NETWORK SCIENCE OF ONLINE INTERACTIONS

Chapter 6: Communities

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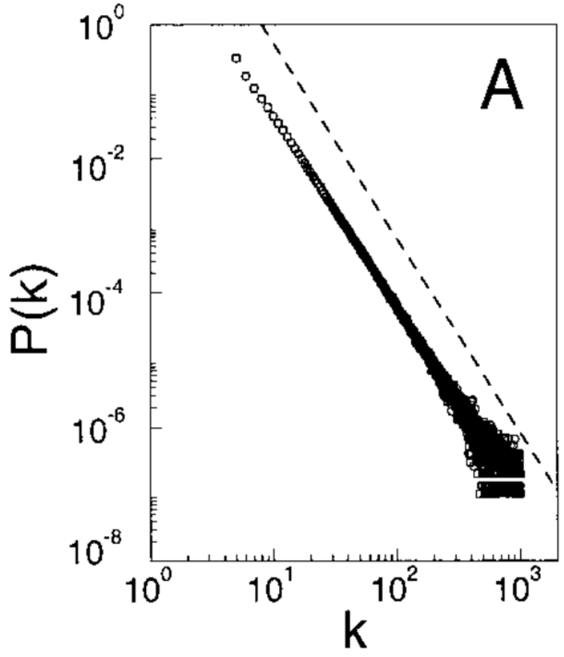
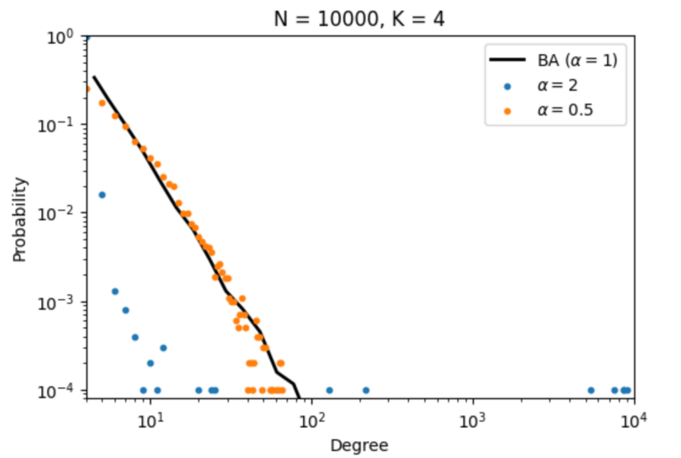
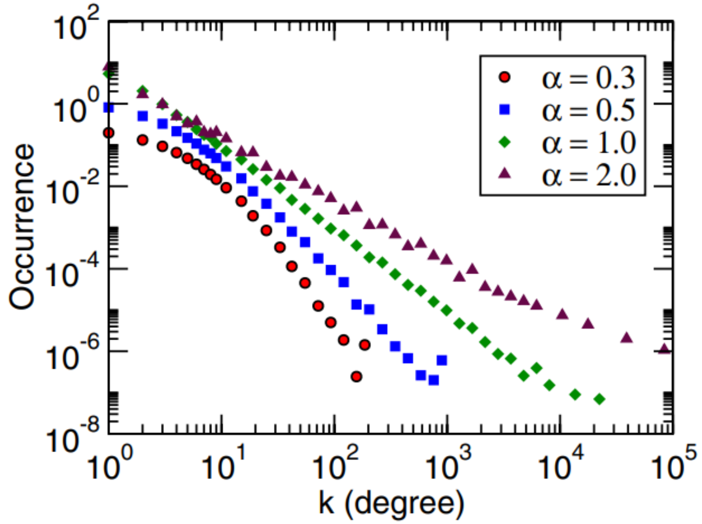
SUMMARY

- Many models focusing on emulating certain properties
  - Degree distribution, clustering, triadic closure, etc
- No single "best model"
- Preferential attachment is a key mechanism
  - Can create heavy-tailed distributions
  - If unbalanced, can create hyper-concentrated hubs
- Variations of PA models can create a variety of degree distributions







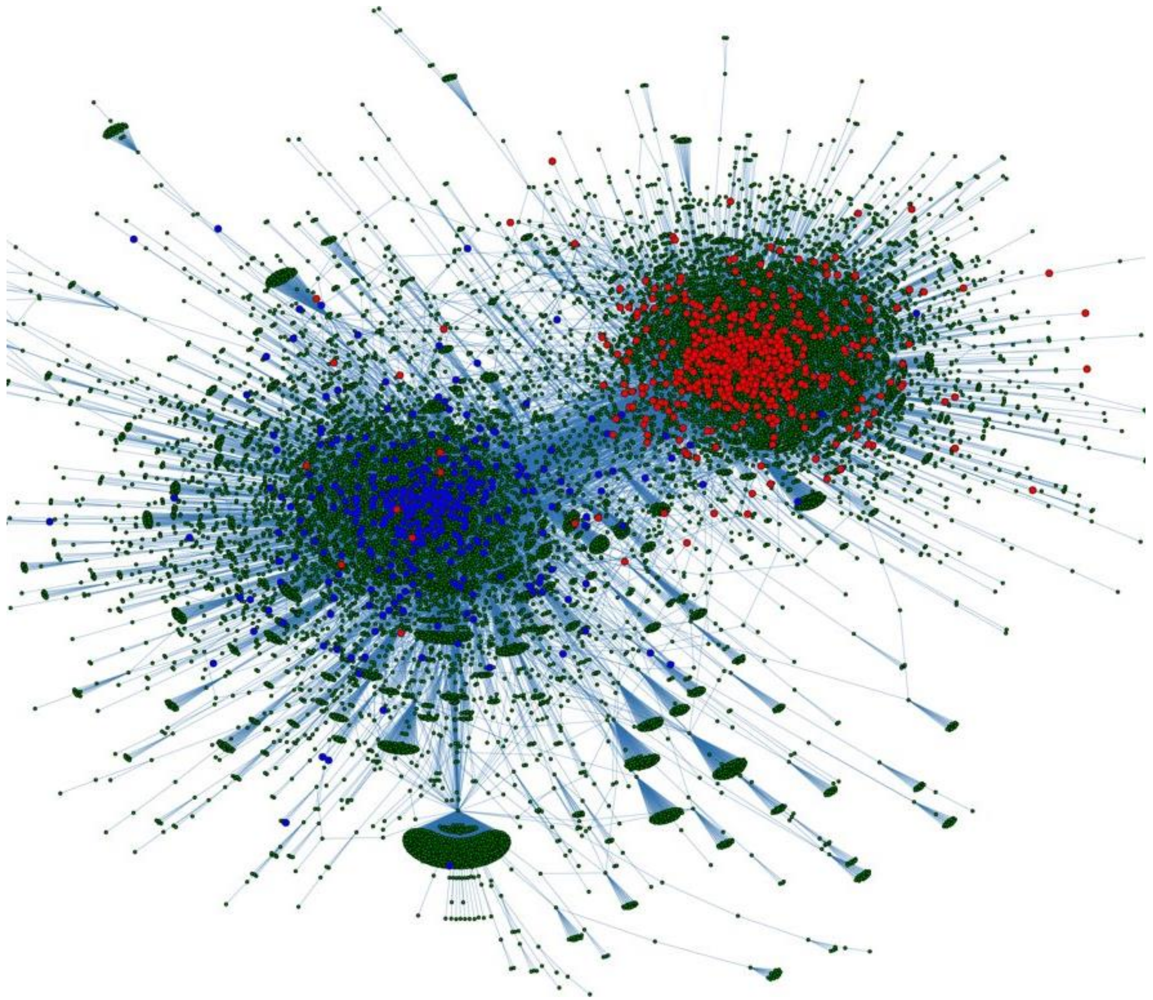
INTRODUCTION

- Community detection is one of the main fields of network theory

- Many reasons:
  - Uncover network structure
  - Identify node affiliation
  - Find missing links ↔ predict links

