \binom{m}{(m+h)/2} F_+$ if $m+h$ is even, and $\gamma_h(m, F_+) = \binom{m}{(m+h+1)/2} \sqrt{F_+(1-F_+)}$ otherwise.

Topological Evolution

During development, the brain loses synapses in a process called synaptic pruning. Inspired by this, we propose a minimal adaptive model capable of controlling the activity of a RTN. The algorithm is the following:

- Run RTN dynamics for τ timesteps
- Choose a node i & calculate $\langle \sigma_i \rangle = \sum_t \sigma_i(t) / \tau$
- If $\langle \sigma_i \rangle > \alpha$, remove a positive in-link. Otherwise, remove a negative in-link

where $\alpha \in [0,1]$ is an evolutionary parameter. By setting the number of times the algorithm is run we can set the network degree K .

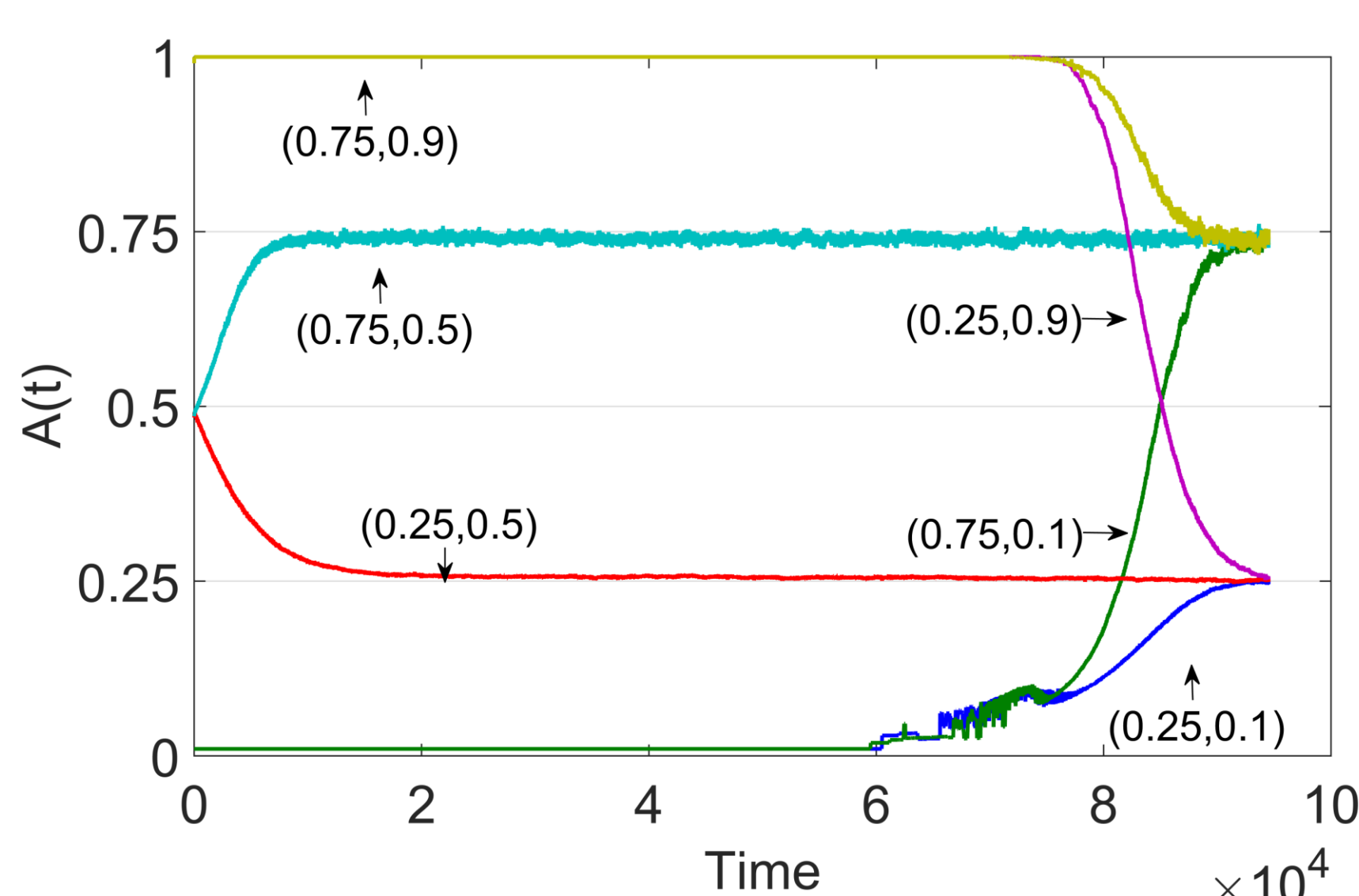


Fig. 4: Activity of evolving networks. The arrows indicate the parameters (α , F_+)

The algorithm is able to drive the network towards $A_\infty = \alpha$. This effect is robust, provided the initial network is densely connected. Starting from a fully connected network, the degree distribution of the evolved network is Poissonian. This is a minimal model, and can be extended in many ways. One is to give each node i a particular α_i instead of a single α for all nodes.

