NETWORK SCIENCE OF ONLINE INTERACTIONS

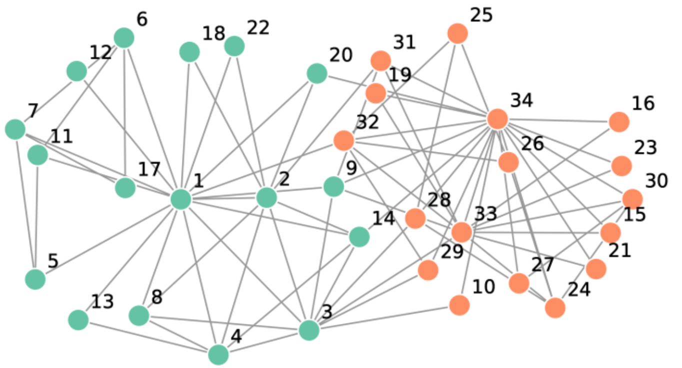
Chapter 7: Dynamics

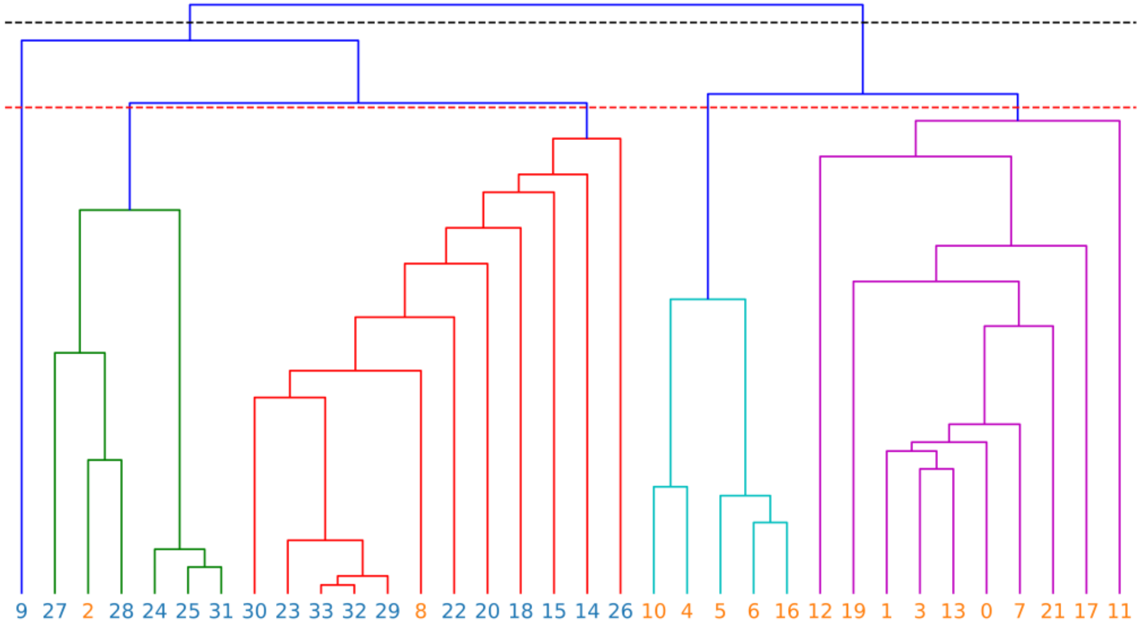
Joao Neto

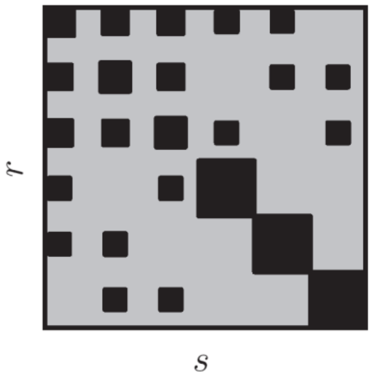
02/Jun/2023

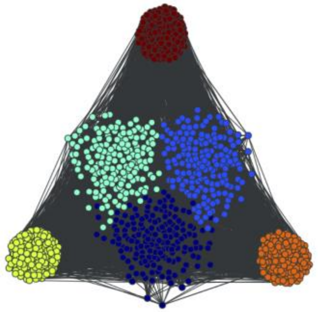
SUMMARY

- Community detection is a huge part of network theory
- Best method depends on community definition
- Dendogram is useful for small networks
- Modularity is very popular, but comes with caveats
- Stochastic block models solve some of those caveats









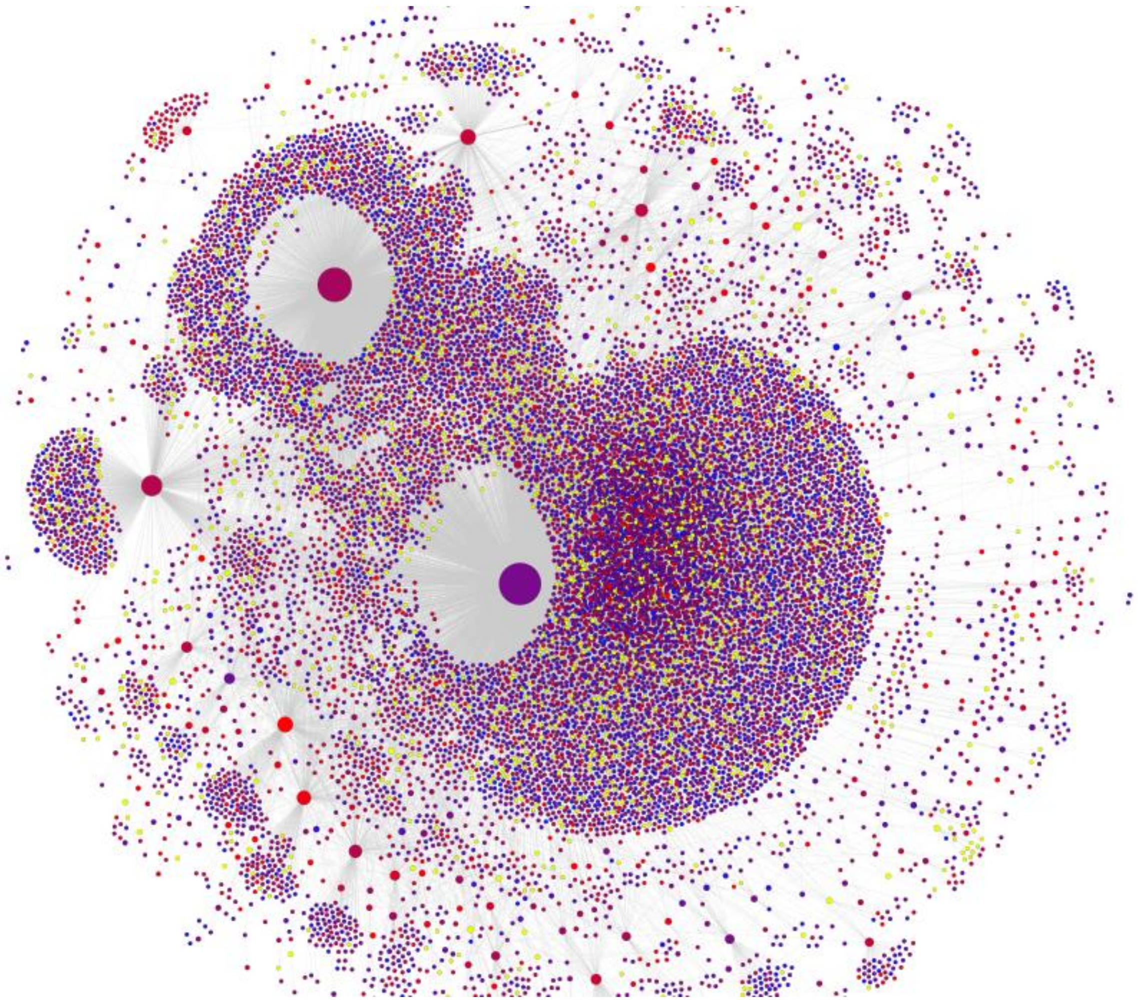
DYNAMICS ON NETWORKS

- Dynamics
  - Models on networks
    - Threshold models
    - Cascade models
    - Epidemic spreading
  - Opinion dynamics

