Towards best-case response times of real-time tasks under fixed-priority scheduling with preemption thresholds

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***Abstract*—Fixed-priority scheduling with preemption thresholds (FPTS) is supported by the AUTOSAR and OSEK standards as a scheduling policy. Since FPTS is a generalization of fixed-priority preemptive scheduling (FPPS) and fixed-priority non-preemptive scheduling (FPNS), it aims to improve schedulability. In this paper, we prove, as an intermediate step towards the exact best-case response time analysis for FPTS, that the best-case computation time of a non-preemptive task scheduled under FPTS or FPNS is a tight lower bound for its response time. In addition, we illustrate by means of an example that the best-case response time analysis for FPTS is most likely not a straight forward extension of the current best-case analysis for FPPS.**

1. Introduction

Fixed-priority scheduling with preemption thresholds (FPTS) was first introduced in the ThreadX kernel [1] to limit the number of preemptions that a task may experience with the aim to improve schedulability and reduce preemption overheads compared to fixed-priority preemptive scheduling (FPPS). It is currently also supported by the automotive standards AUTOSAR and OSEK [2, 3]. Although the worst-case response time analysis for FPTS has been studied extensively in the literature [4-6], little attention has been paid to its best-case counterpart.

FPTS is a generalization of FPPS and fixed-priority non-preemptive scheduling (FPNS). Therefore, it could be desirable to consider first the best-case response time analysis for FPPS and FPNS before investigating the corresponding analysis for FPTS. In particular, the best-case response time analysis for FPPS has been addressed in [7] for constraint deadlines and in [8] for arbitrary deadlines. On the other hand, the best-case response time for FPNS has not been formally studied.

FPNS is widely used in message-based protocols under a network environment. For example, the Controller Area Network (CAN) uses a priority based arbitration and non-preemptive scheduling for message transmission. Furthermore, special attention has been paid to the worst-case analysis for CAN in [9], and for FPNS in [10] to guarantee real-time requirements. However, when considering jitter induced by a network, it becomes relevant to study best-case response times next to worst-case response times of messages scheduled under FPNS.

In [11], a lower bound for the best-case response time under fixed-priority scheduling with deferred preemptions (FPDS) is presented. Since FPNS is a special case of FPDS, this analysis also applies for FPNS. In particular, this analysis gives the trivial result that the best-case computation time of a task is a lower bound for the best-case response time of such a task under FPNS. However, we would like to investigate whether this lower bound is tight for all cases.

The rest of this paper is organized as follows. Section 2 introduces the basic real-time scheduling model for FPTS as well as some related notions. The best-case response time for non-preemptive tasks is presented in Section 3. Section 4 introduces an example that shows that the best-case response time analysis for FPTS is most likely not a trivial extension of the current best-case analysis for FPPS. Finally, we conclude this paper in Section 5.

2. Real-time scheduling model

In this section, we present the real-time scheduling model for FPTS along with some related notions.

2.1. Basic model for FPTS

We assume a single processor and a set $\mathcal{T}$ of n independent periodic tasks $\tau_1, \tau_2, ..., \tau_n$ with unique and fixed priorities π_1, π_2, ..., π_n. We also assume that tasks are given in order of decreasing priority, i.e. τ_1 has the highest priority whereas τ_n has the lowest priority. A higher priority is represented by a higher value.

Each task τ_i generates an infinite sequence of jobs ι_{ik} with $k \in \mathbb{Z}$. In addition, each task is characterized by a *period* $T_i \in \mathbb{R}^+$, a *worst-case computation time* $WC_i \in \mathbb{R}^+$, a *best-case computation time* $BC_i \in \mathbb{R}^+$, where $BC_i \leq WC_i$, a *phasing* $\phi_i \in \mathbb{R}$, a (relative) *worst-case deadline* $WD_i \in \mathbb{R}^+$, and a (relative) *best-case deadline* $BD_i \in \mathbb{R}^+ \cup \{0\}$, where $BD_i \leq WD_i$. We assume arbitrary phasings. Moreover, the set of phasings ϕ_i is termed the phasing ϕ of the task-set $\mathcal{T}$.