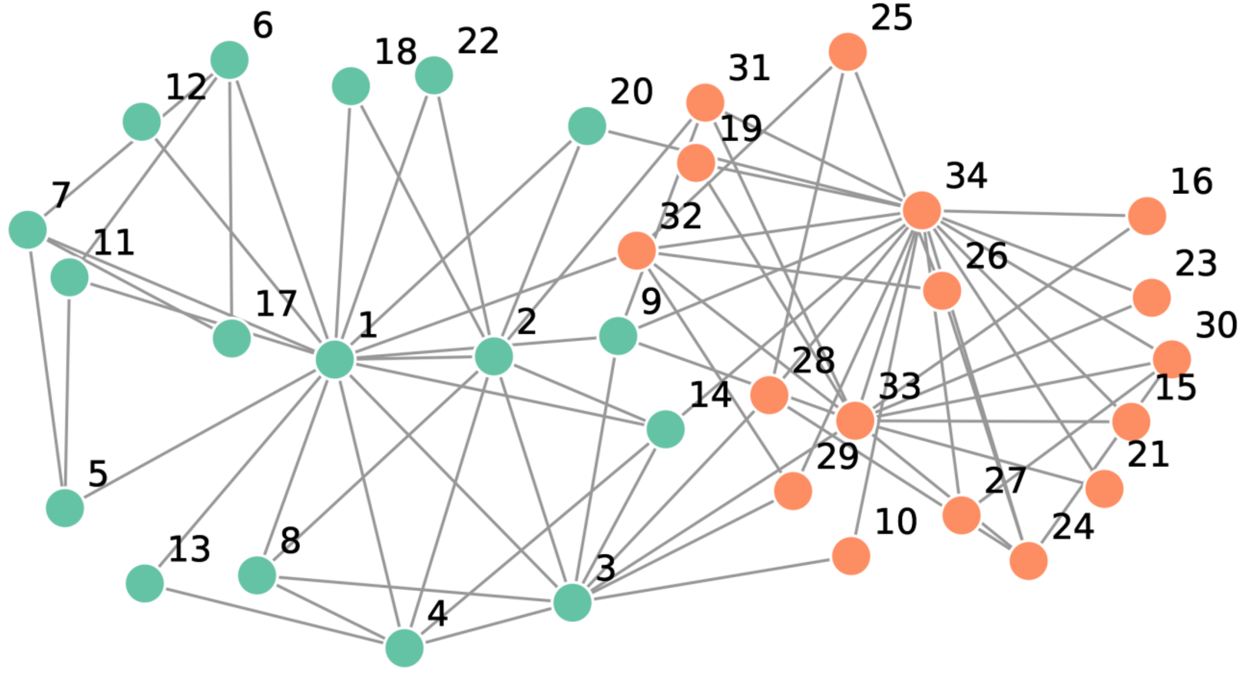
- Different algorithms for different definitions of a community

- Two categories
  - Descriptive algorithms
  - Inferential algorithms



INTRODUCTION

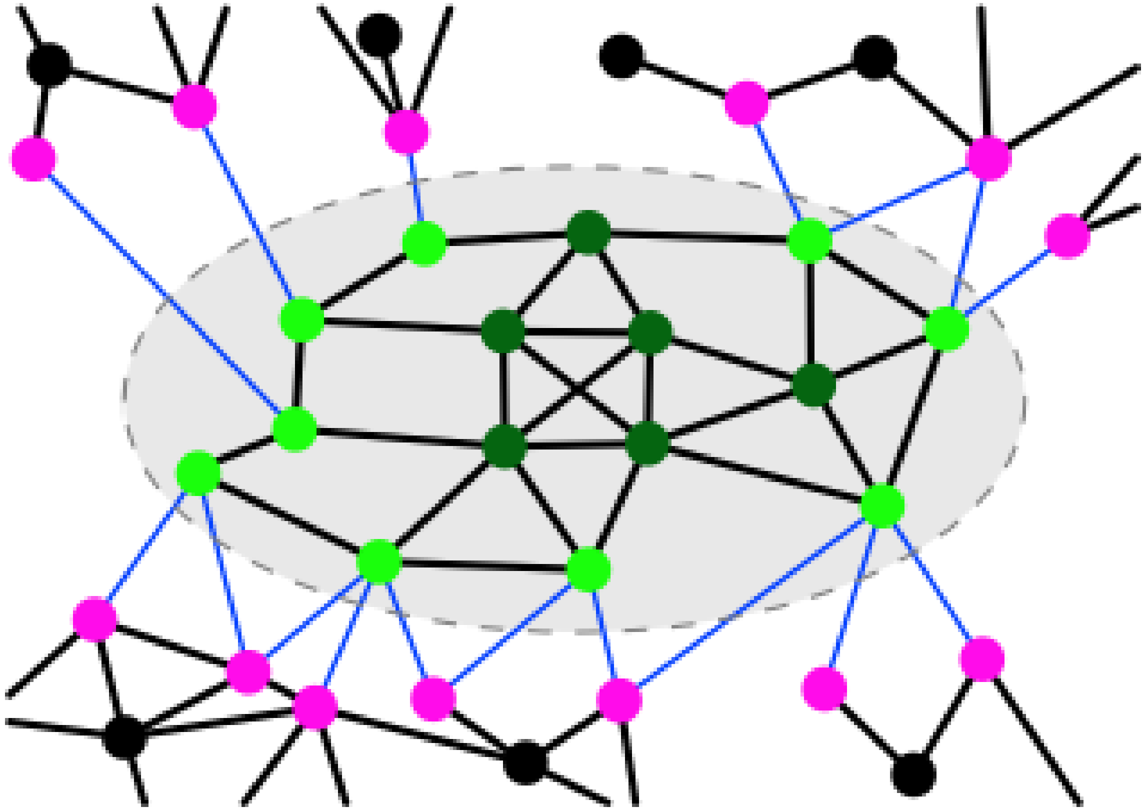
- Basic example: Zachary's Karate club
  - 34 individuals
  - Mapped external relationships
  - Disagreement between 2 instructors
  - Break in two groups
  - Standard test of community detection methods
    - Find two communities
    - Assign nodes to the correct community



5.1 BASIC DEFINITIONS

- Community variables
  - Node degree
    - Internal links k_i^{int} (black)
    - External links k_i^{ext} (blue)
  - Internal links L_C, internal nodes N_C
  - Internal link density $\delta_C^{int} = 2L_C/N_C(N_C-1)$
  - Community degree $k_C = \sum_{j \in C}\left(k_j^{int} + k_j^{ext}\right)$

- Valid for undirected, unweighted networks
  - Undirected, weighted: degree → strength
  - Directed: split in in- and out-links, harder to interpret