- Critical Phenomena on Networks
  - Adaptive networks
  - Phase Transitions
  - Branching and autoregressive processes
  - Properties of networks at criticality
  - Self-organized criticality

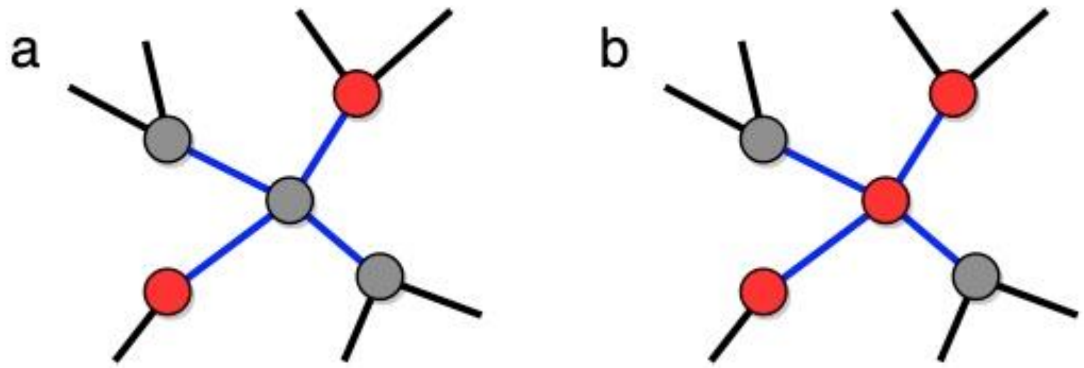
7.1 IDEAS, INFORMATION, INFLUENCE

- Things can spread on networks
  - Social networks: ideas, links
  - Epidemic spreading: diseases
- Definition: nodes have internal state(s)
  $x_i(t) = f\big(\boldsymbol{x}(t)\big)$
- What is the influence of the network structure on the dynamics?
- Example: Fake news spreading on twitter during the 2016 US Election
  - Over 30K tweets + retweets
  - Red ones flagged as bots



7.1 IDEAS, INFORMATION, INFLUENCE

- How do we model how a node state (retweeting, sick, etc) spreads?
  - Deterministic: activation rule
  - Probabilistic: each active node has a probability of activating another



- **Threshold networks**
  - Deterministic dynamics
  - A node is activated if more than θ_i neighbours are

$$x_i(t+1) = \begin{cases} 1 & ,\text{if } \sum_j w_{ij}\, x_j(t) > \theta_i \\ 0 & ,\text{otherwise} \end{cases}$$

or

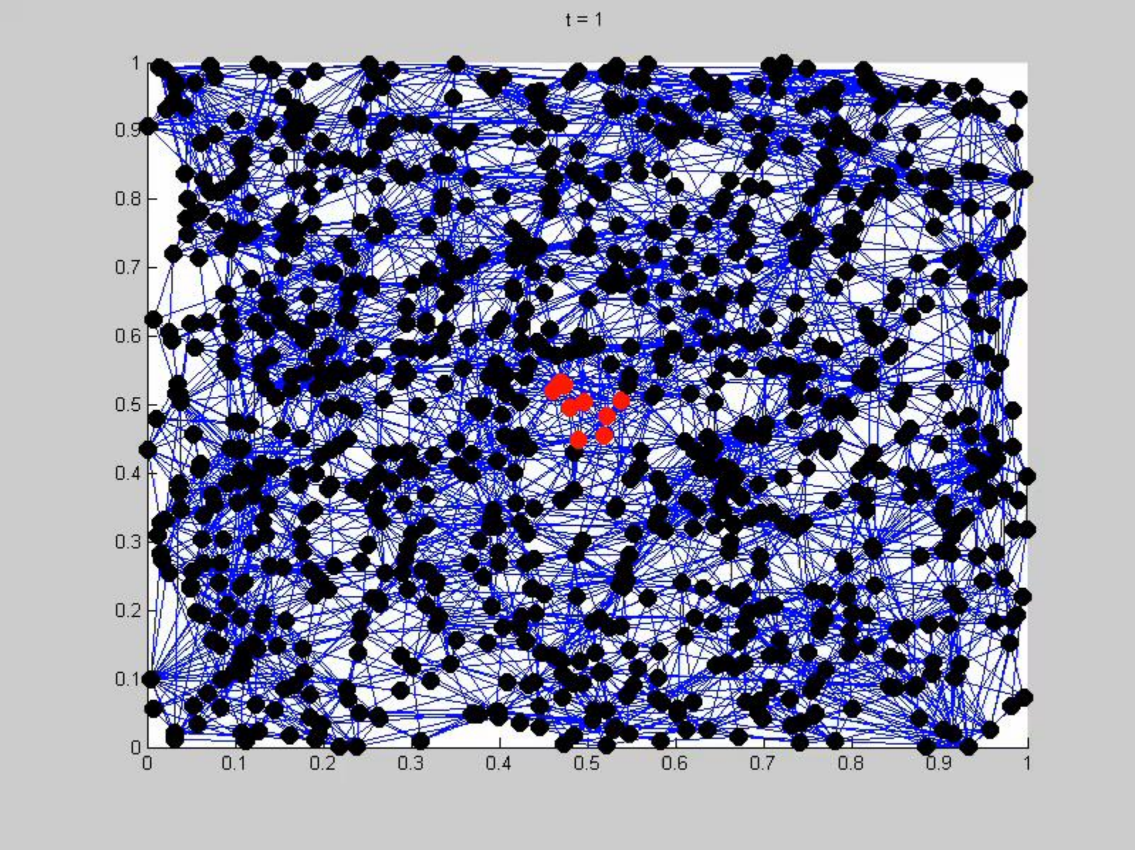
$$x_i(t+1) = \begin{cases} 1 & ,\text{if } \sum_j w_{ij}\, x_j(t) \geq \theta_i \\ 0 & ,\text{otherwise} \end{cases}$$

7.1 IDEAS, INFORMATION, INFLUENCE

- Threshold networks
  - $x_i(t) = H\left(\sum_j x_j(t) - \theta_i\right)$

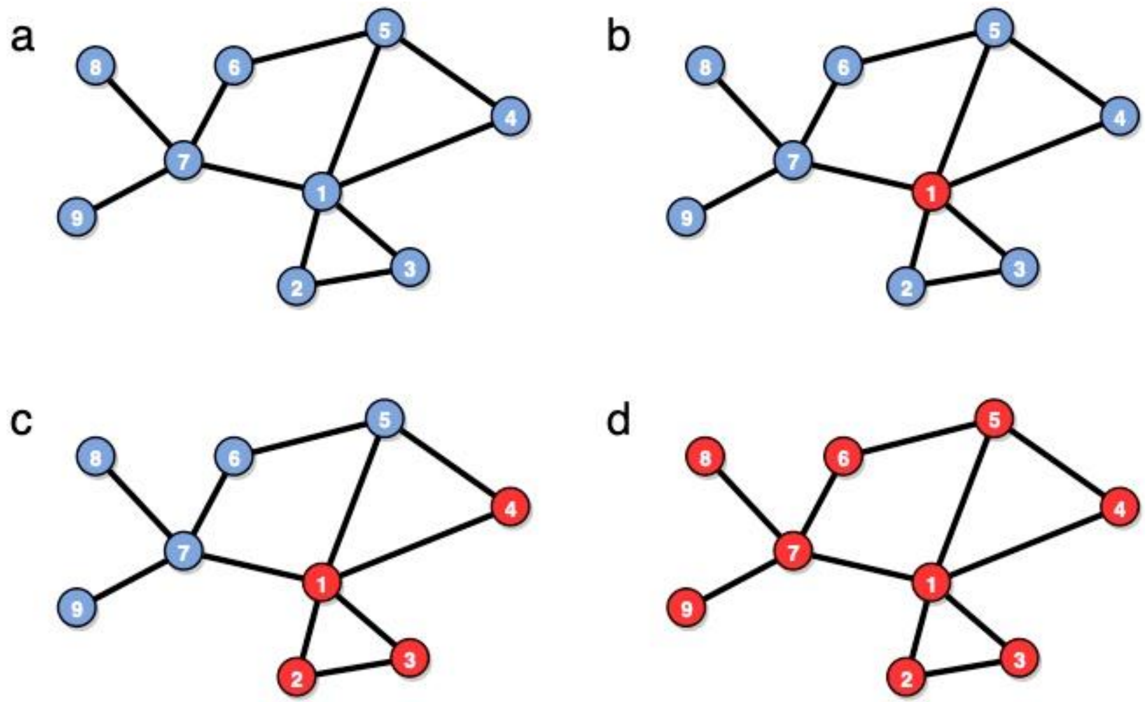
  - Generalization: **Boolean networks**
    - All Boolean functions instead of just $H(x)$
    - Extensively used to model gene regulatory networks