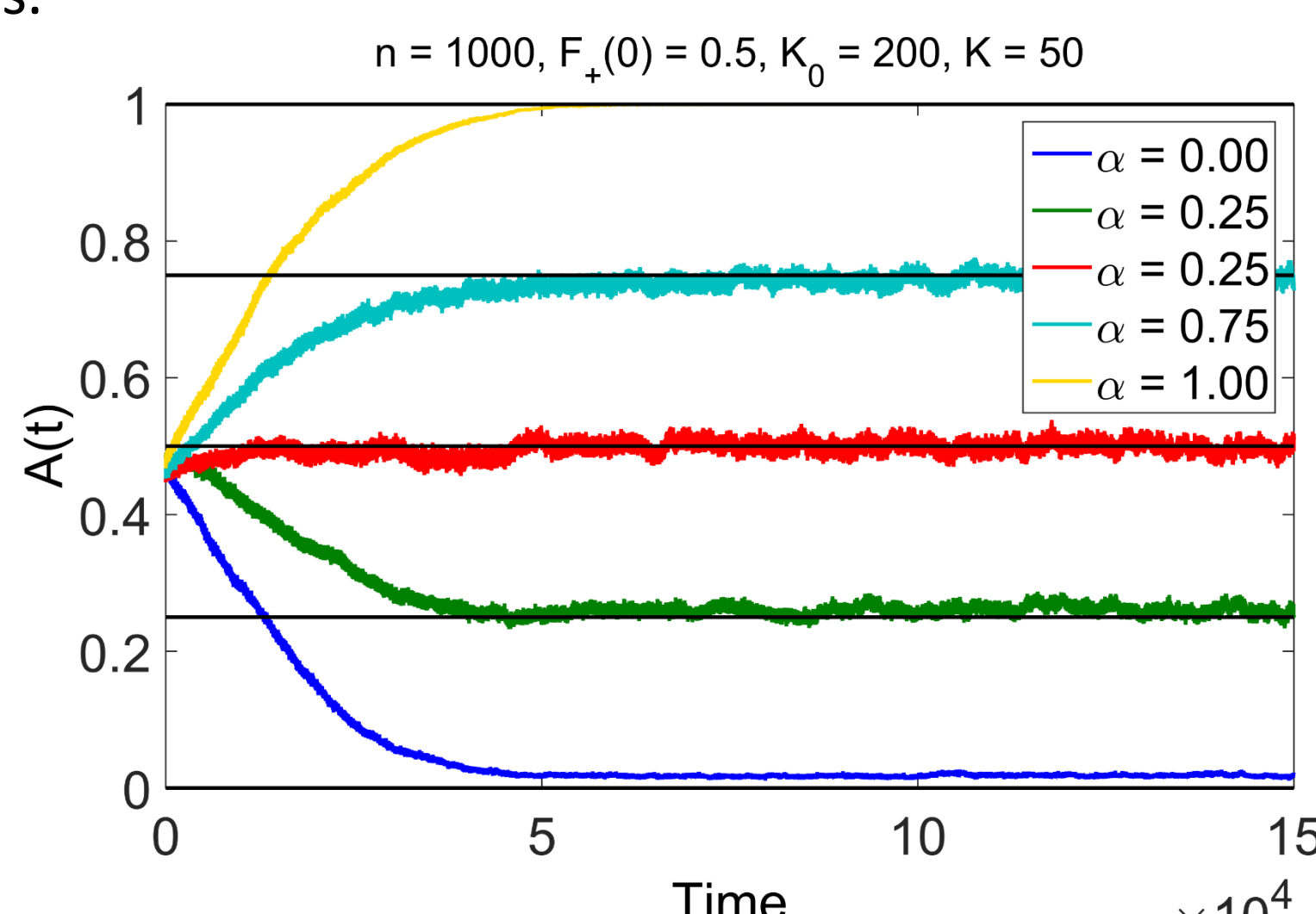


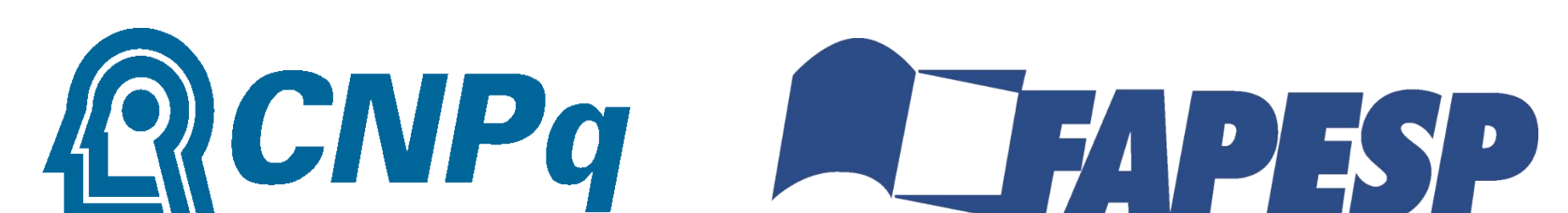
Fig. 5: Network activity for groups with different $\alpha_i \in \{0, 0.25, 0.5, 0.75, 1\}$ within the same network.

The evolution still drives $A(t) \rightarrow \alpha_i$ for every group of nodes with evolutionary parameter α_i . In other words, it is able to produce a topology where nodes with different activity levels coexist. Many other extensions are possible. For instance, a preferential depletion rule can be used to drive the network to a scale-free topology with degree distribution exponent 5/3 [5].

Conclusion

At the core of our model is the idea that the collective dynamics of RTNs is robust, and that it can be controlled by balancing excitation and inhibition. By varying this balance, we can obtain networks with activity in almost the entire range $0 \leq A_\infty \leq 1$. Activity and sensitivity are not independent in RTNs, and a critical network can only be obtained for specific combinations of parameters. We then proposed a minimal model of an adaptive threshold network. In this model we are able to control A_∞ with a single parameter α , driving the network towards $A_\infty = \alpha$. This evolution is robust, and can be extended to control the activity of distinct groups within the same network. The model can also be extended in other ways in order to generate specific topologies and dynamics.

Acknowledgements



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