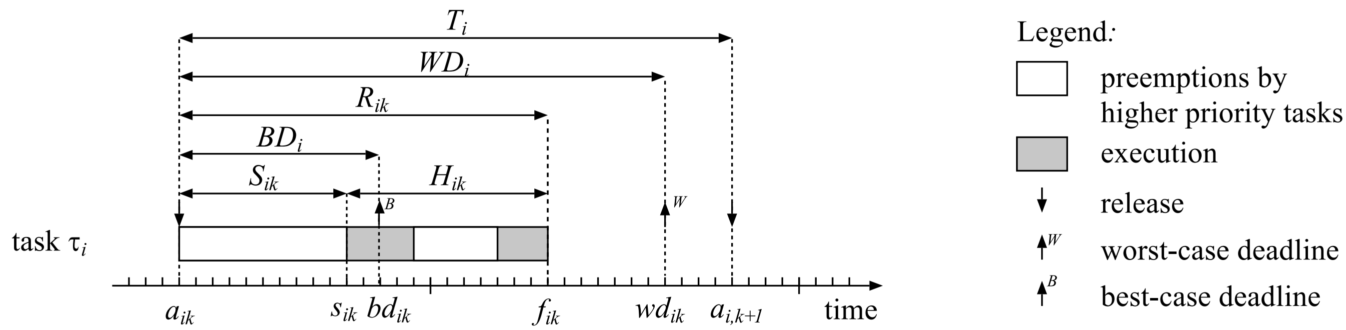

Figure 1. Basic model for a periodic task τ_i.

Furthermore, we assume arbitrary deadlines; hence, deadline WD_i may be smaller than, equal to, or larger than period T_i. In addition, we assume that tasks do not suspend themselves, jobs do not start before the completion of previous jobs of the same task, and the overhead of context switching and task scheduling is ignored.

For FPTS, each task τ_i has an additional property called *preemption threshold* denoted by θ_i, where $\pi_1 \geq \theta_i \geq \pi_i$. A task τ_i can be preempted by a higher priority task τ_h if and only if $\pi_h > \theta_i$. FPPS is a special case of FPTS when $\forall_{1 \leq i \leq n} \theta_i = \pi_i$, and FPNS is a special case when $\forall_{1 \leq i \leq n} \theta_i = \pi_1$.

It is worth noting that activation jitter is ignored in the real-time scheduling model under study. We aim to first investigate the best-case response times analysis for FPTS for the basic model before refining the model with the notion of activation jitter.

2.2. Derived notions

Given the *activation time* $a_{i,k}$ of the k^{th} job of a task τ_i and its (absolute) *finalization time* $f_{i,k}$, the *response time* $R_{i,k}$ of such a job is defined as the time elapsed between its finalization and activation, i.e. $R_{i,k} = f_{i,k} - a_{i,k}$. The *absolute start time* $s_{i,k}$ is the time at which job k of τ_i starts its execution. Furthermore, the *relative start time* $S_{i,k}$ is the start time relative to the activation of a job, i.e. $S_{i,k} = s_{i,k} - a_{i,k}$. Finally, we define the *hold time* $H_{i,k}$ as the time elapsed between the start of a job and its completion, this is expressed as $H_{i,k} = f_{i,k} - s_{i,k}$. In general, under this model, it always holds that $R_{i,k} = S_{i,k} + H_{i,k}$. Figure 1 shows the basic task model with these notions.

The *worst-case response time* WR_i and the *best-case response time* BR_i of a task τ_i are defined as the longest and the shortest response times of its jobs respectively, i.e.

$$WR_i \stackrel{\text{def}}{=} \sup_{\phi,k} R_{i,k}(\phi) \tag{1}$$

$$BR_i \stackrel{\text{def}}{=} \inf_{\phi,k} R_{i,k}(\phi), \tag{2}$$

where $R_{i,k}(\phi)$ denotes a dependency of response time $R_{i,k}$ on phasing ϕ.