  - *Many* variations have been studied
    - Synchronous vs asynchronous updates
    - Varying threshold/threshold distribution
    - **Inhibition and excitation**
    - Long-term memory effects



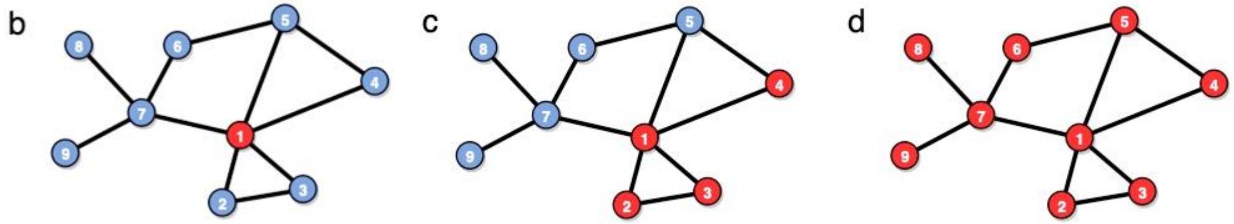
7.1 IDEAS, INFORMATION, INFLUENCE

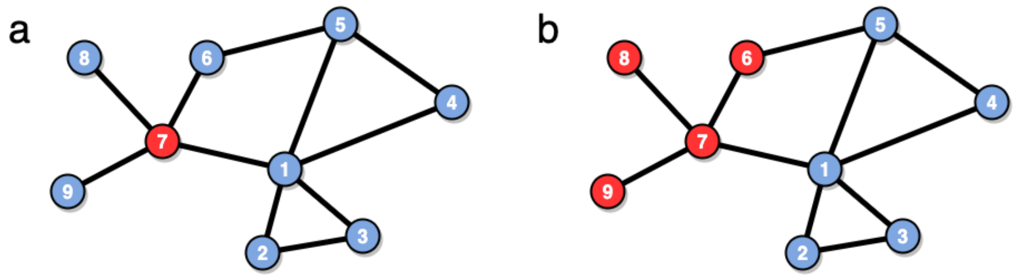
- Thresholds can be
  - Absolute, e.g. $\theta_i = 2$
  - Fractional, e.g. $\theta_i = 0.5k_i$
- Running the model
  - Starts with activating some nodes(s)
  - The dynamics can
    - get frozen (e.g. dying out): $\boldsymbol{x}(t+1) = \boldsymbol{x}(t)$
    - Stay active forever in a loop $\boldsymbol{x}(t + t_{recurrence}) = \boldsymbol{x}(t)$
    - $t_{recurrence}$ can be extremely long (up to 2^N)
  - Example: $\theta_i = k_i/2$
    - Evolves to all active
    - Frozen in that state



7.1 IDEAS, INFORMATION, INFLUENCE

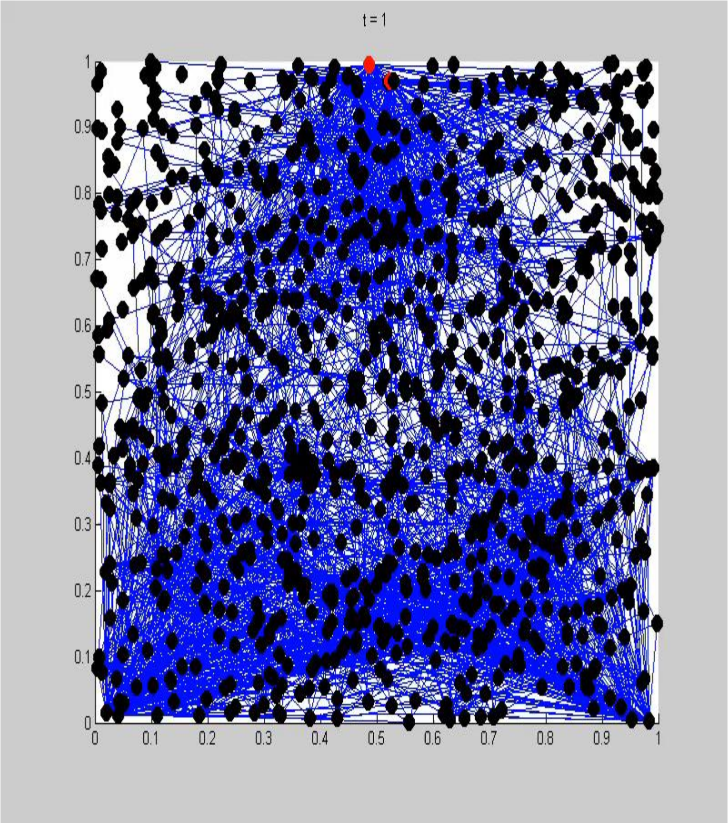
- Evolution of the spreading cascade depends on network structure
- If fractional threshold $\theta_i \sim k_i$, low degree nodes are more vulnerable
- If $\theta_i = \theta$, high degree nodes are both more vulnerable and better *drivers*
- A cascade initiated on the core is more likely to succeed
- Final state can depend on initial state
  - Not only degree, but network positioning

