The Role of Beliefs in Asset Prices: Evidence from Exchange Rates

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Abstract

Motivated by evidence of systematic forecast errors by market participants and professional forecasters, we construct a model of exchange rate determination where short-lived investors each (1) receive noisy private signals about the future path of interest rate differentials between the US and other countries and (2) overestimate the persistence of interest rate differentials. Our model is able to explain the forward premium puzzle, a well-known failure of the uncovered interest rate parity condition implied by traditional models (UIP), in a manner consistent with the survey evidence, in addition to a number of additional puzzles that existing models have struggled to simultaneously explain. These include the initial underreaction and delayed overreaction of currencies in response to monetary news; positive short-horizon and negative long-horizon autocorrelations of currency excess returns; and the lower return predictability of interest rate differentials for UIP trades implemented with longer maturity bonds. Our model is also useful for understanding the strong relationship between survey-based measures of macroeconomic news and exchange rates despite the weak relationship between macroeconomic fundamentals and exchange rates, the persistence of subjective beliefs, and the seeming reversal of the failure of UIP in recent years. Our results highlight the important role that investors' beliefs may play in exchange rate behavior.

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1 Introduction

Underreaction and overreaction to news are pervasive features of asset price behavior across a variety of settings, but a long-standing challenge in the literature remains understanding when and why asset prices sometimes appear to underreact and sometimes appear to overreact to news (Barberis (2018)).[1] A growing body of work in macroeconomics and finance has focused on survey data as a means to understand the beliefs of forecasters and market participants, documenting substantial deviations from the traditional Full-Information Rational Expectations (FIRE) paradigm, and particularly suggesting that underreaction and overreaction to news also feature prominently in people's beliefs (Mankiw et al. (2003), Coibion and Gorodnichenko (2015), Bordalo et al. (2020b), Kohlhas and Walther (2020), and Angeletos et al. (2020b)). Motivated by the evidence, in this paper, we seek to construct a model disciplined by survey data, and study it's ability to qualitatively and quantitatively explain asset price behavior.[2]

We focus on currency markets, where there are rich historical international survey data on the expectations of macroeconomic fundamentals and exchange rates.[3] Moreover, currency markets have a number of well-documented puzzles that appear consistent with underreaction and overreaction to news. Notably, currencies only gradually appreciate in response to an increase in interest rates, rather than immediately reflecting the news (the *delayed overshooting puzzle*, Eichenbaum and Evans (1995));[4] and while a high interest rate positively predicts a currency's excess returns in immediately subsequent quarters, it negatively predicts quarterly excess returns for the currency after eight quarters (the *predictability reversal puzzle*, Bacchetta and Van Wincoop (2010)). Related, but not commonly discussed in conjunction with these facts, currencies exhibit time-series *momentum* and *reversal*, where excess returns twelve months prior positively predict monthly excess returns for a currency, and excess returns one to five years prior negatively predict monthly excess returns (Moskowitz et al. (2012)). As we discuss further, the time-series predictability of short-term interest rate differentials for the returns to borrowing in US bonds and investing in foreign bonds at short maturities (*the forward premium puzzle*, Hansen and Hodrick (1980); Fama (1984)), which represents a well-known failure of the uncovered interest rate parity (UIP) condition implied by traditional monetary models, can also be rationalized by underreaction to news, as can the fact that this return predictability is *declining* in the maturity of bonds used in the transactions (*the*

[1] Significant theoretical work on underreaction and overreaction includes (Barberis et al. (1998), Daniel et al. (1998), Hong and Stein (1999)). Barberis (2018) provides a recent survey on the topic.

[2] In this regard, our paper is highly related to a related strand of literature documents patterns consistent with extrapolation in survey data on expected stock market returns (e.g., Greenwood and Shleifer (2014)), and works to build models consistent with the survey evidence and observed asset price behavior across asset classes (e.g., Barberis et al. (2015), Glaeser and Nathanson (2017)).

[3] There is a notable literature using survey data to study expectations of exchange rates movements, both historical (Frankel and Froot (1987, 1990), Froot and Frankel (1989), Ito (1990)), and more recent (Stavrakeva and Tang (2020a,b) and Kalemli-Ozcan and Varela (2021)). A consensus emerges from this literature that deviations from full information rational expectations may play an important role in explaining exchange rate behavior.

[4] Despite its name, the delayed overshooting puzzle refers to the *underreaction* of exchange rates to news. It draws its name from the fact that under the full information rational expectations benchmark, a currency immediately appreciates coincident with a positive monetary shock, before depreciating over the course of several subsequent periods, behavior that Dornbusch (1976) denotes as *overshooting*.

downward-sloping term structure of UIP violations, Lustig et al. (2019)).[5]

We begin our analysis by presenting three pieces of empirical evidence from survey data. First, in time-series regressions, the coefficient in regressions of market participants' expectations of next quarter's currency excess returns on current interest rate differentials is zero, consistent with belief in the UIP condition holding, and in contrast with the empirical failure of UIP in the data. Second, survey-based forecasts of short-term interest rates underreact in response to monetary shocks, and then subsequently overreact, a pattern which holds *both* for interest rate forecasts and for forecasts of interest rate differentials of the US versus other countries. Third, the underreaction of survey-based forecasts to news about interest rates, and of macroeconomic quantities related to interest rates, is primarily a feature of *consensus* forecasts (the average forecast reported across participants), and is substantially more muted when analyzing individual-level forecasts.

These three facts present additional evidence to understand the behavior of exchange rates, particularly highlighting the role that the beliefs of market participants may play in explaining the facts. The fact that market participants report forecasts of exchange rates that are aligned with UIP, in contrast with the robust failure of UIP, suggests that errors in expectations, rather than risk premia, may play a dominant role in explaining exchange rate behavior. Moreover, the evidence on interest rate forecasts suggests that forecasters make systematic mistakes about the fundamental piece of macroeconomic information for exchange rates - interest rates (facts 2 and 3) - with substantial heterogeneity in forecasters' information (fact 3).

In order to explain the exchange rate facts in a manner consistent with the survey evidence, we construct a small open-economy, overlapping generations model of exchange rate determination. In the model, the interest rate differential between countries is determined by macroeconomic fundamentals, which follow an exogenous AR(1) process. The equilibrium exchange rate is determined by short-lived investors' relative demand for home versus foreign currency bonds.

There are two key frictions in the model that help capture the survey evidence. First, investors receive noisy private signals about macroeconomic fundamentals, and do not learn about other investors' private signals from exchange rates. Though they observe the interest rate differential each period, investors believe that the short-term interest rate differential may deviate from fundamentals in a transitory way (e.g., due to a belief that monetary authorities may not correctly perceive the state of the economy), driving disagreement regarding the future path of interest rate differentials on the basis of private information. Second, investors are *extrapolative*; they uniformly overestimate the persistence of fundamentals. The frictions in the model, relative to a benchmark of Full Information Rational Expectations (FIRE), allow the model to capture the survey evidence. We calibrate the model guided by the three motivating pieces of empirical evidence, and evaluate its ability to explain the exchange rate facts. We find the calibrated model is also able to qualita-

[5]The facts that we discuss here, and the focus of the paper, are primarily facts concerning *time-series* variation of the US dollar versus foreign currencies. These can be drawn in contrast with a literature concerned with *cross-sectional* patterns in exchange rate returns, e.g., concerning whether some characteristics may explain why certain currencies tend to earn higher excess returns than other currencies; prominent work in this line includes Lustig and Verdelhan (2007) and Lustig et al. (2011). Hassan and Mano (2019) discuss the distinction between cross-sectional and time-series predictability of currency returns in more detail, particularly as pertains to the failure of UIP.

tively and quantitatively capture exchange rate behavior.

In the model, when the interest rate differential increases, it takes a few quarters for consensus beliefs to fully internalize the news of higher future interest rate differentials that this increase conveys. The sluggish reaction of consensus beliefs to monetary news stems from noisy private information. Investors only modestly underreact to the news that they observe, but the noise in their signals prevents them from immediately observing, and updating their beliefs in response to, the 'true' monetary news in a given period. In turn, consensus expectations strongly underreact to monetary news. The exchange rate reflects expectations of the sum of all future interest rate differentials. Accordingly, an increase in the interest rate differential also leads the exchange rate to appreciate for a few quarters following the increase, as consensus expectations sluggishly incorporate news of higher future interest rate differentials (the delayed overshooting puzzle).

The initial underreaction of the exchange rate to news of higher future interest rate differentials also drives the time-series return predictability of currency excess returns by interest-rate differentials in the model. On average, a period where the interest rate differential is high is one in which the interest rate differential has either increased, or has increased in a recent past period. Following such a period, the model predicts that the exchange rate will appreciate, or depreciate less than predicted by UIP, as the market continues to incorporate information about higher future interest rate differentials that arrived in the past. The model also predicts more muted predictability for long-maturity bonds relative to short-maturity bonds, as the importance of consensus underreaction to short-rate news for bond prices declines with bond maturity, with the market expecting short-term interest rate differentials to mean-revert in the long run (the downward-sloping term structure of UIP violations).[6]

Co-existing with underreaction driven by dispersed private signals, investors' extrapolation leads them to overestimate the persistence of the interest rate differential. Once consensus expectations fully internalize past monetary news, investors believe that the interest rate differential will remain at its current level longer than it actually does. This mistaken perception leads exchange rates to eventually overreact; currencies experience low excess returns several periods after they have high interest rates, as investors eventually realize that interest rate differentials will be lower than they expected (predictability reversal). Given the relationship between exchange rates and interest rates, the above patterns also manifest in positive autocorrelations of currency excess returns at short-horizons (momentum) and negative autocorrelations at longer-horizons (reversal).

In addition to demonstrating the model's ability to explain outstanding exchange rate puzzles, we also explore several other implications of the model, and find it helpful in understanding a number of additional facts. First, while exchange rates do not co-move with macroeconomic fundamentals, survey-based expectations of exchange rates, and exchange rates themselves, do move in response to survey-based measures of macroeconomic news (Engel et al. (2007), Stavrakeva and Tang (2020b)). This result is exactly consistent with our model, and the role it suggests for

[6]The insight that underreaction to interest rate news may contribute to the downward-sloping term structure of UIP violations is shared with Granziera and Sihvonen (2021), who suggest more broadly that sluggish consensus expectations of short-rates may help explain why short rates and yield spreads predict bond and currency returns.

biased beliefs. In our model, when macroeconomic news arrives, investors learn from that news, and accordingly update their beliefs about fundamentals. This results in a relationship between macroeconomic surprises and exchange rates. However, because they do not fully incorporate news into their expectations, investors have biased beliefs about macroeconomic fundamentals, and there is a disconnect between exchange rates and the true macroeconomic fundamentals.

Second, survey data of investors suggests strong persistence in individual beliefs; pessimists are persistently pessimistic and optimists are persistently optimistic (Giglio et al. (2021)). In our model, because investors never observe the 'true' macroeconomic fundamentals, private information received in a given period influences investor beliefs for several subsequent periods. In turn, investors that receive a positive signal about the future interest rate differential, relative to investors that receive a negative signal, may hold onto the belief of relatively higher future interest rate differentials and exchange rates for several periods. In the calibrated model, it takes more than two years for the average belief of investors in the top or bottom deciles of the belief distribution of exchange rates to converge to the average belief of the population. This result, which is not explicitly targeted in the model, is remarkably consistent with what we find when analyzing forecasts of interest rates from the Survey of Professional Forecasters, and suggests a potentially important role for dispersed information in the persistence of subjective beliefs.

And third, in the post-financial crisis period, the forward premium puzzle appears to have become substantially weaker (Bussiere et al. (2018); Engel et al. (2019, 2021)). In our model, the failure of UIP is driven by underreaction to interest rate news, stemming from dispersed private information. As we reduce the dispersion of private information, extrapolation leads consensus expectations to overreact to interest rate news, reversing the sign of the predictability of interest rate differentials for currency excess returns. Consistent with this channel, we find suggestive evidence that the dispersion of beliefs about future interest rates has decreased in recent times, and consensus forecasts appear to overreact to news about interest rate differentials, in contrast with the full sample evidence.

In the literature on exchange rates, the closest antecedent to our work is Gourinchas and Tornell (2004). In their model, investors are homogeneous, and, as in our model, believe interest rates may temporarily deviate from their fundamental values, which induces each investor to underreact to interest rate changes due to confusion about whether such changes are persistent or transitory. They find this underreaction can explain the failure of UIP, as well as the delayed overshooting puzzle. In contrast, in our model, the belief that interest rate differentials may temporarily deviate from fundamentals is primarily used to provide scope for investors' private information to matter for exchange rates. Individuals in our model only modestly underreact to the interest rate news they receive, as potential underreaction is muted by extrapolative beliefs. However, the presence of dispersed private information leads *consensus* expectations of interest rate differentials to substantially underreact to monetary news. Our model more closely matches the survey data, and also allows us to simultaneously explain a number of puzzles (predictability reversal puzzle, the failure of UIP, and the downward-sloping term structure of UIP violations), which previous models have struggled to do (Engel (2016)).[7]

In independent and contemporaneous work, Candian and De Leo (2021) extend the model of Gourinchas and Tornell (2004) by introducing extrapolation of the level of fundamentals that govern the interest rate. Similar to our model, their model is able to rationalize the failure of UIP, and patterns of initial underreaction and delayed overreaction of exchange rates in response to interest rate news. Nevertheless, there are a number of important differences between the papers, and we believe they are complementary. For instance, Candian and De Leo (2021) embed their framework into a two-country general equilibrium model that endogenizes the interest rate, and turn their focus to the relationship between consensus expectations of macroeconomic quantities and exchange rates. In contrast, we focus on additional puzzles (exchange rate momentum and reversal, and the downward-sloping term-structure of UIP violations), and also focus on more closely understanding the role that belief heterogeneity may play, disciplining our study with individual-level survey data.[8]

2 Motivating Empirical Evidence

We begin our analysis in the paper by presenting three stylized facts using survey data on expectations. These facts are largely consistent with results in prior work using survey-based expectations data. Each of the facts showcases a distinct deviation from Full Information Rational Expectations that serves as motivation for the assumptions we make in our model.

Our sample for this analysis consists of liquidly traded currencies versus the US dollar: the G11 developed market currencies, and 16 emerging market currencies. We obtain data on exchange rates and forward rates from Refinitiv Datastream. Survey data on forecasts of interest rates and macroeconomic fundamentals for the US are from the Survey of Professional Forecasters. Interest rate and exchange rate forecast data are from FX4casts for other countries in our sample. We describe the sample and data in more detail in Appendix A.

Fact 1: UIP and Consensus Exchange Rate Expectations

The forward premium puzzle has been an established fact in the academic literature dating back to Fama (1984). One way to observe the puzzle is via regressions of the form

[7]Our paper also relates to Molavi et al. (2021), who argue that limited information-processing capacity, in the form of only being able to process k of $N > k$ factors that drive the true data generating process, leads some investors to misperceive the process that interest rate differentials follow. This can contribute to the failure of UIP and the predictability reversal puzzle. However, their model does not capture the difference between individual and consensus underreaction to interest rate news or the downward-sloping term structure of UIP violations, as ours is able to.

[8]Due to the role of dispersed private information, exchange rates in our model take the form of a Keynes (1936) beauty contest, with investors formulating their demand based on higher order expectations of other investors' demands. Bacchetta and Van Wincoop (2006) similarly suggest a role for dispersed private information about fundamentals, and higher-order expectations, as drivers of exchange rate behavior. However, they focus on the disconnect of exchange rates from macroeconomic fundamentals, but do not focus on the exchange rate puzzles of interest to us. More broadly, our work adds to a literature on higher-order beliefs and beauty contests in asset pricing; see Singleton (1987) and Allen et al. (2006) for prominent work in this literature.

$$\lambda_{j,t+1} = \alpha_j + \beta i^d_{j,t} + \epsilon_{j,t+1} \tag{1}$$

where $\lambda_{j,t+1}$ are the excess returns of borrowing at short-term interest rates in country j and lending at US short-term interest rates in dollars, i^d_j is the interest rate differential (the US short-term interest rate minus the foreign short-term interest rate), and β is the coefficient of interest. The UIP condition implies that $\beta = 0$, while empirical work has consistently reported estimates of $\beta > 0$.

While the UIP condition appears to fail spectacularly in the data, consensus (average) forecasts of currencies across market participants appear to align much more closely with UIP. That is, when we run the regression in Equation (1) replacing the independent variable with $\bar{\mathbb{E}}_t \lambda_{j,t+1}$, where $\bar{\mathbb{E}}$ captures the average expectation reported in forecasts, we find a coefficient β that is much closer to zero.[9]

Figure 1 plots the average beta from estimating Equation (1) for each country in our sample, using quarterly forecasted and realized excess returns as the dependent variables, and interest rate differentials implied by 3-month forward rates as the independent variables. Betas for individual countries are weighted by the total number of observations that we have for the country in our sample. The figure reports average betas for all countries together, as well as splitting countries into 'Developed' and 'Emerging' market countries. The figure also plots 95% confidence intervals, computed using HAC-panel standard errors. The sample is from August 1986 through December 2019.

The figure reveals a consistent failure of UIP across developed and emerging markets, with average coefficients of 1.34, 1.46, and 1.20 for all countries, developed market countries, and emerging market countries, respectively. The betas for forecasted excess returns are 0.41, 0.22, and 0.63 for all, developed market, and emerging market countries, indicating that the coefficients for forecasted excess returns are much closer to zero. The average developed market coefficient for forecasted excess returns is statistically indistinguishable from zero. The average forecasted coefficient for emerging market countries is significantly greater than zero, consistent with a potential role for risk premia in these markets, as argued by Kalemli-Ozcan and Varela (2021). However, the size of the average coefficient is nearly twice as large where realized returns are the dependent variable rather than expected returns, indicating a substantial role for potential errors in expectations in emerging markets as well.[10] Additionally, the emerging markets sample begins in 2001; hence, the regression evidence indicates that even in the second half of the sample, forecasters have continued to report forecasts of currency excess returns that are closer to UIP than the realized excess returns found in the data.

[9] The conclusions we draw from these regressions are similar to those found in Froot and Frankel (1989) for a sample of five currencies in the 1970s and 1980s. We extend the results to an additional set of currencies, including emerging markets, and a longer and later sample period, and find consistent evidence.

[10] This result differs from Kalemli-Ozcan and Varela (2021), who find similar regression coefficients in emerging markets in regressions of expected and realized excess returns on interest rate differentials. The reason for the difference is that the coefficients we report are the average coefficients in time-series regressions for currencies. In contrast, their regressions are panel regressions that implicitly give more weight to currencies with more volatile interest rate differentials.

The fact that market participants report forecasts of excess returns that are much closer to UIP is important for two reasons. The first is that it suggests that incorrect beliefs may play an important role in explaining the exchange rate puzzles of interest to us. If the failure of UIP were entirely driven by risk premia, we might expect survey-based expectations to capture these risk premia; we find that they only appear partially consistent with risk premia, at best. And second, it provides us with a useful stylized fact to consider in formulating an explanation for the puzzles.

Fact 2: Initial Underreaction and Subsequent Overreaction in Consensus Interest Rate Forecasts

Our second piece of motivating empirical evidence is that, in response to a piece of monetary news, consensus expectations of short-term interest rates reported in surveys initially underreact and subsequently overreact. In particular, following the arrival of monetary news indicating higher short-term interest rates, survey-based forecasts of short-term interest rates are lower than realized interest rates for an initial period, indicating *underreaction*. Following this initial underreaction, in subsequent periods, forecasts of interest rates are higher than realized interest rates, indicating *overreaction*.

To capture the arrival of monetary news, we use a time-series of monetary shocks constructed by Angeletos et al. (2020a).[11] In particular, the shocks are constructed by running a VAR of ten US macroeconomic variables, including the US Federal Funds rate, and extracting the linear combination of residuals in the VAR that explains the most quarterly variation of federal funds rate for 6 to 32 quarters ahead.

Figure 2 plots impulse response functions, at the quarterly frequency, of US Treasury Bill rates, consensus SPF forecasts of US Treasury Bill rates from four quarters prior, and consensus forecast errors (defined as the difference between the realized and forecasted values) of US Treasury Bill rates. The impulse response functions are estimated from regressions of the form

$$x_{t+h} = \alpha_h + \beta_h \epsilon_t + \gamma C_t + u_{t+h} \tag{2}$$

where x_{t+h} is the variable of interest, C_t are lagged values of forecasts and outcomes used as controls, and ϵ_t are the monetary shocks. The variables of interest are i_{t+h} (the Treasury Bill rate h quarters after the shock), $\bar{\mathbb{E}}_{t+h-4} i_{t+h}$ (the period $t+h-4$ consensus forecast of the period $t+h$ Treasury Bill rate rate), and $i_{t+h} - \bar{\mathbb{E}}_{t+h-4} i_{t+h}$ (the consensus forecast error of the interest rate). The sample runs from 1981 to 2017. The figure also plots plus and minus one standard error for the impulse response functions.

The impulse response functions reveal that, for four to six quarters after the arrival of a monetary shock, consensus forecasts of interest rates are persistently lower than the realized interest rate, indicating underreaction to monetary news. However, for seven to eighteen quarters after the shock, forecasted interest rates exceed the realized interest rate, indicating the subsequent overreaction of interest rates. These patterns are captured by the initial positive forecast errors, followed

[11] We download data on shocks from George-Marios Angeletos' website.

by negative forecast errors.

For exchange rates, and the puzzles of interest to us in this paper, the behavior of the short-term interest rate differential between the US interest rate and other currencies is of particular interest to us, not just the behavior of the US short-term interest rate. Using Equation (2), we estimate impulse response functions where the variables of interest are interest rate differentials (the US interest rate minus the foreign interest rate), the consensus forecast of interest rate differentials, and forecast errors of the interest rate differentials. The data on interest rate differential forecasts and realizations span all the countries in our sample and are from FX4casts, and the sample runs from October 2001 through December 2019.

Figure 3 plots impulse response functions where observations are at the quarterly frequency. The figure reveals a similar pattern in initial underreaction and subsequent overreaction of forecasts to positive US monetary shocks as found when focusing on US interest rates. The consistent reflection of initial underreaction and subsequent overreaction in survey-based forecasts of interest rate differentials indicates the potential importance of these features for understanding the behavior of exchange rates.

In Appendix C, we analyze the patterns of underreaction and overreaction in a number of different ways than presented here. This includes using monetary shocks following the methodology in Romer and Romer (2004) (compiled by Wieland and Yang (2020)), computing bias coefficients following the methodology proposed by Kucinskas and Peters (2019), and tests for underreaction and overreaction suggested by Coibion and Gorodnichenko (2015) and Kohlhas and Walther (2020). Across all of our tests, we find consistent evidence of underreaction and overreaction of survey-based consensus expectations to interest rate news.

Other work has shown that short-term interest rate forecasts reported in surveys underreact to monetary news, both in the US and in other countries (e.g., see Cieslak (2018), Brooks et al. (2019), Schmeling et al. (2020) and Wang (2020)). Underreaction to interest rate news also serves as the motivation for Gourinchas and Tornell (2004) in explaining the failure of UIP in exchange rates. But the result on overshooting of interest rates expectations following monetary shocks is new. The broader patterns of initial underreaction and subsequent overreaction of expectations are consistent with similar patterns in survey-based expectations of macroeconomic variables found in other work (see e.g., Angeletos et al. (2020b)).

Fact 3: Underreaction of Interest Rate Forecasts is Primarily a Consensus Phenomenon

Our third piece of motivating empirical evidence is that the underreaction of short-term interest rate forecasts to monetary news appears to be a phenomenon primarily found in consensus forecasts; when focusing on individual forecasts, underreaction to monetary news is much less pronounced.

We show that underreaction of short-term interest rate forecasts is primarily a consensus-level phenomenon. We follow Coibion and Gorodnichenko (2015) and Bordalo et al. (2020b), and regress forecast errors on forecast revisions, using both individual forecaster-level and consensus-level observations. In particular, regressions are of the form

$$x_{t+k} - \mathbb{E}_t x_{t+k} = \alpha + \beta_{CG}(\mathbb{E}_t x_{t+k} - \mathbb{E}_{t-k} x_{t+k}) + \epsilon_{t+k} \tag{3}$$

where x_{t+k} is the variable of interest, $\mathbb{E}_t$ is the period t expectation of x in period $t+k$, and β_{CG} is the coefficient of interest in the regressions. The primary variable of interest is the US Treasury Bill rate, but we also show regression results using unemployment and inflation, variables which we expect to be closely related to monetary policy and short-term interest rates. As Coibion and Gorodnichenko (2015) note, $\beta_{CG} > 0$ corresponds to forecasters underreacting to information that arrives in period $t-k$, and $\beta_{CG} < 0$ corresponds to forecasters overreacting to information that arrives in $t-k$, with larger magnitude coefficients indicating more underreaction or overreaction. A positive coefficient indicates that the forecast error is positively correlated with changes in forecasters' expectations in $t-k$. This reflects that forecasters' beliefs did not move sufficiently to capture information that arrived in period $t-k$, consistent with underreaction. Conversely, a negative coefficient indicates that forecasters' beliefs moved too much in period $t-k$, consistent with overreaction.