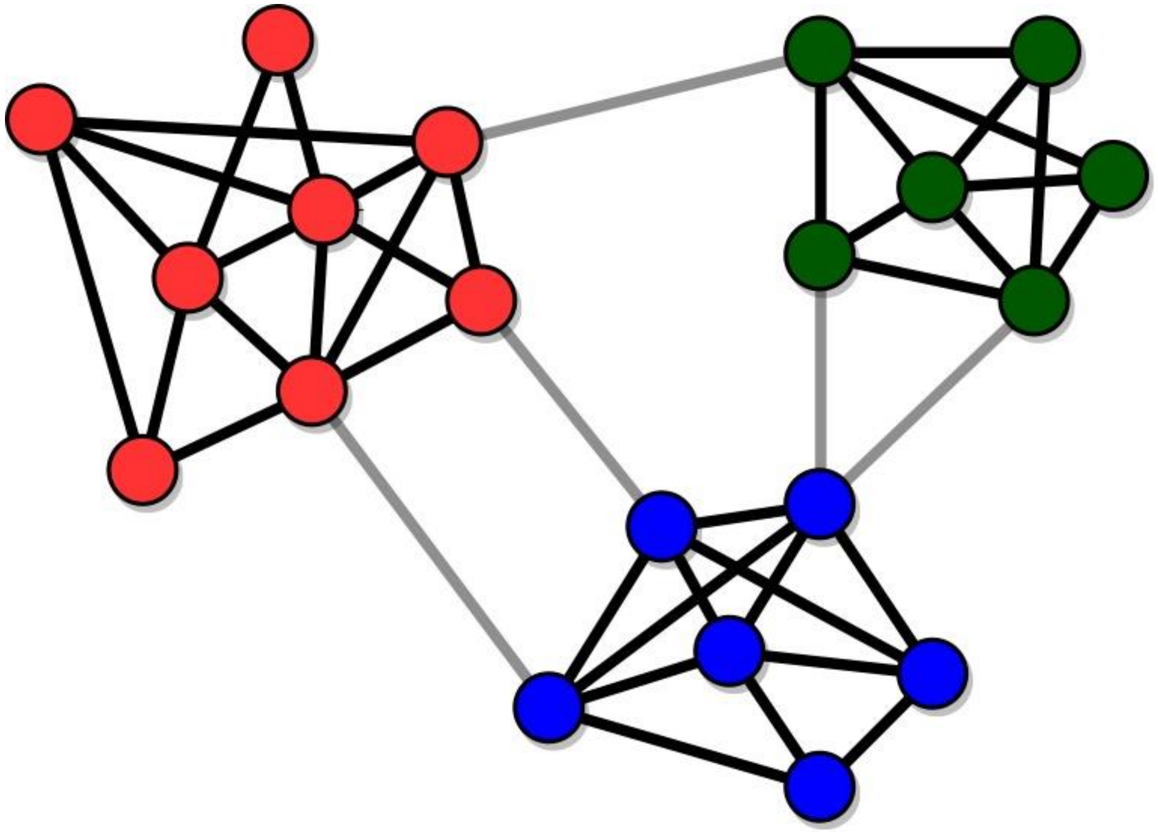


5.1 BASIC DEFINITIONS

- Intuitive idea:
  - Many internal links (cohesion)
  - Few external links (separation)

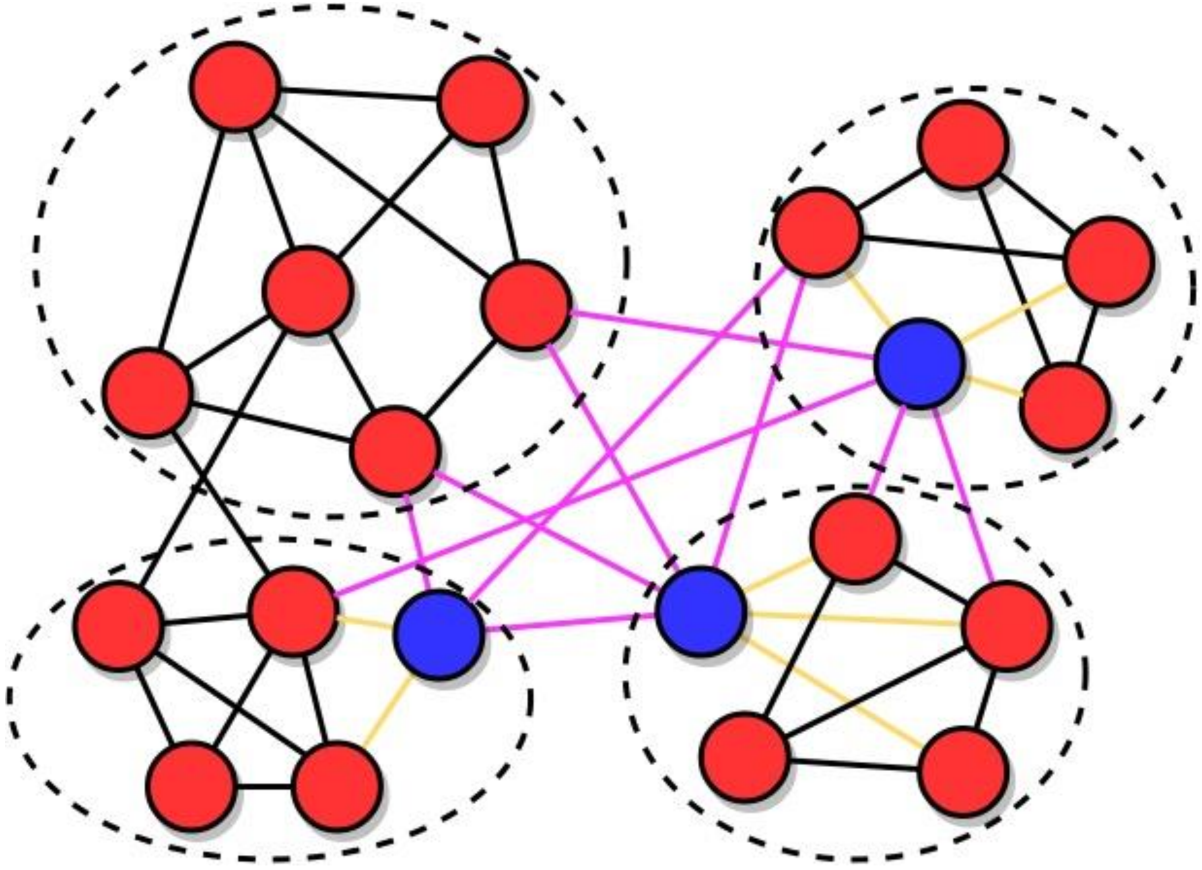
- A community should have more internal than external links

- Definition #1
  - **Strong community**: subnetwork where $k_i^{int} > k_i^{out}$ for *each node*
  - **Weak community**: $\sum_i k_i^{int} > \sum_i k_i^{out}$
  - Problem: compares the community to the entire network



5.1 BASIC DEFINITIONS

- Definition #2
  - Compares communities to *other communities*
  - Strong community: internal degree of each node higher than to any other single community (red)
  - Weak community: sum of internal degrees greater than sum of external to any other community (red + blue)
- Current definitions: counting links
  - Bias against small communities
- Alternative: compare link probabilities
  - Requires a network model
  - Inferential methods (more later)