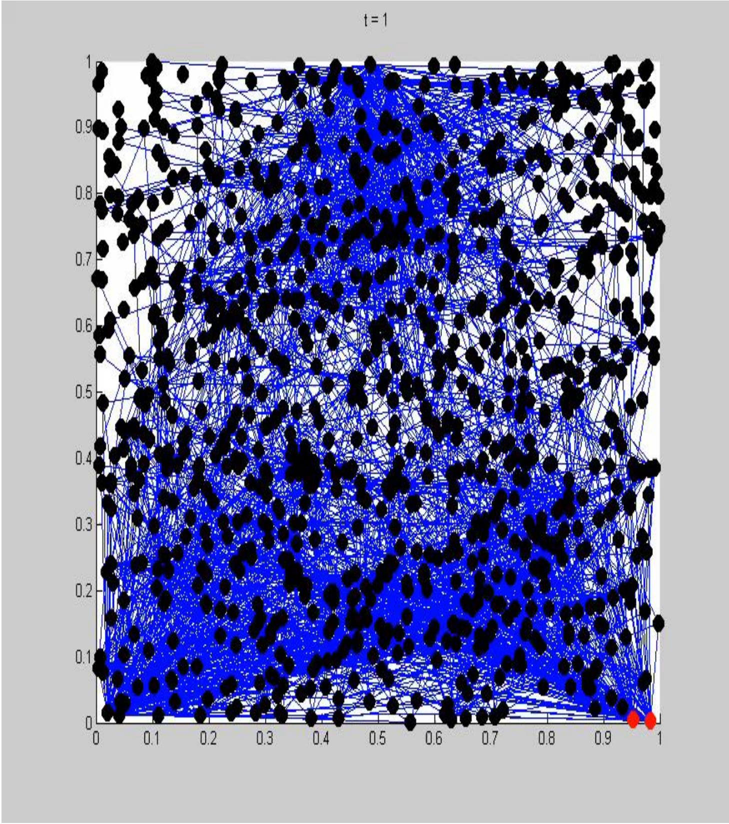


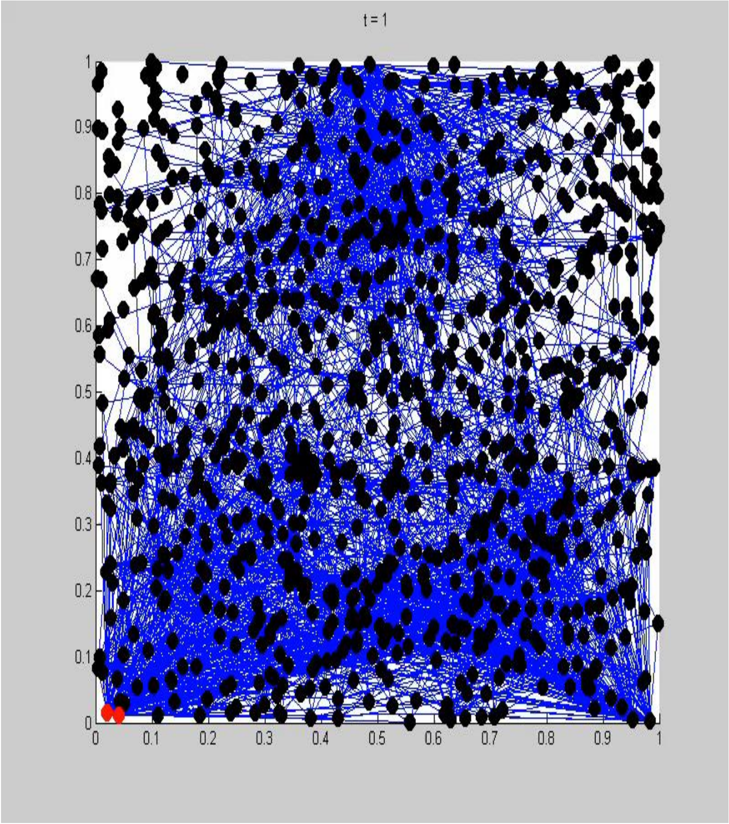


7.1 IDEAS, INFORMATION, INFLUENCE

- Community structure can lead to cascade patterns

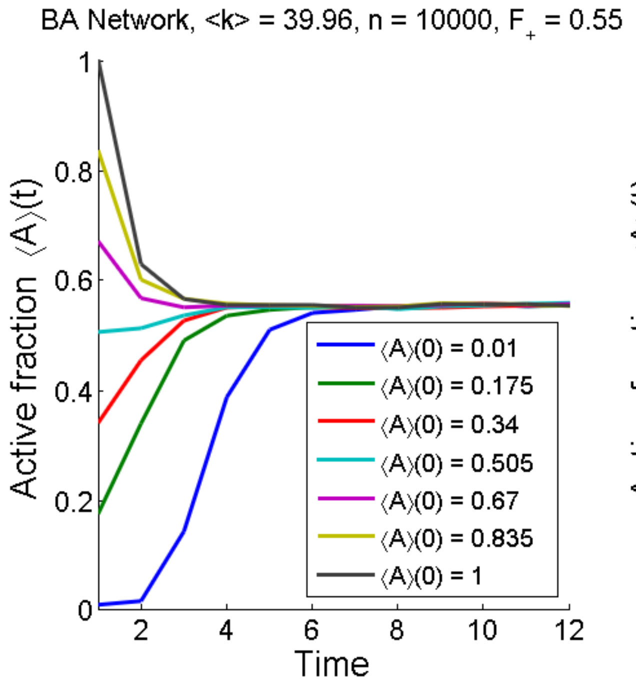


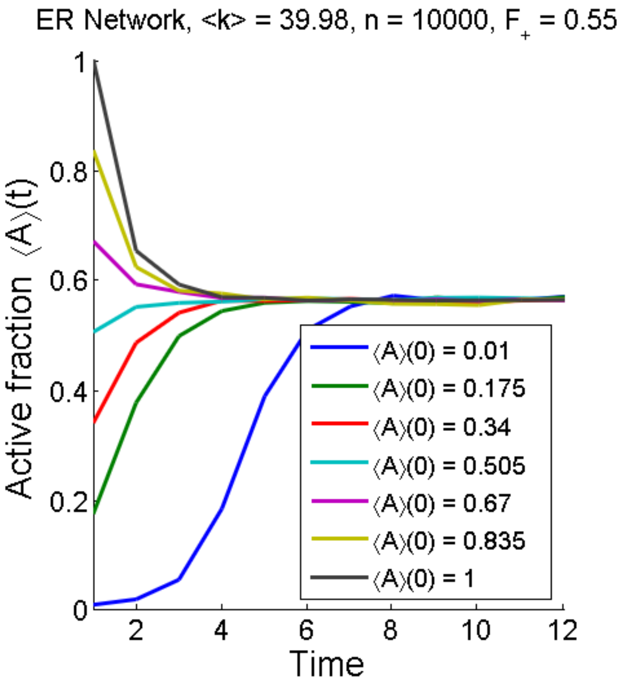




7.1 IDEAS, INFORMATION, INFLUENCE

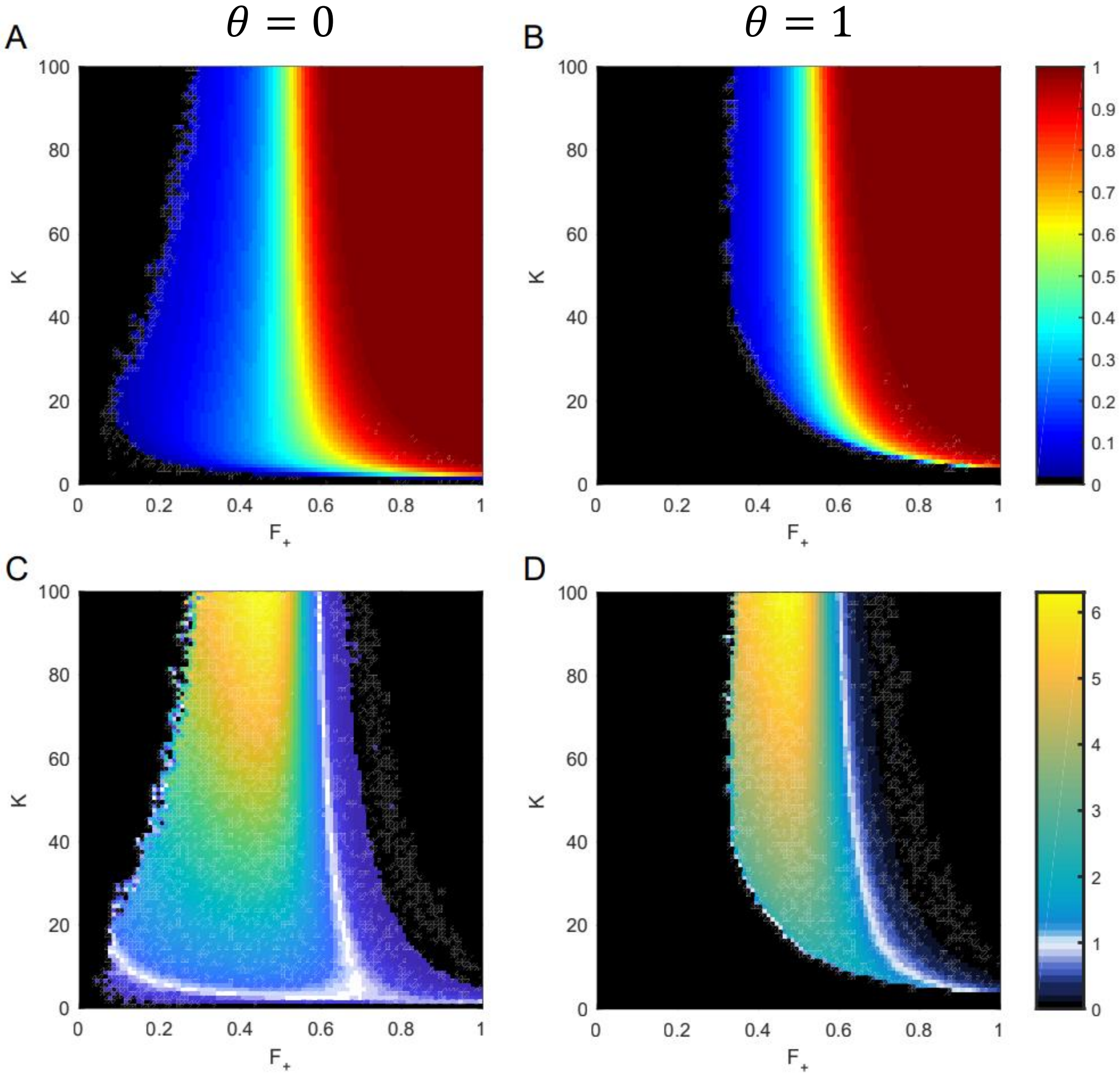
- Extension: inhibition
  - Support of someone you dislike has a negative effect
- **Random Threshold Network** (RTN) with inhibition
  - Random network
  - A fraction F_+ of nodes are positive
  - Parameters:
    - Average degree K
    - Fraction of positive nodes F_+
    - Fraction of initial active nodes $A(0)$
  - Network quickly converges to a fraction A_∞ of active nodes
  - Little dependence on topology (ER vs BA)