Figure 4 plots coefficients from the regressions. For all of the variables, the coefficient estimated using consensus-level observations have more positive coefficients than the observations estimated using individual forecaster-level observations, indicating that underreaction is substantially more pronounced at the individual level than it is as the forecaster level. Focusing on the regression for Treasury Bills, the regression coefficients for one-, two-, and three-quarter ahead forecast errors are (0.22, 0.34, 0.60) at the consensus level, while they are (0.02, 0.11, 0.20) at the individual level, suggesting that while underreaction is substantial in consensus-level forecasts, it is much more muted in individual-level forecasts.

We also conduct a bootstrap analysis, where we sample observations with replacement from the original data and re-run the individual- and consensus-level regressions.[12] The figure reports the proportion of bootstrap samples where the consensus level regression coefficient is greater than the individual level regression coefficient. For Treasury Bill forecasts, the proportion of bootstrap samples where the consensus coefficients are larger are 90%, 93% and 98% for one-, two-, and three-quarter ahead forecasts. For unemployment and inflation forecasts, the consensus coefficients are greater in more than 98% of samples for all horizons.

The regression results indicate that underreaction is much more pronounced at the consensus level than at the individual level. In Appendix Table C.2, we follow an approach similar to Angeletos et al. (2020b), and run multivariate regressions of individual forecast errors on consensus and individual forecast revisions. That analysis similarly reveals that underreaction is primarily a feature of consensus expectations.

The fact that underreaction is primarily a feature of consensus expectations, and not individual expectations, suggests that information heterogeneity across forecasters may play an important

[12] In particular, for the re-sampling procedure, one observation corresponds with all forecasts of and the realization of the variable of interest for a given time period.

role in explaining underreaction, as argued by Bordalo et al. (2020b).[13]

3 Baseline Model

We construct a model of exchange rate determination, which features agents with noisy private information and potentially biased beliefs about the macroeconomic fundamentals that determine interest rates, with the goal of explaining exchange rate behavior in a manner consistent with the motivating empirical evidence. The model is intentionally stylized in order to focus on the frictions of interest for our study. We calibrate the model using moments estimated from data on interest rate forecasts and interest rates. We evaluate the model based on its ability to explain the behavior of exchange rates, and find that the frictions we introduce are able to qualitatively and quantitatively reproduce the patterns of interest in the data.

3.1 Preliminaries

Time is discrete and is indexed by $t \in \{0, 1, 2, ...\}$. There are two countries, the Home country and the Foreign country; variables from the latter are starred. We assume a small open-economy setting, where the Home country is large and the Foreign country is infinitesimally small. The log nominal exchange rate between the two countries in period t is denoted as s_t, expressed in units of foreign currency per one unit of home currency.

There are two assets, a one period bond for each country, which are both in zero net supply. Investors may take short positions (borrow) or take long positions (lend) in each of the bonds. The interest rates of the bonds are given by i_t and i_t^*. We denote the interest rate differential between the two countries as $i_t^d = i_t - i_t^*$. The interest rate differential is generated by a macroeconomic fundamental, which is unobserved. The fundamental follows an $AR(1)$ process,

$$\xi_t = \rho \xi_{t-1} + \eta_t \text{ or } \xi_t = \frac{1}{1 - \rho L} \eta_t, \text{ where } \eta_t \sim \mathcal{N}(0, 1). \tag{4}$$

The interest differential is equal to the fundamental plus an idiosyncratic error term.

$$i_t^d = \xi_t + \varepsilon_t, \text{ where } \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2). \tag{5}$$

Because the Foreign country is infinitesimal, only the Home country investors matter for the bond market equilibrium. Each period, a unit mass of short-lived, Home country investors with

[13] Bordalo et al. (2020b) generally find evidence that individual expectations *overreact* to macroeconomic news. However, for news about short-term interest rates, we (and, in fact, they) find evidence that individual expectations seem to slightly *underreact*, though in a less pronounced way than consensus expectations. Consensus underreaction to interest rate news is consistent with evidence in other work (e.g., Cieslak (2018); Wang (2020); Schmeling et al. (2020)). One rationalization for underreaction to interest rate news present in some of these papers is that forecasters did not have knowledge of Central Banks' reaction functions, and in particular, underestimated how quickly central banks have been willing to cut interest rates in recessionary periods or following poor stock market performance. While this likely contributes to underreaction, we note that our results suggest that heterogeneous private information also plays an important role in underreaction to news about short-term interest rates over the sample period.

exponential utility is born, indexed by $i \in [0, 1]$. Each investor i receives a noisy private signal about the fundamental in period t,[14] given by

$$x_{it} = \xi_t + u_{it}, \text{ where } u_{it} \sim \mathcal{N}(0, \sigma_u^2).$$

Investors born in period t receive a unit endowment, which they invest. In period $t+1$, each investor i consumes her investment return, passes on her private information to the new investor i born in that period, and dies. Investor i's problem is given by

$$\begin{aligned} & \max_{\alpha^i} -\mathbb{E}_{i,t}(e^{-\gamma c^i_{t+1}}) \\ \text{subject to} \quad & c^i_{t+1} = \alpha^i(-s_{t+1} + s_t - i^d_t) + (1 - \alpha^i)(1 + i_t) \end{aligned} \tag{6}$$

where α^i is her allocation to the foreign bond, and $\mathbb{E}_{i,t}$ captures her subjective expectations. Solving Equation (6), investor i's demand for the foreign bond is

$$\alpha_i = \frac{\mathbb{E}_{i,t}(-s_{t+1}) + s_t - i^d_t}{\gamma \sigma^2_t} \tag{7}$$

where σ^2_t is the conditional variance of next period's exchange rate, which is the same for all investors in equilibrium. Each investor's demand for foreign currency bonds is proportional to her expected returns, which are comprised of two components: expectations of foreign currency appreciation, $\mathbb{E}_{i,t}(-s_{t+1}) + s_t$, and the interest rate differential, i^d_t. Investor i's expectation of currency appreciation depends upon her expectation of next period's exchange rate, which is a function of the foreign currency bond demand of every other investor. Accordingly, higher-order beliefs about other investors' beliefs enter into her and every other investor's demand in equilibrium.

We assume that investors *do not* extract information about fundamentals from the equilibrium exchange rate, s_t, when formulating their demand. In the context of our calibrated model, which treats beliefs reported in surveys as investors' true beliefs, this assumption means that any learning from prices on the part of investors is considered part of their noisy private signals. The assumption that investors do not learn from prices also has other motivations in both the noisy rational expectations literature and the behavioral economics literature.[15]

Additionally, we permit investors' beliefs to deviate from the standard framework in the following way. Investors may perceive ($\hat{\rho}$ and $\hat{\sigma}_\varepsilon$), rather than the true parameter values (ρ, and σ_ε), and all investors share the same (potentially distorted) belief about these parameters. $\hat{\rho} > \rho$

[14] Noisy private signals can be interpreted literally as corresponding with dispersed information (as in Lucas Jr (1972), Morris and Shin (2002)), or as emerging from rational inattention (Mankiw and Reis (2002), Sims (2003, 2010), Woodford (2003)).

[15] This type of assumption is motivated in the noisy rational expectations literature by introducing noise traders or noisy asset supply (Allen et al. (2006)), or corresponding with privately informed investors submitting market orders to a centralized limit order book (as in Bacchetta and Van Wincoop (2006)). An alternative motivation for this assumption from behavioral economics is that investors may be "cursed"; they do not fully appreciate that they can invert prices to learn other investors' information (Eyster and Rabin (2005); Eyster et al. (2019)).

indicates that investors are extrapolative, and believe the interest rate differential process is more persistent than it is in reality, which we find when we calibrate the model. The assumption that investors may perceive $\hat{\sigma}_\varepsilon$ differently than the true σ_ε follows Gourinchas and Tornell (2004), though it plays a different role here. In our calibration, we estimate $\sigma_\varepsilon \approx 0$ and $\hat{\sigma}_\varepsilon > \sigma_\varepsilon$, indicating that investors believe the interest rate differential may deviate from the fundamentals in a transitory way each period. This incorrect belief may stem, for example, from market participants disagreeing with central banks about the state of the economy, and hence the future path of interest rates (e.g., as discussed in Caballero and Simsek (2020)), or relatedly, from investors not understanding central banking authorities' reaction functions. In the context of the model, this assumption provides scope for investors' private information about fundamentals to enter into their valuations, which plays an important role in consensus underreaction to interest rate news.

Defining the precision of the innovations as $\tau_\varepsilon = \sigma_\varepsilon^{-2}$ and $\tau_u = \sigma_u^{-2}$ (with corresponding hatted variables indicating investors' perceived precisions), we can write the investors' perceived processes for variables in the economy as

$$\begin{bmatrix} \hat{i}_t^d \\ \hat{x}_{it} \end{bmatrix} = \begin{bmatrix} \hat{\tau}_\varepsilon^{-1/2} & 0 & \frac{1}{1-\hat{\rho}L} \\ 0 & \tau_u^{-1/2} & \frac{1}{1-\hat{\rho}L} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ u_{it} \\ \eta_t \end{bmatrix}.$$

The market clearing condition for foreign bonds is

$$0 = \int \alpha_i di \propto \int \mathbb{E}_{i,t} \left[-s_{t+1} | \mathcal{I}_{i,t} \right] + s_t - i_t^d \tag{8}$$

which in turn yields

$$s_t - i_t^d = \bar{\mathbb{E}}_t[s_{t+1}] \tag{9}$$

where $\bar{\mathbb{E}}_t$ is the average belief across all agents. Note that because each investor's demand for foreign bonds is linear in her expected returns, the equilibrium condition coincides with the UIP condition holding for consensus expectations, corresponding with the first piece of motivating evidence we present in the paper. Solving for the equilibrium exchange rate reduces to solving for the average expectation of next period's exchange rate across investors.

3.2 Interest Rate Expectations and Forecast Errors in the Model

Before solving the model for the equilibrium exchange rate, we discuss the behavior of interest rate expectations in the model, which are key to understanding the behavior of equilibrium exchange rates. The proofs for this section, and for the rest of the paper, are presented in the appendix.

Proposition 1 (**Investors' Expectations of Fundamentals**). *Investor i's expectation of the fundamental* *in period t is*

$$\mathbb{E}_{i,t}\left[\xi_t\right] = \lambda \mathbb{E}_{i,t-1}\left[\xi_{t-1}\right] + \left(1 - \frac{\lambda}{\hat{\rho}}\right)\xi_t + \frac{\lambda \hat{\tau}_\varepsilon \sigma_\varepsilon}{\hat{\rho}\left(1-\hat{\rho}\lambda\right)}\varepsilon_t + \frac{\lambda \tau_u \sigma_u}{\hat{\rho}\left(1-\hat{\rho}\lambda\right)}u_{i,t}, \tag{10}$$

and the consensus expectation of the fundamental in period t is

$$\bar{\mathbb{E}}_t\left[\xi_t\right] = \lambda \bar{\mathbb{E}}_{t-1}\left[\xi_{t-1}\right] + \left(1 - \frac{\lambda}{\hat{\rho}}\right)\xi_t + \frac{\lambda \hat{\tau}_\varepsilon \sigma_\varepsilon}{\hat{\rho}\left(1-\hat{\rho}\lambda\right)}\varepsilon_t, \tag{11}$$

where λ is defined as

$$\lambda = \frac{1}{2}\left(\hat{\rho} + \frac{1}{\hat{\rho}} + \frac{\hat{\tau}_\varepsilon + \tau_u}{\hat{\rho}} - \sqrt{\left(\hat{\rho} + \frac{1}{\hat{\rho}} + \frac{\hat{\tau}_\varepsilon + \tau_u}{\hat{\rho}}\right)^2 - 4}\right). \tag{12}$$

The expressions for individual and consensus interest rate expectations in Proposition 1 illustrates how frictions enter into investor expectations of future interest rates. Under FIRE, $\lambda = 0$ and $\hat{\rho} = \rho$, and investors hold accurate expectations regarding fundamentals. However, $\lambda \neq 0$ corresponds with information processing frictions entering into investors' beliefs. Investors place a weight of $1 - \frac{\lambda}{\hat{\rho}}$ (their Kalman gain) on the true period t fundamental, but also (imperfectly) incorporate their present and past private signals, as well as past interest rate differentials, into their expectations. At the consensus level, private information cancels out to zero across investors.

To better understand the influence of the frictions we introduce on interest rate forecasts, we focus on the consensus forecast error of interest rate differentials, defined as $FE_{t,t+1} \equiv i^d_{t+1} - \bar{\mathbb{E}}_t\left[\xi_{t+1}\right]$. We can study the consensus forecast error to understand *underreaction* and *overreaction* to interest rate news in the model. In the model, the interest rate news that arrives in period t that is relevant to future interest rates is $\eta_t = \xi_t - \rho\xi_{t-1}$, the persistent shock to fundamentals. A positive relationship between the period t consensus forecast error and news that arrived δ periods previously, $\eta_{t-\delta}$, indicates *underreaction* to the past news, while a negative relationship indicates *overreaction*, using similar logic as the tests we implement in the previous section.

The covariance between the period $t+1$ forecast error and period $t-\delta$ interest rate news, $\eta_{t-\delta}$, is

$$cov(FE_{t,t+1}, \eta_{t-\delta}) = \lambda^\delta + (\rho - \hat{\rho})\frac{\lambda^\delta - \rho^\delta}{\lambda - \rho} \tag{13}$$

Under FIRE, $\lambda = 0$ and $\rho = \hat{\rho}$, so Equation (13) reduces to zero, i.e., forecast errors are unforecastable by past news. More generally, however, Equation (13) tells us that interest rate forecast errors *are* predictable. For $\delta = 1$, Equation (13) reduces to

$$cov(FE_{t,t+1}, \eta_{t-1}) = \underbrace{\lambda}_{\text{Information Frictions}} - \underbrace{(\hat{\rho} - \rho)}_{\text{Extrapolation}}. \tag{14}$$

Ceteris paribus, increased extrapolation ($\hat{\rho} > \rho$) generates *overreaction* to period $t-1$ interest rate news, as $\frac{\partial cov(FE_{t,t+1},\eta_{t-1})}{\partial\hat{\rho}} < 0$. Noisy private information, on the other hand, generates underreaction of the consensus interest rate expectation to news. In particular, increasing the dispersion of investors' private signals (smaller τ_u) or the perceived noise in interest rate differentials relative to fundamentals (smaller τ_ε) increases underreaction to short-term interest rate news, as $\frac{\partial cov(FE_{t,t+1},\eta_{t-1})}{\partial\tau_u} < 0$ and $\frac{\partial cov(FE_{t,t+1},\eta_{t-1})}{\partial\tau_\varepsilon} < 0$. Whether overreaction or underreaction dominates depends upon the relative strength of extrapolation versus investors' informational frictions.

Equation (13) also suggests that interest rate expectations may display initial underreaction (to news that arrived in period $t-\delta$ for small values of δ) and delayed overreaction (for larger values of δ), as we empirically observe in the data. The covariance between past news and forecast errors in Equation (13) has indeterminate sign when investors are extrapolative ($\hat{\rho} > \rho$), and can change sign for different values of δ.

Proposition 2 (Initial Underreaction and Delayed Overreaction). *Interest rate expectations display initial underreaction and delayed overreaction for $\hat{\rho} - \lambda < \rho < \hat{\rho}$ and $\rho \neq \lambda$, where initial underreaction indicates $cov(FE_{t,t+1},\eta_{t-1}) > 0$, and delayed overreaction indicates $cov(FE_{t,t+1},\eta_{t-\bar{\delta}}) < 0$, for some $\bar{\delta} > 1$.*

Proposition 2 formally provides conditions under which consensus interest rate expectations underreact to interest rate news that arrived in the recent past, and overreact to interest rate news that arrived further in the past. In particular, extrapolation ($\hat{\rho} > \rho$) helps to generate overreaction to news, and can co-exist with underreaction to recent interest rate news as long as it is not so strong as to dominate the influence of informational frictions in the model.

Lastly, we can also how individuals and consensus expectations respond to news by analyzing the relationship between forecast errors and forecast revisions at the consensus and individual levels.

Proposition 3 (Individual- and Consensus-level Underreaction). *If $\hat{\rho} - \lambda < p$, then $\beta_{CG} > \beta_{BGMS}$, where β_{CG} and β_{BGMS} are coefficients from regressions of forecast errors on forecast revisions at the consensus and individual levels, i.e.,*

$$\underbrace{\xi_{t+1} - \bar{\mathbb{E}}_t\left[\xi_{t+1}\right]}_{\text{consensus forecast error}} = \alpha + \beta_{CG}\underbrace{\left(\bar{\mathbb{E}}_t\left[\xi_{t+1}\right] - \bar{\mathbb{E}}_{t-1}\left[\xi_{t+1}\right]\right)}_{\text{consensus forecast revision}} + e_{t+1}$$

$$\underbrace{\xi_{t+1} - \bar{\mathbb{E}}_{i,t}\left[\xi_{t+1}\right]}_{\text{individual forecast error}} = \alpha + \beta_{BGMS}\underbrace{\left(\bar{\mathbb{E}}_{i,t}\left[\xi_{t+1}\right] - \bar{\mathbb{E}}_{i,t-1}\left[\xi_{t+1}\right]\right)}_{\text{individual forecast revision}} + e_{i,t+1}$$

Proposition 3 indicates that the same condition that captures initial underreaction in Proposition 2 also means that consensus expectations underreact *more* than individual expectations do, as measured by the relationship between forecast errors and revisions (and studied by Coibion and Gorodnichenko (2015), Bordalo et al. (2020b), and Angeletos et al. (2020b)). The intuition behind this result is simple, and follows the discussion in Bordalo et al. (2020a) and Angeletos et al. (2020b). Individuals each underreact less (or perhaps even overreact) to the information they receive. However, the noise in their signals prevents them from immediately observing, and updating their beliefs in response to, the true news in a given period. Hence, at the consensus level, where noisy private signals cancel out, we observe strong underreaction.

3.3 Exchange Rates in the Model

Investors' demand for foreign bonds, and accordingly the equilibrium exchange rate, depend both on investors' beliefs about the future path of interest rate differentials (captured by their beliefs about fundamentals, ξ_t), and, because of their short investment-horizons, also upon their higher order uncertainty regarding other investors' beliefs. To separately understand the influence of belief biases about future interest rate differentials, and of higher-order uncertainty, we first present a solution for the exchange rate in the absence of higher-order uncertainty, which serves as a benchmark. Then we proceed to the solution for the equilibrium exchange rate in the model.

Proposition 4. *In the absence of higher-order uncertainty, the log equilibrium exchange rate is the consensus expected sum of all future interest rate differentials. This log exchange rate can be expressed as*

$$\begin{aligned}\tilde{s}_t =& i_t^d + \frac{\rho}{1-\hat{\rho}}\bar{\mathbb{E}}_t[\xi_t] \\ =& i_t^d + \frac{\hat{\rho}}{1-\hat{\rho}}\left(1-\frac{\lambda}{\hat{\rho}}\right)\frac{1}{1-\lambda L}\xi_t + \frac{\lambda\hat{\tau}_\varepsilon\sigma_\varepsilon}{(1-\hat{\rho})(1-\lambda L)(1-\hat{\rho}\lambda)}\varepsilon_t.\end{aligned}$$

Proposition 4 is noteworthy for two reasons. First, it describes how (errors in) expected interest rate differentials enter into exchange rates. In the absence of higher-order uncertainty, the exchange rate is the sum of expected interest rates differentials, and accordingly inherits the properties of interest rate forecast errors discussed in the previous section. Extrapolation ($\hat{\rho} > \rho$) will tend to generate overreaction of exchange rates to news, and noisy private information (smaller τ_u and $\hat{\tau}_\varepsilon$) will tend to generate underreaction, just as they do for interest rate expectations. Second, the proposition illustrates that in the absence of higher-order uncertainty, the exchange rate can be written as a sum of this period's interest rate differential, past fundamentals (ξ_t), and past transitory wedges between the interest rate differential and macroeconomic fundamentals (ε_t). This analytical expression is useful for understanding the influence of interest rate expectations and higher-order uncertainty in the model, as we discuss further.

We next present the solution for the unique equilibrium exchange rate in the model. To solve for the exchange rate, we broadly follow the methodology outlined in Huo and Takayama (2018) for solving models with dispersed information and strategic complementarity. Relative to previous work solving similar models, which adapts the solution method in Townsend (1983),[16] this

[16] Townsend (1983) points out that a difficulty in solving these types of models is the 'infinite regress' problem, where, due to the role of higher order beliefs, if an agent believes that other agents keep track of n state variables, she, in turn, must keep track of $n+1$ state variables. Iterating ad infinitum, there is no finite-state representation of the equilibrium policy rule. Townsend (1983) deals with this problem by assuming that information becomes common knowledge after a (small) number of periods, a strategy followed and built upon in other subsequent works, including work on asset pricing (e.g., Singleton (1987) and Bacchetta and Van Wincoop (2006)). However, the infinite regress problem can be

solution method has the advantage of providing an exact analytical solution.

Proposition 5 (Equilibrium Exchange Rate). *The log exchange rate in the model is*

$$s_t = i_t^d + \frac{\hat{\rho}}{1-\hat{\rho}}\left(1-\frac{\vartheta}{\hat{\rho}}\right)\frac{1}{1-\vartheta L}\xi_t + \frac{\hat{\tau}_\epsilon \sigma_\epsilon \vartheta}{(1-\hat{\rho})\,(1-\vartheta L)(1-\hat{\rho}\vartheta)}\varepsilon_t, \tag{15}$$

where ϑ^{-1} is the outside root of the equation

$$\lambda\hat{\rho}k^2 - \hat{\rho}(1+\lambda^2)k + \lambda\tau_u + \lambda\hat{\rho} = 0.$$

Proposition 5 shows that the expression for the equilibrium exchange rate presented in Equation (15) is *exactly* identical to the consensus expectation of the sum of all future interest rate differentials (the exchange rate in Proposition 4), except for the fact that λ is replaced by ϑ, where $\vartheta > \lambda$. As with λ, ϑ also increases with noisy private information (it is decreasing in $\hat{\tau}_\varepsilon$ and τ_u), as we show in the appendix.

The similarity of the expressions for exchange rates in Propositions 4 and 5 suggest that exchange rates by-and-large inherit the properties of interest rate expectations. They similarly may overreact to news due to the role of extrapolation, and underreact due to noisy private information. Because both expressions represent exchange rates as a function of the current interest rate differential, past fundamentals, and past transitory wedges between the interest rate differential and fundamentals, the inclusion of higher order uncertainty primarily influences the speed with which information is incorporated into prices.

In Proposition 4, the coefficient on ξ_t can be expressed as

$$\underbrace{\frac{\hat{\rho}}{1-\hat{\rho}}}_{\text{Perceived Fundamental Persistence}} \times \underbrace{\left(1-\frac{\lambda}{\hat{\rho}}\right)}_{\text{Kalman Gain of Fundamental}} \times \underbrace{\frac{1}{1-\lambda L}}_{\text{Reaction to Older Fundamental News}}. \tag{16}$$

Substituting ϑ for λ can be understood in the context of Equation (16). Because $\vartheta > \lambda$, the Kalman gains from current news are smaller, and the reaction to older news that arrived in the past is stronger. Put differently, higher-order uncertainty induces a more sluggish exchange-rate reaction to news about future interest rate differentials, consistent with the long-standing results found in other work regarding the role of higher-order uncertainty.

To summarize, the equilibrium exchange rate in the model can be thought of as having two drivers: (1) investors' expectations of the sum of all future interest rate differentials and (2) each investor's higher order uncertainty regarding all other investors' (higher-order) beliefs about future interest rate differentials. Because of (1), investors may underreact to recent interest rate news (because of noisy private information), but may also overreact to older interest rate news by overestimating the persistence of interest rate differentials. Higher-order uncertainty in (2)

avoided by transforming the problem into a tractable problem of finding analytic functions. This is the approach taken by Kasa et al. (2014) and Huo and Takayama (2018), the latter whom we follow in our solution.

induces sluggishness of exchange rates to interest rate news, on top of the impact of investors' noisy private signals. The relative importance of interest rate forecast errors versus higher-order uncertainty is an empirical question that we evaluate later in the paper; we find that errors in consensus expectations of interest rates play a substantially larger quantitative role than higher-order uncertainty.

3.4 Calibration

With the model solution in hand, we next turn to calibrating the model. The model has four parameters: ρ, $\hat{\rho}$, σ_ϵ, and σ_u. We calibrate these parameters to match the dynamics of the interest rate process and survey-based forecasts of interest rates and exchange rates. We use quarterly data in the calibration, so that one period in the model corresponds with one quarter.

We calibrate ρ, the persistence of the fundamental shock, to match the persistence of US three-month Treasury Bill rates ($\rho = 0.88$), using data from 1981-2017.

We calibrate the precision of investors' signals, σ_u^{-2}, to match the average cross-sectional dispersion of exchange rate forecasts in the data. In particular, the FX4casts dataset provides data on the 5th and 95th percentile forecasts of the one quarter ahead exchange rate, for each time period and each currency versus the USD. Imposing that the distribution of forecasts is normally distributed, we extract an implied standard deviation of beliefs about exchange rates from the data, which has a one-to-one mapping with σ_u in the model.