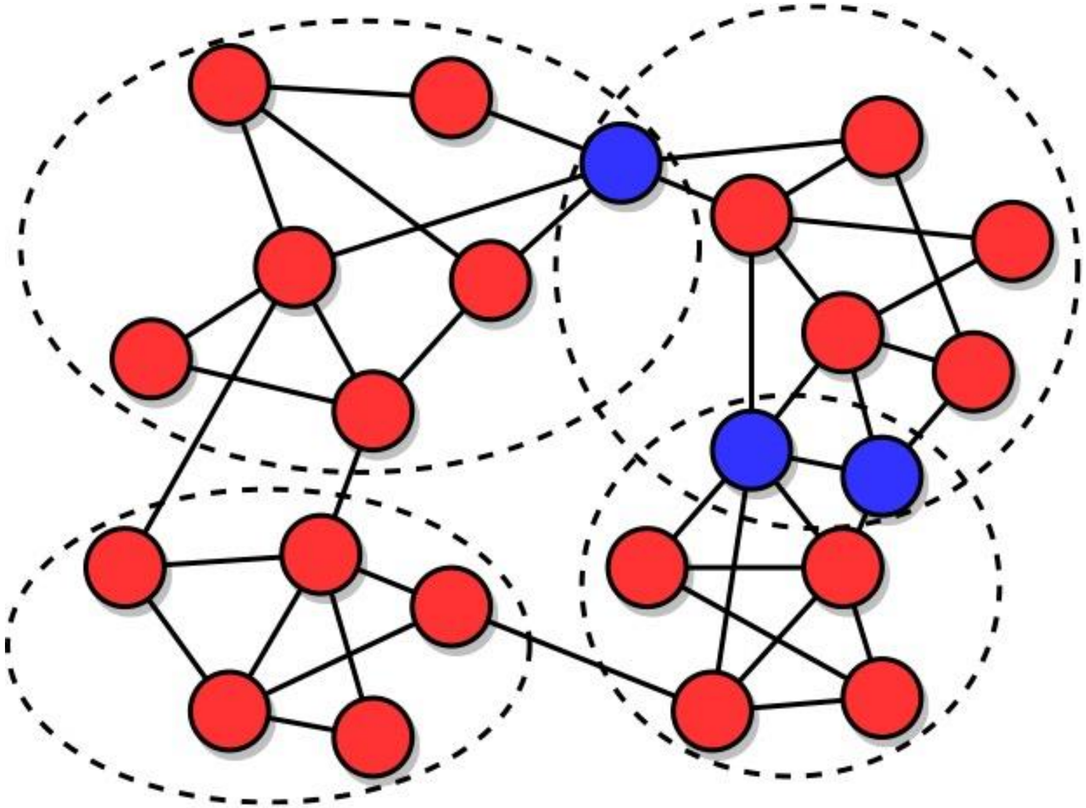


5.1 BASIC DEFINITIONS

- Communities can be
  - Non-overlapping: **partition**
  - Overlapping: **cover**

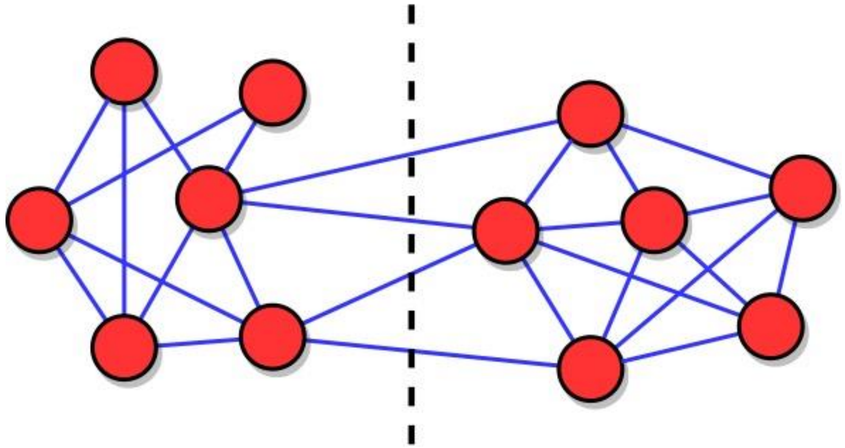
- Number of communities
  - Number of possible partitions grows super-exponentially (Bell's number)
  - $N = 15 \rightarrow$ 1.3 billion possible partitions
  - Number of possible covers is even worse
  - Need for a heuristic algorithm to detect communities

- Graph partitioning is well-studied
  - Community detection in networks
  - Task parallelization in computer science



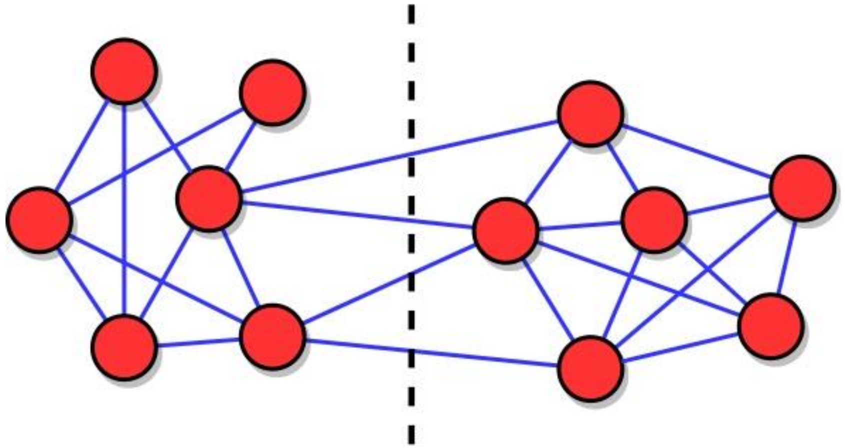
6.2 RELATED PROBLEMS

- Network partitioning with fixed group size
  - **Cut size**: number of links between partitions
  - Good partitions: minimum cut size
  - **Graph bisection**: two partitions with equal size
    - How to do it?
- **Kerninghan-Lin** algorithm
  - The idea: minimize cut size
  - The algorithm
    1. Start from a random bisection
    2. For each pair of nodes from the two groups, compute the swap that would result in the largest decrease in cut size
    3. Swap those nodes and **lock** them in place
    4. Repeat from 2 until cut size cannot be decreased



6.2 RELATED PROBLEMS

- **Kerninghan-Lin** algorithm
  - Greedy algorithm: always tries to minimize/maximize by step
  - May get stuck in local minima/maxima
    - Variants implement random changes to avoid this problem
  - Depends on initial condition
    - Run many in parallel and choose the one with lowest cut size
  - Widely used as post-processing to improve partitioning from other methods (e.g. expert knowledge)
- Limitations of partitioning
  - Partitioning maximizes separation, not internal link density: **not necessarily good communities**
  - Requires giving the number of communities

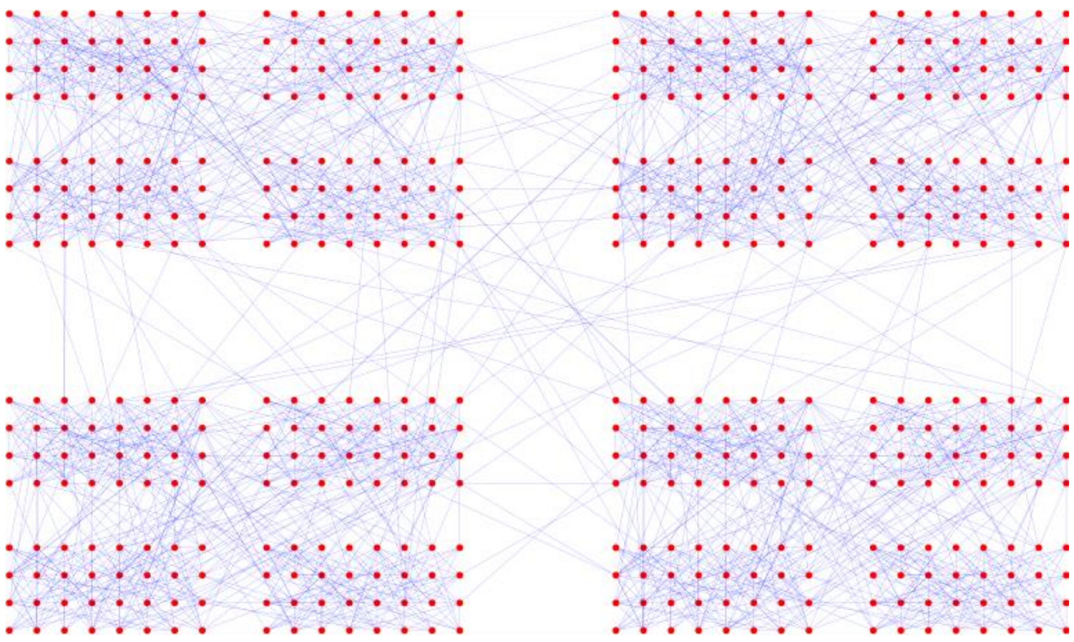


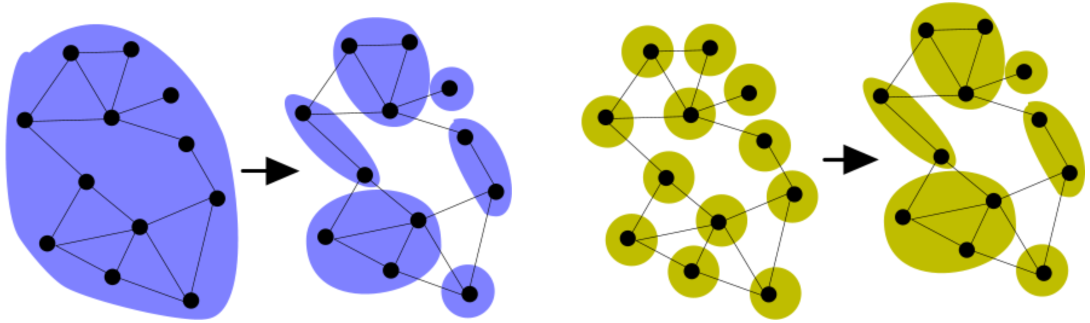
```
partition = nx.community.kernighan_lin_bisection(G)
```

6.2 RELATED PROBLEMS

- Partitions can be hierarchical
  - Subdivisions in companies
  - Classes in school
  - How to detected them?