7.1 IDEAS, INFORMATION, INFLUENCE

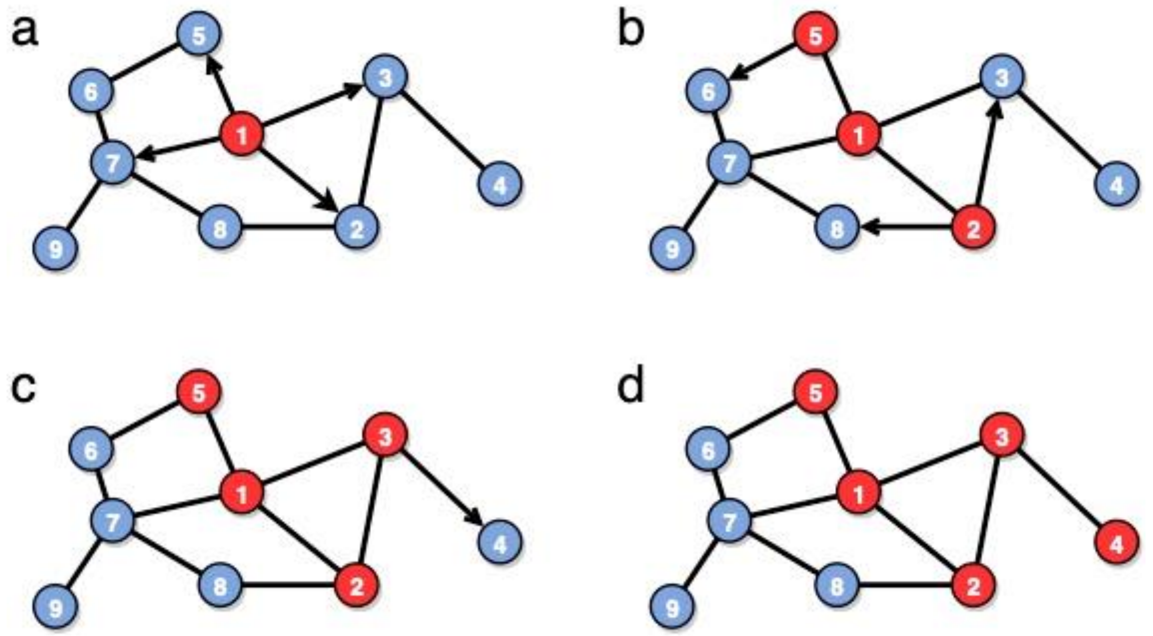
- More activity if
  - Larger F_+
  - Lower θ
  - Effect of K?
- Phase space analysis
  - By varying the parameters, all levels of activity can be obtained
- Sensitivity analysis
  - Due to deterministic dynamics, we can easily check the effect of changing a single node state (Hamming distance)
  - Some network structures are much more susceptible to manipulation than others



JPN et al, arXiv: 1712.08816

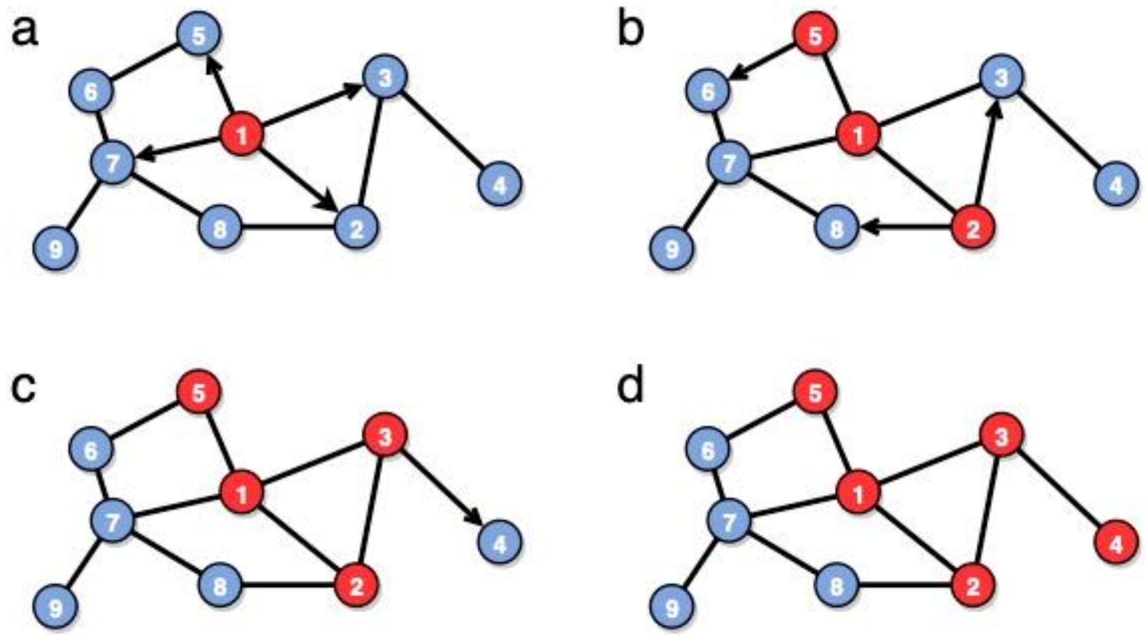
7.1 IDEAS, INFORMATION, INFLUENCE

- **Independent cascade models**
  - Each active node has a probability p_{ij} of activating another
  - Individual node-to-node interaction
  - Many names and variations
    - Independent cascade
    - Percolation models
    - Contact processes
    - Branching models
  - Probabilistic:
    - harder to control cascade
    - Only expected results (e.g. cascade size distribution)