We calibrate the remaining two parameters in the model, ($\hat{\rho}$ and $\hat{\sigma}_\epsilon$) to match the impulse response function of forecast errors. In particular, we estimate these parameters by minimizing the weighted distance between the model-implied and empirical impulse response functions of forecast errors to monetary shocks.[17] The minimization problem is given in Equation (17),

$$\min_{\hat{\rho},\hat{\sigma}_\epsilon} \left(\hat{\Theta} - \Theta(\hat{\rho},\hat{\sigma}_\epsilon)\right)' \Omega^{-1} \left(\hat{\Theta} - \Theta(\hat{\rho},\hat{\sigma}_\epsilon)\right) \tag{17}$$

where Ω is a diagonal matrix containing the sample variances of the empirical impulse responses, $\Theta(\hat{\rho},\hat{\sigma}_\epsilon)$ denotes a function that maps the parameters to the model-implied impulse responses, and $\hat{\Theta}$ is a vector of empirical US interest rate consensus forecast errors' impulse responses to monetary shocks. The system is overidentified, as we use impulse responses from 4 to 20 periods after a monetary shock, meaning there are 16 target moments to estimate two parameters.

The calibrated parameters are $(\rho, \hat{\rho}, \hat{\sigma}_\epsilon, \sigma_u) = (0.88, 0.93, 3.1, 3.6)$. With the calibrated parameters in hand, we evaluate how well the model matches our motivating empirical evidence, and find that the model does a reasonably good job.

The first panel of Figure 5 plots the model-implied impulse response function of interest rate forecast errors in response to a monetary shock, computed by simulating 5000 economies for 144 periods and taking the average IRF computed for each simulation, compared with the same impulse response function estimated directly in the data. The model forecast errors capture the pat-

[17] This approach follows Christiano et al. (2005).

tern of initial underreaction and subsequent overreaction reported in our second motivating fact. Despite only using two parameters to capture the dynamics of forecast errors, the model is able to reasonably accurately capture the impulse response function. Almost all points of the model's IRF are inside the one-standard error confidence interval of the IRF estimated from the data.

The second panel in Figure 5 plots the model-implied regression coefficients for regressions of errors in period t forecasts of period $t+3$ interest rate differential on the change in expectations from $t-3$ to t of the period $t+3$ interest rate differential, computed analytically at both at the consensus level and the individual level. We plot the coefficients alongside the same coefficients estimated using SPF forecasts of US Treasury Bill rates, as reported in Figure 4, with positive coefficient values indicating underreaction. As in the data, the model-implied regression coefficients suggest substantially stronger underreaction at the consensus level than at the individual level, indicating that the model also captures our third motivating piece of evidence. Relative to the data, the model-implied coefficients suggest similar levels of underreaction at the consensus level to the amount we estimate in the data, though they suggest slightly less underreaction at the individual level.

We provide some intuition on the behavior of interest rate forecast errors and the ability of the calibrated model to match the motivating evidence, which is also relevant for understanding the behavior of the exchange rate in the model. Consensus forecast errors are initially positive in response to a monetary shock, due to the relatively high values of $\hat{\sigma}_\epsilon$ (the amount of perceived noise driving a wedge between interest rate differentials and fundamentals) and σ_u (the amount of noise in signals investors receive). Each investor's belief only modestly underreacts to the monetary news she observes. However, noise in private signals leads consensus expectations to substantially underreact, because it prevents investors from immediately observing, and updating their beliefs in response to, the 'true' monetary news that arrives in a given period. Over time, investors observe subsequent realizations of interest rate differentials, and consensus expectations adjust to reflect the monetary news that arrived in the past. Eventually, consensus forecast errors switch to being negative, as over-extrapolation of fundamentals ($\hat{\rho} > \rho$) begins to dominate the initial underreaction, and investors believe that the higher interest rate differential will last longer than it does in the data. While the estimated $\rho = 0.88$ implies a very persistent process for the fundamentals that govern the interest rate differential, our estimate of $\hat{\rho} = 0.93$ suggests that, on average, investors believe the fundamental process is even more persistent.

In Figure 6, we illustrate how each of the ingredients of the model contributes to the dynamics of interest rate differential forecasts and forecast errors. In the figure, we plot the impulse response functions of interest rate differential forecasts and forecast errors in the model to a one standard deviation shock to fundamentals in four scenarios: (1) full information rational expectations (FIRE), (2) a version of the model where there is no noise in investors' private signals (all investors' signals correspond with the true fundamental) (3) a version of the model where $\hat{\rho} = \rho$ (there is no extrapolation) (4) the fully calibrated model. Under FIRE, investors perfectly forecast the interest rate differential process, and understand that monetary shocks eventually mean-revert via standard autoregressive dynamics. Introducing noisy private information, the consensus forecast of interest rate differentials underreacts to the shock to fundamentals, and converges to FIRE, but never overreacts (interest rate differential forecasts are never higher than the realized value in the next period). In the third scenario, where investors are extrapolative but do not receive noisy private signals, investors overreact to the shock to fundamentals, and consistently overestimate next period's interest rate differential. The model does not capture the initial underreaction of the consensus interest rate forecast in this scenario. Combining noisy private information and extrapolation in the full calibrated model, the consensus belief about the interest rate differential initially underreacts and then subsequently overreacts, consistent with the data.

4 Exchange Rate Puzzles

4.1 Baseline Model Predictions

With the calibrated model in hand, we next turn to evaluate the model's ability to explain the forward premium puzzle, and the related exchange rate puzzles previously documented in the literature. As we discuss, our baseline model is able to explain the failure of UIP, as well as the delayed overshooting and predictability reversal puzzles, which other models have struggled to simultaneously explain (Engel (2016)).

Prediction 1: The Forward Premium Puzzle and the Exchange Rate Disconnect

We first assess the model's ability to explain the forward premium puzzle. To do so, we simulate the calibrated model 5,000 times for 140 periods. For each simulation, we run Fama (1984) regressions of the form

$$\lambda_{t+1} = \alpha + \beta i_t^d + \epsilon_{t+1} \tag{18}$$

where λ_{t+1} is the excess return of borrowing using foreign currency bonds and lending with home currency bonds in period t, i_t^d is the interest rate differential, and β is the coefficient of interest.

Figure 7 plots the average coefficients from the regressions, alongside regression coefficients reported of the panel regressions of excess currency returns on interest rate differentials from the data (also reported in Figure 1). The full model yields a strong quantitative fit for the UIP regression coefficients. The average coefficient across the simulations is 1.2, which is of a similar magnitude to the regression coefficients we find in the data.

To understand the importance of the various frictions for explaining the failure of UIP in the model, we also perform a similar simulation and regression exercise for versions of the model, turning off some of the frictions in the model. Figure 7 also plots the corresponding regression coefficients. When investors do not have extrapolative expectations, but do receive noisy private information, the failure of UIP persists, and we find similar coefficients in the regression to the full model. However, when investors have extrapolative expectations, but no private information, we obtain coefficients with the *opposite* sign of the data, indicating that currencies with higher interest rate differentials *depreciate* more than implied by UIP, rather than appreciating. And under FIRE, the regression coefficient in the regressions is zero, consistent with UIP holding.

The evidence suggests that underreaction to interest rate news, stemming from dispersed private information, plays the key role in the forward premium puzzle in the model. This can be understood by the fact that a high interest rate differential generally corresponds with a period in which the interest rate differential has either increased, or had increased in a recent past period. Because they underreact to news, consensus expectations only fully internalize that a higher interest rate differential reflects higher *future* interest rate differentials over the course of multiple periods. The exchange rate largely reflects expectations of the future sum of interest rate differentials. Hence, in periods immediately following high interest rate differentials, the exchange rate tends to appreciate, or to depreciate less than the interest rate differential, and investing in the home currency earns positive excess returns, as investors continue to incorporate past monetary news into their valuations.

While we do not endogenize monetary policy in our model for simplicity, the disconnect of exchange rates from other macroeconomic variables related to monetary policy can be understood in the framework of our model. For example, if interest rates are set by banking authorities that follow a Taylor rule, then exchange rates in the model will naturally be disconnected from macroeconomic fundamentals. This is because movements in exchange rates are driven by investor *perceptions* of future interest rates, and not interest rates (and the corresponding macroeconomic fundamentals) themselves. Trading by investors with mistaken beliefs serves to disconnect exchange rates from fundamentals in the same vein as noise traders, as proposed by Jeanne and Rose (2002) and Devereux and Engel (2002), though here, we endogenize the source of the disconnect as coming from traders with systematically incorrect beliefs.

Prediction 2: Delayed Overshooting

As noted by Eichenbaum and Evans (1995), the delayed overshooting puzzle refers to the fact that "a contractionary shock to US monetary policy leads to (1) persistent, significant appreciations in US nominal and real exchange rates and (2) significant, persistent deviations from uncovered interest rate parity in favor of US interest rates." More broadly, this puzzle can be discussed in terms of interest rate differentials. A positive shock to the US interest rate versus foreign interest rates result in appreciation and positive excess returns for the USD versus foreign currencies for several periods after the monetary shock.

Figure 8 plots the model-implied impulse response function of exchange rates, and of the excess returns to borrowing in the foreign currency and lending in the home currency, in response to a one standard deviation shock to the interest rate differential. We compute the impulse response functions analytically and report derivations in the appendix. The exchange rate appreciates for three quarters after a one standard deviation shock to the interest rate differential, after which it begins to depreciate, with positive excess returns associated with the home currency for five quarters after the shock. These patterns are consistent with the delayed overshooting puzzle, and can be drawn in contrast with the behavior we would expect under FIRE in the model. Under FIRE, we expect the home currency exchange rate to appreciate when home interest rates increase relative to foreign interest rates, but the home currency should subsequently depreciate, and there should be no excess returns from investing in the home currency.

Appendix Figure D.1 presents impulse response functions of exchange rates and excess returns in response to a shock to the interest rate differential when turning off different frictions in the model. The figure reveals that noisy private information is the primary driver of delayed overshooting. This is intuitive; noisy private information leads interest rate expectations to underreact to monetary news, which, in turn leads exchange rates to underreact to monetary news. The mechanism for generating this result is nearly identical to the one that generates the forward premium puzzle.

Prediction 3: Predictability Reversal

Documented by Bacchetta and Van Wincoop (2010), the predictability reversal puzzle refers to the fact that, in contrast to the fact that currencies with higher interest rates earn positive excess returns in the short-horizon, currencies with high interest rate differentials earn lower returns after several periods.[18] That is, when running Fama (1984) regressions of the form

$$\lambda_{t+k} = \alpha + \beta_k i_t^d + \epsilon_{t+k} \tag{19}$$

where λ_{t+k} is the excess return from borrowing in foreign currency bonds and investing in home currency bonds from period $t+k-1$ to $t+k$, and i_t^d is the period t interest rate differential, $\beta_k > 0$ for k less than 8 quarters, and $\beta_k < 0$ for k greater than 8 quarters.

In Figure 9, we plot the model-implied values of β_k for various values of k. The model produces the predictability reversal puzzle, with positive values β_k for $k < 5$, and negative values for $k > 5$. In Appendix Figure D.2, we present the β_k coefficients turning off different frictions in the model. Extrapolation plays a crucial role in predictability reversal; in the model with only noisy private information, β_k coefficients are always positive. Again, the model predictions can be drawn in contrast with FIRE, where we expect coefficients to be zero for all values of k.

The intuition behind this prediction of the model is as follows. When the interest rate differential increases, consensus expectations are initially slow to incorporate that this means that future interest rate differentials will also be higher. However, once they internalize this information, consensus expectations reflect the belief that interest rate differentials will remain high for longer than they actually do, because extrapolation leads investors to believe that interest rates are more persistent than they really are. Hence, several periods after the interest rate differential is high, investors tend to overvalue the home currency on average. In subsequent periods, as the interest rate differential turns out to be lower than expected, the home currency depreciates and home currency excess returns are negative, as investors internalize that future interest rate differentials

[18] See also Engel (2016) and Valchev (2020) for additional discussion of predictability reversal.

will not be as high as they thought.

Predictability reversal also manifests in a related way in Figure 8. A positive shock to the interest rate differential also predicts negative returns for a currency more than a year after the shock, suggesting delayed overreaction. Once again, extrapolation is the driver of this feature.

Prediction 4: Time-Series Momentum and Reversal

The first three predictions capture the relationship between exchange rates and interest rates. A related fact is that currencies display time-series momentum and reversal (Moskowitz et al. (2012)). Currency excess returns over the previous twelve months predict monthly currency excess returns (momentum), and currency excess returns from one to five years prior negatively predict currency excess returns.

Figure 10 plots regression betas computed from regressing period t returns on period $t-k$ returns for various values of t, for $k \in \{1, \ldots, 20\}$. The figure reveals strong evidence of time-series momentum and reversal. Excess returns from one to four quarters prior are strongly positively correlated with quarterly returns; and excess returns from more than four quarters prior are negatively correlated with quarterly returns. The especially strong performance of time-series momentum using a look-back period of one quarter, and the return predictability of past returns using lookback periods of up to four quarters are remarkably consistent with the evidence reported by Moskowitz et al. (2012).

Time-series momentum is driven by underreaction from noisy private information in the model, which can be seen in Appendix Figure D.3, where we plot the regression betas when turning off frictions in the model. Extrapolation shortens the horizon of past returns that are positively correlated with present quarter returns, and is also responsible for generating reversals.

Time-series momentum and reversal are natural features of the model, given the relationship between exchange rates and interest rate differentials; beliefs (and higher-order beliefs) about interest rate differentials are the sole driver of exchange rates in the model. Because exchange rates largely reflect the expected sum of future interest rate differentials, increases in expected future interest rate differentials correspond with positive excess returns for the home currency. Consensus expectations are slow to reflect news about future interest rate differentials, so changes in expectations of future interest rate differentials are positively autocorrelated at short-horizons, leading to positive autocorrelations in currency excess returns. At longer horizons, changes in expectations of future interest rate differentials are negatively autocorrelated. This is because extrapolation leads consensus expectations to eventually reflect the belief that an increased interest rate differential will last for longer than it does; this belief is revised downwards in the future when investors eventually observe lower interest rate differentials. In turn, currency excess returns are negatively autocorrelated at longer horizons.

4.2 The Term Structure of UIP Violations

Lustig et al. (2019) study the term structure of UIP violations, and find that is downward-sloping. That is, while it is profitable to borrow at short-maturities in foreign currency bonds to invest in short-maturity US bonds when the US interest rate is higher than foreign interest rates, the one-period return of executing this trade is decreasing with maturity. For example, it is less profitable to borrow with 10-year maturity foreign bonds and invest in 10-year Treasury bonds when US interest rates are high than to execute a similar trade by borrowing and lending at 3-month interest rates. As Lustig et al. (2019) document, leading no-arbitrage models in international finance are unable to match this downward-sloping term structure. We introduce additional bonds of longer maturity into our model in order to understand the model's ability to explain the term structure of UIP violations.

4.2.1 Preliminaries

The structure of the extended model is identical to the baseline model, except for the traded assets. Each country offers n-period maturity zero coupon bonds for $n = 1, \ldots, N$, which each pay off one unit of local currency at maturity and are each in zero net supply. We denote the log price of the n-period home country bond in period t as $p_t^{(n)}$, and the one period return from holding this bond as $r_{t+1}^{(n)} \equiv p_{t+1}^{(n-1)} - p_t^{(n)}$. Starred variables represent the corresponding quantities for the foreign bond (expressed in foreign currency).

Investor i's problem is given by

$$\begin{aligned} &\max_{\boldsymbol{\alpha}^i} -\mathbb{E}_{i,t}(e^{-\gamma c_{t+1}^i}) \\ &\text{subject to} \quad c_{t+1}^i = \sum_{n=1}^{N} \alpha_i^{(n)} r_{t+1}^{(n)} + \sum_{n=1}^{N} \alpha_i^{(n)*} \left(r_{t+1}^{(n)*} - (s_{t+1} - s_t) \right) \end{aligned} \tag{20}$$

where $\boldsymbol{\alpha}^i = \left[\alpha_i^{(1)}, \ldots, \alpha_i^{(N)}, \alpha_i^{(1)*}, \ldots, \alpha_i^{(N)*}\right]^T$ is a $2N \times 1$ vector of her asset allocations, and $\mathbb{E}_{i,t}$ captures her subjective expectations. Solving Equation (20), investor i's allocations are given by:

$$\boldsymbol{\alpha}_i = \frac{\mathbb{E}_{i,t}\mathbf{r_t}\, \Sigma_t^{-1}}{\gamma} \tag{21}$$

where $\mathbf{r_t} = \left[r_{t+1}^{(1)}, \ldots, r_{t+1}^{(N)}, r_{t+1}^{(1)*}, \ldots, r_{t+1}^{(N)*}\right]$ is a $2N \times 1$ vector capturing the returns from investing in each of the available bonds, and Σ is the $2N \times 2N$ covariance matrix of returns (which all investors agree on).

Because each investor has a unit endowment, $1 = \sum_{n=1}^{N} (\alpha_i^{(n)} + \alpha_i^{(n)*})$. We express $\alpha_i^{(1)} = 1 - \sum_{n=2}^{N} \alpha_i^{(n)} - \sum_{n=1}^{N} \alpha_i^{(n)*}$ for each investor.

4.2.2 Equilibrium Exchange Rate and Bond Prices

Equilibrium consists of the market clearing exchange rate, s_t, prices for each home currency bond, $\left\{p_t^{(n)}\right\}_{n=1}^{N}$, and for each foreign currency bond, $\left\{p_t^{(n)*}\right\}_{n=1}^{N}$. The market clearing condition for home country bonds is

$$\begin{aligned} 0 &= \int \alpha_i^{(n)} di \\ &\propto \int \mathbb{E}_{i,t} r_{t+1}^{(n)} - r_{t+1}^{(1)} di \\ &= \bar{\mathbb{E}}_t(p_{t+1}^{(n-1)}) - p_t^{(n)} - i_t \end{aligned}$$

where $\bar{\mathbb{E}}$ is the average expectation across investors. This yields the market clearing prices

$$p_t^{(n)} = \bar{\mathbb{E}}_t(p_{t+1}^{(n-1)}) - i_t \tag{22}$$

The market clearing condition for foreign country bonds is

$$\begin{aligned} 0 &= \int \alpha_i^{(n)*} di \\ &\propto \int \mathbb{E}_{i,t} r_{t+1}^{(n)*} - (s_{t+1} - s_t) - r_{t+1}^{(1)} \\ &= \int \mathbb{E}_{i,t} p_{t+1}^{(n)*} - p_t^{(n-1)*} - (s_{t+1} - s_t) - i_t \end{aligned}$$

yielding the market clearing prices $p_t^{(n)*} = \bar{\mathbb{E}}_t p_{t+1}^{(n-1)*} - i_t - (\bar{\mathbb{E}}_t s_{t+1} - s_t)$. Solving for the local currency price of the one period foreign currency bond yields the expression $s_t - i_t^d = \bar{\mathbb{E}}_t s_{t+1}$, which is exactly the same UIP condition as in the baseline model, and yields the same expression for exchange rates. We use the UIP condition to re-write the market clearing price of the foreign currency bond as

$$p_t^{(n)*} = \bar{\mathbb{E}}_t p_{t+1}^{(n-1)*} - i_t^* \tag{23}$$

To compute bond prices using Equations (22) and (23), we use a recursive computation method that we outline in the Appendix B.3.[19] We use the same calibrated parameters as the baseline model. We analyze the term structure of UIP violations by simulating the model. In each simulation, we simulate two independent economies, a home and foreign economy, where shocks in both economies have the same magnitude, but are scaled such that the distribution of the difference of shocks in the economies matches the distribution of shocks in the baseline model.

[19]The expressions for bond prices in Equations (22) and (23) are the same as those in Barillas and Nimark (2017). The recursive solution we use is similar in spirit to the approach they follow, though our solution method differs.

Prediction 5: The Downward-Sloping Term Structure of UIP Violations

The one period return to a UIP trade that borrows in units of foreign currency and invests in units of home currency using n-period maturity bonds can be written as

$$r^{(n)}_{UIP,t+1} = \underbrace{r^{(n)}_{t+1} - r^{(n)*}_{t+1} + i^d_t}_{\text{Bond Excess Return Differential}} + \underbrace{(s_{t+1} - s_t)}_{\text{Exchange Rate Change}} \tag{24}$$

The returns of the trade consist of two pieces: the excess return differential of n-period maturity bonds, and the change in exchange rates.

Figure 11 plots the model-implied regression coefficient from regressions of the form

$$r^{(n)}_{UIP,t+1} = \gamma^{(n)} + \beta^{(n)} i^d_t + \epsilon^{(n)}_t \tag{25}$$

with $\beta^{(n)}$ on the y-axis, and bond-maturities, n, on the x-axis. The regressions are calculated by simulating the model 5,000 times for 140 periods. The regression coefficients are decreasing with maturity. This captures the downward-sloping term structure of UIP violations found in Lustig et al. (2019) - a higher US short-term interest rate relative to foreign interest rates positively predicts the returns to borrowing in foreign bonds and investing in US bonds, but this predictability is declining in the maturity of bonds used for borrowing and lending.

The exchange rate component of the UIP trades is the same regardless of the maturity of the traded bonds. Hence, the downward-sloping term structure of UIP violations is primarily driven by the fact that the short-term interest rate differential has more return predictability for the home minus foreign return differential of short-maturity bonds versus long-maturity bonds.

Why is this the case? The return predictability of interest rate differentials for home minus foreign bond return differentials primarily stems from dispersed private information, as we observe in Appendix Figure D.4. When interest rate differentials increase, consensus expectations are slow to internalize that they will remain high in the short-term future. In subsequent periods, the consensus belief adjusts to reflect the high interest rate differential. As this happens, the returns of the higher interest rate home currency bonds exceed the returns of the lower interest rate foreign currency bonds. However, this effect is weaker for long maturity bonds. This is because the prices of longer maturity bonds are less sensitive to movements in the consensus belief about short-term interest rate differentials than the prices of shorter maturity bonds, as investors expect interest rate differentials to revert to the mean over long horizons.[20] This results in more muted return predictability for interest rate differentials for UIP trades in long-maturity bonds.

We can contrast the model predictions with the predictions under FIRE. When investors all have accurate beliefs about the current interest rate differential and the future path of interest rate differentials, there is no excess return predictability in bonds of any maturity. The term structure

[20] While long maturity bonds are *less* sensitive than short maturity bonds to short rate news, long maturity bonds in the model do still display excess sensitivity to short rate movements, consistent with the empirical evidence (see, e.g., Gürkaynak et al. (2005) and Giglio and Kelly (2018)). This is because of extrapolation.

of excess returns for the UIP trade is flat at zero.

4.3 Taking Stock and Comparisons with Other Work

Our model's ability to explain exchange rate behavior stems from connecting the pattern of initial underreaction and delayed-overreaction of consensus expectations about interest rate news to exchange rates. Underreaction to interest rate news drives the forward premium puzzle, the delayed overshooting puzzle, and time-series momentum, and delayed overreaction is responsible for reversals of exchange rates.

The idea that incorrect beliefs about interest rates may be at the heart of some exchange rate puzzles is shared with Gourinchas and Tornell (2004) and Molavi et al. (2021). Distinct from the sources suggested in these other works, the primary source of underreaction in our model is dispersed private information, which is uniquely consistent with the survey evidence that underreaction to short-rate news is primarily a feature of consensus expectations and not individual expectations.[21] A secondary source of underreaction comes from higher-order uncertainty. As discussed by Woodford (2003) and Morris and Shin (2006), higher-order beliefs adjust more sluggishly than first-order beliefs.[22] This can be elucidated concretely in our setting, where an investor may believe the interest rate differential will be high next period, but uncertainty about whether or not *other* investors will agree attenuates the investor's belief about next period's exchange rate appreciation, and causes her to temper her demand for home versus foreign bonds. Each investor is influenced by this higher order uncertainty, further contributing to the sluggish response of exchange rates to monetary news.

A model that only features underreaction, such as the one in Gourinchas and Tornell (2004), is unable to capture the delayed overreaction that is reflected in currency excess returns, in the form of the predictability reversal puzzle and negative autocorrelations of currency excess returns at long horizons. This delayed overreaction emerges due to extrapolation in our model.[23]

In addition to patterns of initial underreaction and delayed overreaction of exchange rates, our model also matches the downward-sloping term structure of UIP violations. This result is also primarily driven by underreaction to interest rate news. Because interest rate differentials follow an AR(1) process, investors expect interest rate differentials to mean-revert over longer horizons. This

[21] Gourinchas and Tornell (2004) study a model in which all investors are homogeneous, and each investor *individually* underreacts to interest rate news. Molavi et al. (2021) simultaneously capture underreaction and overreaction by assuming that exchange rates follow an ARMA(p,q) process. They embed investors that are only able to perceive one factor of the data generating process, due to limited information processing capacity, into an economy where all other investors having rational expectations. In our model, the patterns of initial underreaction and delayed overreaction simultaneously occur even when exchange rates follow a simpler AR(1) process.

[22] This idea is also discussed in more detail, for example, in Kasa et al. (2014) and Angeletos and Huo (2021).