We say that a set $\mathcal{T}$ of n periodic tasks is schedulable if and only if

$$\forall_{1 \leq i \leq n} (BD_i \leq BR_i \wedge WR_i \leq WD_i). \tag{3}$$

Finally, we introduce the notion of level-i active period and best-case utilization. In order to define the notion of level-i active period [10], we first introduce the notion of pending load. The *pending load* $P_i(t)$ of a task τ_i is defined as the amount of processing at time t that still needs to be completed for the jobs with a priority higher than or equal to task τ_i that are activated before time t. A level-i active period is then defined as the interval $[t_s, t_e)$ such that $P_i(t_s) = 0$, $P_i(t_e) = 0$, and $P_i(t) > 0$ for all $t \in (t_s, t_e)$.

We define the best-case utilization $BU^{\mathcal{T}}$ as the best-case fraction of the processor time spent on the execution of $\mathcal{T}$, i.e.

$$BU^{\mathcal{T}} \stackrel{\text{def}}{=} \sum_{1 \leq i \leq n} \frac{BC_i}{T_i}. \tag{4}$$

3. Best-case response time for non-preemptive tasks

Since FPNS is a special case of FPTS when thresholds of tasks are set to the highest priority, we first investigate the best-case response time for non-preemptive tasks as an intermediate step towards the best-case analysis for FPTS.

Lemma 1. *Let τ_i be a non-preemptive task, i.e. $\theta_i = \pi_1$, and let $BU^{\mathcal{T}} < 1$ or $BU^{\mathcal{T}} = 1$ and the lcm of the periods exists. The best-case response time of τ_i is always equal to its best-case computation time, i.e. $BR_i = BC_i$.*

Proof. Given a task-set $\mathcal{T}$ and a non-preemptive task $\tau_i \in \mathcal{T}$, assume that all jobs of τ_i have a computation time equal to BC_i. Since τ_i is non-preemptive, the hold time of all its jobs is simply equal to BC_i. Recall that the response time of a job k of τ_i is given by $R_{i,k} = S_{i,k} + H_{i,k}$. Therefore, in order to prove the lemma, it is sufficient to show that it is always possible to schedule task τ_i in such a way that the relative start time of a job k of τ_i is zero, i.e. $S_{i,k} = 0$. We divide this proof in two cases:

$\{$*Case* $BU^{\mathcal{T}} < 1\}$. Given an arbitrary schedule for $\mathcal{T}$, let $[t_s, t_e)$ with $t_e > t_s$ be an idle interval in such a schedule, where an idle interval is a time interval in which there is no pending load. Note that it is always possible to find an idle interval because $BU^{\mathcal{T}} < 1$. Furthermore, let $\iota_{i,k}$ be the first job of τ_i activated at or after time t_e, i.e. $t_e \leq a_{i,k}$. We now show that, by pushing the activation of $\iota_{i,k}$ to occur in the interval $[t_s, t_e)$, the relative start time of $\iota_{i,k}$ becomes zero.

First observe that there is no other job of τ_i between the idle interval $[t_s, t_e)$ and $\iota_{i,k}$. Hence, after pushing the activations of τ_i till $a_{i,k}$ occurs in $[t_s, t_e)$, job $\iota_{i,k}$ does not experience blocking by a previous job. Furthermore, since $[t_s, t_e)$ was originally idle, job $\iota_{i,k}$ can start upon activation, leading to $S_{i,k} = 0$.

$\{$*Case* $BU^{\mathcal{T}} = 1$ *and the lcm of the periods exists*$\}$. In order to prove this case, first recall that the hyperperiod is defined as the length of the shortest time interval in which the schedule repeats itself after an initial start-up. For a set of periodic tasks, Leung and Merrill proved in [12] that the hyperperiod H can be calculated as the least common multiple of the periods, i.e. $H = lcm(T_1, ..., T_n)$ for n periodic tasks.

Let $n_i = H/T_i$ be the number of jobs of τ_i in every hyperperiod and assume that no job of τ_i starts upon activation. Hence, for each interval $[a_{i,k'}, s_{i,k'})$ with $1 \leq k' \leq n_i$, there are only higher priority tasks of τ_i and/or at most one blocking lower priority task executing within such an interval. As an example of this situation, consider the timeline depicted in Figure 2 for task-set $\mathcal{T}_1$ described in Table 1. As can be seen, no job of τ_2 can start upon activation.