- **Hierarchical clustering**
  - Main ingredient: **similarity measure**
  - Approaches:
    - Agglomerative hierarchical clustering
    - Divisive hierarchical clustering





6.2 RELATED PROBLEMS

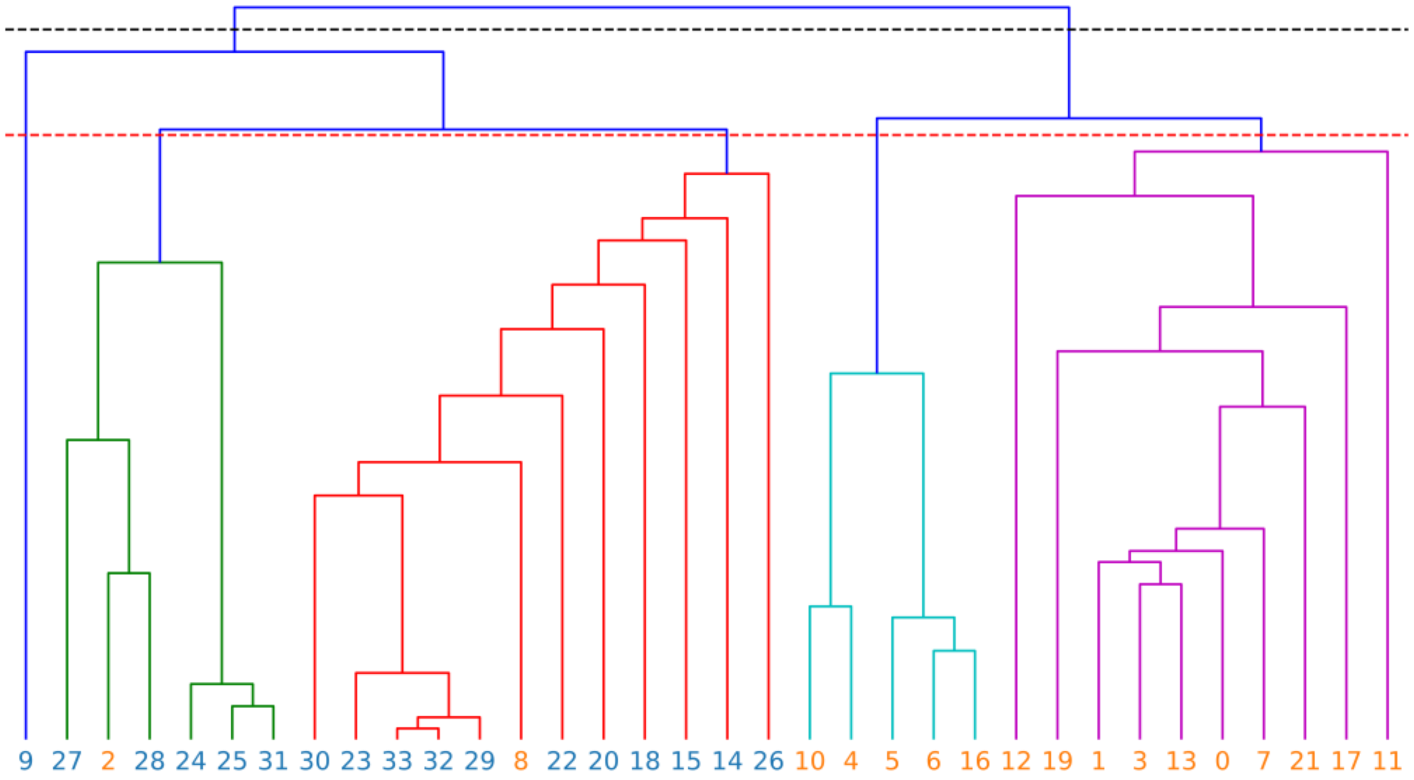
- Example similarity: **structural equivalence**
  - Idea: nodes are similar if their neighbours are similar

$$S_{ij}^{SE} = \frac{\text{number of neighbors shared by } i \text{ and } j}{\text{total number of nodes neighboring only } i\text{, only } j\text{, or both}}$$

- How do define similarity between *partitions*?
  - **Single linkage**: take the maximum pairwise similarity $S_{G_1,G_2} = \max_{i,j} S_{i,j}$
  - **Complete linkage**: take the minimum pairwise similarity $S_{G_1,G_2} = \min_{i,j} S_{i,j}$
  - **Average linkage**: take the average pairwise similarity $S_{G_1,G_2} = \langle S_{ij} \rangle_{i,j}$

6.2 RELATED PROBLEMS

- Result of clustering: **dendrogram**
  - Summary of similarity between nodes
  - Shows the partitioning for each value of number of partitions (horizontal cuts)
  - Each partition includes the partitions lower in the hierarchy