[23] The mechanism for generating initial underreaction and delayed overreaction in our model bears some resemblance to Hong and Stein (1999), who present a model populated by 'news-watchers' who receive noisy private information, and 'momentum traders' who trade using univariate strategies based on past price trends. However, our model differs in that all investors are ex-ante homogeneous, and extrapolate fundamentals (here, the level of the interest rate differential), rather than trading based on past price changes. This difference is manifested, for example, in our model's ability to capture patterns in the term structure of asset prices, which is not a result that can be captured by only extrapolating past price changes.

means underreaction of investor beliefs to news about short-term interest rate differentials plays a larger role for return differences across short-maturity bonds than for return differences across long-maturity bonds. One additional point is that in our model, the difference in returns between short- and long-maturity bonds transpires entirely because of changes in expectations of future short rates; there are no *term premia*. Accordingly our model presents a distinct, but complementary explanation for the downward-sloping term structure of UIP violations to Greenwood et al. (2020) and Gourinchas et al. (2021), who focus on the potential role of term premia for explaining the facts.[24]

Our model is complementary to, but distinct from, another mechanism that has been proposed to explain the puzzles of interest to us, delayed portfolio adjustment - the idea that investors may only rebalance their portfolios in response to information with a delay. Bacchetta and Van Wincoop (2010, 2021) suggest that delayed portfolio adjustment might explain the failure of UIP, predictability reversal, delayed overshooting, and the downward-sloping term structure of UIP violations. Our model has the advantage of being consistent with evidence from survey data, but does implicitly assume that expectations reported in surveys coincide with the actual beliefs of market participants, and that market participants dynamically trade based on these beliefs. In comparison, as Bacchetta and Van Wincoop (2021) note, empirically verifying the existence of delayed portfolio adjustment is difficult. Giglio et al. (2021) combine data on the portfolios of individual investors with survey evidence of those investors. They find that changes in beliefs have a weak relationship with the decision to trade; but, conditional on trading, trades are executed in the direction of changes in beliefs. This evidence suggests that both errors in expectations, as well as the types of portfolio rebalancing frictions suggested by Bacchetta and Van Wincoop (2010, 2021), may play complementary and important roles in explaining the facts.

Our evidence that underreaction to monetary news might be a driver of time-series momentum in exchange rates is related to a similar idea present in Brooks et al. (2019). Analyzing the return response of bonds and government bond funds to FOMC announcements, they find evidence consistent with return predictability stemming from initial underreaction followed by delayed overreaction occurring over the same time-horizon as time-series momentum in bond markets. Our work adds additional evidence to suggest that monetary news might be a driver of time-series momentum *across* asset classes.

[24]In particular, Greenwood et al. (2020) and Gourinchas et al. (2021) focus on specialized bond investors that absorb demand for bonds (as in Vayanos and Vila (2021)), and also absorb currency risk (as in Gabaix and Maggiori (2015)). The downward-sloping term structure of UIP violations emerges in these models because, relative to the short-maturity UIP trade, the currency exposure of global bond investors in the long-maturity UIP trade helps offset the interest rate risk the investors face in long-term bonds. Greenwood et al. (2020) and Gourinchas et al. (2021) also use their models to understand the relationship between exchange rates, term premia, and policies that affect term premia such as quantitative easing.

5 Other Implications of the Model

In addition to major exchange rate puzzles, we also explore some other empirical facts through the lens of our model. We find that the frictions we introduce, and particularly our focus on deviations from FIRE in investor beliefs, are helpful for understanding a number of additional facts in the data.

5.1 Macroeconomic News and Exchange Rate Expectations

One of the puzzles in the international finance literature is the exchange rate disconnect: the seeming lack of relationship between macroeconomic fundamentals and exchange rates in the directions predicted by standard models (Meese and Rogoff (1983)). But exchange rates do appear to strongly co-move with survey-based measures of macroeconomic news, in the directions predicted by standard models (Engel et al. (2007), Stavrakeva and Tang (2020b)).

We illustrate the explanatory power of survey-based measures of macroeconomic news via contemporaneous regressions of the form

$$y_{t,j} = \alpha_j + \beta \left(\text{macro news}_{t,j}\right) + \epsilon_{t,j} \tag{26}$$

where $y_{t,j}$ is a variable capturing the exchange rate of country j versus the USD, and β is the coefficient of interest. For y_t, we use two measures: changes in exchange rates ($s_t - s_{t-1}$) and changes in the consensus expectation of future exchange rates ($\bar{\mathbb{E}}_t s_{t+1} - \bar{\mathbb{E}}_{t-1} s_{t+1}$). We use three measures of macroeconomic news: interest rate revisions $\left(\bar{\mathbb{E}}_t i^d_{t+1,j} - \bar{\mathbb{E}}_{t-1} i^d_{t+1,j}\right)$, interest rate surprises $\left(i^d_{t,j} - \bar{\mathbb{E}}_{t-1} i^d_{t,j}\right)$, and a macroeconomic surprise index $\left(\sum_{i=1}^{n} \omega_{k,j}\left(x_{k,t,j} - \bar{\mathbb{E}}_{t-1} x_{k,t,j}\right)\right)$, where $x_{k,t,j}$ is a macroeconomic series k captures GDP, Employment, and Industrial Production information for country k, and $\omega_{k,j}$ is a weight computed to give equal weight to GDP, employment and IP news. Interest rate differential data are from FX4Casts and macroeconomic surprise data are from Bloomberg. Observations are quarterly, and independent variables are scaled to have zero mean and unit standard deviation, and signed such that we expect a positive coefficient in the regressions (e.g., an increase in the interest differential is signed positively, corresponding with the expected contemporaneous increase in the exchange rate).

Figure 12 plots the regression coefficients from the regressions, with the 95% confidence intervals. Across each of the regressions, we find significant evidence that exchange rates, and the beliefs of market participants about future exchange rates, move in the predicted directions. A one standard deviation change in the macroeconomic surprise index corresponds with a 30 basis point quarterly change in exchange rates, while a one standard deviation short-rate forecast revision or surprise move exchange rates by 50 to 60 basis points.[25]

Our proposed framework is useful for simultaneously reconciling the exchange rate discon-

[25] Using similar data and a VAR approach, Stavrakeva and Tang (2020b) find macroeconomic surprises may explain explain nearly half of nominal exchange rate variation.

nect with the fact that exchange rates appear to co-move with survey-based measures of macroeconomic news. In particular, in our model, investors have the 'correct' belief in mind regarding the relationship between macroeconomic variables and exchange rates (consistent with the evidence in Figure 12). However, investors make systematic errors in appropriately processing macroeconomic news, leading to predictable forecast errors and the exchange rate disconnect.[26]

Consider interest rate differentials: the standard prediction of macroeconomic models is that a country's exchange rate should contemporaneously increase when the interest rate differential increases. In our model, because beliefs deviate from FIRE, a disconnect emerges because consensus forecasts of the interest rate differential, and correspondingly, exchange rates, do not move *sufficiently* when news arrives. This leads interest rate differentials to predict subsequent exchange rate movements in the wrong direction. Additionally, an especially strong contemporaneous relationship between survey-based interest rate 'surprises' and exchange rates arises in our model. This is because survey-based surprises measures include both unpredictable news that moves exchange rates and expectations, as well as a predictable component that reflects investor beliefs not having sufficiently moved in the previous period. When predictable forecast errors occur, investors update their beliefs (in a predictable way) and trade, further driving the relationship between survey-based measures of news and exchange rates.

Though we do not explicitly microfound monetary policy, a broad interpretation of our model's logic also explains the link between exchange rates, and surprises for non-interest rate variables. For example, interpreting ξ_t as the fundamental level of the interest rate differential implied by macroeconomic fundamentals (e.g., when monetary policy is determined by a Taylor rule), we expect surprises in macroeconomic variables, such as those in the macroeconomic surprise index we study, to be related to exchange rate movements.

5.2 Persistence of Subjective Beliefs and Belief Convergence

A notable fact in survey data of individual investors is the persistence of subjective beliefs. Optimists are persistently optimistic, and pessimists are persistently pessimistic (Giglio et al. (2021)).

This feature arises in our model due to the persist impact of private information. A private signal received in a given period remain important for an investor's beliefs for several periods. To better understand this feature in the model, we simulate the calibrated model 5,000 times and record the beliefs of 1000 investors in the model in each period in each simulation. We rank investors based on their beliefs about the fundamental ξ_t in each period. Each investor's expected interest rate differential, and expected returns from borrowing in foreign bonds and purchasing home bonds, are determined by ξ_t, so this ranking also ranks investors on the basis of their beliefs about fundamentals and expected returns.[27]

[26] This framework is also consistent with the empirical evidence presented, for example, in Engel et al. (2021), suggesting that errors in understanding monetary policy may be useful for understanding exchange rate behavior.

[27] Here, we refer to investor i as being the same investor over time, given that i's belief is based on the sequence of signals observed by all agent i's in the past.

The first panel in Figure 13 takes investors ranked in the top and bottom quartiles based on their beliefs in period zero, and plots the average percentile rank of these investors in subsequent periods. For example, in period one, the figure plots the average percentile rank of investors whose beliefs ranked in the top quartile in period zero, and also plots the average percentile rank of investors whose belief ranked in the bottom quartile in period zero. A value of 0.5 indicates that average belief of investors in a particular group are at the average of the overall population, and values greater than or less than 0.5 indicate that the average investor in the group has a higher or lower belief about ξ than the average investor in the population. The panel reveals that there is substantial persistence of individual beliefs. It takes more than two years for the average belief of investors in the top and bottom quartiles of the belief distribution in period zero to converge to the average belief of the population.

To better understand the driver of the dynamics of beliefs, the second panel in Figure 13 plots the model-based impulse response function of subjective expectations of the interest rate four periods ahead, $\mathbb{E}_{i,t-4} i_t^d$, in response to a one standard deviation shock to the fundamental, η_t, and in response to a one-standard deviation private information shock, $u_{i,t}$. η shocks are commonly observed across all agents, and influence expectations of interest rate differentials for several quarters, consistent with the high degree of persistence of interest rate differentials. Private information shocks are specific to individual investors, and drive disagreement. A one standard deviation private information shock initially influences an investor's beliefs more than a one standard deviation shock to fundamentals; however, the importance of the private information shock fades bit more quickly. Private information shocks cease to become important after 15 quarters. This is consistent with time it takes the beliefs of optimists and pessimists to converge towards the average belief in the first panel.

How does the degree of persistence of individual beliefs in the model compare with the data? Alongside the model-generated values, Figure 13 also plots corresponding values based on interest rate forecasts in the SPF data. The model based values track the values in the data remarkably well. However, notably, in the SPF data, even after five years, the average belief of period-zero optimists and pessimists does not converge completely towards the population average.

Our results suggest that noisy private information may play an important role in explaining the persistence of individual beliefs and disagreement. However, we also highlight that while our calibrated model captures one dimension of the persistence of beliefs, it does not capture other features of the persistence of subjective beliefs that have been documented elsewhere. Giglio et al. (2021) document that individual fixed effects explain a substantial amount of belief disagreement about stock market returns. This is true in our model in short samples, but does not hold over longer samples, given that our model predicts the average beliefs of optimists and pessimists eventually converge to the population average. This difference may stem from a few different sources. First, we focus on beliefs about interest rate differentials, which may be more fast-moving than investors' beliefs about stock market returns in Giglio et al. (2021). Second, our focus is on survey-based expectations of professional forecasters, while Giglio et al. (2021) study survey-based expectations of retail investors; it is possible that beliefs may be slower moving for the latter group versus the former. Third, other features that we do not capture in our model, for example the importance of individual experiences for beliefs, may be highly relevant for explaining the importance of individual fixed effects in ways that our model does not capture.[28]

5.3 Exchange Rates During and After the Financial Crisis

Recent empirical work has documented that the time-series relationships between exchange rates and interest rate differentials has substantially attenuated, and even reversed, in developed markets in recent years, when interest rates have been at the zero lower bound (Bussiere et al. (2018), Engel et al. (2021)). We more closely study some of the empirical evidence, and seek to understand the facts through the lens of our model.

We follow Bussiere et al. (2018) and regress realized and forecasted monthly excess returns on the lagged interest rate differential from September 2006 through the end of our sample for each country. Panel A of Table 1 reports the average coefficients across countries, and also reports results separately for developed and emerging markets. Consistent with Bussiere et al. (2018), with realized excess returns as the dependent variable, we find a negative coefficient for developed market countries (-1.94). For emerging market countries, we find a small and positive coefficient (0.19), and the average coefficient across all countries is -0.61. These coefficients are substantially lower than the full sample coefficients of 1.46, 1.20, and 1.34 for developed, emerging, and all countries reported in Figure 1. The coefficients can also be drawn in contrast to the average coefficients where forecasted excess returns are the dependent variable; these coefficients are 0.70, 0.66, and 0.73 for all, developed, and emerging countries. The results indicate that the realized relationship between currency excess returns and interest rate differentials is more negative than the forecasted relationship. Put differently, they suggest that forecast errors of exchange rates are negatively related to interest rate differentials in recent times, while they were previously positively related.

How can our model help understand these facts? In our model, the reason we have positive coefficients in the UIP regressions (and more positive coefficients where realized rather than forecasted excess returns are the dependent variable) is that dispersed private information leads consensus expectations of interest rates, and hence, exchange rates, to *underreact* to interest rate news. As captured by Figures 6 and 7, when we reduce the dispersion in private information in the model, investors *overreact* to interest rate news rather than underreacting, as extrapolation of the level of interest rate differentials dominates (Figure 6), and the coefficient in the UIP regression flips sign to being negative (Figure 7). Therefore, our model may rationalize the changing sign of

[28] Malmendier and Nagel (2011) document the role of experience effects in individual beliefs about stock market returns and portfolio allocations to the stock market. The importance of past experiences for belief formation about financial variables may be grounded in psychological evidence, particularly the availability heuristic (Tversky and Kahneman (1974)), the tendency of people to overweight information that is most readily 'available' to them when making forecasts. Barberis and Jin (2021) suggest that a commonly used framework in psychology and neuroscience based on "model-free" and "model-based" learning can help capture the role of individual fixed effects in explaining variation in investor beliefs.

the UIP regression in recent times, if investors have begun to overreact to news about short-term interest rates, stemming from reduced dispersion of beliefs about interest rate differentials.

We find suggestive evidence of overreaction to interest rate news in the post-GFC sample, with less dispersion in beliefs reported in surveys. Panel B of Table 1 reports the average regression coefficient from regressing consensus forecast errors of interest rate differentials on the previous period's forecasted revision, using quarterly data from FX4casts for each country from September 2006 through December 2019. Regression coefficients are consistently negative in these regressions (-0.03 for Developed, and -0.06 for Emerging and all countries), consistent with consensus forecasts overreacting to interest rate news in the later part of the sample.

In addition to observing overreaction to news about interest rate differentials in the later part of the sample, there is also less dispersion of beliefs about interest rates in the later part of the sample as well. The first panel of Figure 14 plots the cross-sectional standard deviation of forecasts for 1-, 2-, and 3-quarters ahead of the Treasury Bill rate, from the Survey of Professional Forecasters. For each horizon of forecast, we observe a particularly low dispersion in forecasts of Treasury Bill rates in the post-financial crisis period. The second panel in the figure plots the average cross-sectional standard deviation of forecasts for the next policy rate announcement for developed and emerging countries in the sample, using data from Bloomberg from 2000 through the end of our sample. Here, again, we observe lower dispersion in beliefs, in both emerging and developed markets.

Taken together, our model and these additional facts suggest a potential resolution to the behavior of exchange rates in the post financial crisis period, when US and other developed market interest rates have been at the zero lower bound - there is less dispersion in information about interest rates. Accordingly, interest rate forecasts and exchange rates may overreact to interest rate news, leading the relationship between interest rate differentials and subsequent currency excess returns to reverse.

This analysis also speaks to a point raised in Engel et al. (2021); when running UIP regressions, the coefficients in the regressions tend to vary over time and across different countries. While the focus of this paper is not to dig more deeply into this idea, our results do suggest that the relative magnitude of public information about interest rates, which investors may overreact to, versus private information, which consensus expectations may underreact to, may be useful for better understanding time-variation in the relationship between interest rate differentials and currency excess returns.

6 Conclusion

In this paper, we propose an explanation for the failure of UIP and related exchange rate puzzles, motivated by three facts from surveys of professional forecasters and market participants. First, despite the failure of UIP, we find that market participants report forecasts of exchange rates that are closely aligned with UIP. Second, consensus forecasts of interest rates and interest rate differentials appear to initially underreact, and subsequently overreact to monetary news. And third, the underreaction of forecasts to interest rate news is primarily a feature of consensus forecasts, and is substantially muted when we analyze individual forecaster level data.

We propose a parsimonious model that matches the facts that we document in survey data, with investors who each extrapolate the level of interest rates and receive noisy private signals about interest rates. We find that the model can qualitatively and quantitatively match a number of facts in the data, such as the failure of UIP, patterns of underreaction and overreaction of currencies in response to interest rate news (the delayed overshooting and predictability reversal puzzles), the positive autocorrelation of currency excess returns at short horizons (time-series momentum), the negative autocorrelation of currency excess returns at longer horizons (reversal), and the fact that the profitability of borrowing in foreign currency bonds and investing in US bonds when the US interest rate differential is high is decreasing in the maturity of bonds used to borrow and lend (the downward-sloping term structure of UIP violations). Our model is also helpful for understanding the strong contemporaneous relationship between survey-based measures of macroeconomic news and changes in exchange rates *despite* the weak relationship between macroeconomic fundamentals and exchange rates, the persistence of subjective beliefs, and the seeming reversal of the relationship between exchange rates and interest rate differentials in recent times.

We conclude with some thoughts on further directions for work suggested by our analysis. Our paper highlights dispersed private information about the future path of interest rates as playing an important role in explaining exchange rate puzzles. But we do not take a stance on the source of this dispersed private information. A deeper understanding and analysis of *when* and *why* investors disagree about interest rates may help us further understand patterns in exchange rates. Such an understanding of the nature of dispersed information can be applied more broadly, towards further understanding the well-established but still puzzling fact that asset prices sometimes appear to underreact to information and sometimes appear to overreact to information.

Tables and Figures

FIGURE 1: UIP REGRESSIONS USING REALIZED AND EXPECTED CURRENCY RETURNS

The figure presents regression coefficients from two sets of panel regressions: (a) $\lambda_{j,t+1} = \alpha_j + \beta i^d_{j,t} + \epsilon_{j,t+1}$ and $\bar{\mathbb{E}}_t \lambda_{j,t+1} = \alpha_j + \beta i^d_{j,t} + \epsilon_{j,t+1}$, where $\lambda_{j,t+1}$ are the excess returns from borrowing in currency j and lending in USD, $\bar{\mathbb{E}}_t \lambda_{j,t+1}$ is the expected excess return measured using consensus forecasts of the period $t+1$ exchange rate, and $i^d_{j,t}$ is the short-term interest rate differential between the US and country j at period t. The sample is from August 1986 to through December 2019.

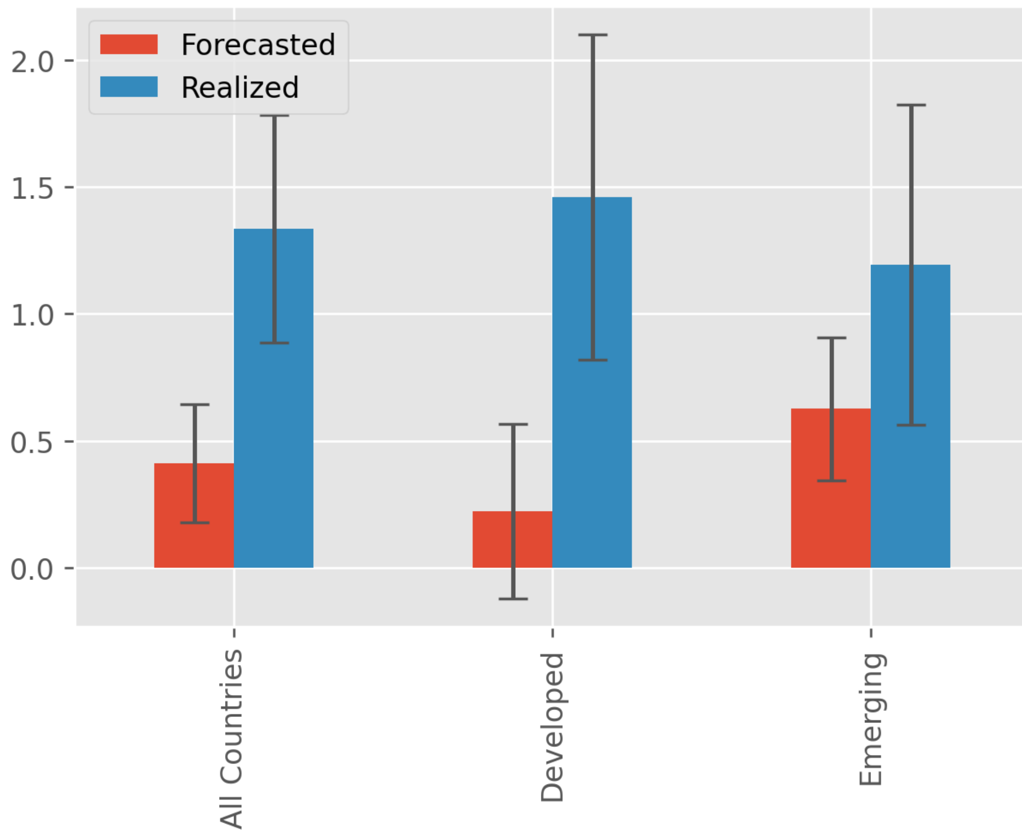
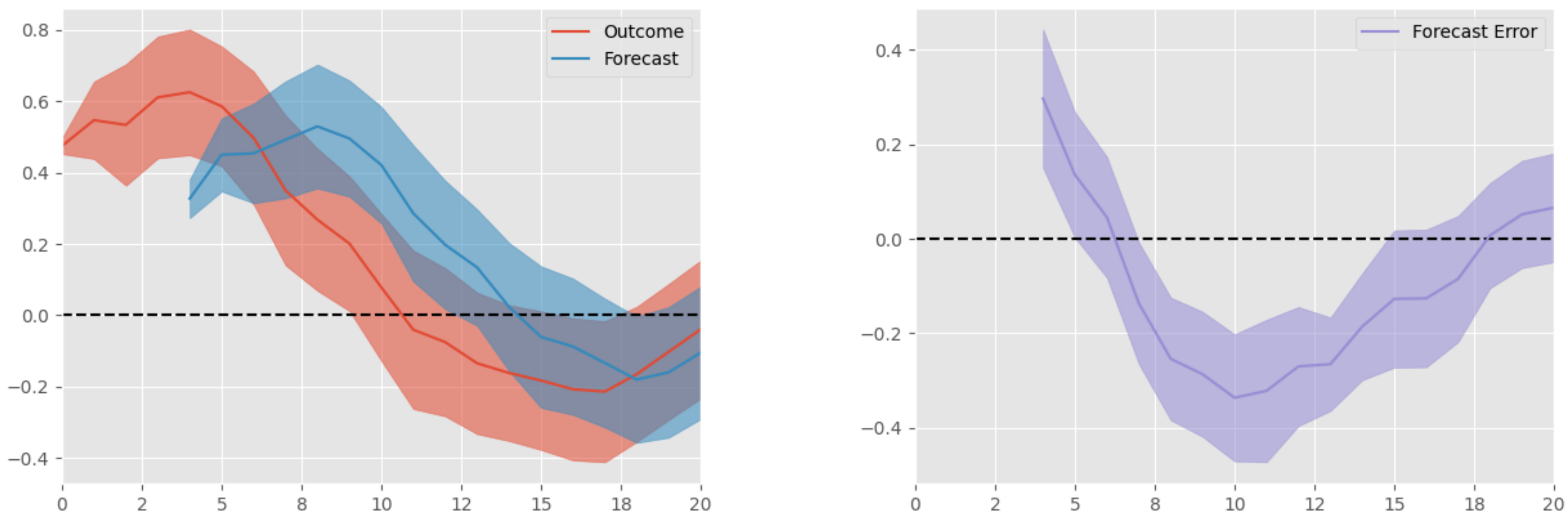


FIGURE 2: UNDERREACTION AND OVERREACTION IN US INTEREST RATE RESPONSE TO MONETARY SHOCKS

The figure plots impulse response functions (IRFs) of US Treasury Bill rates, US Treasury Bill rate consensus forecasts, and US Treasury Bill rate consensus forecast errors in response to monetary shocks. The IRFs are estimated from regressions of the form $x_{t+h} = \alpha_h + \beta_h \epsilon_t + \gamma C_t + u_{t+h}$, where $x_{t+h} \in (i_{t+h}, \bar{\mathbb{E}}_{t+h} i_{t+h+k}, i_{t+h+k} - \bar{\mathbb{E}}_{t+h} i_{t+h+k})$, C_t are lagged values of forecasts and outcomes used as controls, and ϵ_t are the estimated monetary shocks. The estimated monetary shocks come from Angeletos et al. (2020a). Forecast data are from the Survey of Professional Forecasters and the sample runs from 1981 to 2017.

