Throwing Dice — Método Matrix exponentiation

$f(n) = f(n-1) + f(n-2) \cdots f(n-6)$ ⇝ recorrência (1)

↳ $f(n+1) = f(n) + f(n-1) \cdots + f(n-5)$

↳ Valores iniciais

$$P_{6\times1} = \begin{bmatrix} f(n-1) \\ \vdots \\ f(n-6) \end{bmatrix} \qquad A_{6\times6} \cdot P_{6\times1} = P' = \begin{bmatrix} f(n) \\ \vdots \\ f(n-5) \end{bmatrix}$$

⇝ Precisamos achar A

$$A \begin{bmatrix} x_{11} \cdots x_{16} \\ \vdots \\ x_{61} \cdots x_{66} \end{bmatrix} \cdot P = P'$$

$- x_{11} f(n-1) + x_{12} f(n-2) + x_{13} f(n-3) + x_{14} f(n-4) + x_{15} f(n-5) + x_{16} + f(n-6)$
$= f(n)$ ⇝ Expande por 1 logo $x_{11} \cdots x_{16} = 1$

$- x_{21} f(n-1) + x_{22} f(n-2) + x_{23} f(n-3) + x_{24} f(n-4) + x_{25} f(n-5) + f(n-6) \cdot x_{26}$
$= f(n-1)$ ⇝ $x_{21} = 1$, $x_{22} \cdots x_{26} = 0$

$\vdots$

$- x_{61} \cdot f(n-1) + x_{62} \cdot f(n-2) \cdots + x_{65} \cdot f(n-5) + x_{66} \cdot f(n-6) = f(n-5)$
$\parallel x_{65} = 1 \parallel$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad P = \begin{bmatrix} 32 \\ 16 \\ 8 \\ 4 \\ 2 \\ 1 \end{bmatrix}$$

$$P'_m = A^{m-1} \cdot P \qquad O(\log m)$$