TABLE 1. TASK CHARACTERISTICS OF $\mathcal{T}_1$.

task	T_i	WD_i	C_i	π_i	θ_i
τ_1	4	4	2	2	2
τ_2	6	6	3	1	2

The hyperperiod is $H = 12$ and $BU^{\mathcal{T}_1} = 1$.

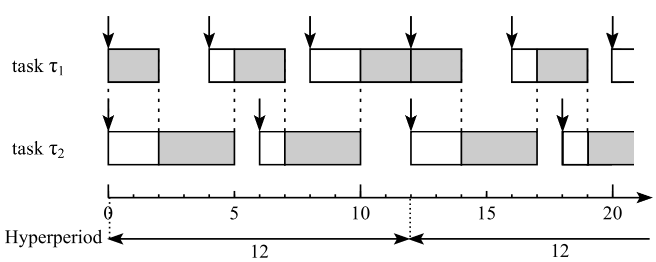

Figure 2. A timeline for $\mathcal{T}_1$. The activations of the tasks are repeated in every interval with length $H = 12$.

Now let $\iota_{i,k}$ be the job of τ_i with the shortest relative start time among all jobs of τ_i in an hyperperiod. Since no job of τ_i starts upon activation, we can push the activation of all jobs of τ_i to a later moment in time, without modifying the schedule, till job $\iota_{i,k}$ starts upon activation. After pushing the activations of τ_i, it holds that $S_{i,k} = 0$ therefore concluding the proof. Figure 3 shows an example where the schedule shown in Figure 2 is preserved after pushing the activations of τ_2 till its second job starts upon activation. □

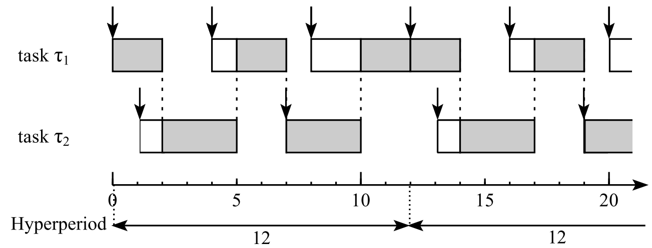

Figure 3. A timeline for $\mathcal{T}_1$. After pushing the activations of τ_2, the second job can immediately start.

4. Towards best-case analysis for FPTS

In this section, we illustrate by means of an example that the best-case response time analysis for FPTS is most likely not a trivial extension of the existing best-case analysis for FPPS.

4.1. Recap of optimal instant for FPPS

An optimal instant is defined as the (hypothetical) instant that leads to the best-case response time of a task. For FPPS, the notion of optimal instant is the following [8]:

Theorem 1. *An optimal instant of a task τ_i under FPPS and arbitrary deadlines occurs when the completion of a job of task τ_i coincides with the simultaneous activation of all its higher priority tasks.*

It is worth noting that, for FPPS, the job experiencing the optimal instant is the job that assumes the best-case response time. Furthermore, such a job is also the last job in its level-i active period.

4.2. An introductory example

From Lemma 1, we can conclude that the best-case response time of a non-preemptive task is independent of higher priority tasks. Based on this, it could be tentative to think that, given a task τ_i scheduled under FPTS, the best-case response time of τ_i would be independent of all its higher priority tasks that cannot preempt it, i.e. the tasks in $\{\tau_h | \theta_i \geq \pi_h > \pi_i\}$. However, this is not the case for FPTS as we show in the following example.

TABLE 2. TASK CHARACTERISTICS OF $\mathcal{T}_2$.

task	T_i	WD_i	$WC_i = BC_i$	π_i	θ_i	WR_i	BR_i
τ_1	35	20	10	3	3	10	10
τ_2	50	65	20	2	2	62	20
τ_3	70	70	22	1	2	66	32

The *least common multiple* of the periods is 350 and $BU^{\mathcal{T}_2} = 1$.