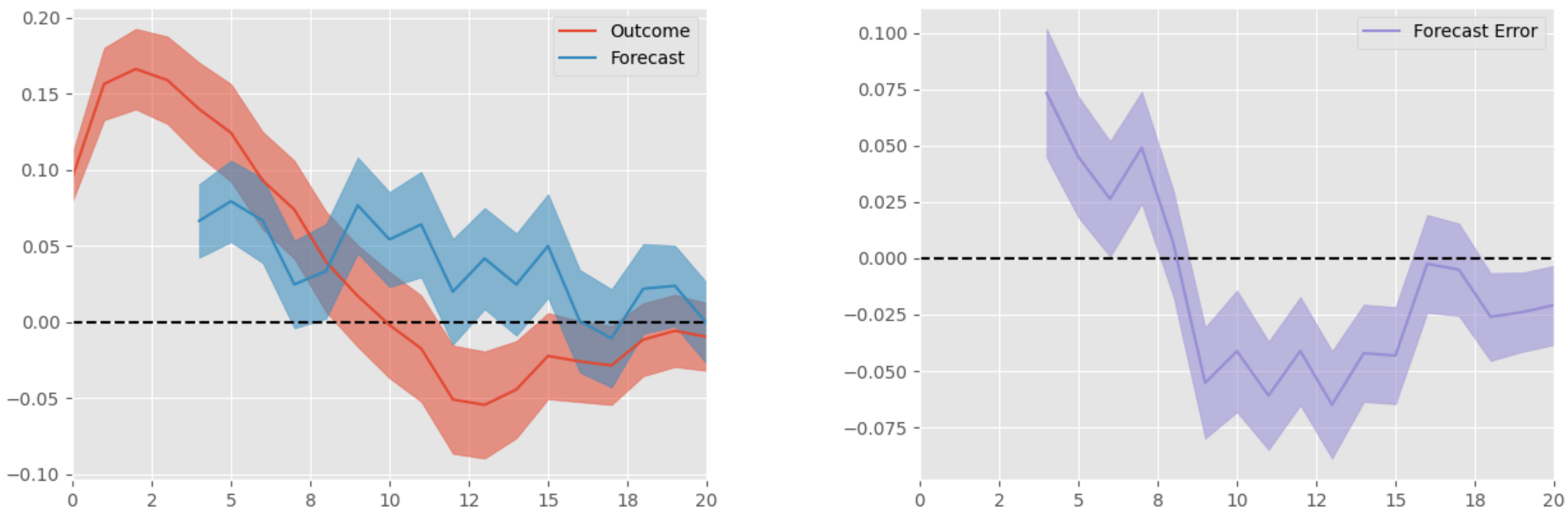


(a) US interest rate IRF to monetary shock

(b) US interest rate forecast errors IRF to monetary shocks

Figure 3: Underreaction and Overreaction in Interest Rate Differential response to Monetary Shocks

The figure plots impulse response functions (IRFs) of interest rate differentials, interest rate differential consensus forecasts, and interest rate differential forecast errors between the US and international countries, in response to monetary shocks. The IRFs are estimated from regressions of the form $x_{j,t+h} = \alpha_{j,h} + \beta_h \epsilon_t + \gamma C_{j,t} + u_{j,t+h}$, where corresponds to country j, $x_{j,t+h} \in (i_{j,t+h}, \bar{\mathbb{E}}_{j,t+h} i_{j,t+h+k}, i_{j,t+h+k} - \bar{\mathbb{E}}_{j,t+h} i_{j,t+h+k})$, $C_{j,t}$ are lagged values of forecasts and outcomes used as controls, and ϵ_t are the estimated monetary shocks. The estimated monetary shocks come from Angeletos et al. (2020a). Data on interest differential forecasts are from FX4casts. The sample consists of 27 countries, with data from October 2001 to December 2019.

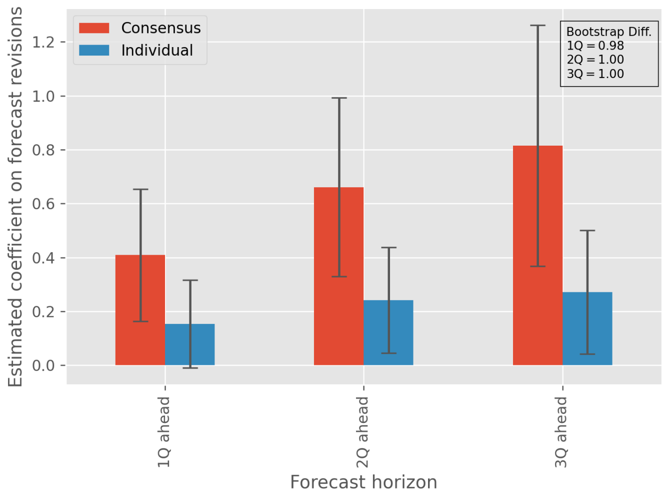


(a) Interest rate differential IRF to monetary shock

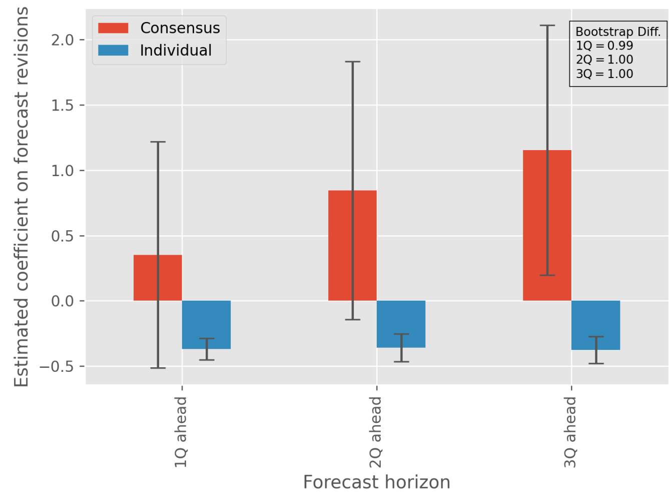
(b) Interest rate differential forecast errors IRF to monetary shocks

FIGURE 4: CONSENSUS VS. INDIVIDUAL DEVIATIONS FROM FIRE

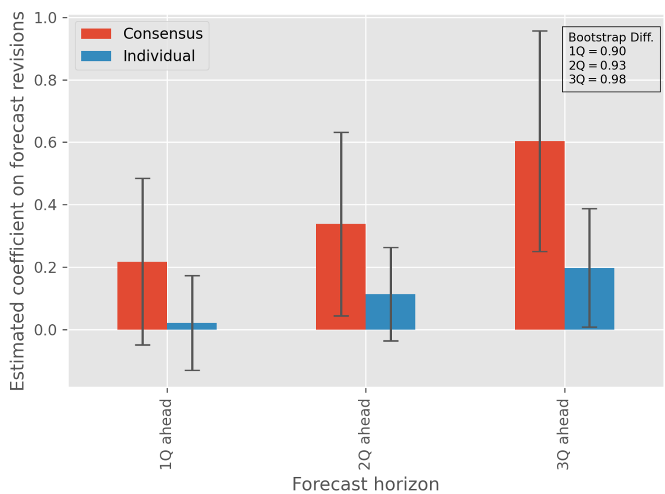
The figure reports regression coefficients from regressions of the form $x_{t+k} - \mathbb{E}_t x_{t+k} = \alpha + \beta_{CG}(\mathbb{E}_t x_{t+k} - \mathbb{E}_{t-k} x_{t+k}) + \epsilon_{t+k}$, where x_{t+k} is the realized outcome, and $\mathbb{E}_t x_{t+k}$ is the forecast at period t of the realized outcome at period $t+k$. The red bars correspond with regressions where observations correspond with consensus forecasts and the blue bars correspond with regressions where observations correspond with individual forecasts. Standard errors for panel regressions are two-way clustered by forecaster and time period and reported in parentheses. The sample consists of quarterly observations from 1969 through 2018 from the Survey of Professional Forecasters.



(a) Unemployment (SPF)



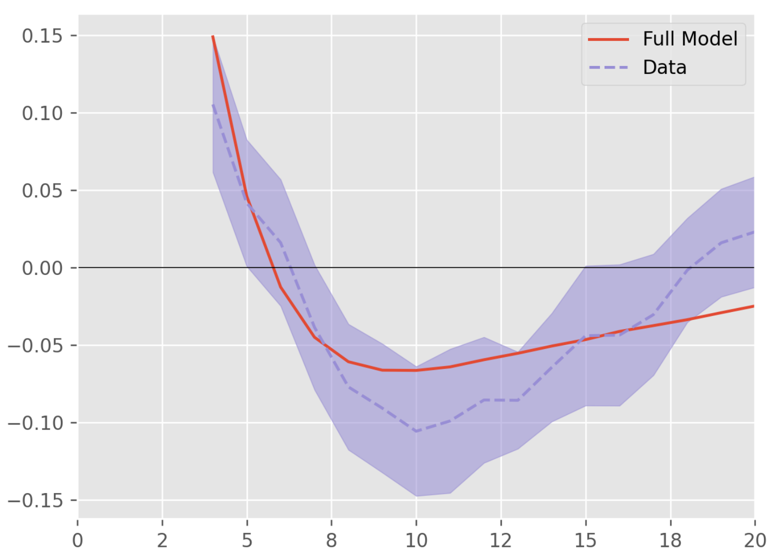
(b) Inflation (SPF)



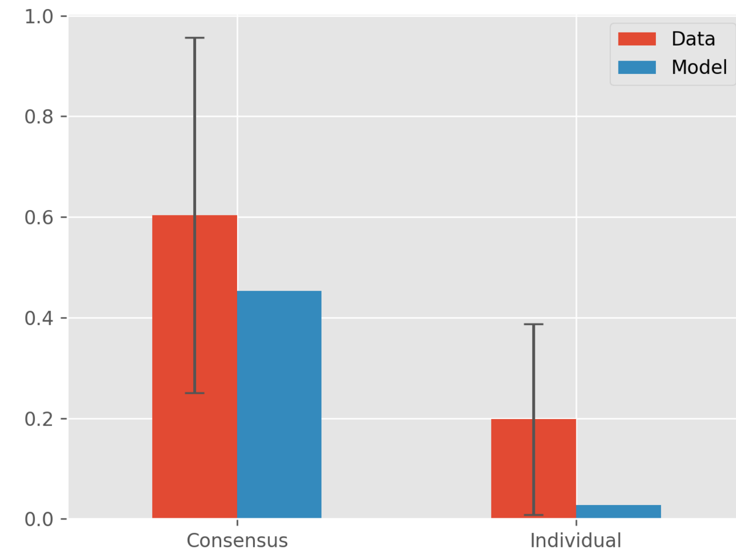
(c) T-Bill (SPF)

FIGURE 5: MODEL CALIBRATION OF INTEREST RATE FORECAST ERRORS

The figure displays information about the model calibration of interest rate forecast errors. The first panel plots impulse response functions (IRFs) of forecast errors generated by the calibrated model (Full Model) and compares it with the empirical IRF (Data). The Full Model IRF is computed by simulating 5,000 economies for 144 periods, computing the IRF for each simulated economy, and computing the average across each simulation. The second panel plots regression coefficients from regressions of the form $x_{t+3} - \bar{\mathbb{E}}_t x_{t+3} = \alpha + \beta_{CG}(\bar{\mathbb{E}}_t x_{t+3} - \bar{\mathbb{E}}_{t-3} x_{t+3}) + \epsilon_{t+3}$, where x is the variable of interest, F captures (subjective) expectations, and each time period corresponds with one quarter. The Data bars, in red, corresponds with regression coefficients estimated using US Treasury Bill rate forecasts from the Survey of Professional Forecasters from 1969 through 2018. The Model bars, in blue, correspond with regression coefficients implied by the model calibration for the interest rate differential. The panel presents regression coefficients where observations are at the consensus forecast level (averaged across individuals), as well as regression coefficients where observations are at the individual forecast level.



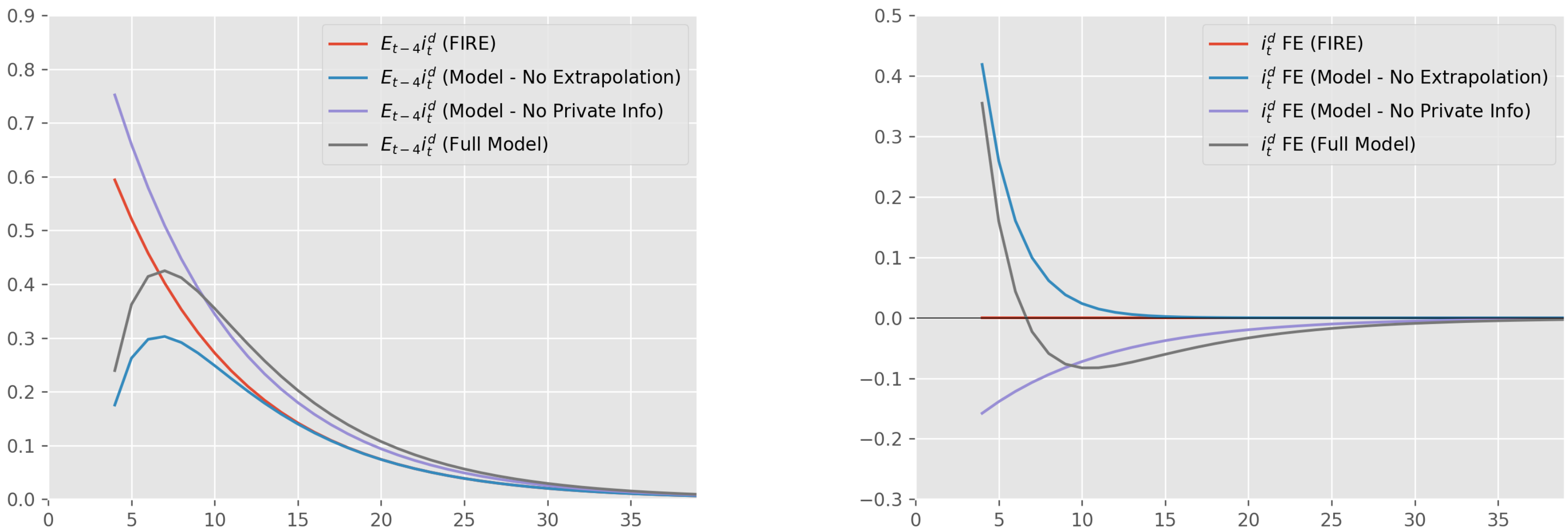
(a) Forecast Error IRFs



(b) Model Bias Coefficients

FIGURE 6: MODEL IMPULSE RESPONSE FUNCTIONS TO A FUNDAMENTAL SHOCK

The first panel in the figure plots the model-implied impulse response function of the calibrated model for consensus interest rate differential forecasts ($\bar{\mathbb{E}}_{t-4}i_t^d$) in response to a one standard deviation shock to fundamentals, $\tilde{\xi}_t$. The second panel in the figure plots the model-implied impulse response function of interest rate differential forecast errors ($i_t^d - \bar{\mathbb{E}}_{t-4}i_t^d$) in response to a one standard deviation to fundamentals, $\tilde{\xi}_t$. Both panels include IRFs corresponding with the full calibrated model, as well as IRFs for models with a subset of frictions included.



(a) Interest Rate Differential Forecast

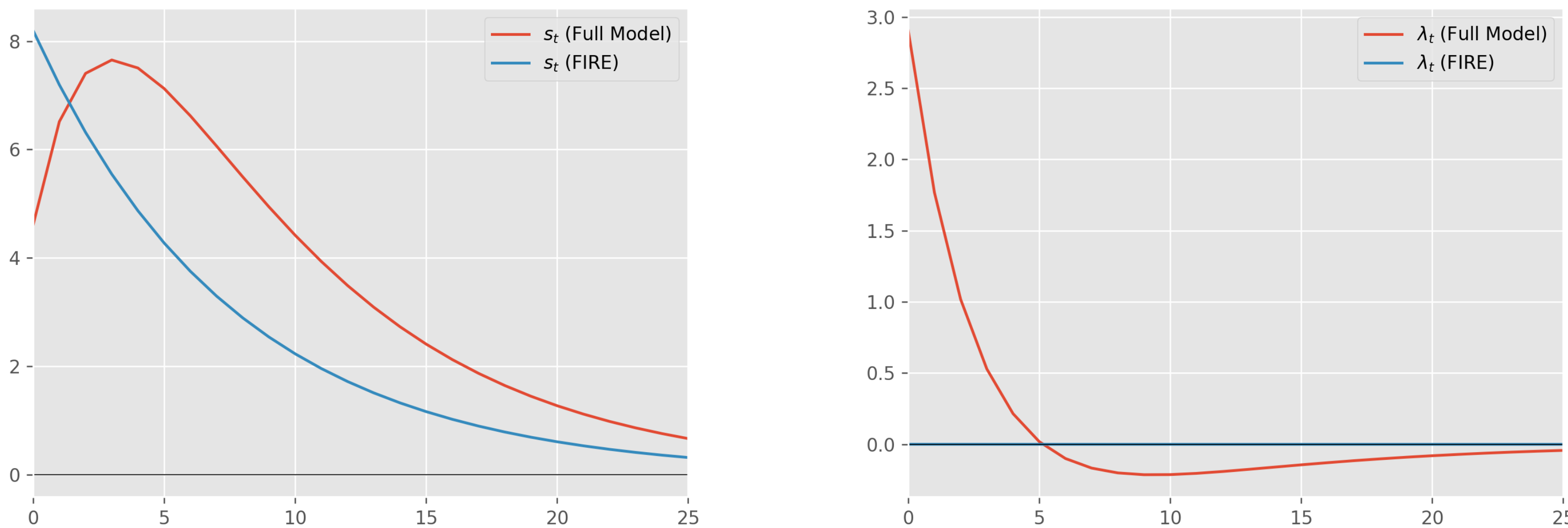
(b) Interest Rate Differential Forecast Error

FIGURE 7: CARRY TRADE REGRESSION

The figure compares the empirical estimates of the UIP regression coefficients with an equivalent measure in the calibrated model. The empirical measure is obtained from the following panel regression: $\lambda_{j,t+1} = \alpha_j + \beta i^d_{j,t} + \epsilon_{j,t+1}$, where $\lambda_{j,t+1}$ is the excess returns of borrowing in foreign currency short-term rates and lending in US short-term rates. The sample is from August 1986 to through December 2019. To compute the model's coefficient, we simulate the calibrated model 5,000 times for 140 periods. We report the average regression coefficient of all simulations and its standard error.

FIGURE 8: DELAYED OVERSHOOTING

The figure plots exchange rates and currency excess returns in response to a one standard deviation shock to the fundamental process, ξ_t, as implied by the fully calibrated model. The plots capture the *delayed* overshooting puzzle, which is the fact that exchange rates gradually response to the arrival of monetary news, rather than immediately responding. For comparison, the figure also includes exchange rates and currency excess returns in a Full-Information Rational Expectations model.



(a) Exchange Rate

(b) Excess Return

FIGURE 9: PREDICTABILITY REVERSAL

The figure reports model's UIP regression for different k-period ahead horizons. We simulate the calibrated model 5,000 times for 140 periods. For each simulation and k-period ahead horizon, we estimate the following regression $\lambda_{t+k} = \alpha + \beta i_t^d + \epsilon_{t+k}$, where λ_{t+k} is the excess return between period $t+k-1$ and $t+k$, and i_t^d is the interest rate differential at period t. We report the average regression coefficient of all simulations.

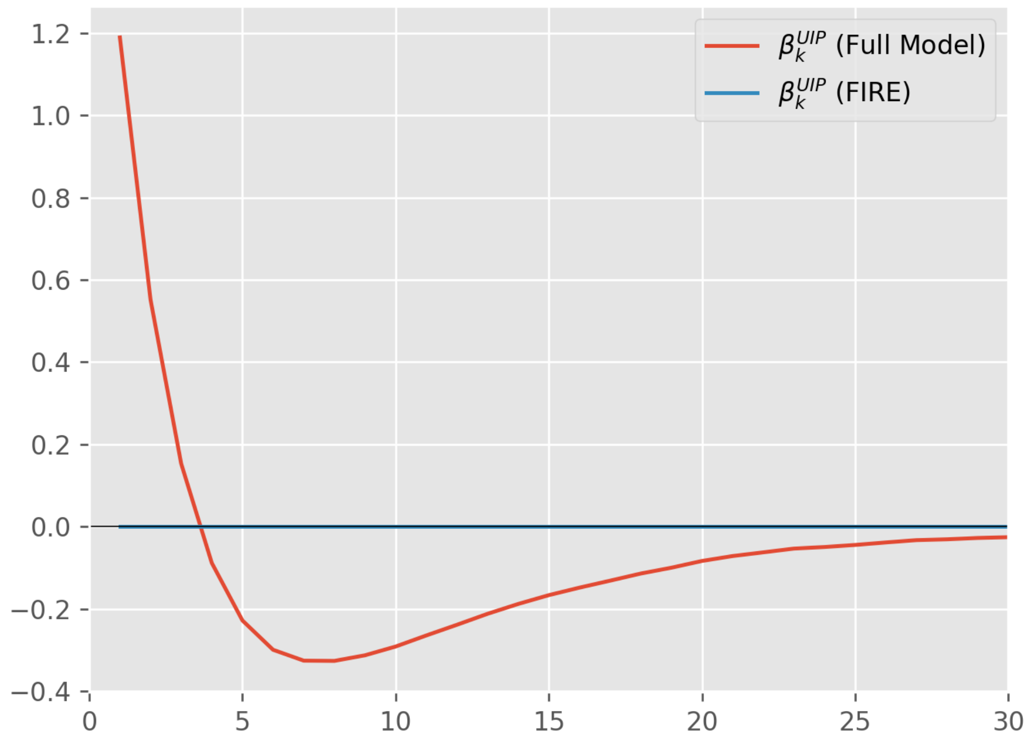


FIGURE 10: TIME-SERIES MOMENTUM AND REVERSAL

The figure plots autocorrelations of currency excess returns in the model. The k-period autocorrelation is calculated by simulating the calibrated model 5,000 times for 140 periods, and taking an average autocorrelation of currency excess returns with k-period lagged excess returns in each simulated sample.

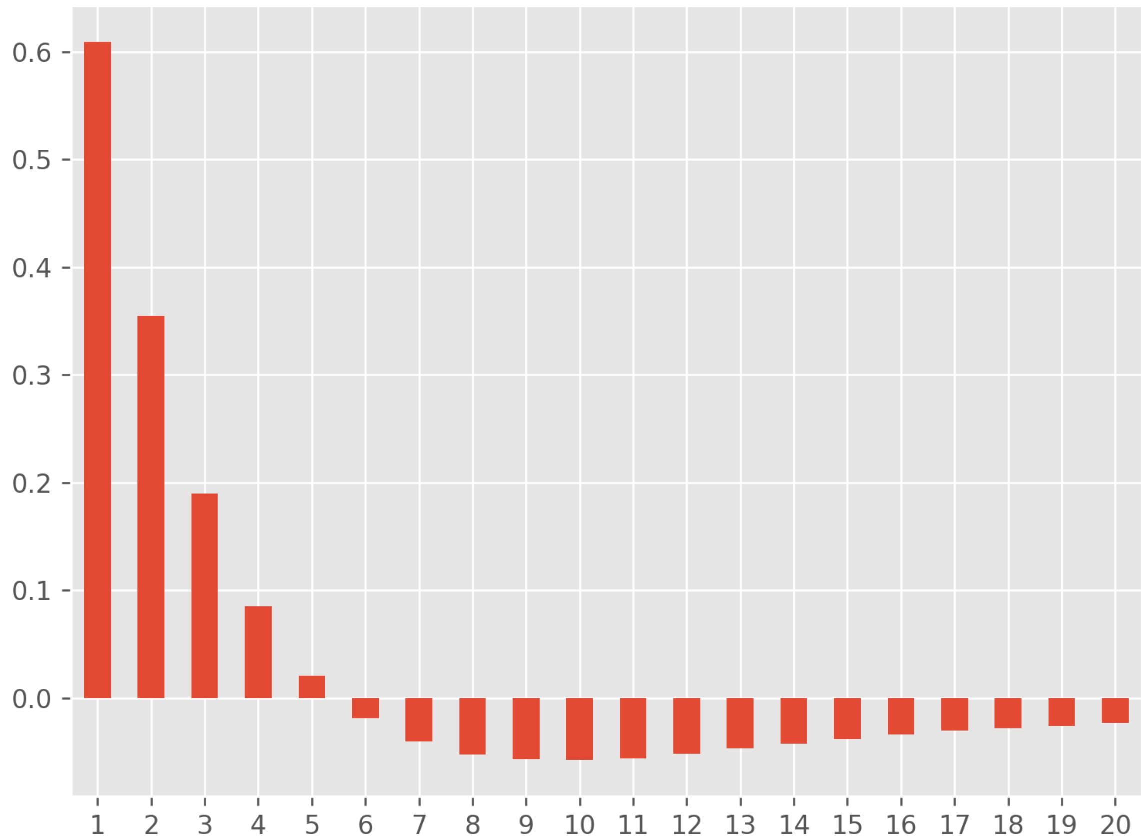


Figure 11: The Downward-Sloping Term Structure of UIP Violations

The figure plots the model-implied regression coefficients from regressing the returns to borrowing in n-period maturity foreign bonds and investing in n-period maturity home country bonds on the interest rate differential (the home currency interest rate minus the foreign country interest rate), for different values of n. The coefficients are computed by simulating the model 5,000 times for 140 periods. The figure plots regression coefficients for the full calibrated model and under FIRE.