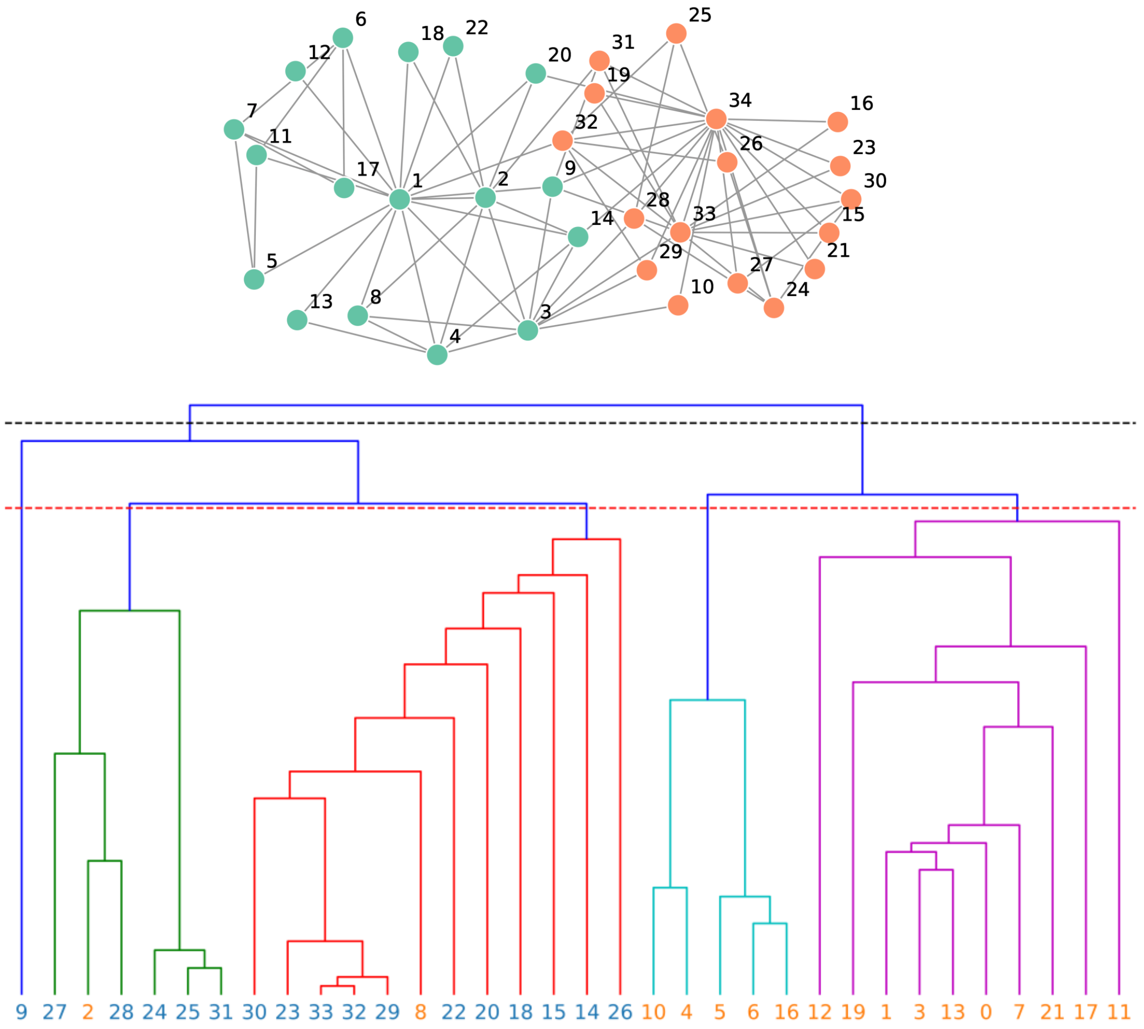


6.2 RELATED PROBLEMS

- Result of clustering: **dendrogram**

- Benefits
  - Clear picture of node affiliation at different levels
  - Different view from the graph

- Caveats
  - No criteria to select ideal partitioning
  - Depends heavily on similarity measure
  - Rather useless for large networks (slow, too many lines)

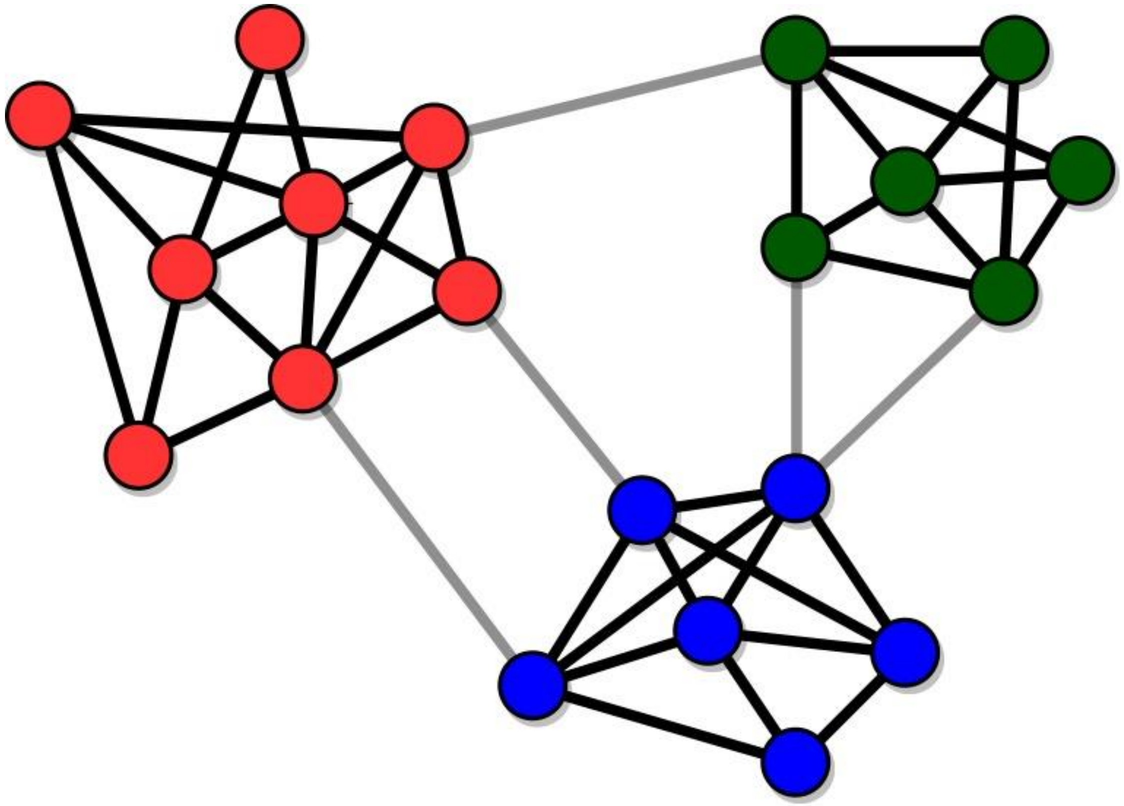


6.3 COMMUNITY DETECTION

- Many different methods

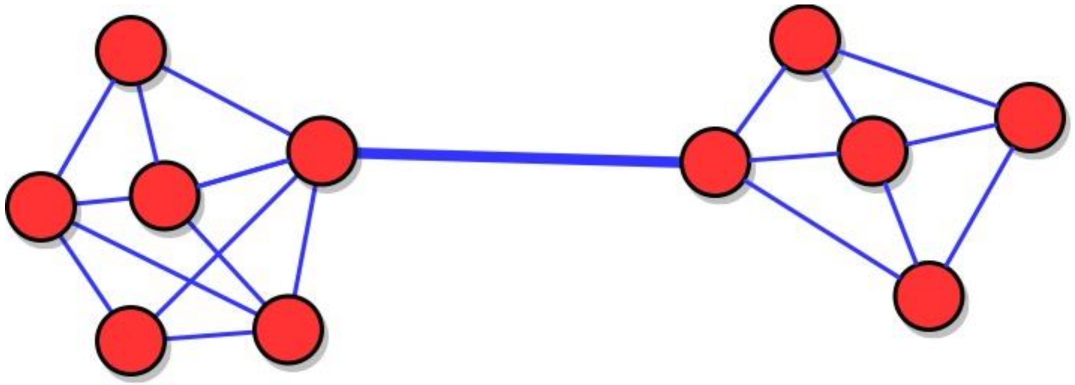
- Today
  - Bridge removal
  - Modularity maximization