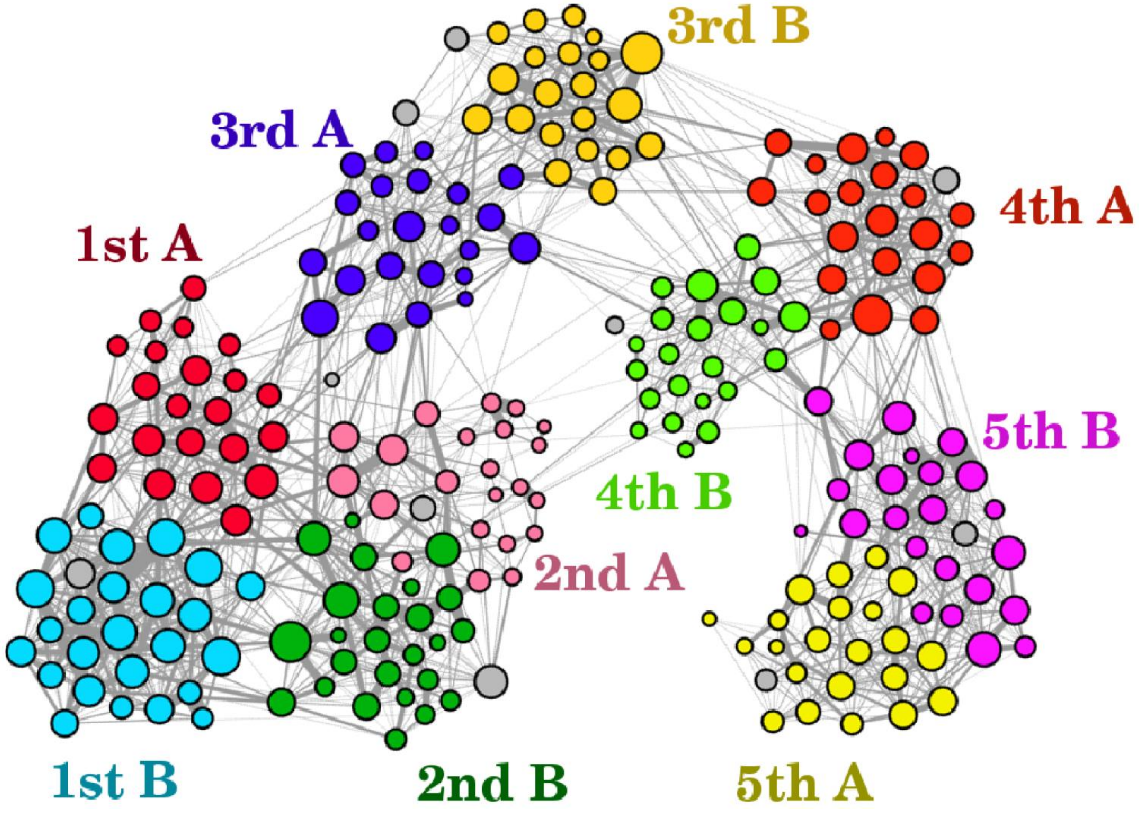
7.1 IDEAS, INFORMATION, INFLUENCE

- **Independent cascade models**
  - Many variants
  - **Complex contagion:** models in which the spreading requires multiple exposures to succeed
    - E.g. spread of innovation in technological networks



7.2 EPIDEMIC SPREADING

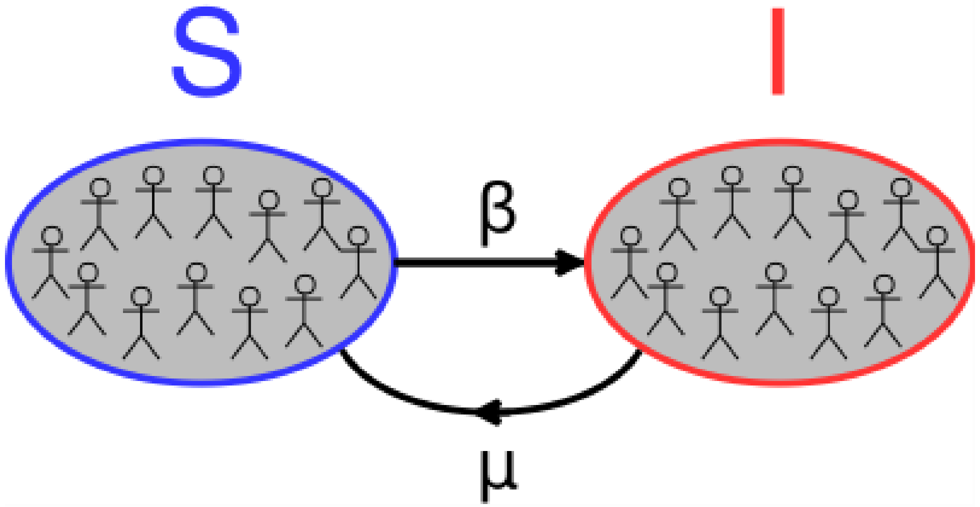
- Models of disease spreading borrow many of the concepts seen so far
- Diseases can spread faster on networks that are
  - denser
  - Small-world
  - with community structure
- COVID-19
  - Fast-spreader
  - Happened during the largest migratory events of mankind (Chinese New Year)
  - Happened at a busy airport (Wuhan)

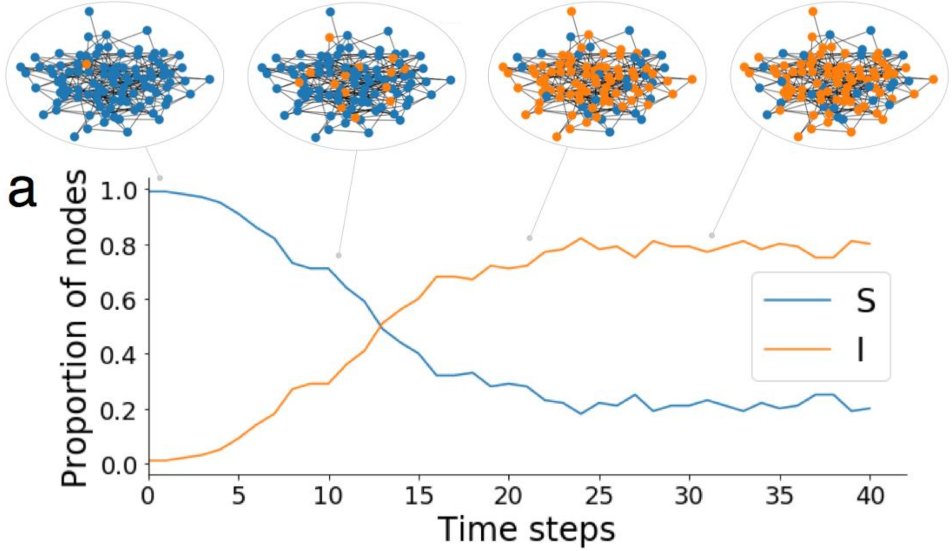


Primary school contact network

7.2 EPIDEMIC SPREADING

- **Compartment models**
  - Blocks and connection between blocks
- The **SIS model**
  - Nodes are **S**usceptible or **I**nfected
  - S gets infected with rate β
  - I recovers with rate μ
  - Good if there is no immunity effect (e.g. common cold)
  - Analytically: $\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{\beta SI}{N} + \mu I \qquad \frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\beta SI}{N} - \mu I$
  - Steady state that depends on the parameters
    - $dS/dt = 0 \rightarrow S = \mu N/\beta, I = 1 - S$