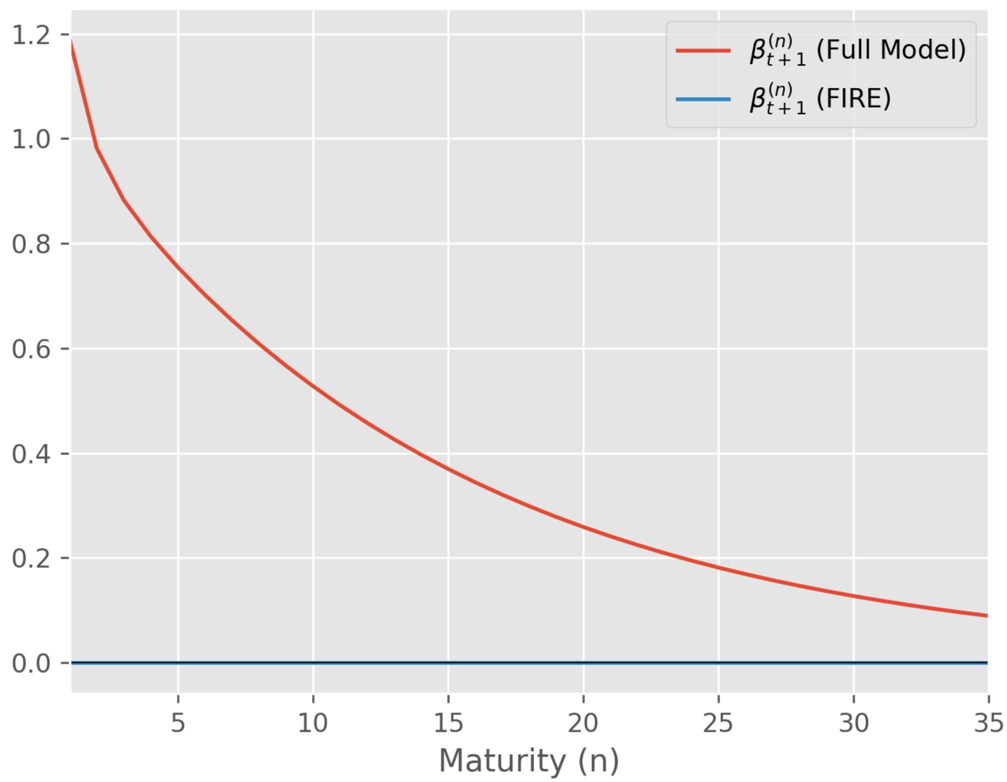


FIGURE 12: MACROECONOMIC NEWS AND EXCHANGE RATES

The figure plots regression results from panel regressions of the form $y_{t,j} = \alpha_j + \beta\left(\text{macro news}_{t,j}\right) + \epsilon_{t,j}$ where $y_{t,j}$ is a variable capturing the exchange rate of country j versus the USD, and β is the coefficient of interest. There are two measures of y_t: changes in exchange rates $(s_t - s_{t-1})$ and changes in the consensus expectation of future exchange rates $(\bar{\mathbb{E}}_t s_{t+1} - \bar{\mathbb{E}}_{t-1} s_{t+1})$. There are three measure of macroeconomic news: interest rate revisions $\left(\bar{\mathbb{E}}_t i^d_{t+1,j} - \bar{\mathbb{E}}_{t-1} i^d_{t+1,j}\right)$, interest rate surprises $\left(i^d_{t,j} - \bar{\mathbb{E}}_{t-1} i^d_{t,j}\right)$, and a macroeconomic surprise index $\left(\sum_{i=1}^{n} \omega_{k,j}\left(x_{k,t,j} - \bar{\mathbb{E}}_{t-1} x_{k,t,j}\right)\right)$, where $x_{k,t,j}$ is a macroeconomic series k captures GDP, Employment, and Industrial Production information for country k, and $\omega_{k,j}$ is a weight computed to give equal weight to GDP, employment and IP news. Interest rate differential data are from FX4Casts and macroeconomic surprise data are from Bloomberg. Observations are quarterly, and independent variables are scaled to have zero mean and unit standard deviation, and signed such that we expect a positive coefficient in the regressions.

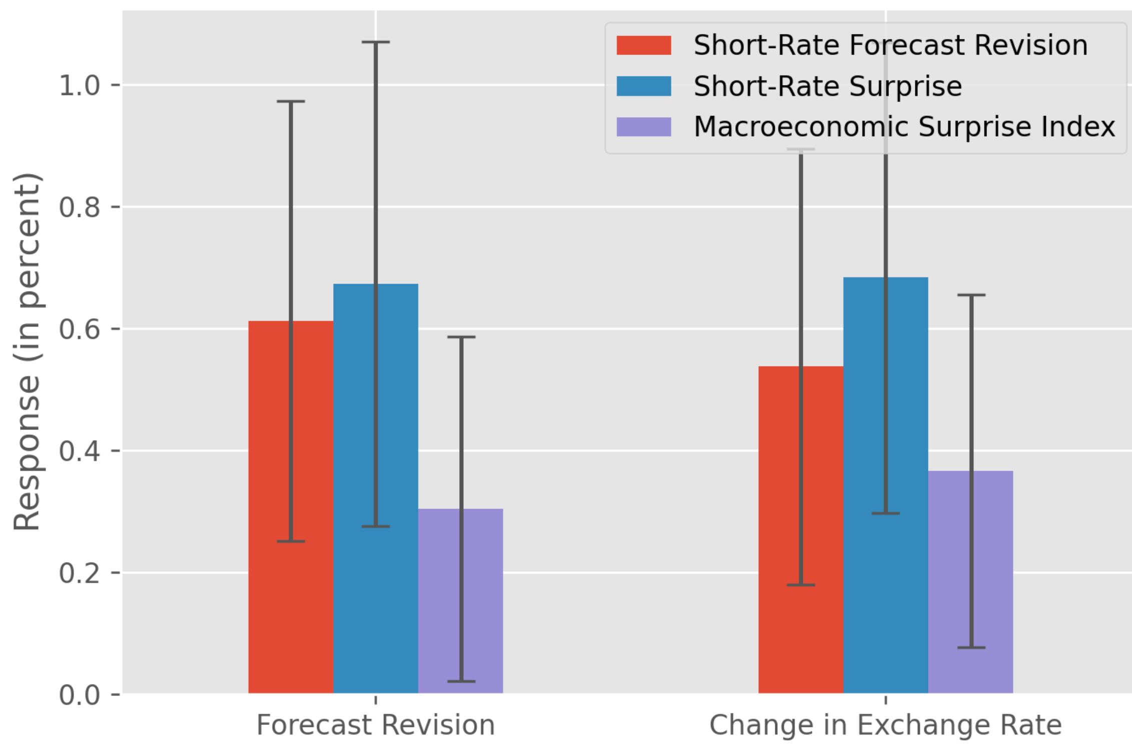


Figure 13: Persistence of Subjective Beliefs

The first panel in the figure plots the persistence of subjective beliefs in the model and in the data. The model results are computed by simulating the model 5,000 times for 140 periods. For each simulation, we compute the beliefs of 1000 investors in each period of the simulation. We rank investors based on their beliefs about the fundamental, ξ_t, in each period. The panel plots the average percentile ranks of investors in the top and bottom quartile of the belief distribution in subsequent periods. The data lines in the plot are computed by ranking forecasters in the SPF data based on their beliefs about short-term interest rates, and computing the average percentile rank of the forecasters in the top and bottom quartiles of the belief distribution in subsequent periods. The second panel in the figure plots the impulse response function of expected interest rate differentials four periods ahead, $\mathbb{E}_{i,t-4} i_t^d$, in response to a one-standard deviation shock to private information ($u_{i,t}$), and in response to a one-standard deviation shock to fundamentals (η_t).

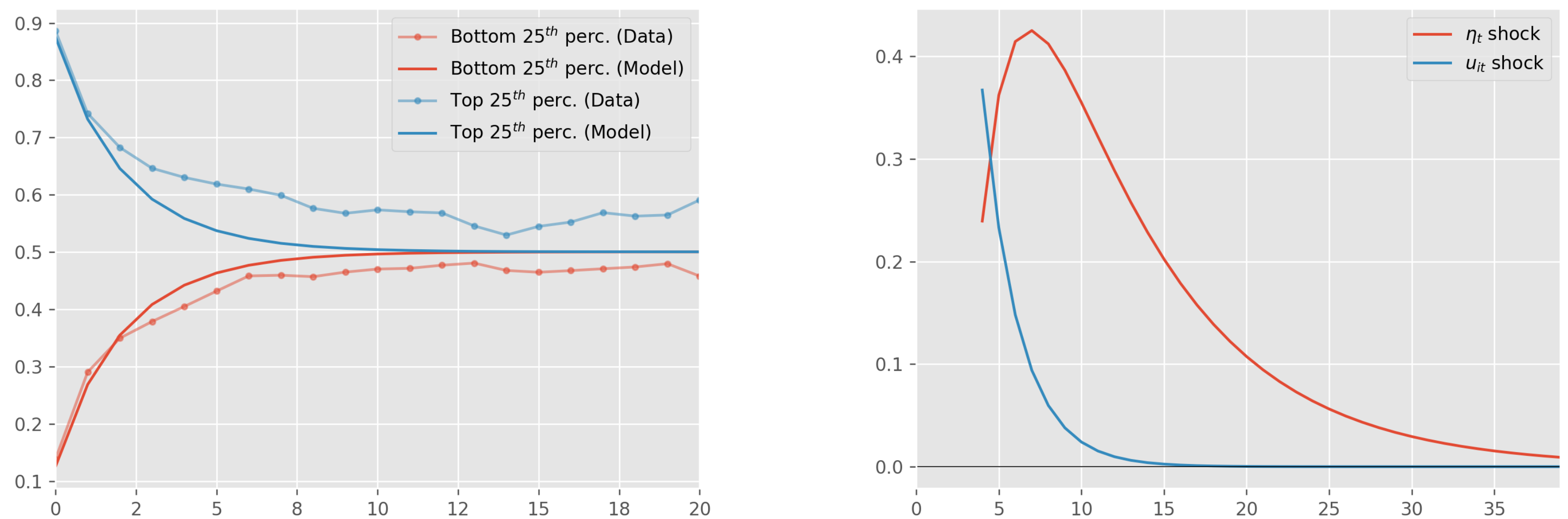
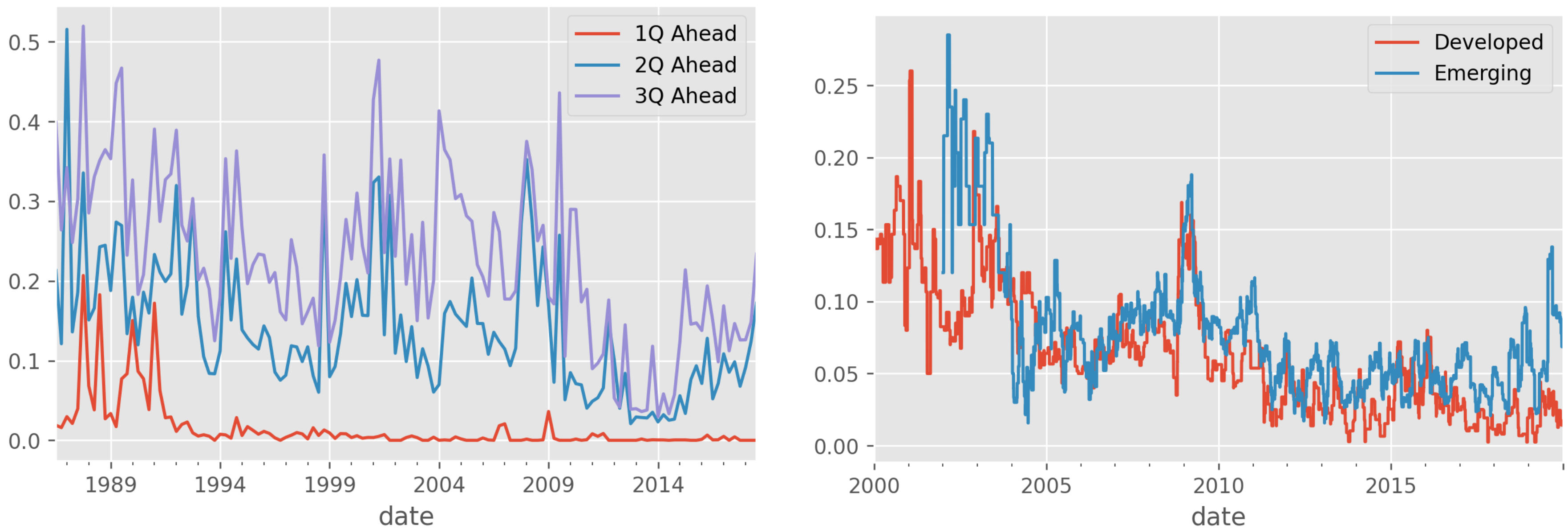


FIGURE 14: DISPERSION OF BELIEFS ABOUT INTEREST RATES OVER TIME

The first panel in the figure plots the cross-sectional standard deviation of forecasts of Treasury Bill rates from the Survey of Professional Forecasters. The second panel in the figure plots the cross-sectional standard deviation of forecasts of next announced central bank policy rate, average across developed and emerging market countries, using data from Bloomberg.



(a) Standard Deviation of T-Bill Rate Forecasts

(b) Standard Deviation of Policy Rate Forecasts

TABLE 1: THE FAILURE OF UIP IN THE POST-FINANCIAL CRISIS ERA

Panel A reports results from time-series regressions of the form $\lambda_{t+1} = \alpha + \beta_{UIP} i_t^d + \epsilon_t$, where λ_{t+1} is either the realized or forecasted excess returns for borrowing in a foreign currency and purchasing US bonds, and and i_t^d is the interest rate differential. The panel reports the average coefficient for regressions across individual countries, and also separately reports the average coefficient for developed and emerging market countries. Panel B reports results from regressions of the form $i_t^d - \bar{\mathbb{E}}_{-\mathbb{1}} i_t^d = \alpha + \beta_{CG} i_t^d + \epsilon$, where $\bar{\mathbb{E}}$ is the consensus expectation, and i_t^d is the interest rate differential. In both panels, the sample consists of quarterly observations from September 2006 through December 2019. Standard errors are HAC-Panel standard errors and are reported in parentheses.

Panel A: UIP Regressions

	All		Developed		Emerging	
	Realized	Forecasted	Realized	Forecasted	Realized	Forecasted
β_{UIP}	-0.61	0.70	-1.94	0.66	0.19	0.73
	(0.38)	(0.17)	(0.75)	(0.30)	(0.39)	(0.21)
N	1417	1417	529	529	888	888

Panel B: Interest Rate Expectations

	All	Developed	Emerging
β_{CG}	-0.049	-0.034	-0.057
	(0.025)	(0.040)	(0.032)
N	1417	529	888

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Appendices For

The Role of Beliefs in Asset Prices: Evidence from Exchange Rates

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A Sample and Data Sources

In this section, we describe the sample of currencies and the data used in the empirical analysis conducted in the paper.

Sample

The developed market currencies in the sample consist of the G11 currencies: the Australian dollar, the Canadian dollar, the Danish krone, the Euro, the Japanese yen, the New Zealand dollar, the Norwegian Krone, the Swedish krona, the Swiss franc, and the British pound sterling. The emerging market currencies are the Brazilian real, the Chilean peso, the Colombian peso, the Czech koruna, the Indian rupee, the Malaysian ringgit, the Mexican peso, the Philippine peso, the Polish zloty, the Russian ruble, the Singaporean dollar, the South African rand, the South Korean won, the New Taiwan dollar, the Thai bhat, and the Turkish lira.

Survey of Professional Forecasters

We use data on forecasts (and the corresponding realizations) of US Treasury Bill rates, US unemployment, and US inflation from the Survey of Professional Forecasters from the Philadelphia Fed, a commonly used data source to study macroeconomic forecasting. The data include quarterly data on forecasts and realizations of macroeconomic series.

FX4Casts

We obtain data on exchange rate and interest rate forecasts for the full sample of countries from FX4casts. For each month, the dataset provides the average forecast of exchange rates and interest rates from a number of large financial institutions that actively participate in foreign exchange markets.

The data on exchange rate forecasts include 1-, 3-, 6-, 12-, and 24-month ahead forecasts of the spot exchange rates for 32 currencies, along with the 5th and 95th percentile of the distribution of forecasts made for each currency at each point in time. The data begin in August 1986 for developed market currencies, and begin in October 2001 for the remaining currencies in our sample. The datasets also contains data on interest rate forecasts beginning in October 2001 for all countries in the sample. We construct interest rate differential forecasts for each country by subtracting the United States forecasted interest rates from the consensus interest rate forecast for that country.

Bloomberg Economic Forecasts

We obtain data on macroeconomic announcements for the countries in our sample from Bloomberg. The dataset includes macroeconomic release for a number of series, such as CPI, GDP, Employment, the central bank policy rate, etc. Among other quantities, the data include the median forecast, the standard deviation of forecasts, and the actual released number for each macroeconomic series. We particularly focus on series related to GDP, employment, and industrial production.

The data begin in Q4 2001. The exact availability of data varies series-by-series and country-by-country. The periodicity of the series also vary across series and by country; some release occur on a monthly frequency, while others occur on a quarterly frequency.

B Proofs and Derivations

The Wold representation theorem and the Wiener-Hopf prediction theorem are used to prove the propositions in the paper; they can be found in Huo and Takayama (2018). For completeness, we reproduce the details below.

Signal Process. The signals observed by investor i follow

$$\boldsymbol{x}_t = \begin{bmatrix} \hat{i}_t^d \\ \hat{x}_{it} \end{bmatrix} = \begin{bmatrix} \hat{\tau}_\varepsilon^{-1/2} & 0 & \frac{1}{1-\hat{\rho}L} \\ 0 & \tau_u^{-1/2} & \frac{1}{1-\hat{\rho}L} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ u_{it} \\ \eta_t \end{bmatrix} = \boldsymbol{M}(L)\boldsymbol{e}_t.$$

Wold Representation. Suppose the signal's state-space representation is

$$\mathbf{x}_t = \mathbf{H}\boldsymbol{\xi}_t + \mathbf{R}\mathbf{u}_t \text{ and } \boldsymbol{\xi}_t = \mathbf{F}\boldsymbol{\xi}_{t-1} + \boldsymbol{Q}\boldsymbol{\nu}_t$$

where $\boldsymbol{v}_t$ and $\boldsymbol{u}_t$ are standard normal shocks. If all the eigenvalues of $\boldsymbol{F}$ lie inside the unit circle, the Wold representation is

$$\mathbf{x}_t = \boldsymbol{B}(L)\boldsymbol{w}_t.$$

$\boldsymbol{B}(L)$ is given by

$$\mathbf{B}(L) = \mathbf{I} + \mathbf{H}(\mathbf{I} - \mathbf{FL})^{-\mathbf{1}}\mathbf{FKL}$$

the inverse of $\mathbf{B}(L)$ is

$$\mathbf{B}(L)^{-1} = \mathbf{I} - \mathbf{H}[\mathbf{I} - (\mathbf{F} - \mathbf{FKH})\mathbf{L}]^{-\mathbf{1}}\mathbf{FKL}$$

and the co-variance matrix $\boldsymbol{V}$ is

$$\boldsymbol{V} = \mathbf{HPH}' + \mathbf{RR}'$$

where define the $\mathbf{P}$ matrix as the one solves

$$\mathbf{P} = \mathbf{F}[\mathbf{P} - \mathbf{PH}'(\mathbf{HPH}' + \mathbf{RR}')^{-\mathbf{1}}\mathbf{HP}]\mathbf{F}' + \mathbf{QQ}'.$$

The Kalman gain matrix is $\mathbf{K} = \mathbf{PH}'(\mathbf{HPH}' + \mathbf{RR}')^{-\mathbf{1}}$.

Wiener-Hopf Prediction. Suppose the original representation of the signal process is $\boldsymbol{x}_t = \boldsymbol{M}(L)\boldsymbol{e}_t$, and a stationary process $\boldsymbol{f}_t = \varphi(L)\boldsymbol{e}_t$, then the prediction formula is

$$\mathbb{E}[\boldsymbol{f}_t|\boldsymbol{x}_t] = \left[\varphi(L)\boldsymbol{M}'(L^{-1})\boldsymbol{B}'(L^{-1})^{-1}\right]_+ \boldsymbol{V}^{-1}\boldsymbol{B}(L)^{-1}\boldsymbol{x}_t.$$

In order to apply the prediction formula, we need to find the Wold representation of our signal process. Define

$$\lambda = \frac{1}{2}\left(\hat{\rho} + \frac{1}{\hat{\rho}} + \frac{\hat{\tau}_\varepsilon + \tau_u}{\hat{\rho}} - \sqrt{(\hat{\rho} + \frac{1}{\hat{\rho}} + \frac{\hat{\tau}_\varepsilon + \tau_u}{\hat{\rho}})^2 - 4}\right).$$

In our setting,

$$\boldsymbol{B}^{-1}(L) = \frac{1}{1-\lambda L}\begin{bmatrix} 1 - \frac{\hat{\tau}_\varepsilon\hat{\rho}+\lambda\tau_u}{\hat{\tau}_\varepsilon+\tau_u}L & \frac{\tau_u(\lambda-\hat{\rho})}{\hat{\tau}_\varepsilon+\tau_u}L \\ \frac{\hat{\tau}_\varepsilon(\lambda-\hat{\rho})}{\hat{\tau}_\varepsilon+\tau_u}L & 1 - \frac{\tau_u\hat{\rho}+\lambda\hat{\tau}_\varepsilon}{\hat{\tau}_\varepsilon+\tau_u}L \end{bmatrix},$$

and

$$\boldsymbol{V}^{-1} = \frac{\hat{\tau}_\varepsilon\tau_u}{\hat{\rho}(\hat{\tau}_\varepsilon+\tau_u)}\begin{bmatrix} \frac{\tau_u\hat{\rho}+\lambda\hat{\tau}_\varepsilon}{\tau_u} & \lambda-\hat{\rho} \\ \lambda-\hat{\rho} & \frac{\hat{\tau}_\varepsilon\hat{\rho}+\lambda\tau_u}{\hat{\tau}_\varepsilon} \end{bmatrix}.$$

B.1 Interest Rate Expectations and Errors

We use the Wiener-Hopf prediction formula to derive the expectation of the fundamental variable (and accordingly, expectations of future interest rate differentials), which we in turn use to prove Proposition 1.

$$\begin{aligned}\mathbb{E}_{it}[\xi_t|\mathcal{I}_{it}] =& [\begin{bmatrix}0 & 0 & \frac{1}{1-\rho L}\end{bmatrix}\boldsymbol{M}'(L^{-1})\boldsymbol{B}'(L^{-1})^{-1}]_+\boldsymbol{V}^{-1}\boldsymbol{B}(L)^{-1}\begin{bmatrix} i_t^d \\ x_{it}\end{bmatrix} \\ =& \frac{\lambda(\hat{\tau}_\varepsilon i_t^d + \tau_u x_{it})}{\hat{\rho}(1-\lambda L)(1-\hat{\rho}\lambda)}\end{aligned}$$

Therefore $\mathbb{E}_{it}[\xi_{t+j}|\mathcal{I}_{it}] = \hat{\rho}^{j-1}\frac{\lambda(\hat{\tau}_\varepsilon i_t^d+\tau_u x_{it})}{(1-\lambda L)(1-\hat{\rho}\lambda)}$. At the consensus level

$$\bar{\mathbb{E}}_t[\xi_t] = \int \mathbb{E}_{it}[\xi_t|\mathcal{I}_{it}]di = \frac{\lambda\hat{\tau}_\varepsilon\varepsilon_t + \lambda(\hat{\tau}_\varepsilon+\tau_u)\xi_t}{\hat{\rho}(1-\lambda L)(1-\hat{\rho}\lambda)} \text{ and } \bar{\mathbb{E}}_t[\xi_{t+j}] = \hat{\rho}^{j-1}\frac{\lambda\hat{\tau}_\varepsilon\varepsilon_t + \lambda(\hat{\tau}_\varepsilon+\tau_u)\xi_t}{(1-\lambda L)(1-\hat{\rho}\lambda)}.$$

Proof of Proposition 1

Proof. The period t expectation of the fundamental variable in $t+1$ can be written as

$$\bar{\mathbb{E}}_t[\xi_{t+1}] = \hat{\rho}\left(1-\frac{\lambda}{\hat{\rho}}\right)\frac{1}{1-\lambda L}\xi_t + \frac{\lambda\hat{\tau}_\varepsilon\sigma_\varepsilon}{(1-\lambda L)(1-\hat{\rho}\lambda)}\varepsilon_t.$$

The forecast error is therefore

$$FE_{t,t+1} = i_{t+1}^d - \hat{\rho}(1-\frac{\lambda}{\hat{\rho}})\frac{1}{1-\lambda L}\xi_t - \frac{\lambda\hat{\tau}_\varepsilon\sigma_\varepsilon}{(1-\lambda L)(1-\hat{\rho}\lambda)}\varepsilon_t$$

$$
\begin{aligned}
&=\xi_{t+1}+\sigma_\varepsilon\varepsilon_{t+1}-\hat{\rho}(1-\frac{\lambda}{\hat{\rho}})\frac{1}{1-\lambda L}\xi_t-\frac{\lambda\hat{\tau}_\varepsilon\sigma_\varepsilon}{(1-\lambda L)(1-\hat{\rho}\lambda)}\varepsilon_t \\
&=\frac{1-\hat{\rho}L}{1-\lambda L}\xi_{t+1}+\sigma_\varepsilon\varepsilon_{t+1}-\frac{\lambda\hat{\tau}_\varepsilon\sigma_\varepsilon}{(1-\lambda L)(1-\hat{\rho}\lambda)}\varepsilon_t.
\end{aligned}
$$