- Wednesday:
  - Stochastic block modelling
  - Community validation



6.3 COMMUNITY DETECTION

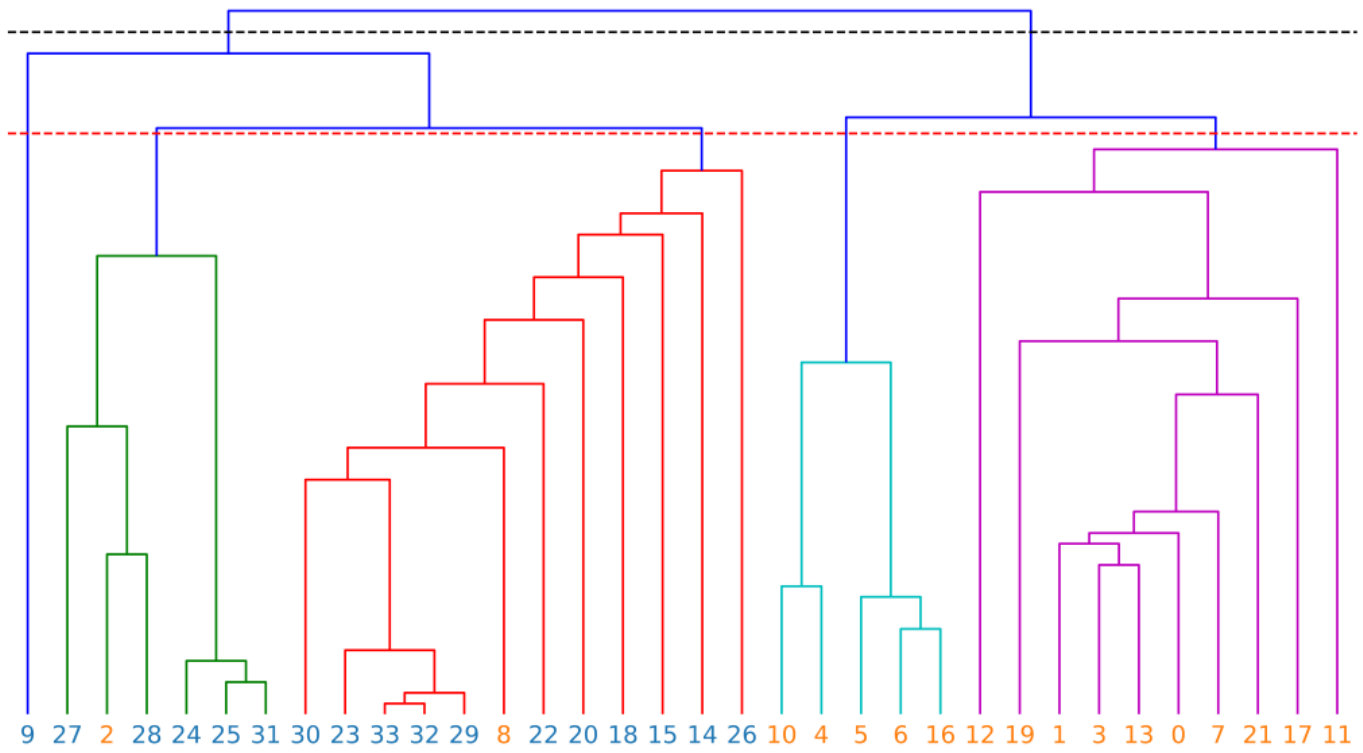
- **Bridge removal**
  - The idea: detects bridges between clusters, removes them until clusters are disconnected
  - Those clusters are communities
  - What is a good bridge metric?



- **Girvan-Newman** algorithm
  - Measure: **link betweenness centrality**
  - Algorithm
    1. Compute betweenness for all links
    2. Removes the link with largest betweenness, recomputes betweenness for all links
    3. Iterate until all links are removed

6.3 COMMUNITY DETECTION

- **Girvan-Newman:** the bad
  - Recalculate betweenness for *all links at each step*: very slow
  - Not feasible for large networks (say, > 10000 nodes)
- **Girvan-Newman:** the good
  - Dendogram
  - If strong community structure, nodes quickly get disconnected
  - Faster variants
    - Computing similarity over a sample
    - Faster similarity than betweenness




```
partition = nx.community.girvan_newman(G)
```

6.3 COMMUNITY DETECTION

- Method finds a partition: how good is it?
  - Quality function
  - Assumption: random networks have no communities
  - Must distinguish between real communities and random fluctuations

- Most famous quality function: **modularity**
  - The idea: compares community structure against randomized networks with equal degree
  - For each community, computes the **difference in number of internal links** between the network and the ensemble of random networks
  - Value $Q \in [-0.5,1]$

Modularity and community structure in networks

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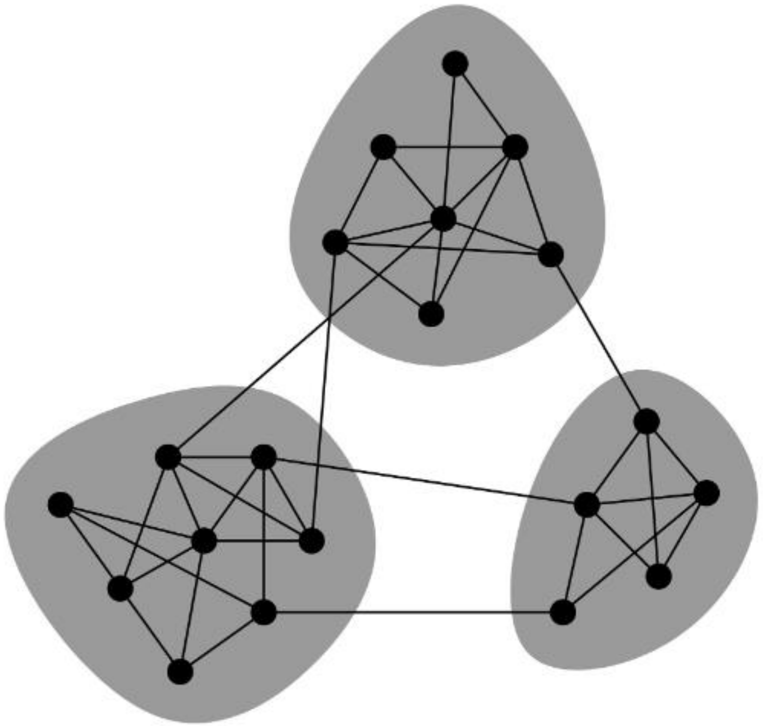
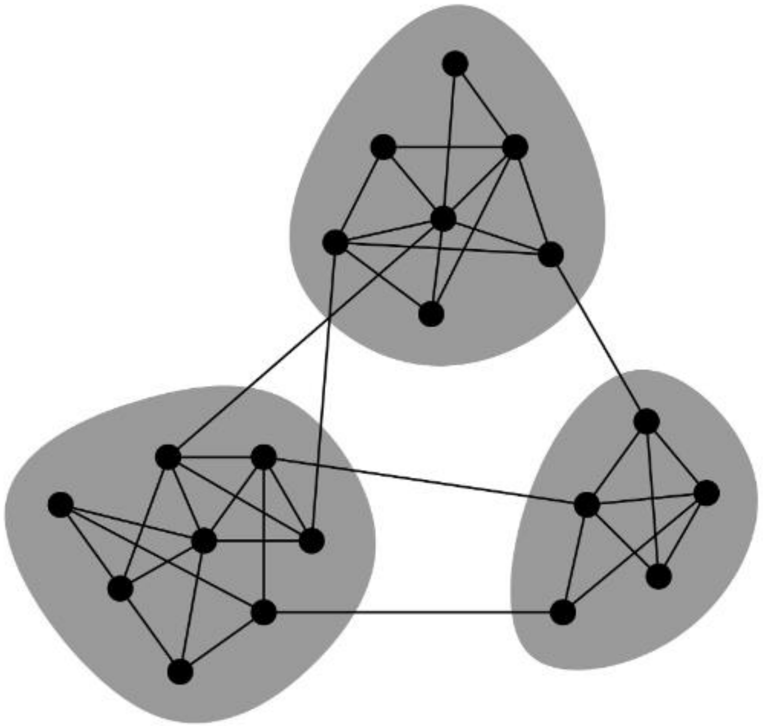