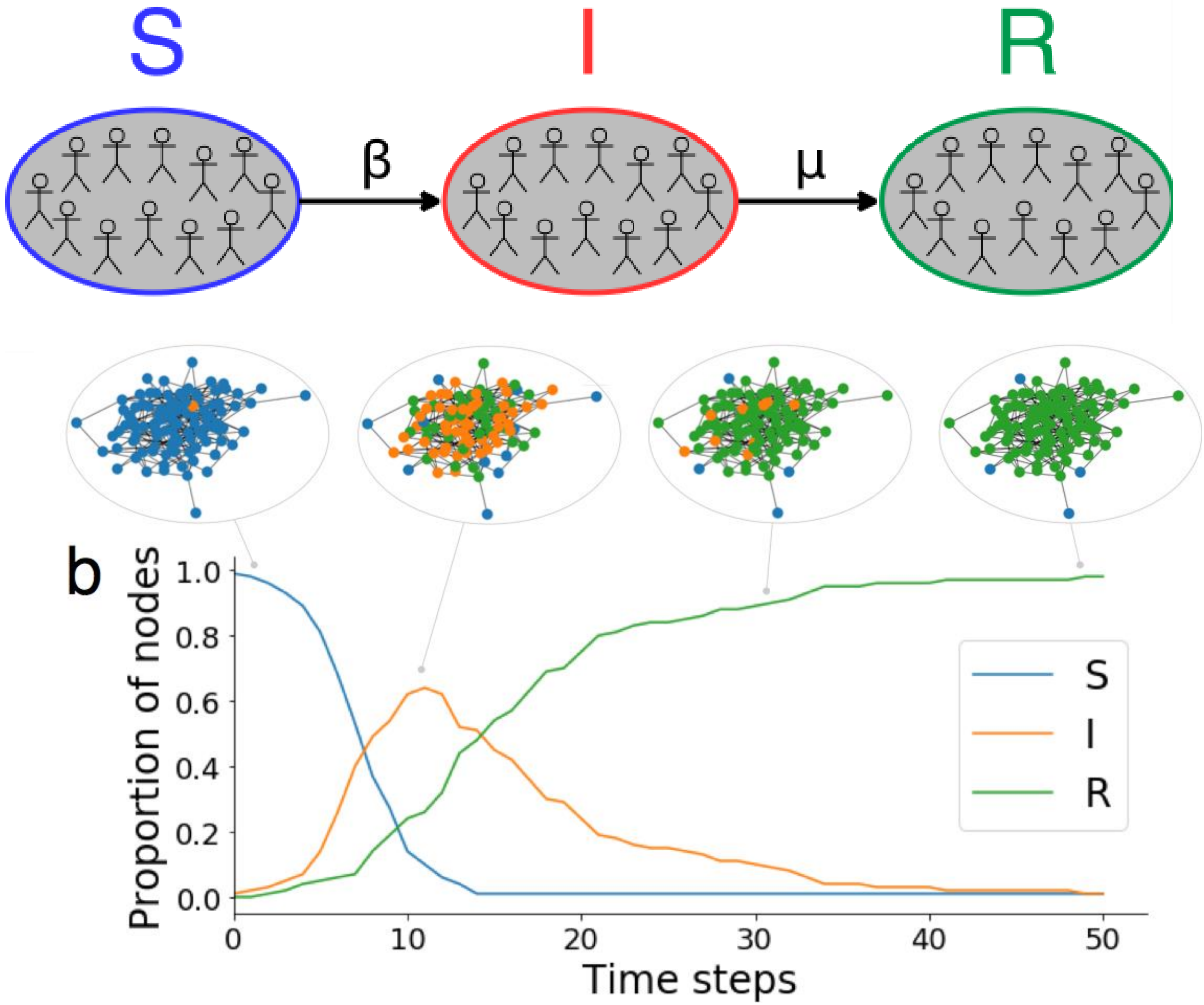


7.2 EPIDEMIC SPREADING

- The **SIR model**
  - Adds a new block: **R**ecovered
  - Models diseases with immunity
  - Analytically

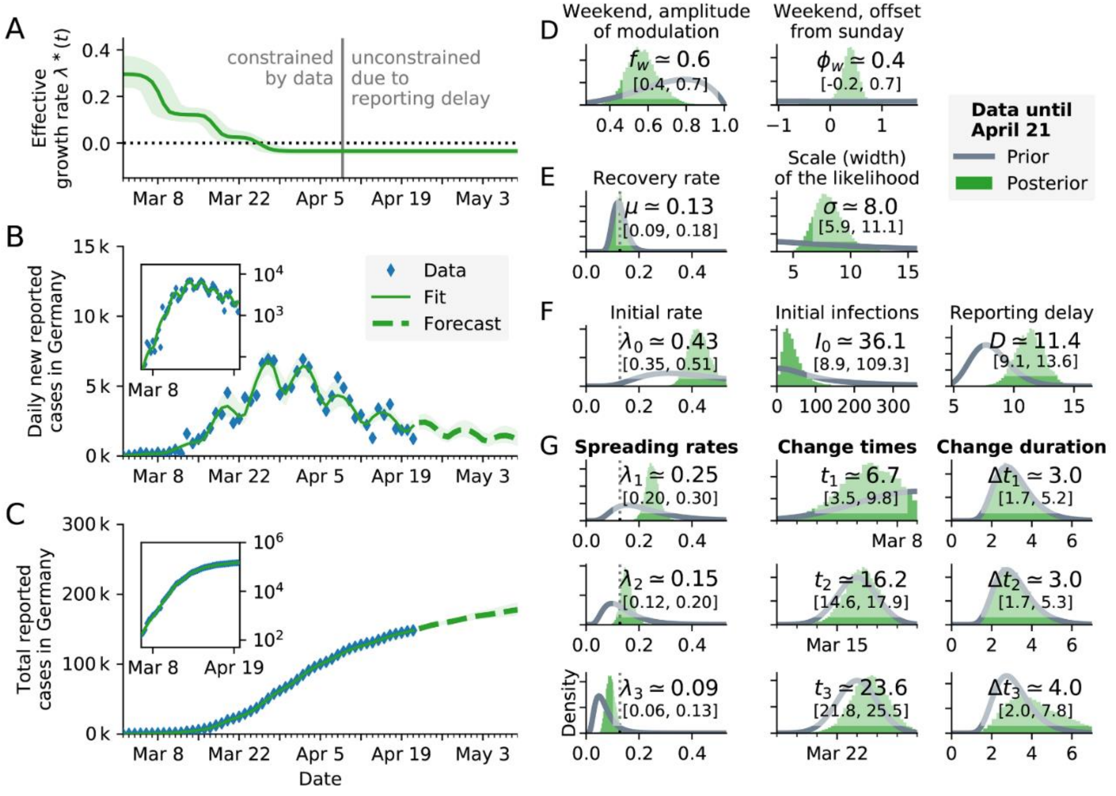
$$\frac{\mathrm{d}S}{\mathrm{dt}} = -\frac{\beta SI}{N} \qquad \frac{\mathrm{d}I}{\mathrm{dt}} = \frac{\beta SI}{N} - \mu I \qquad \frac{\mathrm{d}R}{\mathrm{dt}} = \mu I$$

  - Slow, irregular ramp-up of infected (usually indetectable)
  - Exponential growth of infected
  - Stationary state
    - Endemic $(I > 0)$
    - Eradicated $(I = 0)$



7.2 EPIDEMIC SPREADING

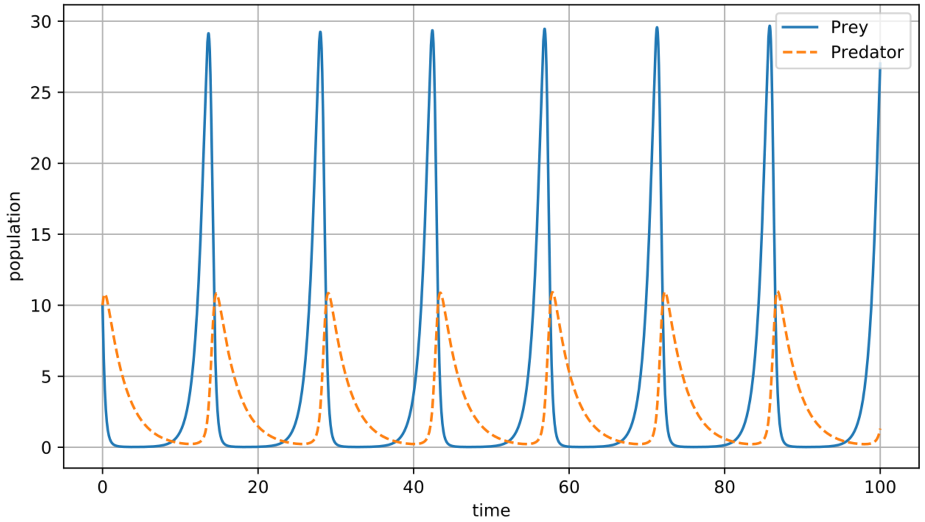
- Many other possible blocks
  - **SIRS**: Susceptible-Infected-Recovered-Susceptible
    - Immunity wanes with a rate γ
  - **SEIRS**: Susceptible-**Exposed**-Infected-Recovered-Susceptible
    - Models a latent phase where individual are infected but not yet infectious
  - **Vital dynamics**: adds birth-death processes
- Different block configurations have different sampling challenges
  - Exposed population can be undetectable
  - Solution: Bayesian inference



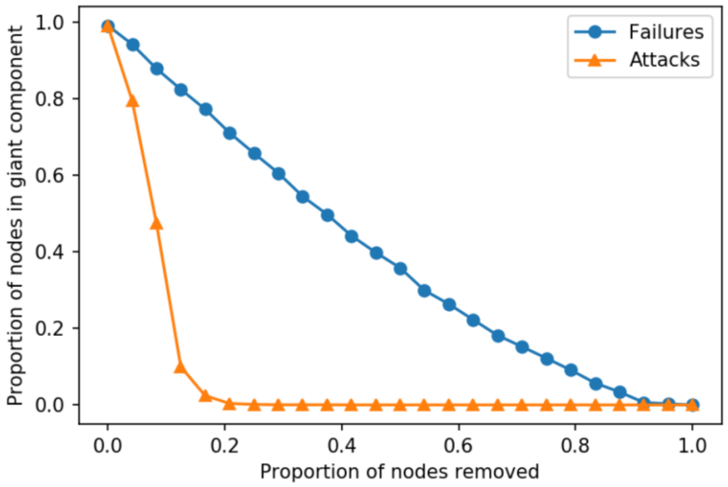
Dehning, J., Zierenberg, J., Spitzner, F. P., Wibral, M., **Neto, J. P.,** Wilczek, M., & Priesemann, V. (2020). *Science*, *369*(6500), eabb9789.