□

Proof of Proposition 2

Proof. To have initial under-reaction, we only need the covariance term to be positive when $\delta=1$. This holds when $\rho>\hat{\rho}-\lambda$. To have delayed overreaction, it's sufficient to show that the function below has a root in the open interval $(0,+\infty)$

$$
g(\delta)=\lambda^\delta(\hat{\rho}-\lambda)+\rho^\delta(\rho-\hat{\rho}).
$$

Suppose such a root exits and we write the root as $\bar{\delta}$. We know

$$
(\lambda/\rho)^{\bar{\delta}}=\frac{\hat{\rho}-\rho}{\hat{\rho}-\lambda}.
$$

For such $\bar{\delta}$ exists, we need $\hat{\rho}-\rho$ to be positive or $\hat{\rho}>\rho$. And when $\lambda<\rho$ ($\lambda>\rho$), the LHS is smaller (greater) than one, and this also implies the RHS is smaller (greater) than one. So $\rho<\hat{\rho}$ is a sufficient condition to have a finite $\bar{\delta}$. □

Proof of Proposition 3

Proof. Denote $\theta=\frac{\lambda}{1-\hat{\rho}\lambda}$, we write the individual level and consensus level forecast as

$$
\begin{aligned}
\mathbb{E}_{it}[\xi_{t+1}] &=\frac{\hat{\rho}-\lambda}{1-\lambda L}\xi_t+\frac{\theta\hat{\tau}_\varepsilon\sigma_\varepsilon}{1-\lambda L}\varepsilon_t+\frac{\theta\tau_u\sigma_u}{1-\lambda L}u_{it} \\
\bar{\mathbb{E}}_t[\xi_{t+1}] &=\frac{\hat{\rho}-\lambda}{1-\lambda L}\xi_t+\frac{\theta\hat{\tau}_\varepsilon\sigma_\varepsilon}{1-\lambda L}\varepsilon_t.
\end{aligned}
$$

The individual and consensus forecast errors are

$$
\begin{aligned}
FE_{it,t+1} &=\frac{1-\hat{\rho}L}{1-\lambda L}\xi_{t+1}+\sigma_\varepsilon\varepsilon_{t+1}-\frac{\theta\hat{\tau}_\varepsilon\sigma_\varepsilon}{1-\lambda L}\varepsilon_t-\frac{\theta\tau_u\sigma_u}{1-\lambda L}u_{it} \\
FE_{t,t+1} &=\frac{1-\hat{\rho}L}{1-\lambda L}\xi_{t+1}+\sigma_\varepsilon\varepsilon_{t+1}-\frac{\theta\hat{\tau}_\varepsilon\sigma_\varepsilon}{1-\lambda L}\varepsilon_t.
\end{aligned}
$$

The individual and consensus forecast revisions are

$$
\begin{aligned}
FR_{it,t+1} &=\frac{(\hat{\rho}-\lambda)(1-\hat{\rho}L)}{1-\lambda L}\xi_t+\frac{\theta\hat{\tau}_\varepsilon\sigma_\varepsilon(1-\hat{\rho}L)}{1-\lambda L}\varepsilon_t+\frac{\theta\tau_u\sigma_u(1-\hat{\rho}L)}{1-\lambda L}u_{it} \\
FR_{t,t+1} &=\frac{(\hat{\rho}-\lambda)(1-\hat{\rho}L)}{1-\lambda L}\xi_t+\frac{\theta\hat{\tau}_\varepsilon\sigma_\varepsilon(1-\hat{\rho}L)}{1-\lambda L}\varepsilon_t.
\end{aligned}
$$

Denote $\kappa_1 = (\hat{\rho} - \lambda)\frac{\lambda}{1-\lambda^2}$, $\kappa_2 = (\hat{\rho} - \lambda)(\rho - \hat{\rho})\frac{(1+\lambda^2)(1-\rho^2)+(\lambda+\rho)(\rho-\hat{\rho})}{(1-\lambda^2)(1-\rho^2)(1-\rho\lambda)}$. The covariance between the consensus forecast error and forecast revision can be written as

$$\begin{aligned}
cov(FR_{t,t+1}, FE_{t,t+1}) =& cov(\frac{1-\hat{\rho}L}{(1-\lambda L)(1-\rho L)}\eta_{t+1}, \frac{(\hat{\rho}-\lambda)(1-\hat{\rho}L)}{(1-\lambda L)(1-\rho L)}\eta_t) \\
& - \theta^2 \hat{\tau}_\varepsilon^2 \sigma_\varepsilon^2 cov(\frac{1}{1-\lambda L}\varepsilon_t, \frac{1-\hat{\rho}L}{1-\lambda L}\varepsilon_t) \\
=& \kappa_1 + \kappa_2 - \theta^2 \hat{\tau}_\varepsilon^2 \sigma_\varepsilon^4 \frac{1-\hat{\rho}\lambda}{1-\lambda^2}.
\end{aligned}$$

The covariance between individual forecast errors and forecast revisions can be written as

$$cov(FR_{it,t+1}, FE_{it,t+1}) = \kappa_1 + \kappa_2 - \theta^2 \hat{\tau}_\varepsilon^2 \sigma_\varepsilon^4 \frac{1-\hat{\rho}\lambda}{1-\lambda^2} - \theta^2 \tau_u^2 \sigma_u^4 \frac{1-\hat{\rho}\lambda}{1-\lambda^2}.$$

The variance of the forecast revisions is

$$\begin{aligned}
var(FR_{t,t+1}) =& (\hat{\rho}-\lambda)^2 \frac{1-\hat{\rho}}{(1-\lambda)(1-\rho)} + \theta^2 \hat{\tau}_\varepsilon^2 \sigma_\varepsilon^4 \frac{1-2\lambda\hat{\rho}+\hat{\rho}^2}{1-\lambda^2} \\
var(FR_{it,t+1}) =& (\hat{\rho}-\lambda)^2 \frac{1-\hat{\rho}}{(1-\lambda)(1-\rho)} + \theta^2 \hat{\tau}_\varepsilon^2 \sigma_\varepsilon^4 \frac{1-2\lambda\hat{\rho}+\hat{\rho}^2}{1-\lambda^2} + \theta^2 \tau_u^2 \sigma_u^4 \frac{1-2\lambda\hat{\rho}+\hat{\rho}^2}{1-\lambda^2}.
\end{aligned}$$

Therefore, we have

$$\frac{cov(FR_{it,t+1}, FE_{it,t+1}) + \theta^2 \tau_u^2 \sigma_u^4 \frac{1-\hat{\rho}\lambda}{1-\lambda^2}}{var(FR_{it,t+1}) - \theta^2 \tau_u^2 \sigma_u^4 \frac{1-2\lambda\hat{\rho}+\hat{\rho}^2}{1-\lambda^2}} = \frac{cov(FR_{t,t+1}, FE_{t,t+1})}{var(FR_{t,t+1})}.$$

As long as $cov(FR_{t,t+1}, FE_{t,t+1}) > 0$, we have

$$\frac{cov(FR_{t,t+1}, FE_{t,t+1})}{var(FR_{t,t+1})} > \frac{cov(FR_{it,t+1}, FE_{it,t+1})}{var(FR_{it,t+1})}$$

provide $\tau_u \neq 0$, $\sigma_u \neq 0$ and $\hat{\sigma}_\varepsilon \neq 0$. The condition $cov(FR_{t,t+1}, FE_{t,t+1}) > 0$ is satisfied when $\hat{\rho} - \lambda < \rho$.

□

B.2 Exchange Rates

Proof of Proposition 4

Proof. The average expectation of the fundamental is

$$\bar{\mathbb{E}}_t[\xi_t] = \frac{\lambda(\hat{\tau}_\varepsilon + \tau_u)}{\hat{\rho}(1-\lambda L)(1-\hat{\rho}\lambda)}\xi_t + \frac{\lambda \hat{\tau}_\varepsilon \sigma_\varepsilon}{\hat{\rho}(1-\lambda L)(1-\hat{\rho}\lambda)}\varepsilon_t.$$

We have

$$\frac{\lambda(\hat{\tau}_\varepsilon + \tau_u)}{(1-\hat{\rho})(1-\lambda L)(1-\hat{\rho}\lambda)}\xi_t = \frac{\hat{\rho}}{1-\hat{\rho}}(1-\frac{\lambda}{\hat{\rho}})\frac{1}{1-\lambda L}\xi_t$$

by using the identity

$$\lambda + \lambda^{-1} = \hat{\rho} + \hat{\rho}^{-1} + (\hat{\tau}_\varepsilon + \tau_u)\hat{\rho}^{-1}.$$

□

Proof of Proposition 5

Proof. We conjecture the exchange rate takes the form $s_t = g(L)\xi_t + h_1(L)\sigma_\varepsilon\varepsilon_t$. Therefore

$$\begin{aligned} s_{t+1} =& g(L)/L\xi_t + h_1(L)\sigma_\varepsilon/L\varepsilon_t \\ =& \frac{g(L)L^{-1}}{1-\hat{\rho}L}\eta_t + \tau_\varepsilon^{-1/2}L^{-1}h_1(L)\varepsilon_t. \end{aligned}$$

Defining $h_2(L) = g(L) - h_1(L)$ and applying the Wiener-Hopf prediction formula and incorporating investors' subjective beliefs, we have the following

$$\begin{aligned} \mathbb{E}_{it}[s_{t+1}|\mathcal{I}_{it}] =& \left[\left[\hat{\tau}_\varepsilon^{-1/2}L^{-1}h_1(L) \quad 0 \quad \tfrac{g(L)L^{-1}}{1-\hat{\rho}L}\right]\boldsymbol{M}'(L^{-1})\boldsymbol{B}'(L^{-1})^{-1}\right]_+ \boldsymbol{V}^{-1}\boldsymbol{B}(L)^{-1}\begin{bmatrix} i_t^d \\ x_{it}\end{bmatrix} \\ =& \begin{bmatrix} -\frac{\lambda(1-\hat{\rho}L)\tau_\varepsilon h_2(\lambda)}{\hat{\rho}(1-\hat{\rho}\lambda)(L-\lambda)(1-\lambda L)} - \frac{(1-\lambda L)\tau_u + (1-\hat{\rho}L)\tau_\varepsilon}{L(\tau_\varepsilon+\tau_u)(1-\lambda L)}h_1(0) + \frac{h_1(L)}{L} + \frac{\lambda\tau_\varepsilon h_2(L)}{\hat{\rho}(L-\lambda)(1-\lambda L)} \\ -\frac{\lambda(1-\hat{\rho}L)\tau_u h_2(\lambda)}{\hat{\rho}(1-\hat{\rho}\lambda)(L-\lambda)(1-\lambda L)} + \frac{(\hat{\rho}-\lambda)\tau_u h_1(0)}{(\tau_u+\tau_\varepsilon)(1-\lambda L)} + \frac{\lambda\tau_u h_2(L)}{\hat{\rho}(L-\lambda)(1-\lambda L)} \end{bmatrix}' \begin{bmatrix} i_t^d \\ x_{it}\end{bmatrix} \\ \equiv& q_1(L)i_t^d + q_2(L)x_{it}. \end{aligned}$$

As a result, we can express the consensus expectation of the period $t+1$ exchange rate as

$$\int \mathbb{E}_{it}[s_{t+1}|\mathcal{I}_{it}]di = (q_1(L) + q_2(L))\,\xi_t + q_1(L)\sigma_\varepsilon\varepsilon_t.$$

Recall the equilibrium condition for the exchange rate:

$$s_t - i_t^d = \int \mathbb{E}_{it}[s_{t+1}|\mathcal{I}_{it}]di.$$

We can re-write this condition as

$$g(L)\xi_t + h_1(L)\sigma_\varepsilon\varepsilon_t - \xi_t - \sigma_\varepsilon\varepsilon_t = (q_1(L) + q_2(L))\,\xi_t + q_1(L)\sigma_\varepsilon\varepsilon_t.$$

Matching coefficients on ξ_t and ε_t yields

$$g(L) - 1 = q_1(L) + q_2(L) \text{ and } h_1(L) - 1 = q_1(L)$$

which can be written as the following functional equations in matrix form

$$\boldsymbol{A}(L)\begin{bmatrix} h_1(L) \\ h_2(L) \end{bmatrix} = \boldsymbol{d}(L)$$

where

$$\boldsymbol{A}(L) = \begin{bmatrix} 1 - L^{-1} & -\frac{\lambda\hat{\tau}_\varepsilon}{\hat{\rho}(L-\lambda)(1-\lambda L)} \\ 0 & 1 - \frac{\lambda\tau_u}{\hat{\rho}(L-\lambda)(1-\lambda L)} \end{bmatrix}$$

$$\boldsymbol{d}_1(L) = -\frac{\lambda(1-\hat{\rho}L)\hat{\tau}_\varepsilon h_2(\lambda)}{\hat{\rho}(1-\hat{\rho}\lambda)(L-\lambda)(1-\lambda L)} - \frac{(1-\lambda L)\tau_u + (1-\hat{\rho}L)\hat{\tau}_\varepsilon}{L(\hat{\tau}_\varepsilon + \tau_u)(1-\lambda L)} h_1(0) + 1$$

and

$$\boldsymbol{d}_2(L) = -\frac{\lambda(1-\hat{\rho}L)\tau_u h_2(\lambda)}{\hat{\rho}(1-\hat{\rho}\lambda)(L-\lambda)(1-\lambda L)} + \frac{(\hat{\rho}-\lambda)\tau_u}{(\tau_u + \hat{\tau}_\varepsilon)(1-\lambda L)} h_1(0).$$

The determinant of $\boldsymbol{A}(L)$ is given by

$$det(\boldsymbol{A}(L)) = \frac{(L-1)(-\lambda\hat{\rho}L^2 + \hat{\rho}(1+\lambda^2)L - \lambda\tau_u - \lambda\hat{\rho})}{\hat{\rho}L(L-\lambda)(1-\lambda L)} = \frac{-\lambda(L-1)(L-\omega)(L-\vartheta^{-1})}{L(L-\lambda)(1-\lambda L)}$$

which has three roots, 1, ω and ϑ^{-1} with $|\omega| < |\vartheta^{-1}|$. The following two identifies hold,

$$\omega\vartheta^{-1} = 1 + \frac{\tau_u}{\hat{\rho}} \text{ and } \omega + \vartheta^{-1} = \lambda + \frac{1}{\lambda} = \hat{\rho} + \frac{1}{\hat{\rho}} + \frac{\tau_u + \hat{\tau}_\varepsilon}{\hat{\rho}}$$

We need to solve two unknowns

$$\varphi_1 = -\frac{\lambda h_2(\lambda)}{\hat{\rho}(1-\hat{\rho}\lambda)} \text{ and } \varphi_2 = \frac{h_1(0)}{\tau_\varepsilon + \tau_u}.$$

Note by Cramer's rule, we know

$$h_1(L) = \frac{\begin{vmatrix} d_1(L) & A_{12}(L) \\ d_2(L) & A_{22}(L) \end{vmatrix}}{det(\boldsymbol{A}(L))} \text{ and } h_2(L) = \frac{\begin{vmatrix} A_{11}(L) & d_1(L) \\ A_{21}(L) & d_2(L) \end{vmatrix}}{det(\boldsymbol{A}(L))}.$$

We choose φ_1 and φ_2 to remove the inside poles of $h_1(L)$. This leads to the following system of equations

$$\begin{aligned} \varphi_1 =& \frac{(\omega-\lambda)(\lambda-\hat{\rho})}{1-\omega\hat{\rho}}\varphi_2 \\ \varphi_2 =& \frac{\omega\left(\lambda\tau_u + \hat{\rho}\left((\omega-\lambda)(\lambda\omega-1) - \hat{\tau}_\varepsilon\varphi_1\right) + \hat{\rho}^2\hat{\tau}_\varepsilon\omega\varphi_1\right)}{\tau_u\left(\lambda\tau_u + \hat{\rho}(\omega-\lambda)(\lambda\omega-1)\right) + \hat{\tau}_\varepsilon\left(\lambda\tau_u + \hat{\rho}(\omega-\lambda)\left(\hat{\rho}\omega-1\right)\right)}. \end{aligned}$$

The policy functions are

$$
\begin{aligned}
h_1(L) =& -\frac{\omega\left(\left(\hat{\rho}-1\right)\left(\tau_u\left(1-\hat{\rho}(L+\omega-1)\right)+\hat{\rho}\left(L\hat{\rho}-1\right)\left(\hat{\rho}\omega-1\right)+\tau_u^2\right)-\hat{\tau}_\varepsilon\left(\hat{\rho}+\tau_u\right)\right)}{\left(\hat{\rho}-1\right)\left(\hat{\rho}+\tau_u-\hat{\rho}^2\omega\right)\left(\hat{\rho}(L\omega-1)-\tau_u\right)} \\
=& -\frac{\vartheta\frac{\hat{\tau}_\varepsilon-(\hat{\rho}-1)\tau_u}{(\hat{\rho}-1)(\hat{\rho}\omega-1)}(\vartheta\tau_u-(1-\hat{\rho}\vartheta))+\vartheta^2\tau_u-\vartheta(1-\hat{\rho}L)(1-\hat{\rho}\vartheta)}{\hat{\rho}(1-\vartheta L)(1-\hat{\rho}\vartheta)} \\
=& -\frac{\frac{\vartheta-\hat{\rho}}{\hat{\rho}-1}(\vartheta\tau_u-(1-\hat{\rho}\vartheta))+\vartheta^2\tau_u-\vartheta(1-\hat{\rho}L)(1-\hat{\rho}\vartheta)}{\hat{\rho}(1-\vartheta L)(1-\hat{\rho}\vartheta)} \\
=& 1+\frac{\hat{\tau}_\varepsilon\vartheta}{(1-\hat{\rho})(1-\vartheta L)(1-\hat{\rho}\vartheta)} \\
h_2(L) =& -\frac{\hat{\rho}\tau_u\omega^2\left(-\hat{\rho}\left(\tau_u+\omega+1\right)+\tau_u+\tau_\varepsilon+\hat{\rho}^2\omega+1\right)}{\left(\hat{\rho}-1\right)\left(\hat{\rho}\omega-1\right)\left(\hat{\rho}+\tau_u-\hat{\rho}^2\omega\right)\left(\hat{\rho}(L\omega-1)-\tau_u\right)} \\
=& \vartheta^2\frac{\tau_u(1+\frac{\hat{\tau}_\varepsilon-(\hat{\rho}-1)\tau_u}{(\hat{\rho}-1)(\hat{\rho}\omega-1)})}{\hat{\rho}(1-\vartheta L)(1-\hat{\rho}\vartheta)} \\
=& \frac{\tau_u\vartheta(1-\vartheta)}{(1-\vartheta L)(1-\hat{\rho}\vartheta)(1-\hat{\rho})}
\end{aligned}
$$

And $g(L) = h_1(L) + h_2(L)$ is

$$
\begin{aligned}
g(L) =& -\frac{\omega\left(-\hat{\rho}\left(L+\tau_u+\omega+1\right)-L\hat{\rho}^3\omega+\hat{\rho}^2(L\omega+L+\omega)+\tau_u+\hat{\tau}_\varepsilon+1\right)}{\left(\hat{\rho}-1\right)\left(\hat{\rho}\omega-1\right)\left(\hat{\rho}(L\omega-1)-\tau_u\right)} \\
=& \vartheta\frac{\frac{\hat{\tau}_\varepsilon-(\hat{\rho}-1)\tau_u}{(\hat{\rho}-1)(\hat{\rho}\omega-1)}+1-\hat{\rho}L}{\hat{\rho}(1-\vartheta L)} \\
=& \frac{\vartheta-\hat{\rho}+\vartheta(1-\hat{\rho}L)(\hat{\rho}-1)}{\hat{\rho}(1-\vartheta L)(\hat{\rho}-1)} \\
=& 1+\frac{\hat{\rho}-\vartheta}{(1-\vartheta L)(1-\hat{\rho})}
\end{aligned}
$$

□

Comparative Statics of ϑ

We prove the following comparative statistics,

$$\frac{\partial\vartheta}{\partial\hat{\rho}} > 0, \frac{\partial\vartheta}{\partial\hat{\tau}_\varepsilon} < 0 \text{ and } \frac{\partial\vartheta}{\partial\tau_u} < 0.$$

Proof. Note ω and ϑ^{-1} are defined as the roots of the following quadratic equation

$$L^2 - (\lambda + \frac{1}{\lambda})L + (1 + \frac{\tau_u}{\hat{\rho}}) = 0.$$

Therefore we have

$$\omega\vartheta^{-1} = 1 + \frac{\tau_u}{\hat{\rho}}$$
$$\omega + \vartheta^{-1} = \lambda + \frac{1}{\lambda}$$

which implies that $0 < \omega < 1 < \vartheta^{-1}$. Define the following function

$$g(x) = x^2 - (\lambda + \frac{1}{\lambda})x + (1 + \frac{\tau_u}{\hat{\rho}}).$$

We first observe that the following holds

$$\frac{\partial\vartheta}{\partial\hat{\rho}} = -\vartheta^2 \frac{\partial\vartheta^{-1}}{\partial\hat{\rho}}$$

where

$$\frac{\partial\vartheta^{-1}}{\partial\hat{\rho}} = -\frac{\partial g(\vartheta^{-1})/\partial\hat{\rho}}{\partial g(\vartheta^{-1})/\partial\vartheta^{-1}}.$$

Using the identify $\lambda + \lambda^{-1} = \hat{\rho} + \hat{\rho}^{-1} + (\hat{\tau}_\varepsilon + \tau_u)\hat{\rho}^{-1}$, we then prove the following

$$\frac{\partial g(\vartheta^{-1})}{\partial\hat{\rho}} = -(1 - \frac{1}{\hat{\rho}^2} - \frac{\hat{\tau}_\varepsilon + \tau_u}{\hat{\rho}^2})\vartheta^{-1} - \frac{\tau_u}{\hat{\rho}^2} > 0$$
$$\frac{\partial g(\vartheta^{-1})}{\partial\vartheta^{-1}} = 2\vartheta^{-1} - (\lambda + \frac{1}{\lambda}) > 0.$$

To prove that $-(1 - \frac{1}{\hat{\rho}^2} - \frac{\hat{\tau}_\varepsilon + \tau_u}{\hat{\rho}^2})\vartheta^{-1} - \frac{\tau_u}{\hat{\rho}^2} > 0$, it is equivalent to showing the following holds,

$$(\hat{\rho}^2 - 1 - \hat{\tau}_\varepsilon - \tau_u) < \tau_u\vartheta$$
$$\Rightarrow \hat{\rho}^2 < 1 + \hat{\tau}_\varepsilon + \tau_u + \tau_u\vartheta$$

which holds because $\hat{\rho}^2 < 1$. To prove $\frac{\partial g(\vartheta^{-1})}{\partial\vartheta^{-1}} > 0$, note

$$2\vartheta^{-1} > \omega + \vartheta^{-1} = \lambda + \frac{1}{\lambda},$$

as a result, $2\vartheta^{-1} > \lambda + \frac{1}{\lambda}$. Therefore

$$\frac{\partial\vartheta^{-1}}{\partial\hat{\rho}} = \frac{2\hat{\rho}\vartheta^{-1} - \vartheta^{-2} - 1}{2\hat{\rho}\vartheta^{-1} - (\lambda + \lambda^{-1})\hat{\rho}} < 0 \text{ and } \frac{\partial\vartheta}{\partial\hat{\rho}} > 0.$$

Similarly, we have

$$\frac{\partial g(\vartheta^{-1})}{\partial \hat{\tau}_\varepsilon} = -\frac{\vartheta^{-1}}{\hat{\rho}} < 0 \text{ and } \frac{\partial g(\vartheta^{-1})}{\partial \tau_u} = \frac{1-\vartheta^{-1}}{\hat{\rho}} < 0.$$

Therefore

$$\frac{\partial \vartheta}{\partial \hat{\tau}_\varepsilon} < 0 \text{ and } \frac{\partial \vartheta}{\partial \tau_u} < 0.$$

□

B.3 Term Structure of UIP Violations

To compute bond prices in the model, we implement an iterative procedure. In particular, we assume that bond prices follow the general relation that $p_t^{(n)} = p_t^{(n-1)} + g(t)$. Then, we know that

$$p_t^{(n+1)} = p_t^{(n)} + \int_i \mathbb{E}_{it}[g(t+1)]di$$

where $g(t+1)$ is a function of f_{t+1} and i_{t+1}^d, where f_t is the average expectation of the fundamental in period t across agents. With this idea in hand, we implement the following steps:

(i) Start from $p_t^{(1)} = -i_t^d = g(t)$. The next period's price is

$$p_t^{(2)} = p_t^{(1)} + \int_i \mathbb{E}_{it}[g(t+1)]di.$$

In computing $\mathbb{E}_{it}[g(t+1)]$, expectation of future interest rates uses f_{it} and past information is perfectly observed. We have

$$p_t^{(2)} = p_t^{(1)} - \hat{\rho} f_t.$$

(ii) Computing $p_t^{(3)}$, the second term is the average forecast of the one-period-forward second term in the last price

$$\int_i \mathbb{E}_{it}[-\hat{\rho} f_{t+1}]di.$$

When computing the integrand, we use the following equation to first replace future forecast

$$f_{t+1} = \lambda f_t + a_\varepsilon i_{t+1}^d + a_u \xi_{t+1}.$$

The expectation of i_{t+1}^d and ξ_{t+1} is easily to derive. For $\mathbb{E}_{it}[f_t]$, we replace f_t by its state-space representation, i.e.,

$$f_t = \frac{a_\varepsilon i_t^d + a_u \xi_t}{1 - \lambda L}$$

and then we use

$$
\begin{aligned}
&\mathbb{E}_{it}[\frac{\xi_t}{(1-\lambda L)^k}] \\
=&\frac{L(1-\hat{\rho}\lambda)(1-\lambda^2)^k-\lambda(1-\hat{\rho}L)(1-\lambda L)^k}{(L-\lambda)(1-\lambda L)^k(1-\lambda^2)^k} f_{it}.
\end{aligned}
$$

(iii) Repeat the procedure, forward one-period, collect future average forecast, use the process of f_t to replace and apply the above prediction formula to aggregate information.