Consider the set $\mathcal{T}_2$ of three tasks with characteristics as described in Table 2. The best-case response time values in this table were found using brute force. This is, by considering the phasings $\phi_1 \in [0, 35)$ and $\phi_2 \in [0, 50)$ with a granularity of one time unit in the simulations, while fixing

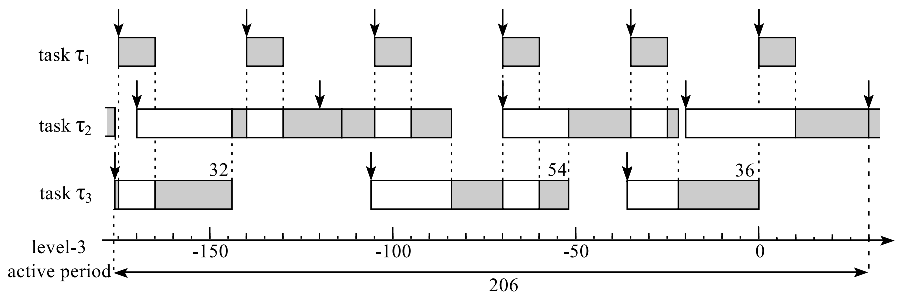

Figure 4. A timeline for $\mathcal{T}_2$ depicting a level-3 active period. The best-case response time of τ_3 is assumed by the first job in the level-3 active period.

the phasing of τ_3 to $\phi_3 = 0$. Furthermore, note that task τ_1 can preempt task τ_3 because $\pi_1 > \theta_3$. On the other hand, task τ_2 cannot preempt task τ_3 because $\pi_2 = \theta_3$.

Figure 4 shows a timeline for task-set $\mathcal{T}_2$ where the best-case response time of task τ_3 is assumed by the first job in the level-3 active period. As can be seen, this best-case response time is assumed when τ_3 experiences one preemption by task τ_1; hence, resulting in $BR_3 = 32$. However, when ignoring task τ_2 from task-set $\mathcal{T}_2$, the best-case response time of task τ_3 is reduced to $BR_3 = 22$. Based on this observation, we conclude that, in general, the best-case response time of a task τ_i is not independent of the higher priority tasks that cannot preempt τ_i. Therefore, we formulate the following fact.

Fact 1. *In FPTS, the best-case response time of a preemptive task τ_i, i.e. $\pi_1 > \theta_i$, can also be affected by a higher priority task that cannot preempt τ_i.*

From Figure 4, we can also observe that an activation of τ_1 coincides with the completion of a job of τ_3 at time $t = 0$. This is similar to the notion of optimal instant for FPPS. However, the best-case response time is not assumed by such a job ending at time $t = 0$. Therefore, we have a first witness of dissimilarity between the best-case response time analysis for FPPS and FPTS.

Fact 2. *In FPTS, a job k of task τ_i does not necessarily assume the best-case response time when the activation of all higher priority tasks that can preempt task τ_i coincides with the completion of job k.*

Furthermore, an important property in the best-case analysis for FPPS and arbitrary deadlines is that the last job in a level-i active period always experiences the shortest response time of all jobs in that level-i active period. This property was highlighted in [8] as a witness of non-duality between best-case and worst-case analysis. This property clearly does not hold for FPTS as shown in Figure 4. As can be seen, the first job in the level-3 active period is the one with the shortest response time. Based on this, we introduce the following fact as second witness of dissimilarity between the best-case behavior of FPPS and FPTS.

Fact 3. *In FPTS, the shortest response time of a task τ_i in a level-i active period is not necessarily assumed by the last job in that level-i active period.*

5. Conclusion

In this paper, we proved that the best-case response time of a non-preemptive task is equal to its best-case computation time when scheduled under FPTS or FPNS. In addition, we showed some witnesses of dissimilarity between the best-case behaviors of FPPS and FPTS. We therefore conclude that the best-case analysis for FPTS is most likely not a trivial extension of the current analysis for FPPS. An exact best-case response time analysis for FPTS is currently under study.

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