6.3 COMMUNITY DETECTION

- Modularity definition:

$$Q = \frac{1}{L} \sum_C \left(L_C - \frac{k_C^2}{4L} \right)$$

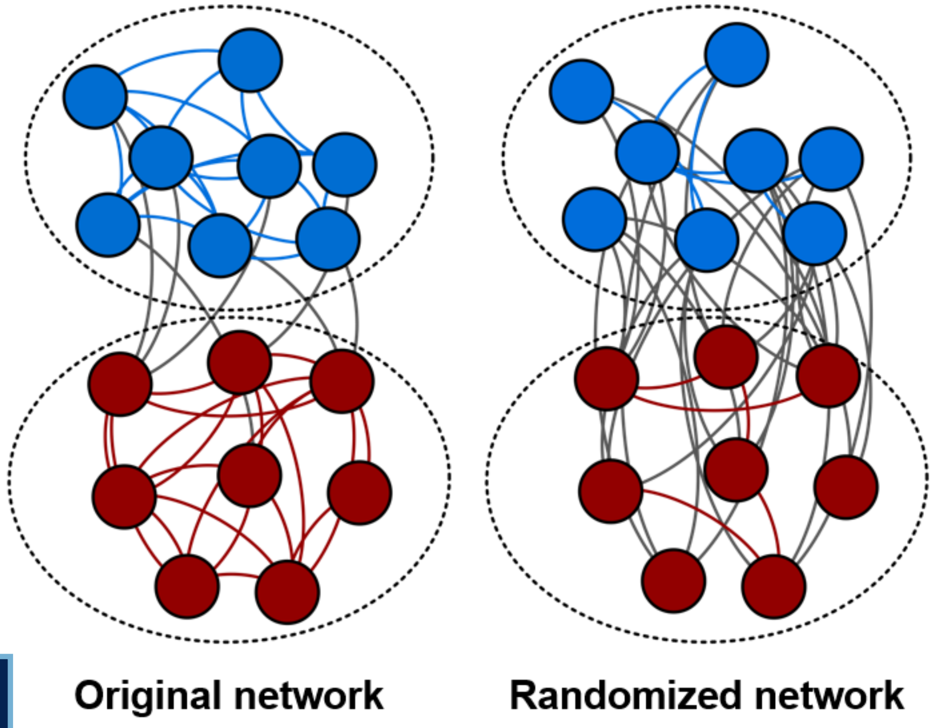
- where
  - L: number of links in the network
  - L_C: number of internal links in community C
  - k_C: degree of community C
  - $k_C^2/4L$: expected number of internal links in community C



6.3 COMMUNITY DETECTION

- Explaining $k_C^2/4L$:
  - Random links by pairing stubs (configuration model)
  - Total number of stubs in C is k_C
  - Probability of selecting a stub is $k_C/2L$
  - Probability of link is $p_C = k_C^2/4L^2$
  - Expected number of internal links is $Lp_C = k_C^2/4L$
- Properties of Q
  - $Q > 0$: some structure
  - $Q = 0$: no structure
  - $Q < 0$ if particularly bad partitioning

$$Q = \frac{1}{L}\sum_C \left(L_C - \frac{k_C^2}{4L}\right)$$




```
modularity = nx.community.quality.modularity(G,partition)
```

6.3 COMMUNITY DETECTION

- Modularity has straightforward extensions
- Directed:

$$Q_d = \frac{1}{L} \sum_C \left(L_C - \frac{k_C^{in} k_C^{out}}{L} \right)$$

- Weighted:

$$Q_w = \frac{1}{W} \sum_C \left(W_C - \frac{s_C^2}{4W} \right)$$

- Directed and weighted:

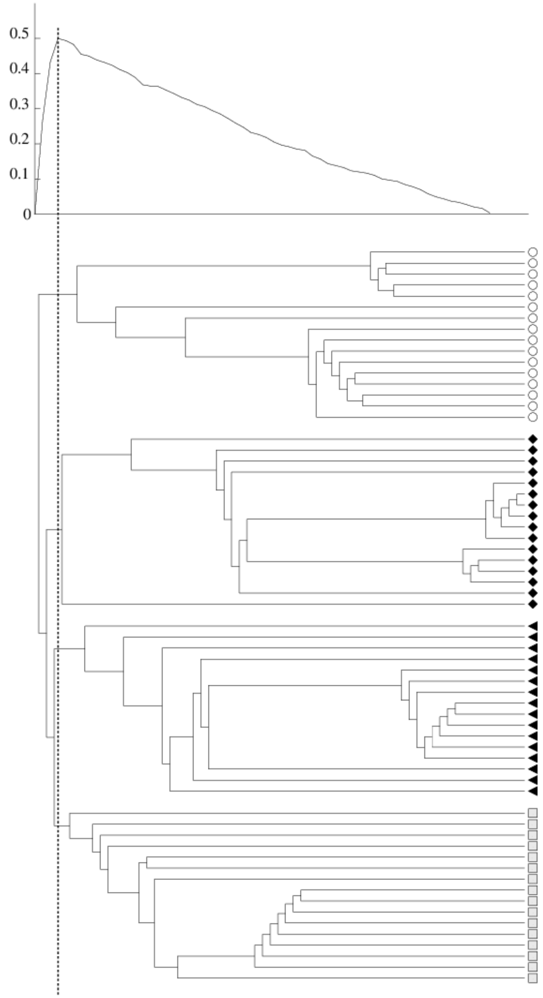
$$Q_{dw} = \frac{1}{W} \sum_C \left(W_C - \frac{s_C^{in} s_C^{out}}{W} \right)$$

6.3 COMMUNITY DETECTION

- Modularity maximization
  - Finding partitioning that maximizes Q

- **Newman's greedy algorithm**
  1. Each node in its community (agglomerative)
  2. Merge nodes that yield the largest increase in Q
  3. Continue until single community
  4. Pick partition with largest Q

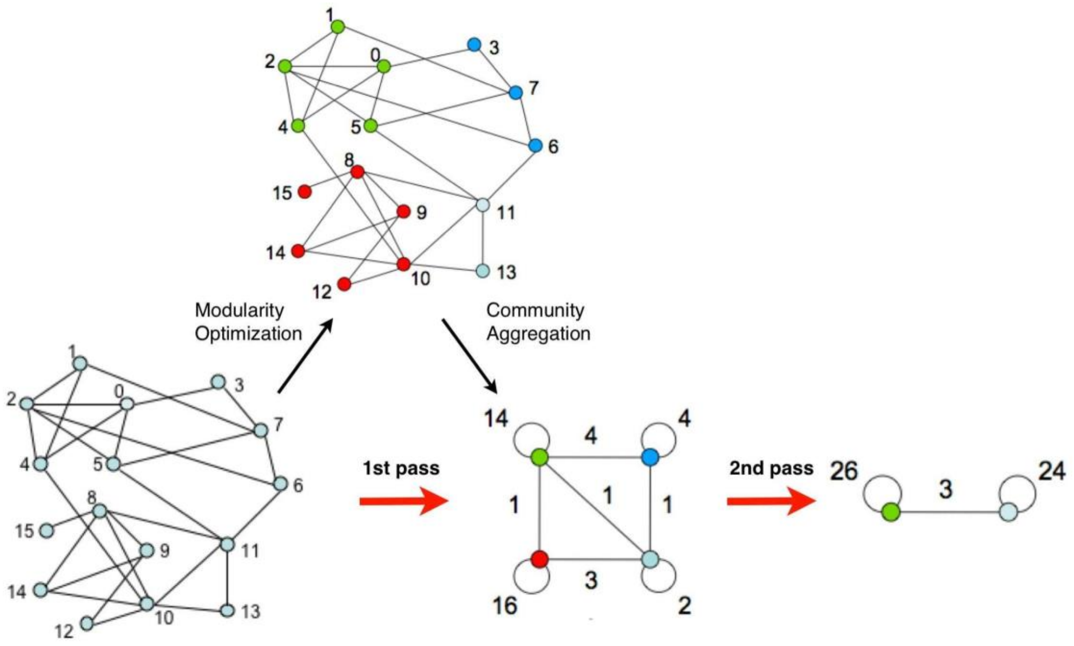
```
partition = nx.community.greedy_modularity_communities(G)
```



pip install python-louvain

6.3 COMMUNITY DETECTION

- Newman's greedy algorithm limitations
  - greedy: gets stuck on local minima
  - tends to generate unbalanced networks
  - Slow
- Standard modularity algorithm: **Louvain algorithm**
  - Similar to Newman's
  - **Very fast**
  - In python




```
#pip install python-Louvain
partition = community.community_louvain.best_partition(G)
```

6.3 COMMUNITY DETECTION

- Modularity maximization is widely used
- Has a few problems
  - **Resolution limit**: small communities are undetected
  - Fails the random network test
- Solution: stochastic blockmodels

SUMMARY

- Community detection is a huge part of network theory
- Best method depends on community definition
- Dendogram is useful for small networks
- Modularity is very popular, but comes with caveats