C Additional Analysis of Underreaction and Delayed Overreaction

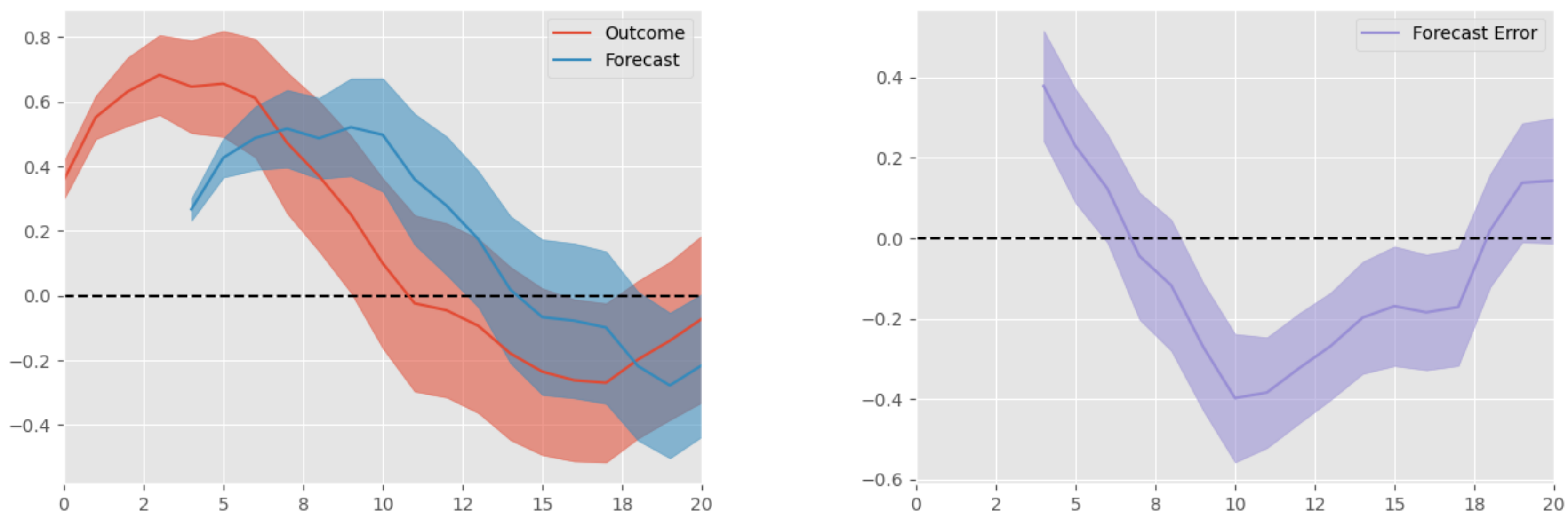
TABLE C.1: UNDERREACTION AND OVERREACTION IN HYBRID REGRESSIONS

This table reports results from hybrid Coibion and Gorodnichenko (2015) and Kohlhas and Walther (2020) regressions for T-Bill (3mo) forecasts (SPF) from January 1985 to December 2018. Coibion and Gorodnichenko (2015) regressions are given by $x_{t+k} - F_t x_{t+k} = \alpha + \beta_{CG}(F_t x_{t+k} - F_{t-k} x_{t+k}) + \epsilon_{t+k}$. Kohlhas and Walther (2020) regressions are given by $x_{t+k} - F_t x_{t+k} = \alpha + \beta_{KW} x_t + \epsilon_{t+k}$. HAC-panel standard errors are reported in parentheses. The table also reports results using Hamilton(2017) and HP filter ($\lambda = 1600$) in the detrend columns to account for potential structural changes.

				No Detrending				Hamilton (2017) Detrending				HP Filter Detrending			
	FE (1Q)	FE (2Q)	FE (3Q)	FE (1Q)	FE (2Q)	FE (3Q)	FE (4Q)	FE (1Q)	FE (2Q)	FE (3Q)	FE (4Q)	FE (1Q)	FE (2Q)	FE (3Q)	FE (4Q)
Constant	-0.11	-0.22	-0.38	-0.09	-0.15	-0.24	-0.35	-0.16	-0.29	-0.44	-0.59	-0.16	-0.29	-0.44	-0.60
	0.04	0.09	0.15	0.06	0.10	0.15	0.21	0.06	0.11	0.16	0.21	0.06	0.11	0.16	0.20
Forecast Revision	0.37	0.52	0.66												
	0.07	0.10	0.18												
Current Realization				-0.02	-0.04	-0.06	-0.07	0.00	-0.01	-0.01	-0.02	-0.09	-0.18	-0.28	-0.40
				0.02	0.03	0.05	0.06	0.02	0.04	0.06	0.08	0.04	0.09	0.13	0.18
R2	0.16	0.12	0.10	0.01	0.02	0.03	0.02	0.00	0.00	0.00	0.00	0.05	0.07	0.08	0.11
F-stat	29.98	25.26	14.34	1.43	1.71	1.68	1.39	0.03	0.04	0.06	0.09	4.15	4.28	4.55	5.17
N	134	134	134	133	132	131	130	133	132	131	130	133	132	131	130

FIGURE C.1: RESPONSE OF US INTEREST RATE FORECASTS TO ROMER AND ROMER (2004) SHOCKS

The figure reports results from regressions of the form $x_{t+h} = \alpha_h + \beta_h \epsilon_t + \gamma C_t + u_{t+h}$, where $x_{t+h} \in (i_{t+h}, \bar{\mathbb{E}}_{t+h} i_{t+h+k}, i_{t+h+k} - \bar{\mathbb{E}}_{t+h} i_{t+h+k})$, C_t are lagged values of forecasts and outcomes used as controls, and ϵ_t are Romer and Romer (2004) monetary shocks, compiled by Wieland and Yang (2020). Expectations are measured as Treasury Bill forecasts from the Survey of Professional Forecasters. The sample consists of quarterly observations from Q3/1981 to Q4/2007.

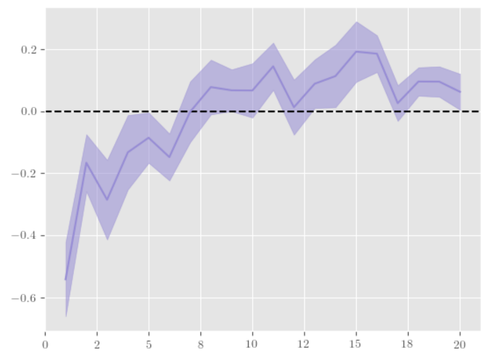


(a) US interest rate IRF to monetary shocks

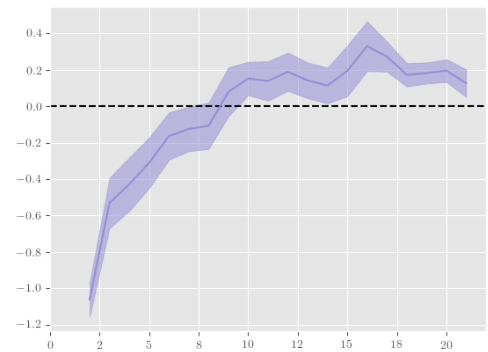
(b) US interest rate forecast errors IRF to monetary shocks

FIGURE C.2: KUCINSKAS AND PETERS (2019) COMPOSITE BIAS COEFFICIENTS FOR INTEREST RATE FORECASTS

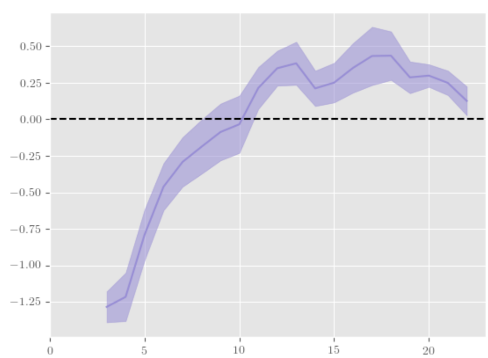
The figure reports composite bias coefficients for interest rate consensus (median) forecasts using Kucinskas and Peters (2019) method. Confidence intervals computed using Newey-West standard errors with max{4, 11} lags. Negative (positive) coefficients suggest under(over)-reaction. The IRF of forecast errors are based on the following regression, estimated via local projection $E_t x_t - F_{t-1} x_t = -b_0 - \sum_{l=1}^{\infty} sgn(\alpha_l) b_l \epsilon_{t-l} + \epsilon_t$, where $b_l = sgn(\alpha_l)(a_l - \alpha_l)$ are the bias coefficients, $x_t = \sum_{l=0}^{+\infty} \alpha_l \epsilon_{t-l}$, and $\mathbb{E}_t x_{t+1} = b_0 + \sum_{l=0}^{+\infty} a_{l+1} \epsilon_{t-l}$. Expectations are measured as Treasury Bill forecasts from the Survey of Professional Forecasters. The sample consists of quarterly observations from the survey of professional forecasters from Q1/1985 to Q4/2017.



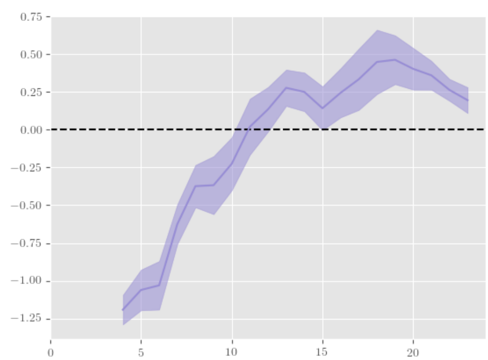
(a) One quarter-ahead interest rate forecasts



(b) Two quarter-ahead interest rate forecasts



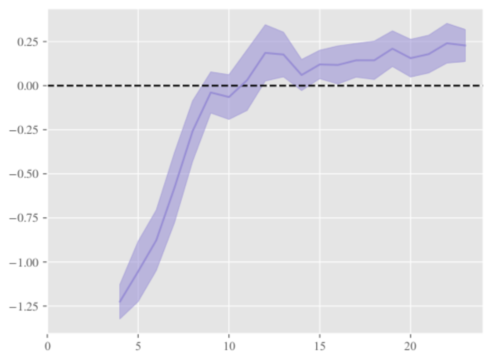
(a) Three quarter-ahead interest rate forecasts



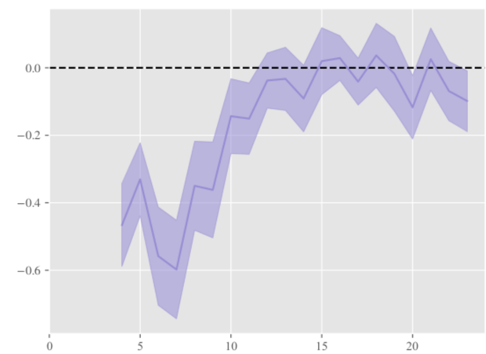
(b) Four quarter-ahead interest rate forecasts

FIGURE C.3: KUCINSKAS AND PETERS (2019) COMPOSITE BIAS COEFFICIENTS FOR INTEREST RATE FORECASTS

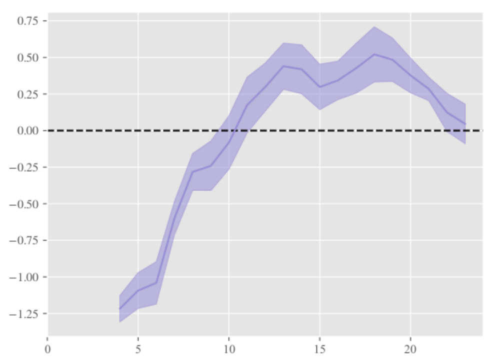
The figure reports composite bias coefficients for interest rate consensus (median) forecasts using Kucinskas and Peters (2019) method. Confidence intervals computed using Newey-West standard errors with max{4, 11} lags. Negative (positive) coefficients suggest under(over)-reaction. The IRF of forecast errors are based on the following regression, estimated via local projection $E_t x_t - F_{t-1} x_t = -b_0 - \sum_{l=1}^{\infty} sgn(\alpha_l) b_l \epsilon_{t-l} + \epsilon_t$, where $b_l = sgn(\alpha_l)(a_l - \alpha_l)$ are the bias coefficients, $x_t = \sum_{l=0}^{+\infty} \alpha_l \epsilon_{t-l}$, and $\mathbb{E}_t x_{t+1} = b_0 + \sum_{l=0}^{+\infty} a_{l+1} \epsilon_{t-l}$. Expectations are measured as Treasury Bill, Treasury Bond, Inflation, and Unemployment from the Survey of Professional Forecasters. The sample consists of quarterly observations from the survey of professional forecasters Q1/1985 to Q4/2017.



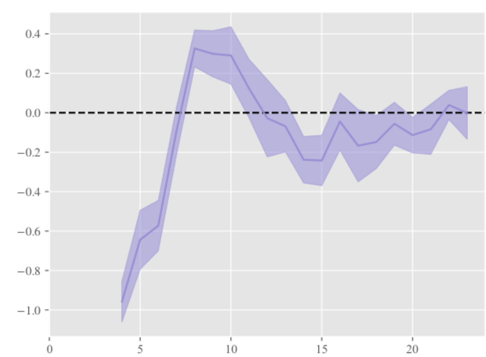
(a) Four quarter-ahead inflation forecasts



(b) Four quarter-ahead inflation forecasts



(a) Four-quarter-ahead T-Bill forecast



(b) Four quarter-ahead Treasury bond forecasts

TABLE C.2: CONSENSUS VERSUS INDIVIDUAL FORECAST ERRORS

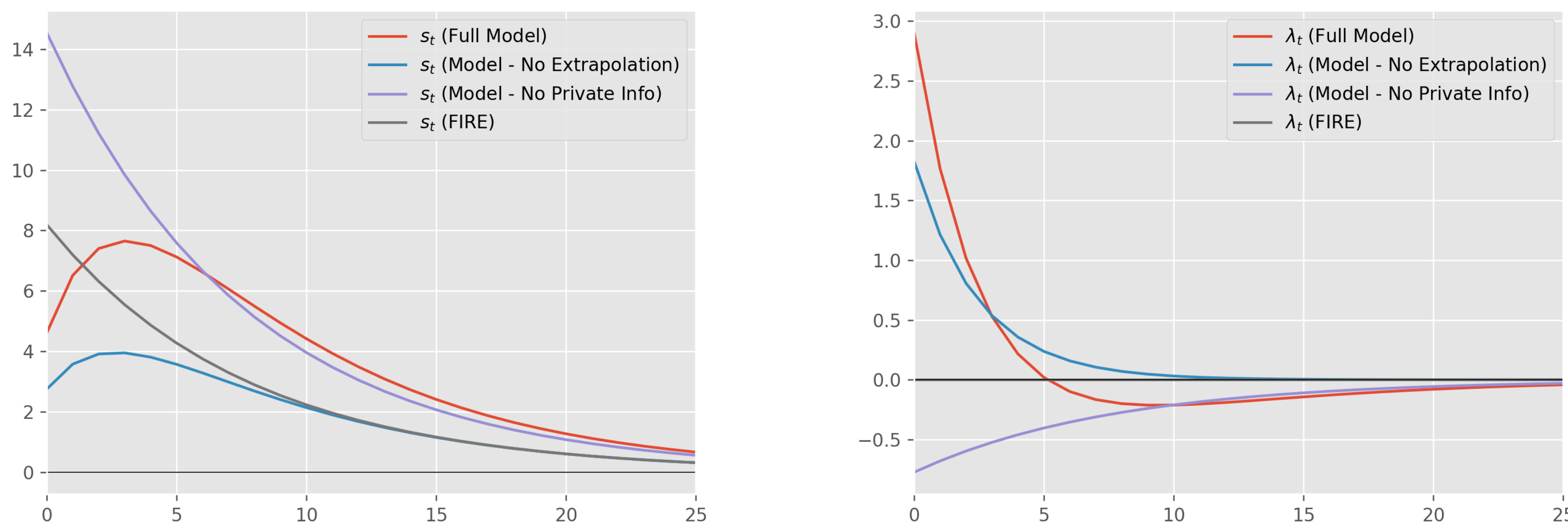
The table reports regression results following the approach of Angeletos et al. (2020b), to analyze the underreaction and overreaction of consensus and individual expectations. Regressions of the form $x_{t+k} - \mathbb{E}_{i,t}x_{t+k} = \beta_{\text{Revision}}(\bar{\mathbb{E}}_t x_{t+k} - \bar{\mathbb{E}}_{t-k}x_{t+k}) + \beta_{\Delta_i\text{Revision}}\left[(\mathbb{E}_{i,t}x_{t+k} - \mathbb{E}_{i,t-k}x_{t+k}) - (\mathbb{E}_t x_{t+k} - \mathbb{E}_{t-k}x_{t+k})\right] + \epsilon_{i,t+k}$, where $\bar{\mathbb{E}}$ captures the average forecast across all forecasters, and $\mathbb{E}_i$ captures the forecast of forecaster i. Positive coefficients in the regressions correspond with *underreaction* to news and negative coefficients correspond with *overreaction* to news. Standard errors are two-way clustered by forecaster and time period. The sample consists of quarterly observations between 1969 to 2018 from the Survey of Professional Forecasters.

	Unemployment			Inflation			Treasury Bill		
	1Q	2Q	3Q	1Q	2Q	3Q	1Q	2Q	3Q
Revision	0.468	0.687	0.813	1.162	1.329	1.482	0.260	0.508	0.823
	(0.125)	(0.151)	(0.197)	(0.348)	(0.473)	(0.692)	(0.142)	(0.147)	(0.204)
Δ_iRevision	-0.224	-0.252	-0.220	-0.471	-0.415	-0.383	-0.239	-0.183	-0.031
	(0.034)	(0.048)	(0.045)	(0.036)	(0.042)	(0.051)	(0.052)	(0.053)	(0.080)
R^2	0.135	0.158	0.118	0.211	0.151	0.129	0.093	0.109	0.128
N	5817	5761	5447	5649	5610	5306	3947	3902	3762

D Exchange Rate Puzzles with Different Frictions

FIGURE D.1: DELAYED OVERSHOOTING

The figure reports model's delayed overshooting and UIP deviations. We plot model's exchange rate and excess return IRFs after a one standard deviation shock to the fundamental process ξ_t.



(a) Exchange Rate

(b) Excess Return

FIGURE D.2: PREDICTABILITY REVERSAL

The figure reports model's UIP regression for different k-period ahead horizons, including different frictions in the model. We simulate the calibrated model 5,000 times for 140 periods. For each simulation and k-period ahead horizon, we estimate the following regression $\lambda_{t+k} = \alpha + \beta i_t^d + \epsilon_{t+k}$, where λ_{t+k} is the excess return between period $t+k-1$ and $t+k$, and i_t^d is the interest rate differential at period t. We report the average regression coefficient of all simulations.

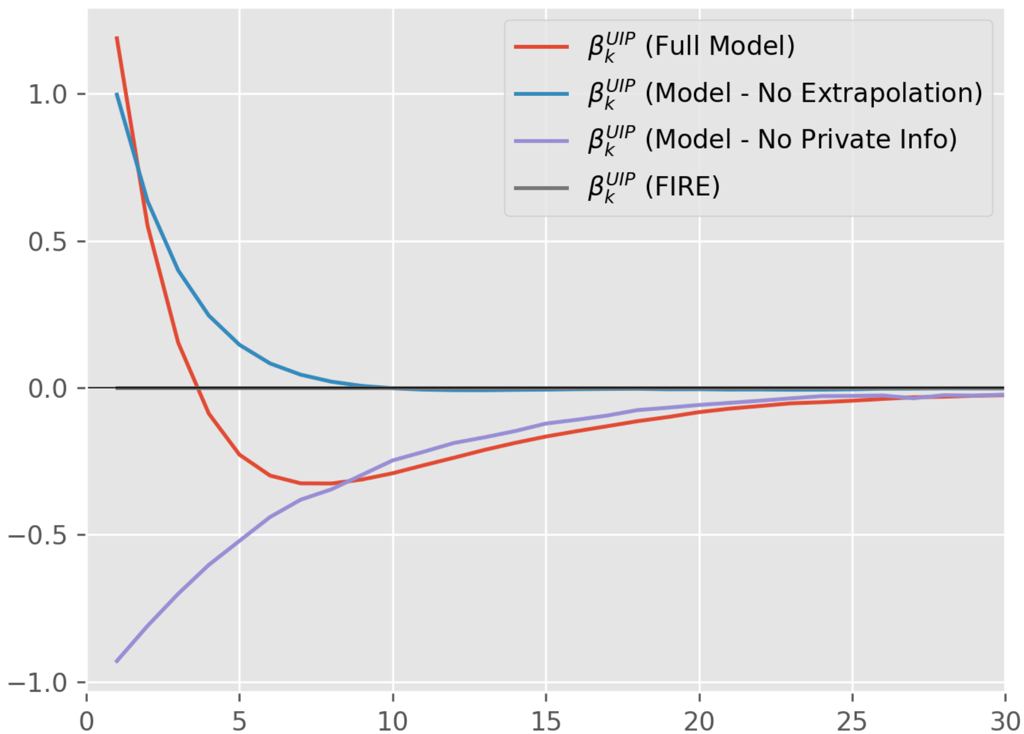


FIGURE D.3: TIME-SERIES MOMENTUM AND REVERSAL

The figure plots autocorrelations of currency excess returns in the model, including different frictions. The k-period autocorrelation is calculated by simulating the calibrated model 5,000 times for 140 periods, and taking an average autocorrelation of currency excess returns with k-period lagged excess returns in each simulated sample.

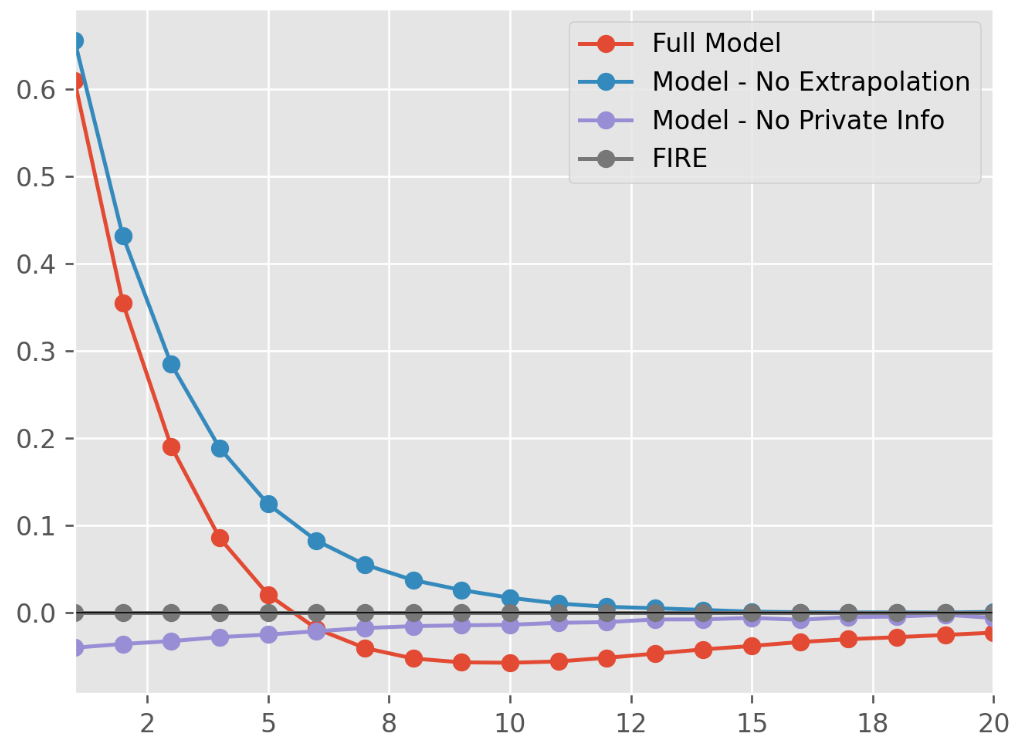


FIGURE D.4: THE DOWNWARD-SLOPING TERM STRUCTURE OF UIP VIOLATIONS

The figure plots the model-implied regression coefficients from regressing the returns to borrowing in n-period maturity foreign bonds and investing in n-period maturity home country bonds on the interest rate differential (the home currency interest rate minus the foreign country interest rate), for different values of n. The coefficients are computed by simulating the model 5,000 times for 140 periods. The figure plots regression coefficients for the full calibrated model, as well as for versions of the model with different frictions.