7.2 EPIDEMIC SPREADING

- Continuous model
  - Models a fully-mixed system (all-to-all)
  - **Mean-field solution**: equivalent to the expected behaviour of all identical nodes in the same compartment
  - Disregards node heterogeneity, and fluctuations
    - "Atto-Foxes": can result in continuous dynamics where the actual system would die out



- Network model
  - Can have very different results from the continuous model depending on topology
  - Important example: contact network
    - **No epidemic threshold if there are hubs**
    - Hubs are likely to get infected, and can infect many in return
    - Vaccination strategy: vaccinate hubs first
    - Similar to network robustness analysis

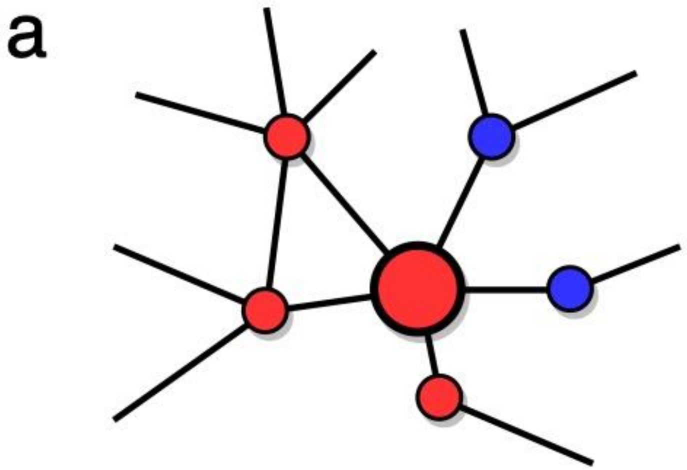


7.3 OPINION DYNAMICS

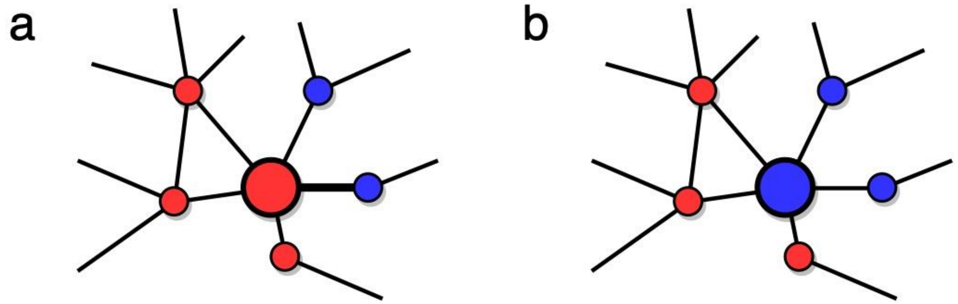
- Opinion spread in social networks borrow from the models seen so far
- Two classes:
  - Discrete opinions
  - Continuous opinions
- Dimensionality:
  - Single topic
  - Multiple topics
- Usual set-up:
  - Random initial opinions (disagreement)
  - Study the rules for consensus-formation
- Possible outcomes
  - Discrete opinions: consensus or polarization
  - Continuous opinion: statistical distribution
  - Unlikely to have a stationary state
  - Other observables (e.g. fraction of system with one opinion) may stabilize

7.3 OPINION DYNAMICS

- **Majority model**
  - Node takes the opinion of the majority of its neighbours
  - Similar to a clustering algorithm
  - Equivalent to a threshold model with $\theta_i = k_i/2$
  - Two ideas competing instead of one spreading
  - Rarely results in consensus, but



- **Voter model**
  - A node may randomly change the opinion to one of its neighbours
  - Consensus is the only stable state
  - Due to being probabilistic, it will eventually happen as long as the network is connected
  - May take an extremely long time



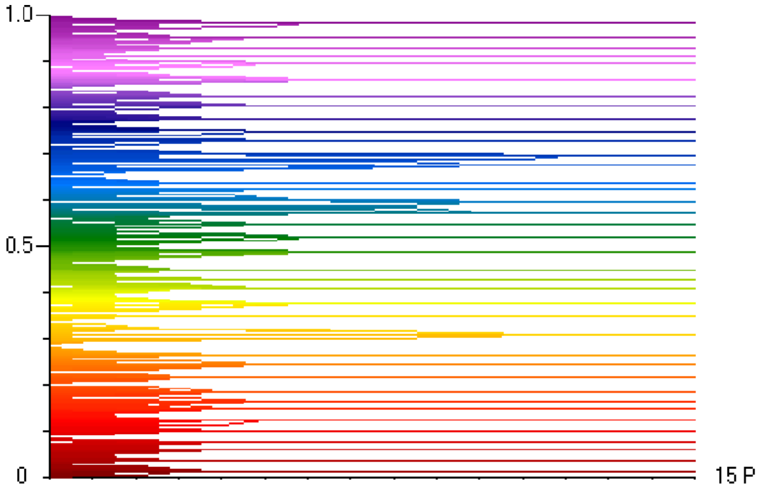
7.3 OPINION DYNAMICS

- Continuous opinions: **Bounded confidence model**
  - The idea: opinions can only influence each other if they are within a distance (confidence bound) ϵ
  - Parameters:
    - Confidence bound ϵ
    - Convergence parameter μ
  - Set-up
    - For two nodes i and j with opinions within ϵ, update their opinions with

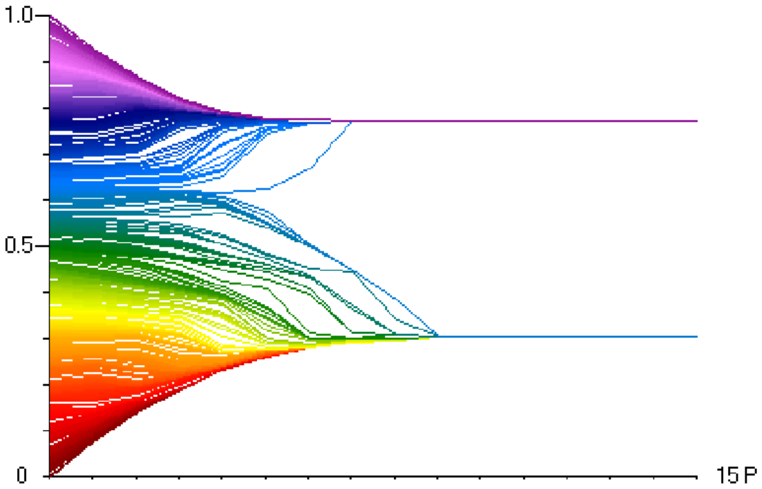
$$o_i(t+1) = o_i(t) + \mu[o_j(t) - o_i(t)]$$
$$o_j(t+1) = o_j(t) + \mu[o_i(t) - o_j(t)]$$

7.3 OPINION DYNAMICS

- Results
  - If $\mu = 1/2$, opinions converge to the average
  - If $\mu = 1$, they switch
  - **Average opinion is conserved**
  - Convergence parameter only affects speed
  - For $\epsilon > 1/2$ the system always reaches consensus, on any network, with the opinions centred around ½
  - For $\epsilon < 1/2$, a varying number of clusters will appear



(a) $\varepsilon_l = \varepsilon_r = 0.01$



(b) $\varepsilon_l = \varepsilon_r = 0.15$

SUMMARY

- Many models with different applicability
- Threshold models
  - Deterministic compounded influence of neighbours
  - Depends on network and starting point
- Cascade models
  - Probabilistic single-node influence
- Epidemic spreading
  - Various components for different types of diseases
- Opinion dynamics
  - Different spreading rules: majority, probabilistic, bounded effect